ELEMENTS OF MODULAR THEORY FOR W^*-ALGEBRAS
PHD COURSE 2020/2021, 25 HOURS

F. FIDALEO

1. MODULAR THEORY

- Tomita theorem for von Neumann algebras equipped with a cyclic and separating vector.
- Case of W^*-algebras equipped with a faithful normal state: the standard representation.
- General case: Left Hilbert algebras and the modular theory associated to a normal semifinite weight (an outline).

2. APPLICATIONS

- Kubo-Martin-Schwinger condition, applications to (quantum) statistical mechanics.
- Conditional expectations and Takesaki existence theorem.
- Haagerup operator-valued weights (an outline).
- Accardi-Cecchini φ-expectation and Longo's canonical endomorphism, applications to quantum probability and theory of subfactors (an outline).
- Connes' classifications of type III W^*-factors (an outline).

3. PREREQUISITES

- Basic elements of functional analysis.
- Basic elements of operator algebras.

4. LECTURES

Eight/nine 3-hour-long lectures from 22.03.'21 to 21.05.'21.

5. REFERENCES

- General:
  (1) O. Bratteli, D. W. Robinson *Operator algebras and quantum statistical mechanics I,II*, Springer.
  (2) Ş. Strătilă *Modular theory in operator algebras*, Abacus press.
  (3) Strătilă, L. Zsidó *Lectures on von Neumann algebras*, Abacus press.
  (4) V. S. Sunder *An invitation to von Neumann algebras*, Springer.
  (5) M. Takesaki *Theory of operator algebras I, II, III*, Springer.
- Section 1:
  (1) O. Bratteli, D. W. Robinson *Operator algebras and quantum statistical mechanics I*, Springer.

(2) S. Doplicher (a cura di Benfatto, D'Antoni, Niccolò, Rossi): *Introduzione alla teoria delle algebre di von Neumann e teorema di Tomita-Takesaki*, quaderni del CNR.
(3) U. Haagerup *The standard form of von Neumann algebras I, II*, Math. Scand. 37 (1975), 271-283.
(4) Ş. Strătilă, *Modular theory in operator algebras*, Abacus press.
(5) Strătilă, L. Zsidó, *Lectures on von Neumann algebras*, Abacus press.
(6) M. Takesaki *Theory of operator algebras II*, Springer.

– Section 2:
(1) L. Accardi, C. Cecchini *Conditional expectations in von Neumann algebras and a theorem of Takesaki*, J. Funct. Anal. 45 (1982), 245-273.
(2) O. Bratteli, D. W. Robinson *Operator algebras and quantum statistical mechanics II*, Springer.
(3) A. Connes *Une classification des facteurs de type III*, Ann. Ec. Norm. Sup. 6 (1973), 133-252.
(4) U. Haagerup *Operator valued weights in von Neumann algebras I, II*, J. Funct. Anal. 32 (1979), 175-206 & 33 (1979), 339-361.
(5) R. Longo *Index of subfactors and statistics of quantum fields I, II*, Commun. Math. Phys. 126 (1989), 217-247 & 130 (1990), 285-309.
(6) Ş. Strătilă, *Modular theory in operator algebras*, Abacus press.
(7) V. S. Sunder *An invitation to von Neumann algebras*, Springer.
(8) M. Takesaki *Conditional expectations in von Neumann algebras*, J. Funct. Anal. 9 (1972), 306-321.
(9) M. Takesaki *Theory of operator algebras II*, Springer.