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“The global quantum duality principle”


ABSTRACT

Let $R$ be an integral domain, let $h \in R \setminus \{0\}$ be such that $k := R/h R$ is a field, and let $\mathcal{H}A$ be the category of torsionless (or flat) Hopf algebras over $R$. We call $H \in \mathcal{H}A$ a “quantized function algebra” (=QFA), resp. “quantized restricted universal enveloping algebras” (=QrUEA), at $h$ if — roughly speaking — $H/h H$ is the function algebra of a connected Poisson group, resp. the (restricted, if $R/h R$ has positive characteristic) universal enveloping algebra of a (restricted) Lie bialgebra. Extending a result of Drinfeld, we establish an “inner” Galois’ correspondence on $\mathcal{H}A$, via two endofunctors, $( )^\vee$ and $( )'$, of $\mathcal{H}A$ such that $H^\vee$ is a QrUEA and $H'$ is a QFA (for all $H \in \mathcal{H}A$). In addition:

(a) the image of $( )^\vee$, resp. of $( )'$, is the full subcategory of all QrUEAs, resp. QFAs;
(b) if $p := \text{Char}(k) = 0$, the restrictions $( )^\vee|_{\text{QFAs}}$ and $( )'|_{\text{QrUEAs}}$ yield equivalences inverse to each other;
(c) if $p = 0$, starting from a QFA over a Poisson group $G$, resp. from a QrUEA over a Lie bialgebra $g$, the functor $( )^\vee$, resp. $( )'$, gives a QrUEA, resp. a QFA, over the dual Lie bialgebra, resp. the dual Poisson group.

Several, far-reaching applications are developed in detail in [Ga2–4].

REFERENCES

