MULTIPARAMETER QUANTUM SUPERGROUPS, DEFORMATIONS AND SPECIALIZATIONS

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Abstract. In this paper we introduce a multiparameter version of the quantum universal enveloping superalgebras introduced by Yamane in [Ya1]. For these objects we consider:

— *(1)* their deformations by twist and by 2–cocycle (both of "toral type"); in particular, we prove that this family is stable under both types of deformations;

— *(2)* their semiclassical limits, which are multiparameter Lie superbialgebras;

— *(3)* the deformations by twist and by 2–cocycle (of "toral type") of these multiparameter Lie superbialgebras: in particular, we prove that this family is stable under these deformations, and that "quantization commutes with deformation".

To the memory of Pierre Cartier,
with our deepest admiration.

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