

DUALITY FOR ACTION BIALGEBROIDS

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ABSTRACT. We study the effect of linear duality on action bialgebroids (also known as smash product or scalar extension bialgebroids) and, for those bearing a quantisation nature, the effect of Drinfeld functors underlying the quantum duality principle. By means of various categorical equivalences, it is shown that any braided commutative Yetter-Drinfeld algebra over any bialgebroid is also a braided commutative Yetter-Drinfeld algebra over the respective dual bialgebroid. This implies that the action bialgebroid of the dual exists, which is then proven to be isomorphic, as a bialgebroid, to the dual of the initial action bialgebroid: in short, (linear) duality commutes with the action bialgebroid construction. Similarly, for quantum groupoids to which the Drinfeld duality functors apply and the quantum duality principle holds, these Drinfeld duality functors are shown to commute with the action bialgebroid construction as well.

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