Introduction to Inverse Hyperbolic Problems by Carleman Estimates

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Abstract

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with boundary $\partial\Omega$. For $T > 0$, we consider the following initial-boundary value problem for a hyperbolic equation:

$$\begin{cases} \partial_t^2 u(x,t) = \Delta u(x,t) + p(x)u(x,t), \quad x \in \Omega,\ 0 < t < T, \\ u|_{\partial\Omega} = 0, \qquad 0 < t < T, \\ u(x,0) = a(x), \quad \partial_t u(x,0) = b(x), \quad x \in \Omega. \end{cases} \tag{$*$}$$

In the forward problem, given initial values a and b, we are requested to solve $u(x,t)$ satisfying (*). Our main interest is a coefficient inverse problem where we are requested to find $p(x)$, $x \in \Omega$ by some extra boundary data of u (e.g., ∇u on some subboundary). The coefficient inverse problem is seriously related to various applications such as medical diagnosis, physical prospecting. Since the available data are limited, it is not easy to prove the uniqueness and the stability in determining $p(x)$ by the extra data.

We can refer to the major two methodologies: by Dirichlet-to-Neumann map and by Carleman estimates.

I give self-contained lectures on Carleman estimates and the applications to hyperbolic coefficient inverse problems. Possible are contents are as follows.

- Derivation of Carleman estimates
- Well-posedness for the initial-boundary value problem

- General method how to apply a Carleman estimate for the stability in the hyperbolic inverse problem
- Inverse problems for heat, elasticity equations

The method by Carleman estimates is very widely applicable to various equations, and can establish also observability inequalities for the exact controllability. I hope that the audiences can understand the essence.

Background knowledge

Calculus (indispensable). Basics of Lebesgue integrals, Soblev spaces, etc. (if possible).

Reference

M. Bellassoued and M. Yamamoto, *Carleman Estimates and Applications to Inverse Problems for Hyperbolic Systems*, Springer-Japan, Tokyo, 2017.