

SUPER KÄHLER STRUCTURES ON THE COMPLEX ABELIAN LIE SUPERGROUPS

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ABSTRACT. We consider a real Abelian Lie supergroup G acting on its complexification M , equipped with a G -invariant super Kähler form. We extend the scheme of classical geometric quantization to this setting and construct a unitary G -representation. We show that the occurrences of its irreducible subrepresentations are governed by the image of the moment map of the super Kähler form. As an application, we construct a Gelfand model of G , namely a unitary G -representation in which every unitary irreducible representation occurs exactly once.

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