

QUANTUM GROUP DEFORMATIONS AND QUANTUM R -(CO)MATRICES VS. QUANTUM DUALITY PRINCIPLE

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ABSTRACT. In this paper we describe the effect on quantum groups — namely, both QUEA’s and QFSA’s — of deformations by twist and by 2-cocycles, showing how such deformations affect the semiclassical limit.

As a second, more important task, we discuss how these deformation procedures can be extended, via a formal variation of the original recipes, using *quasi-twists* and *quasi-2-cocycles*. These new recipes seemingly should make no sense at all, yet we prove that they do work, thus providing more general deformation procedures. Later on, we explain the underlying motivation: this comes from *Quantum Duality Principle*, through which every “quasi-twist/2-cocycle” for a given quantum group can be seen as a standard twist/2-cocycle for another quantum group, associated to the original one via the appropriate Drinfeld functor.

Finally, we consider standard constructions involving R -(co)matrices for Hopf algebras. First we adapt them to quantum groups, then we show that they extend to the case of *quasi- R -(co)matrices*, and finally we discuss how these constructions interact with the Quantum Duality Principle. This also yields new symmetries for the underlying pair of dual Poisson (formal) groups that one gets by specialization.

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