F. Gavarini, Z. Rakić

“$F_q[\text{Mat}_2]$, $F_q[\text{GL}_2]$ and $F_q[\text{SL}_2]$ as quantized hyperalgebras”

to appear in Communications in Algebra (2008)

ABSTRACT

Within the quantum function algebra $F_q[\text{SL}_2]$, we study the subset $\mathcal{F}_q[\text{SL}_2]$ — introduced in [Ga1] — of all elements of $F_q[\text{SL}_2]$ which are $\mathbb{Z}[q,q^{-1}]$–valued when paired with $\mathcal{U}_q(\mathfrak{sl}_2)$, the unrestricted $\mathbb{Z}[q,q^{-1}]$–integral form of $\mathcal{U}_q(\mathfrak{sl}_2)$ introduced by De Concini, Kac and Procesi. In particular, we yield a presentation of $\mathcal{F}_q[\text{SL}_2]$ by generators and relations, and a nice $\mathbb{Z}[q,q^{-1}]$–spanning set (of PBW type) for it. Moreover, we give a direct proof that $\mathcal{F}_q[\text{SL}_2]$ is a Hopf subalgebra of $F_q[\text{SL}_2]$, and that $\mathcal{F}_q[\text{SL}_2]|_{q=1} \cong U_\mathbb{Z}(\mathfrak{sl}_2^*)$. We describe explicitly its specializations at roots of 1, say $\epsilon$, and the associated quantum Frobenius (epi)morphism (also introduced in [Ga1]) from $\mathcal{F}_\epsilon[\text{SL}_2]$ to $\mathcal{F}_1[\text{SL}_2] \cong U_\mathbb{Z}(\mathfrak{sl}_2^*)$. The same analysis is done for $F_q[\text{GL}_2]$, with similar results, and also (as a key, intermediate step) for $F_q[\text{M}_2]$.

References