# A Distributed Protocol for the Bounded-Hops Converge-cast in Ad-Hoc Networks[0]

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**Abstract.** Given a set $S$ of points (*stations*) located in the $d$-dim. Euclidean space and a *root* $b \in S$, the $h$-HOPS CONVERGECAST problem asks to find for a minimal energy-cost range assignment which allows to perform the *converge-cast* primitive (i.e. node accumulation) towards $b$ in at most $h$ hops. For this problem no polynomial time algorithm is known even for $h = 2$.

The main goal of this work is the design of an efficient *distributed* heuristic (i.e. protocol) and the analysis (both theoretical and experimental) of its expected solution cost. In particular, we introduce an efficient parameterized randomized protocol for $h$-HOPS CONVERGECAST and we analyze it on *random instances* created by placing $n$ points uniformly at random in a $d$-cube of side length $L$. We prove that for $h = 2$, its expected approximation ratio is bounded by some constant factor. Finally, for $h = 3, \ldots, 8$, we provide a wide experimental study showing that our protocol has very good performances when compared with previously introduced (centralized) heuristics.

## 1 Introduction

An *ad-hoc* (wireless) network consists of a set of radio stations connected by wireless links. In an ad hoc network, a transmission range is assigned to every station. The overall range assignment determines a transmission (directed) graph since one station $s$, with transmission $r$, can transmit to another station $t$ if and only if $t$ belongs to the *disk* centered in $s$ and of radius $r$. The transmission range of a station depends, in turn, on the energy power supplied to the station. In particular, the power $P_s$ required by a station $s$ to correctly transmit data to another station $t$ must satisfy the inequality

$$\frac{P_s}{d(s,t)^\alpha} > \gamma \tag{1}$$

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where $d(s,t)$ is the Euclidean distance between $s$ and $t$, $\alpha \geq 1$ is the *distance-power gradient*, and $\gamma \geq 1$ is the *transmission quality* parameter. The parameter $\alpha$ may vary from 1 to more than 6 depending on the environment conditions; in the ideal condition (empty space), $\alpha = 2$ (see [21]). Stations of an ad-hoc network cooperate in order to provide specific network connectivity properties by dynamically adapting their transmission ranges. A range assignment $r : S \to \mathcal{R}^+$ determines a directed *transmission graph* $G(S,E)$ where edge $(i,j) \in E$ if and only if $d(i,j) \leq r(i)$. A fundamental problem underlying any phase of a dynamic resource allocation algorithm in ad-hoc wireless networks is the following [10, 16, 7, 15, 19, 23]: find a range assignment such that (1) the corresponding transmission graph $G$ satisfies a given connectivity property $\Pi$, and (2) the overall energy power required to deploy the assignment (according to Inequality (1)) is minimized. The overall energy power (i.e. the *cost*) of a range assignment $r : S \to \mathcal{R}+$ is defined as

$$\mathsf{cost}(r) \;=\; \sum_{s \in S} r(s)^{\alpha} \tag{2}$$

In this work, we address the range assignment problem in which $G$ is required to contain a tree directed towards a given *root* station $b$ (called *root*), spanning $S$ and of depth at most $h$. The relevance of this particular connectivity property is clear: feasible solutions for this problem, denoted as $h$-HOPS CONVERGECAST, allow minimal energy-cost *converge-cast* (i.e. node accumulation) operations towards $b$ in at most $h$-hops.

It is easy to verify that the $h$-HOPS CONVERGECAST problem is a particular case of the well-known *Minimum $h$-hops Spanning Tree* problem ($h$-HOPS MST) defined as follows: given a graph $G(V,E)$ with nonnegative edge weights and a node $b \in V$, find a minimum-cost directed tree rooted at $b$, of depth at most $h$, and spanning $G$. In fact, $h$-HOPS CONVERGECAST corresponds to the restriction of $h$-HOPS MST in which nodes are the stations and there is an edge for any pair of stations $i$ and $j$ whose weight is $d(i,j)^{\alpha}$.

The main goal of this work is to design efficient *distributed* heuristics (i.e. protocols) for $h$-HOPS CONVERGECAST and then analyze (both theoretically and experimentally) their expected solution costs. We intend to investigate the problem, for any constant $h$, on *random* instances created by placing $n$ points uniformly at random in a $d$-cube of side length $L$. Such instances will be simply called *random instances*.

**Previous results.** Almost all previous related works refer to the $h$-HOPS MST problem. It is known that it is MAX SNP-hard even when the edge weights of the input graphs form a *metric* and $h = 2$ [1].

The 2-DIM 2-HOPS MST problem can be easily reduced to the classic Facility Location Problem on the plane. Indeed, the distance of the root from vertex $i$ can be seen as the cost of opening a facility at vertex $i$. It thus follows that all the (centralized) approximation algorithms for the latter problem apply to the 2-DIM 2-HOPS MST as well. In particular, the best result is the PTAS given by Arora et al in [3]. The algorithm works also in higher dimensions;

however, it is based on a complex dynamic programming technique that makes any (distributed) implementation very far to be practical. Several polynomial-time approximation algorithms for the METRIC 2-HOPS MST problem have been presented in the literature. The first constant factor approximation algorithm was given by Shmoys et al in [26], they presented a 3.16 approximation algorithm. After this, a series of constant factor approximation algorithms was published, see [5, 18, 13]. Currently, the best factor is 1.52 due to Mahdian et al [20].
The general $h$-HOPS MST problem was studied in [12, 14, 17] by providing exact but super-polynomial or $O(\log n)$-approximate solutions.
Another series of works have been devoted to evaluate and compare solutions for the $d$-DIM $h$-HOPS MST problem returned by some heuristics on *random* planar instances by performing computer experiments [8, 11, 12, 25, 27]. Almost all such works adopt *random instances*.

More recently, a tight analysis of the expected optimal cost for the $d$-DIM $h$-HOPS MST problem on random instances has been done in [9, 6].
Given a rooted tree $T$ the *cost* of $T$, denoted as $\mathsf{cost}(T)$, is the sum of the edge weights.

**Theorem 1 ([9, 6]).** *Let $h$ and $d$ be fixed positive integers. Let $S$ be a random instance of $n$ points in a $d$-cube of side length $L$ and let $T$ be any tree of height $h$ spanning $S$. Then, it holds that*

$$\mathsf{cost}(T) = \begin{cases} \Theta\left(L \cdot n^{\frac{1}{h}}\right) & \text{if } d = 1,\ \alpha = 1 \\ \Theta\left(L \cdot n^{1-\frac{1}{d}+\frac{d-1}{d^{h+1}-d}}\right) & \text{if } d \geq 2,\ \alpha = 1 \\ \Theta\left(L^2 \cdot n^{\frac{1}{h}}\right) & \text{if } d = 2,\ \alpha = 2 \end{cases} \quad \textit{with high probability.}$$

Here and in the sequel the term *with high probability* (in short, *w.h.p.*) means that the event holds with probability at least $1 - e^{-c \cdot n}$, for some constant $c > 0$. So, according to our input model, claiming that a given bound holds w.h.p. is equivalent to claim that it holds *for almost all* inputs [2]. Theorem 1 shows that the optimal cost *quickly decreases* in $h$ (even for small, constant values of $h$). Actually, an efficient, *centralized Divide and Conquer* heuristic $h$-PARTY (see Figure 2) is introduced in [9, 6] that yields (w.h.p.) a constant approximation ratio.

### 1.1 Our Results

The asymptotically cost-optimal *divide and conquer* heuristic $h$-PARTY (see Figure 2) proposed in [9, 6] requires global knowledge of the network and *centralized* decisions: at each recursive phase, a suitable grid partition of the $d$-cube $Q$ is performed. In each element (*cell*) of the grid, the algorithm runs a *leader election* task in order to select a *base cell*. A cell base, elected in the recursive phase $j$, is connected to the root and becomes itself *root* for the $(h - j)$-HOPS CONVERGECAST problem on the sub-instance restricted to nodes inside the cell. And so on. The key-issue of the optimality of this algorithm is the size of the cells (and, hence, the number of bases selected at each phase). It should be clear that, the above tasks are unfeasible (or extremely expensive) in our distributed model: once a (unique) base cell has been selected, all the other nodes of the cell must agree about that and must know its label and position.

We thus propose a distributed protocol that combines the grid partitioning and the leader election of $h$-PARTY with the greedy approach. Our protocol $h$-PROT "simulates" the "optimal" strategy of $h$-PARTY (based on grid partition and leader election), by using local *independent random choices*: every node (but the root) decides, *independently*, to be a leader (i.e. a base) with probability $p$. Then, we use a *greedy* approach in order to establish nodes-to-leader connections. The choice of parameter $p$ depends on $n$ (and $h$) and it is a key-ingredient in the quality of the returned solution.

We consider the (synchronous) ad-hoc network model in the following *distributed* fashion: at the starting time, every station knows only its label, its geographical position and the parameter $L$ (i.e.a "good" bound on the diameter of region where nodes are located in). Furthermore, no (wireless or not) link exists before that time. All stations are in the quiescent status but the $b$: $b$ starts the protocol by broadcasting a *start* message with 1-hop transmission of range $L$ (as we will see, the cost of this root transmission is negligible w.r.t. the overall protocol and solution costs). We focus on the *global energy-cost* spent by the protocol and on the energy cost of the computed range assignment. So, we will not consider *interference* and *synchronization* problems [21]: we assume they will be eventually solved by using some of the techniques previously introduced in the literature [4, 22, 24].

Our main result is expressed by the following theorem.

**Theorem 2.** *Let $S$ be a random set of $n$ points in a 2-dim square $Q$ of side length $L$ and let* $\mathsf{range}^{Prot}$ *be the range assignment returned by* 2-PROT$(S, b)$. *Then, for $h = 2$, the expected cost of* $\mathsf{range}^{Prot}$ *satisfies the following bounds*

$$\mathbf{E}\left(\mathsf{cost}(\mathsf{range}^{Prot})\right) = O\left(L^{\alpha} n^{\frac{2}{\alpha+2}}\right).$$

By comparing Theorem 1 with Theorem 2 we thus have that for $\alpha = 1$ and $\alpha = 2$, $h$-PROT achieves a constant expected approximation ratio.
We emphasize that the expected cost analysis of our protocol departs significantly from that of 2-PARTY, mainly because of two reasons. Firstly, our protocol makes no cell partition (it cannot!) and so it cannot guarantee that exactly one leader per cell will be selected. The analysis needs to be *amortized* and based on the expected *good* leader distribution on $Q$. On the other hand, we need to deal with *two* probabilistic distributions: the input one and the one determining our leader selection. The resulting random variables are not mutually independent. We also prove that the overall energy cost spent by the stations during the entire protocol is $\Theta(\mathsf{cost}(\mathsf{range}^{Prot}))$.
We believe that Theorem 2 can be extended to any *constant* $h$. This question is still open. However, we performed a large number of computer experiments that strongly support our conjecture. In particular, we have compared the performances of the $h$-PROT with those of the asymptotically-optimal $h$-PARTY and

with those of a prim-based randomized heuristic, named RANDOMIZED $h$-PRIM, that gives good performance in practice (see [8]). The results of this experimental work are summarized in Figure 3.

**Paper's Organization.** In Section 2, we present our protocol. In Section 2.1 we prove that its expected approximation ratio is constant for $h = 2$. In Section 2.2 we provide an experimental analysis that compare the proposed protocol with the $h$-PARTY heuristic. Finally, in Section 3, we briefly address some open problems.

## 2 Distributed Protocol for the Ad-Hoc Model

We consider $h$-HOPS CONVERGECAST on the ad-hoc model and we propose a distributed randomized protocol $h$-PROT that, given a set $S$ and a root $b$, constructs a feasible range assignment $\mathsf{range}^{Prot}$. The protocol works in *phases* and assumes that, at the starting time, every node knows only its label, its geographical position and $L$. Each node (station) is equipped by an omni-directional antenna: it is able to change its transmission range. We will consider the case $d = 2$ (i.e. instances on the plane). The details of the protocol executed by station $s \in S$ are described in Figure 1.

**Init phase (raws 3–10)** The root station $b$ sends a *start* message to all the other stations. All the other stations are waiting for this message.

**Phase $j$ (raws 13–28)** If $j < h$, all the non-`connected` stations flip a bit $x$ with $\mathbf{Pr}[x = 1] = f(n, h, j, \alpha)$. If $j = h$ or $x = 1$ each non-`connected` station $s$ sends a *search* ("search:s") message (containing its label and its coordinates) at increasing range $r$ (raw 23). Station $s$ stops in sending messages as soon as it receives an *echo* message from the closest connected node $v_s$ ("echo:$v_s$") at level $j-1$. Node $s$ chooses $v_s$ as its father by fixing $\mathsf{range}^{Prot}(s) = d(s, v_s)$. Finally $s$ becomes `connected` and sets its level to $j$(raws 18–22). In the meantime, all the connected stations at level $j - 1$ are waiting for messages from non-`connected` stations. If a *search* message from a station $v$ is received ("search:$v$") then, if $s$ is the closest connected node to $v$, it sends an *echo* message to $v$ containing its label and its coordinates (raws 25–28). Notice that, this last step can be easily performed by using the connection between the connected nodes.

The selection probability $\mathbf{Pr}[x = 1]$ is defined as

$$f(h, j, n, \alpha) = n^{-\lambda(h,j,\alpha)} \quad \text{where } \lambda(h, j, \alpha) = \frac{\sum_{i=1}^{h-j-1} (2/\alpha)^i}{\sum_{i=1}^{h-j} (2/\alpha)^i}.$$

This function has been inspired by the almost "optimal" centralized heuristic $h$-PARTY: The expected number of selected nodes during phase $j$ equals the number of bases selected during the $j$-th recursive step of $h$-PARTY.

```
h-Prot(S,b)
begin
  if (s = b) then begin
    send("start",L√2);
    connected := true;
    level := 0;
  end else begin
    wait("start");
    connected := false;
  end

  for j = 1, ..., h do begin
    if (connected = false) then
      if ( j ≤ h − 2 ) then
        randomly choose a bit x with Pr[x = 1] = f(n, h, j, α);
      else x = 1;
    for r = 1, 2, ..., 2^ℓ, 2^{log⌈L√2⌉} do
      if (connected = false and x = 1) then
        if (received("echo:v_s") = true) then begin
          range^{Prot}(s) := d(s, v_s);
          connected := true;
          level = j;
        end else send("search:s", r);
      else
        if (level = j − 1) then
          if (received("search:v") = true) then
            if ( d(s, v) = min{ d(s̃, v) : s̃ received "search:v" }) then
              send("echo:s",d(s, v));
end
```

**Fig. 1.** The protocol $h$-Prot executed by station $s \in S$

### 2.1 Probabilistic analysis

**Theorem 3 (Energy cost of the protocol.).** *Let* $\widehat{r}(s)$ *be the* maximal *transmission range used by* $s \in S$ *during any phase of the protocol. Then, it holds that*

$$\mathsf{cost}(\widehat{r}) = \sum_{s \in S} (\widehat{r}(s))^{\alpha} = \Theta(\mathsf{cost}(\mathsf{range}^{Prot}))$$

*Proof.* When a node $s$ given in a Phase $j$ is selected then its maximal range $\widehat{r}(s)$ will be no larger than twice its final range $\mathsf{range}^{Prot}(s)$. Furthermore, when a node $s \in S$ has been selected in Phase $j-1$ (so it acts like a potential father), it will send *echo* and *father* messages, during Phase $j$, at ranges equal to those of the corresponding sons.[3] Let $\widehat{x_s}$ be the son of $s$ at maximal distance in the final transmission graph. We have that

$$\sum_{s \in S} (\widehat{r}(s))^{\alpha} \leq 2 \sum_{s \in S} (\max\{\mathsf{range}^{Prot}(s), \mathsf{range}^{Prot}(x_s)\})^{\alpha} \leq$$

$$\leq \; 4 \sum_{s \in S} \mathsf{range}^{Prot}(s)^{\alpha} = \Theta(\mathsf{cost}(\mathsf{range}^{Prot})).$$

□

**Theorem 4 (Energy cost of the protocol solution.).** *Energy cost of the protocol. Let* $S$ *be a random instance of* $n$ *nodes selected from a square* $Q$ *of edge size* $L$ *and let* $b \in S$*. Then, for* $h = 2$*, the expected cost of* $\mathsf{range}^{Prot}$ *satisfies the following bounds*

$$\mathbf{E}\left(\mathsf{cost}(\mathsf{range}^{Prot})\right) = O\left(L^{\alpha} n^{\frac{2}{\alpha+2}}\right).$$

*Proof.* Without loss of generality, assume that the root $b$ is the node of index $n$. We denote any node selected in the first phase as *base* and define, for any node $i$, $D_i$ as the minimal distance between node $i$ and the root and any base. We denote as $\mathtt{p_b}$ the probability that a node becomes a base. As for $h = 2$, this probability is equal to

$$f(n, 2, 0, \alpha) = n^{-\lambda(2,0,\alpha)} = n^{-\frac{\alpha}{\alpha+2}}.$$

It holds that,

$$\mathbf{E}\left(\mathsf{cost}(\mathsf{range}^{Prot})\right) \leq \sum_{i=1}^{n-1} [\mathtt{p_b}(\sqrt{2}\mathrm{L})^{\alpha} + (1 - \mathtt{p_b})\overline{\mathbf{E}}\left((\mathtt{D_i})^{\alpha}\right)]$$

$$\leq (n-1)(\sqrt{2}L)^{\alpha} \mathtt{p_b} + \sum_{\mathtt{i=1}}^{\mathtt{n-1}} \overline{\mathbf{E}}\left((\mathtt{D_i})^{\alpha}\right). \qquad (3)$$

[3] Observe that, in this case, we can assume that all such nodes (potential father) known the partial solution constructed so far. So, only the real father will send echo message to a son.

Where $\overline{\mathbf{E}}\left((D_i)^\alpha\right)$ denotes $\mathbf{E}\left((D_i)^\alpha \mid i \text{ is not a base }\right)$. In order to evaluate the value $\overline{\mathbf{E}}\left((D_i)^\alpha\right)$ we define

$$\Delta = \frac{L}{n^{\frac{1}{\alpha+2}}}$$

Then

$$\begin{aligned}
\overline{\mathbf{E}}\left((D_i)^\alpha\right) &\leq \overline{\mathbf{Pr}}\left[D_i < \Delta\right] \Delta^\alpha + \sum_{k=0}^{\infty} \overline{\mathbf{Pr}}\left[2^k \Delta \leq D_i < 2^{k+1}\Delta\right] (2^{k+1}\Delta)^\alpha \\
&\leq \Delta^\alpha \left[1 + 2^\alpha \sum_{k=0}^{\infty} \overline{\mathbf{Pr}}\left[2^k \Delta \leq D_i < 2^{k+1}\Delta\right] 2^{k\alpha}\right] \\
&\leq \Delta^\alpha \left[1 + 2^\alpha \sum_{k=0}^{\infty} \left[\overline{\mathbf{Pr}}\left[D_i \geq 2^k \Delta\right] - \overline{\mathbf{Pr}}\left[D_i \geq 2^{k+1}\Delta\right]\right] 2^{k\alpha}\right] \\
&\leq \Delta^\alpha \left[1 + 2^\alpha \sum_{k=0}^{\infty} \overline{\mathbf{Pr}}\left[D_i \geq 2^k \Delta\right] 2^{k\alpha}\right],
\end{aligned}$$

where $\overline{\mathbf{Pr}}\left[X\right]$ denotes $\mathbf{Pr}\left[X \mid i \text{ is not a base }\right]$. Observe that if $2^k\Delta \geq \sqrt{2}L$ then $\overline{\mathbf{Pr}}\left[D_i \geq 2^k\Delta\right] = 0$. So

$$\overline{\mathbf{E}}\left((D_i)^\alpha\right) \leq \Delta^\alpha \left[1 + 2^\alpha \sum_{k=0}^{\lceil \log(\sqrt{2}L/\Delta)\rceil} \overline{\mathbf{Pr}}\left[D_i \geq 2^k \Delta\right] 2^{k\alpha}\right] \tag{4}$$

We now need an upper bound for the probability that $D_i$ is larger than a given parameter $\rho$, when $\rho \leq 2\sqrt{2}L$. It holds that,

$$\overline{\mathbf{Pr}}\left[D_i \geq \rho\right] = \left(1 - \frac{A_{i,\rho}}{L^2}\right) \sum_{j=0}^{n-2} \mathbf{Pr}\left[N_{i,\rho} = j\right] (1 - \mathtt{p_b})^{\mathtt{j}} \tag{5}$$

where $N_{i,\rho}$ is the number of nodes, different from $i$ and the root, falling into the disk of radius $\rho$ and centered at node $i$, while $A_{i,\rho}$ is the area of the intersection between the disk of radius $\rho$ and centered at node $i$ and the square $Q$. Then, it holds that

$$\mathbf{Pr}\left[N_{i,\rho} = j\right] = \binom{n-2}{j} \left(\frac{A_{i,\rho}}{L^2}\right)^j \left(1 - \frac{A_{i,\rho}}{L^2}\right)^{n-2-j} \tag{6}$$

By combining (5) and (6), we get

$$\begin{aligned}
\overline{\mathbf{Pr}}\left[D_i \geq \rho\right] &= \left(1 - \frac{A_{i,\rho}}{L^2}\right) \sum_{j=0}^{n-2} \binom{n-2}{j} \left(\frac{A_{i,\rho}}{L^2}\right)^j \left(1 - \frac{A_{i,\rho}}{L^2}\right)^{n-2-j} (1 - \mathtt{p_b})^{\mathtt{j}} \\
&= \left(1 - \frac{A_{i,\rho}}{L^2}\right) \left(1 - \mathtt{p_b} \frac{\mathtt{A_{i,\rho}}}{\mathtt{L^2}}\right)^{n-2} \\
&\leq \left(1 - \mathtt{p_b} \frac{\rho^2}{\mathtt{8L^2}}\right)^{n-1}
\end{aligned}$$

where, in the last inequality, we use the fact that $A_{i,\rho} \geq \rho^2/8$ when $\rho \leq 2\sqrt{2}L$. By combining the last inequality with (4), we obtain

$$\overline{\mathbf{E}}\left((D_i)^\alpha\right) \leq \Delta^\alpha \left[1 + 2^\alpha \sum_{k=0}^{\lceil \log(\sqrt{2}L/\Delta) \rceil} \left(1 - \mathtt{p_b} \frac{(2^k\Delta)^2}{8L^2}\right)^{n-1} 2^{k\alpha}\right].$$

By replacing the values of $\mathtt{p_b}$ and $\Delta$, we get

$$\begin{aligned}
\overline{\mathbf{E}}\left((D_i)^\alpha\right) &\leq \frac{L^\alpha}{n^{\frac{\alpha}{\alpha+2}}} \left[1 + 2^\alpha \sum_{k=0}^{\lceil \log(\sqrt{2}n^{\frac{1}{\alpha+2}}) \rceil} \left(1 - \frac{4^k}{8n}\right)^{n-1} 2^{k\alpha}\right] \\
&\leq \frac{L^\alpha}{n^{\frac{\alpha}{\alpha+2}}} \left[1 + 2^\alpha \sum_{k=0}^{\lceil \log(\sqrt{2}n^{\frac{1}{\alpha+2}}) \rceil} e^{-\frac{4^k}{8n}(n-1)} 2^{k\alpha}\right] \\
&\leq \frac{L^\alpha}{n^{\frac{\alpha}{\alpha+2}}} \left[1 + 2^\alpha \sum_{k=0}^{\lceil \log(\sqrt{2}n^{\frac{1}{\alpha+2}}) \rceil} e^{-\frac{4^k}{16}} 2^{k\alpha}\right].
\end{aligned}$$

We observe that there exists a constant $c = c(\alpha)$ (remind that $\alpha$ is a constant) such that

$$1 + 2^\alpha \sum_{k=0}^{\lceil \log(\sqrt{2}n^{\frac{1}{\alpha+2}}) \rceil} e^{-\frac{4^k}{16}} 2^{k\alpha} \leq c.$$

It thus follows that

$$\overline{\mathbf{E}}\left((D_i)^\alpha\right) \leq c \frac{L^\alpha}{n^{\frac{\alpha}{\alpha+2}}}.$$

By replacing this bound in (3), we get

$$\begin{aligned}
\mathbf{E}\left(\mathsf{cost}(\mathsf{range}^{Prot})\right) &\leq n(\sqrt{2}L)^\alpha \mathtt{p_b} + \sum_{\mathtt{i}=1}^{\mathtt{n}} \overline{\mathbf{E}}\left((\mathtt{D_i})^\alpha\right) \\
&\leq (\sqrt{2}L)^\alpha n^{\frac{2}{\alpha+2}} + cn \frac{L^\alpha}{n^{\frac{\alpha}{\alpha+2}}} \\
&\leq cL^\alpha n^{\frac{2}{\alpha+2}} \\
&= O\left(L^\alpha n^{\frac{2}{\alpha+2}}\right).
\end{aligned}$$

□

By comparing Theorem 4 with Theorem 1, we can state the following

**Corollary 1.** *The expected approximation factor yielded by* 2-Prot$(S, b)$ *on random instances, for* $h = 2$ *and* $\alpha = 1, 2$, *is bounded by a constant.*

```
procedure h-PARTY(h, V, p)
    if h = 1 then T ← {{x, p}|x ∈ V − {p}};
    else begin
        k ← ⌊|V|^{η_α(h)}⌋;        T ← ∅;
        Let l be the side length of the smallest square
            containing all points in V;
        Partition the square into a grid of square cells
            of side length l/⌊√k⌋;
        Let k' be the number of cells and let V_i be the
            points of V in the i-th cell, 1 ≤ i ≤ k';
        for i ← 1 to k' do
            if |V_i| ≥ 1 then begin
                a ← a random point in V_i;
                T ← T ∪ {{a, p}};
                if |V_i| > 1 then
                    T ← T∪ h-PARTY(h − 1, V_i, a);
            end;
    end;
    output T
```

**Fig. 2.** The $h$-Party heuristic.

### 2.2 Experimental Evaluations

As mentioned in the Introduction, we believe that Theorem 3 holds for any *constant* $h \geq 1$. However, till now, we have not been able to extend our theoretical analysis to $h > 2$. The goal of this section is thus providing some experimental evidence of our conjecture by considering the cases $h = 2, \ldots, 8$ and $\alpha = 1$.

We compare the costs of the solutions generated by $h$-Prot with those generated by the centralized heuristics $h$-Party (see Figure 2 for a description) and with those of a prim-based randomized heuristic, named Randomized $h$-Prim, that gives good performance in practice (see [8]). For practical reasons, we have implemented a variant of the protocol in which: *(i)* the selected node(s) of phase $j$ may be connected to any already connected node (though not belonging to level $j-1$); *(ii)* the probability according to which a node is selected as a base is scaled by a factor $f$ $(0.009 \leq f \leq 1)$ with respect to that adopted in the theoretical protocol.

All the experiments are carried out for several sizes $n$ of the random instances (between 100 and $10,000$) and for $h = 2, 3, \ldots, 8$. Node positions of any instance are chosen independently and uniformly at random in a square of side length 1. For each $n$ and $h$, the number of instances for each $n$ decreases from 100 to 50 as $n$ grows. Moreover, for every instance five runs are executed (only for randomized algorithms with randomness). Then, we get the average value (on these runs) for a comparison among the algorithms.

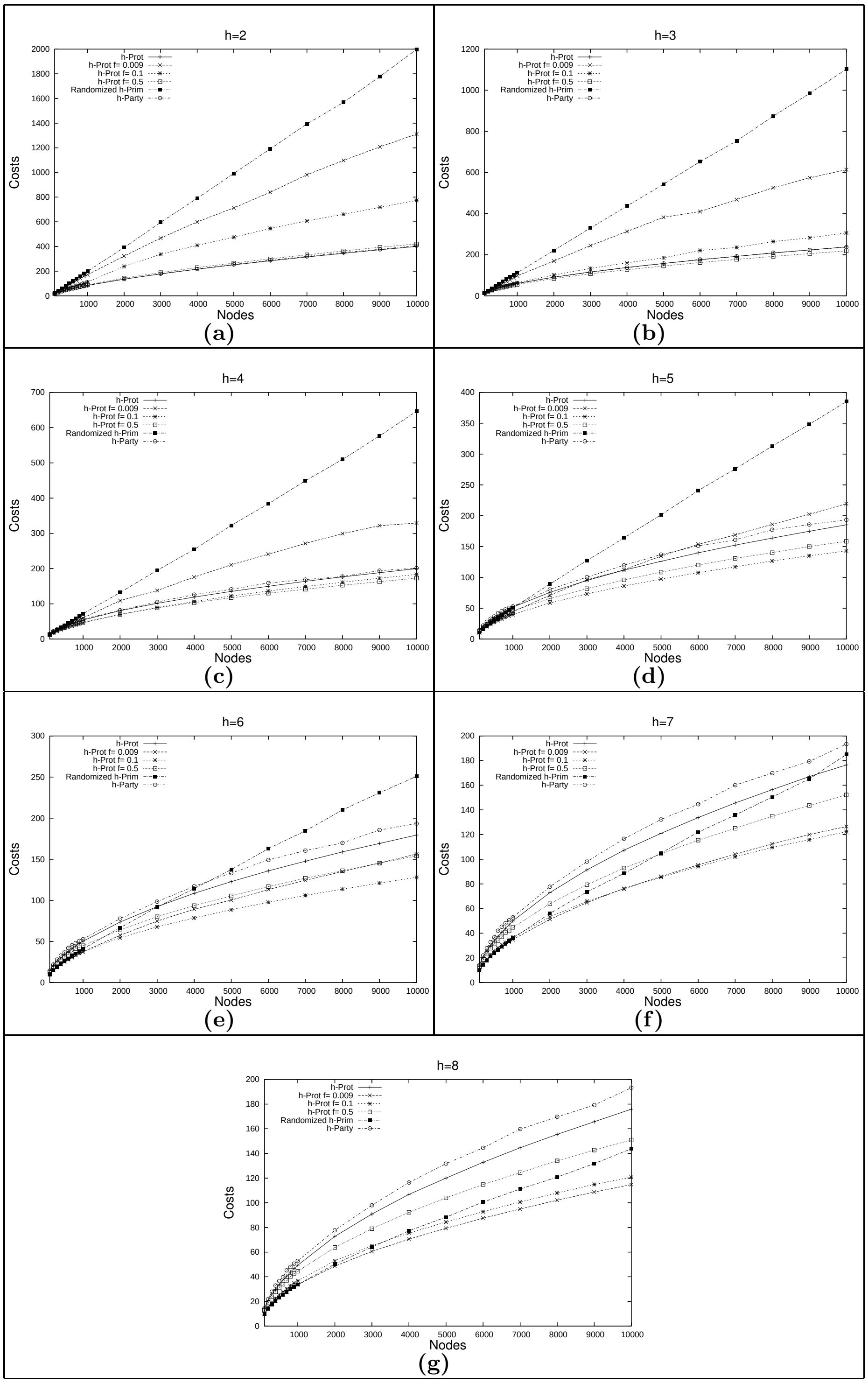


**Fig. 3.** Experimental results.

As we can see from the Tables in Figure 3, $h$-PROT has performances equivalent to $h$-PARTY. This is not surprising since the selection probability in $h$-PROT is defined so as to *simulate* $h$-PARTY. So, it works very well for all $h \leq 5$.
Another important contribution of our experimental work is that of providing a "good" strategy for tuning the selection probability of $h$-PROT in order to overcome $h$-PARTY and reach RANDOMIZED $h$-PRIM's performances when $h$ is large. The strategy relies on the following basic fact. RANDOMIZED $h$-PRIM works better when $h$ is large since it generates much less *bases* (here, one may see a base as a node with a large transmission range) than $h$-PARTY: RANDOMIZED $h$-PRIM indeed chooses *just one* base per phase. The strategy is thus to reduce the selection probability $f$ in order to get closer to RANDOMIZED $h$-PRIM's behavior as $h$ grows. In Figure 3g, we can see the outperforming of our protocol with $f = 0.009$ and $h = 8$ with respect to both $h$-PARTY and RANDOMIZED $h$-PRIM and all implementations of $h$-PROT. Figure 3 indeed shows that, in order to improve the performance of $h$-PROT, $f$ should decrease as $h$ increases.

## 3 Open problems

The most important issue to be addressed is the probabilistic analysis of our protocol for $h \geq 3$. We believe that, for constant values of $h$, it is possible to prove that its expected approximation ratio is constant.

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