

Dynamical instabilities in apsidally aligned systems

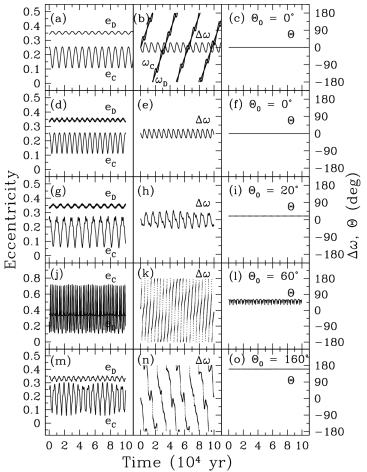
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IX CELMEC, Viterbo, Italy, 13–18/September/2026

14/September/2026

Introduction: Apsidal Alignment

- Apsidal alignment has been studied for roughly 25 years.
- First well measured case: two-planets υ Andromedae:
  - Chiang et al. (2002), Malhotra (2002), Michtchenko & Malhotra (2004)
- Further explorations of three planet systems:
  - Migaszewski & Gozdziewski (2008): HD37124, GJ581

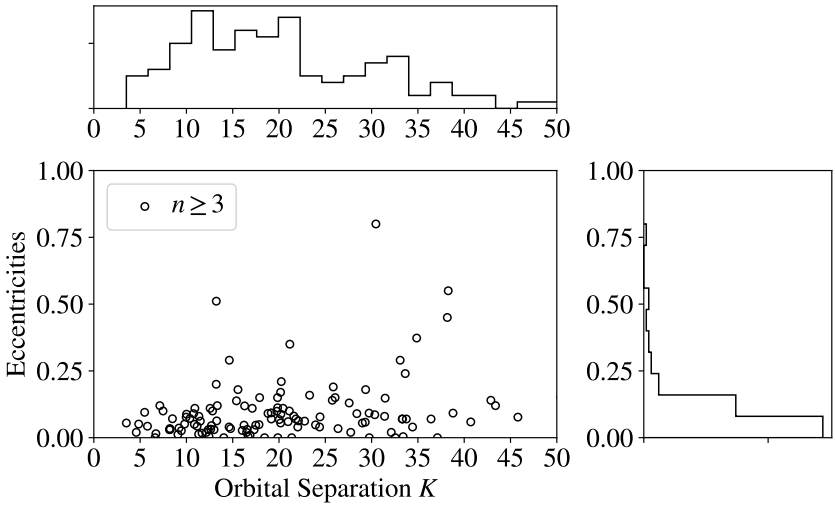


Chiang et al (2002)

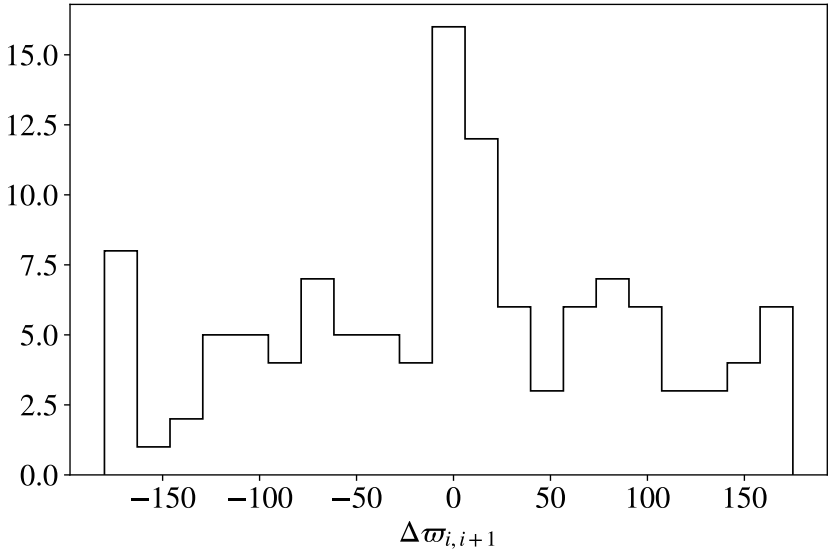
What about Kepler-multis?

- Not much literature: Gratia & Lissauer, 2019 briefly explored the role of $\Delta\varpi_{i,j} = 0$
- But does it make sense to look for it?

Introduction — Compact planetary systems



The role of periapsis: Compact planetary systems



Goals

- We numerically explore the effects of apsidal alignement in the stability of compact multi-planetary systems:
  - How are they different from randomly oriented systems?
  - What's the extent of apsidal alignment as a function of relative longitudes of pericenter $\Delta\varpi_{i,j}$

Methods

Initial conditions

- Planetary systems with $n = 3$ planets.
- Equal orbital separation $K = (a_{i+1} - a_i)/R_{Hill}^{i,i+1}$
- Innermost planet with $a_1 = 0.1au$ ($\approx 12\mathrm{days}$)
- Central star with $1M_{\odot}$
- Equal planetary masses $M_p = 1M_{\oplus}$

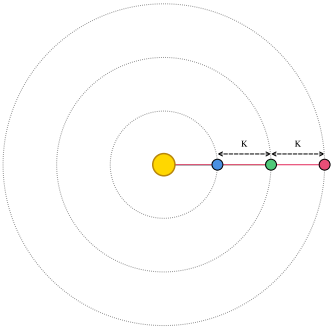
Parameter sampling:

- Different orbital separations $K \in [8, 18]$
- Eccentricities $e_1 = e_2 = e_3 = e \ \in [0, 0.5]$
- Relative longitudes of periapsis were probed:

$$\Delta\varpi_{i,j} = \varpi_i - \varpi_j, \forall \ \mathrm{i} \in [1, 2, 3],$$

$$\Delta\varpi_{i,j} \in [-180^o, 180^o], \varpi_1 = 0$$

Methods

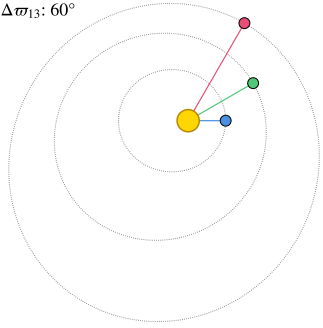


$\Delta\varpi$ (longitude of pericenter diff.):

$\Delta\varpi_{12}$: 30°

$\Delta\varpi_{23}$: 30°

$\Delta\varpi_{13}$: 60°



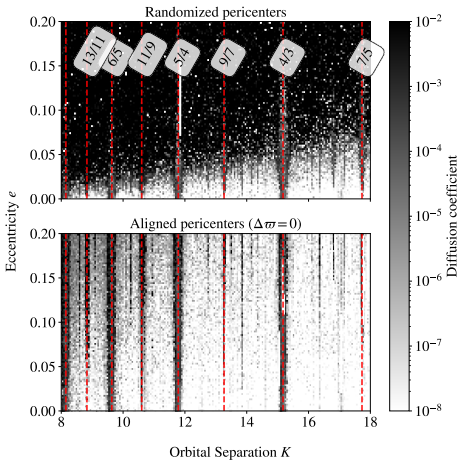
Methods

Numerical setup

- 160.000 integrations with REBOUND (pure N-body, Rein & Liu, 2012).
  - TRACE hybrid integrator (Wisdom-Holman + Bulirsch-Stoer; Lu et al., 2024))
  - NAOJ CfCA GPPC cluster.
- Integration time: $10^6 P_1 \approx 10^{4.5} yrs$.
- Chaos metric: diffusion of orbital period:

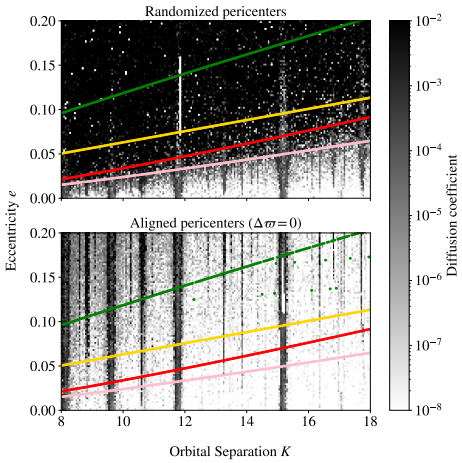
$$D_i^n = \mid n_i^1 - n_i^2 \mid / n_i^1 > 10^{-4} \longrightarrow \mathrm{Chaotic!}$$

Results: aligned × randomized periapsis



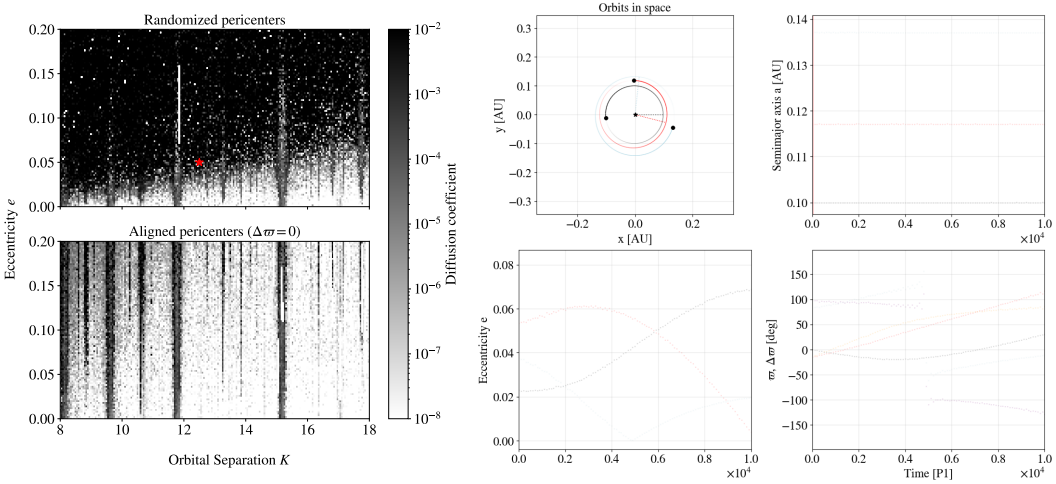
- Randomized pericenters:
  - Most of the parameter space is unstable (high diffusion).
  - Resonances (MMR) are important: bring instability and make stable.
  - Instability mechanism: resonance overlap (variable width).
- Apsidal alignment:
  - Parameter space with low diffusion
  - Resonances (MMR) have fixed widths

Results: aligned $\times$ randomized periapsis

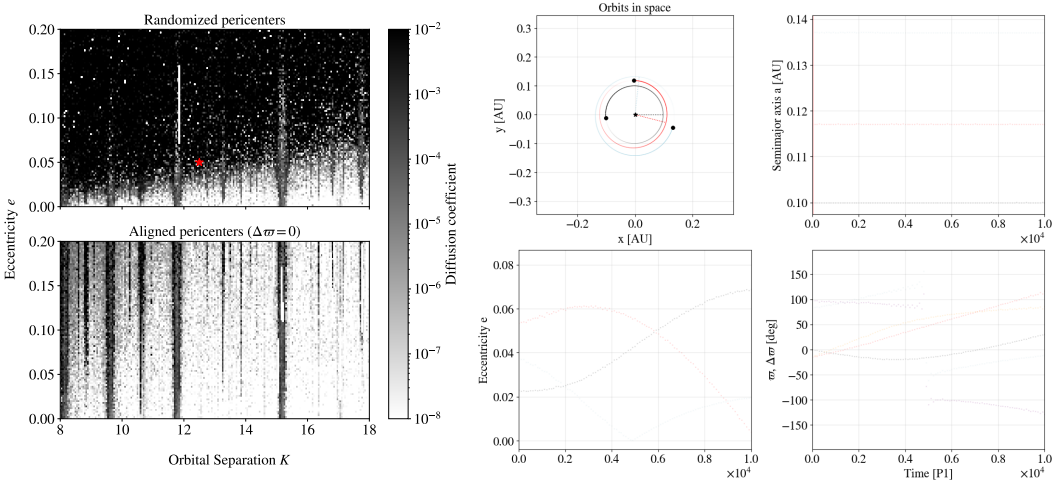


- Analytical estimates for instability onset:
  - Green: Orbit crossing $a_1(1+e_1) \geq a_2(1-e_2)$
  - Yellow: Orbit intersection (Vassiliev and Kholshevnikov, 1999)
  - Red: Updated MMR overlap (Hadden and Lithwick, 2018), random $\Delta\varpi$
  - Pink: Updated MMR overlap , $\Delta\varpi = -\pi$

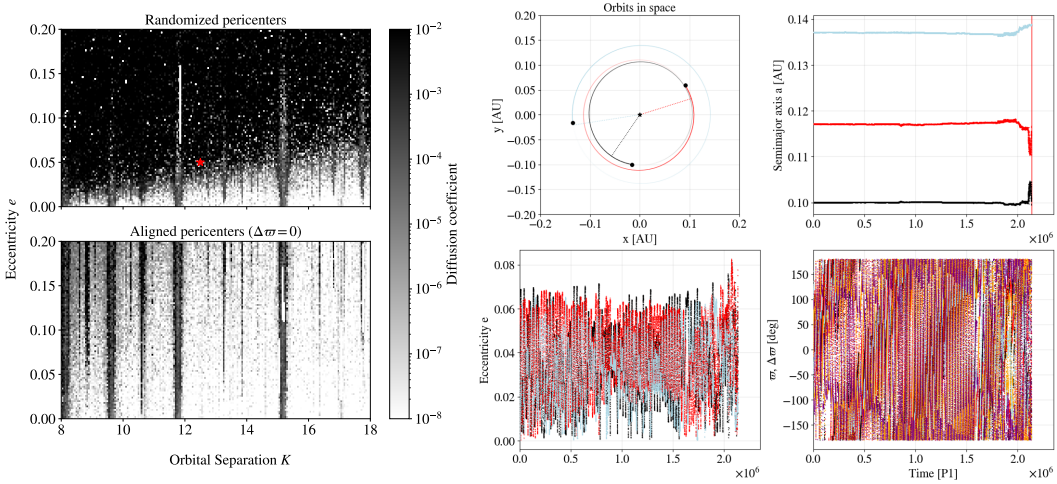
Results: aligned × randomized periapsis



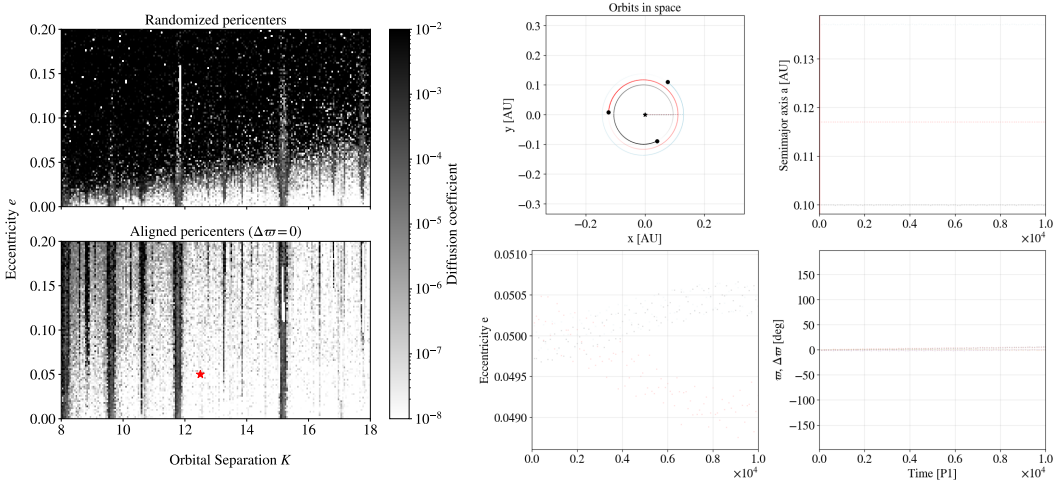
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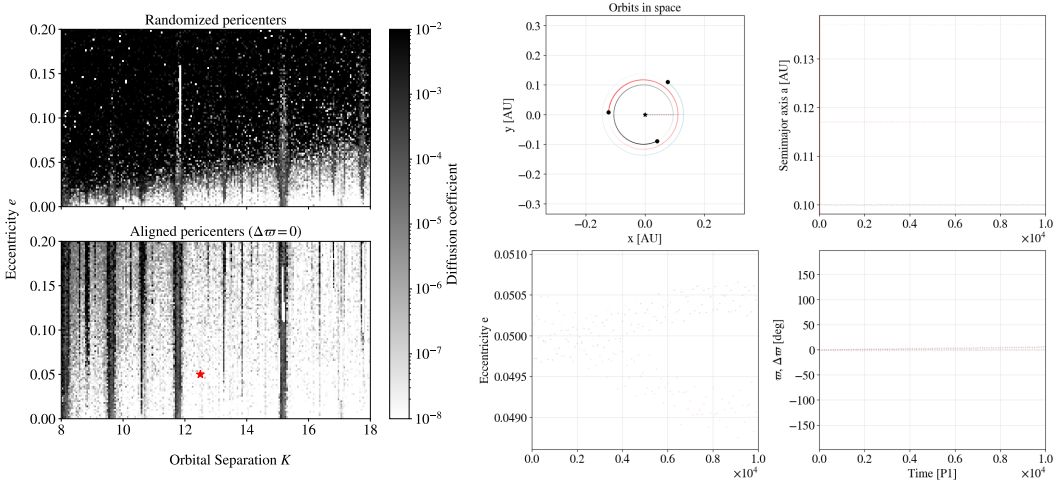
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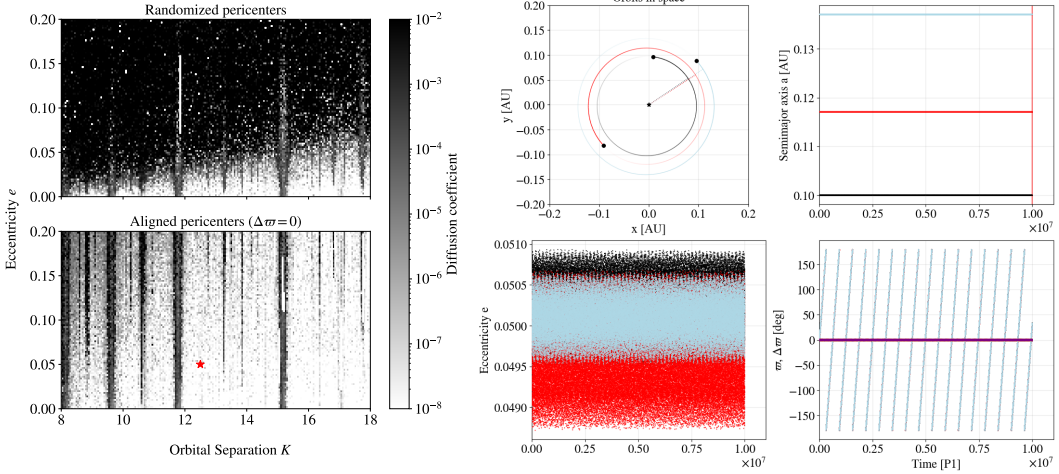
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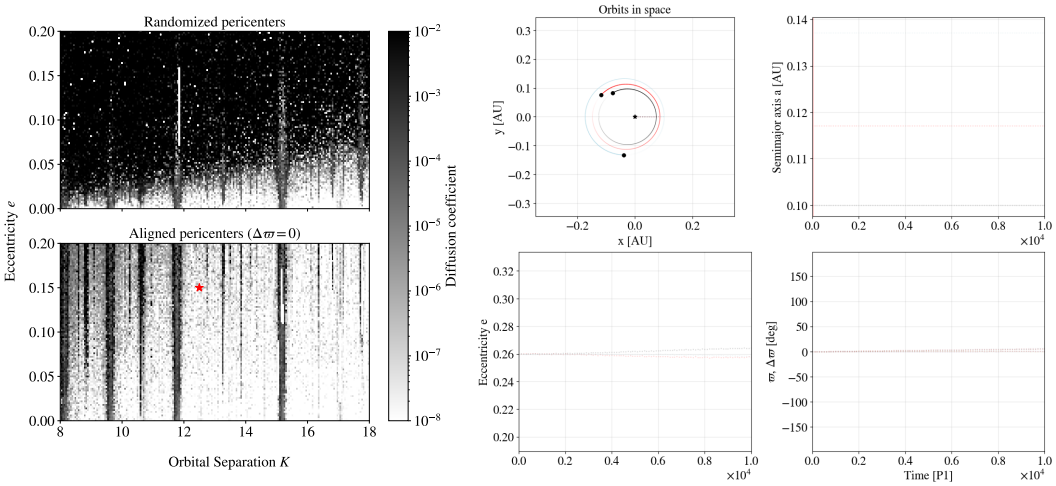
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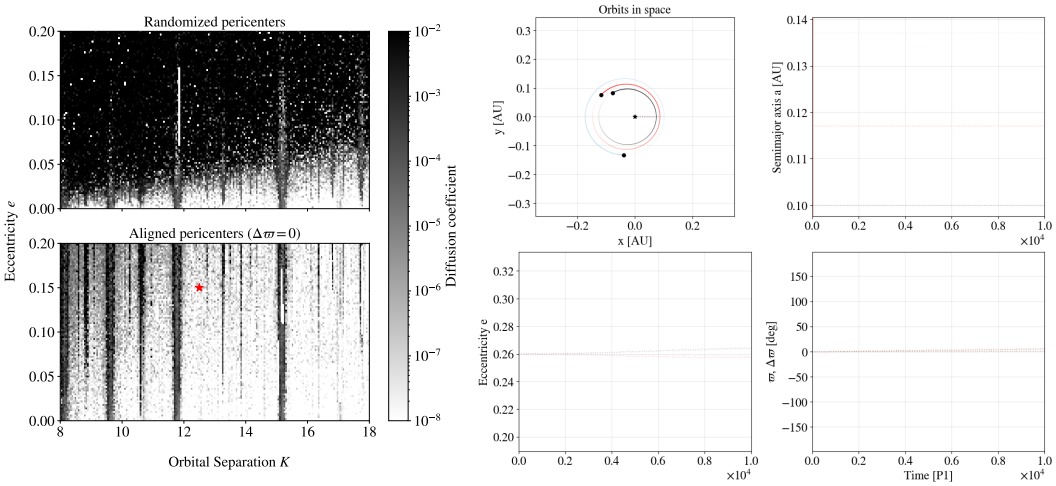
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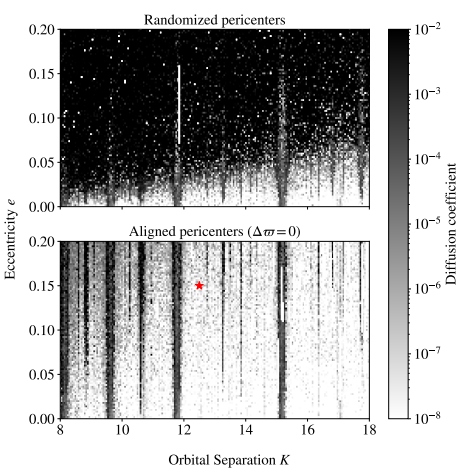
Results: aligned × randomized periapsis

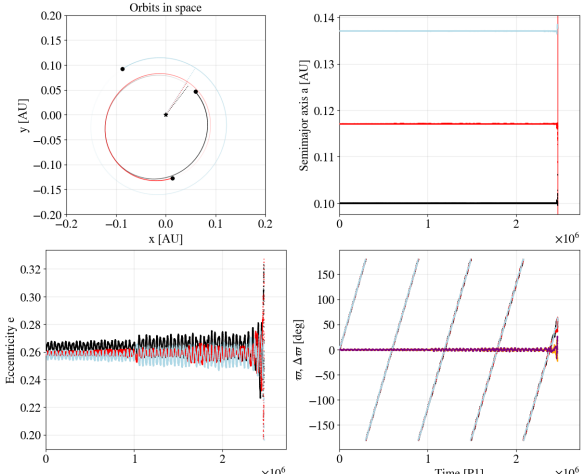


Results: aligned × randomized periapsis



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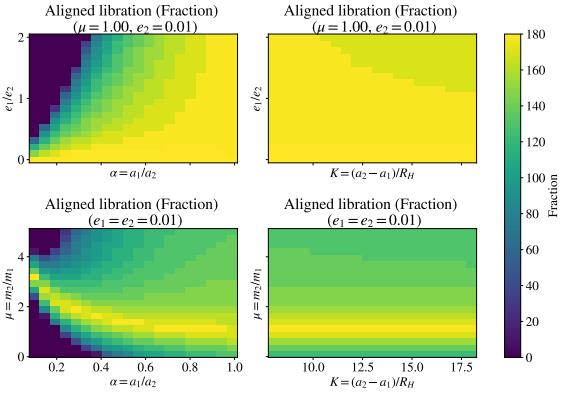




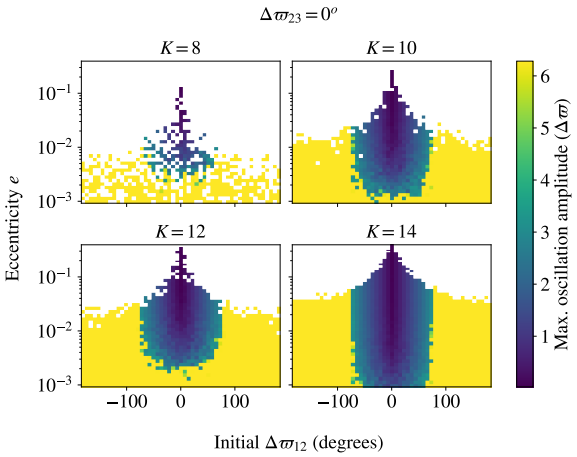
To what extent can the planets be locked in apsidal alignment before they decouple? What's the maximum oscillation amplitude?

Results: Thresholds of apsidal alignment from Lagrange-Laplace ($n = 2$ planet)

- Oscillation amplitude only depends on $\mu = m_1/m_2$, $\alpha = a_1/a_2$ and e_1/e_2
- High sensitivity to hierarchical systems,
- Compact systems have larger $\Delta\varpi$ amplitude:
- Apsidal corotation can take place for $\Delta\varpi \leq 90^o$ uniformly.

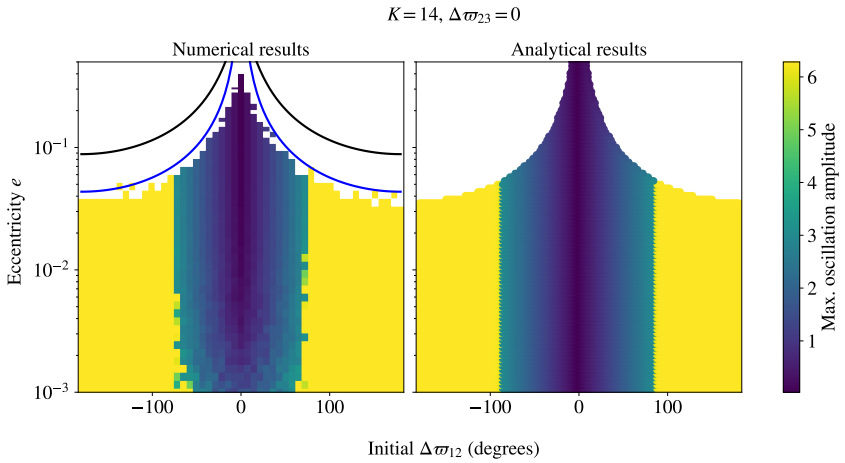


Results: Thresholds of apsidal alignment ($n = 2$ planets)



- Apsidal alignment and stability:
  - High eccentricity stability is due to apsidal corotation
  - Low-eccentricity orbits can remain stable without alignment.

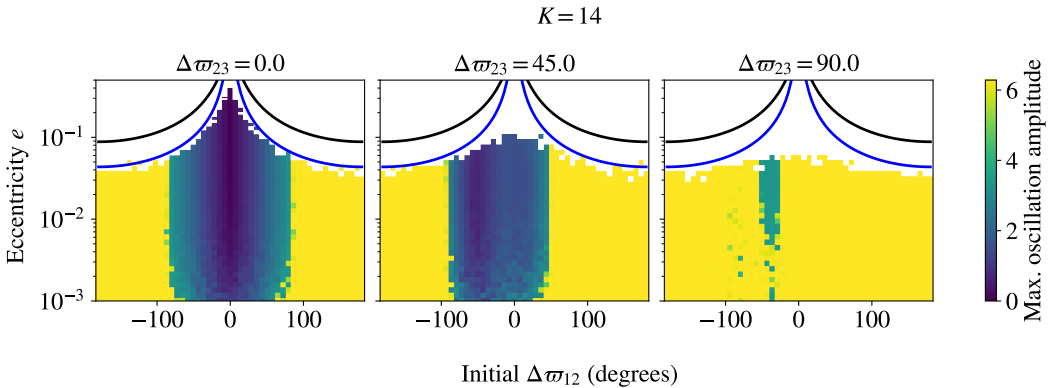
Results: Thresholds of apsidal alignment $n = 2$ planets



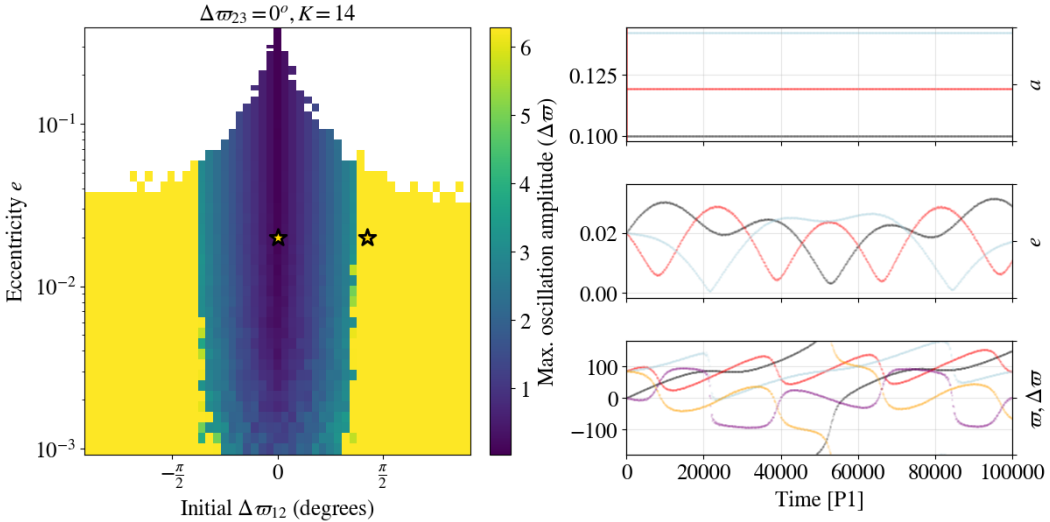
Black: Vassiliev and Kholshevnikov (1999); Blue: Hadden and Lithwick (2018)

Results: Thresholds of apsidal alignment $n = 3$ planets

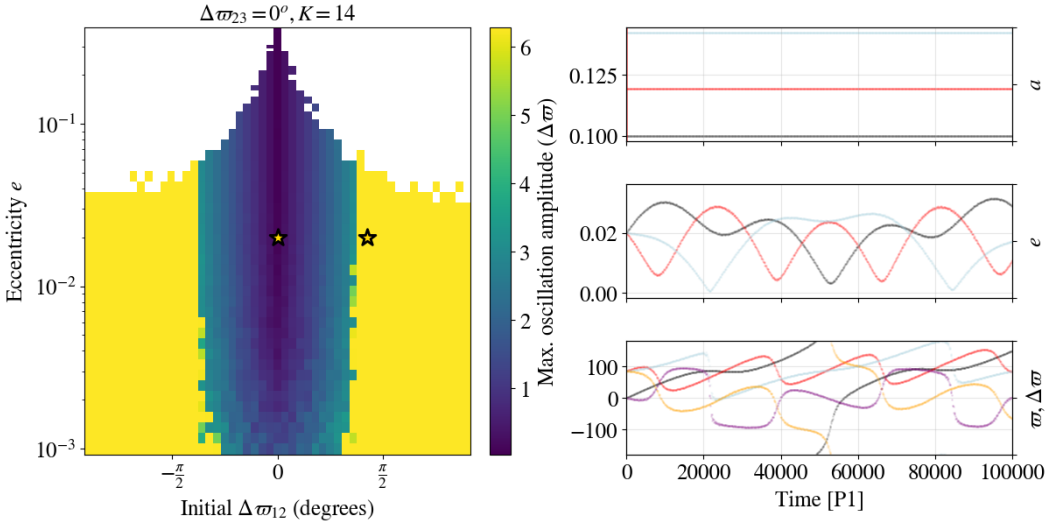
- But, in fact, we have a new degree of freedom: $\Delta\varpi_{23}$.
- Things get more complex.



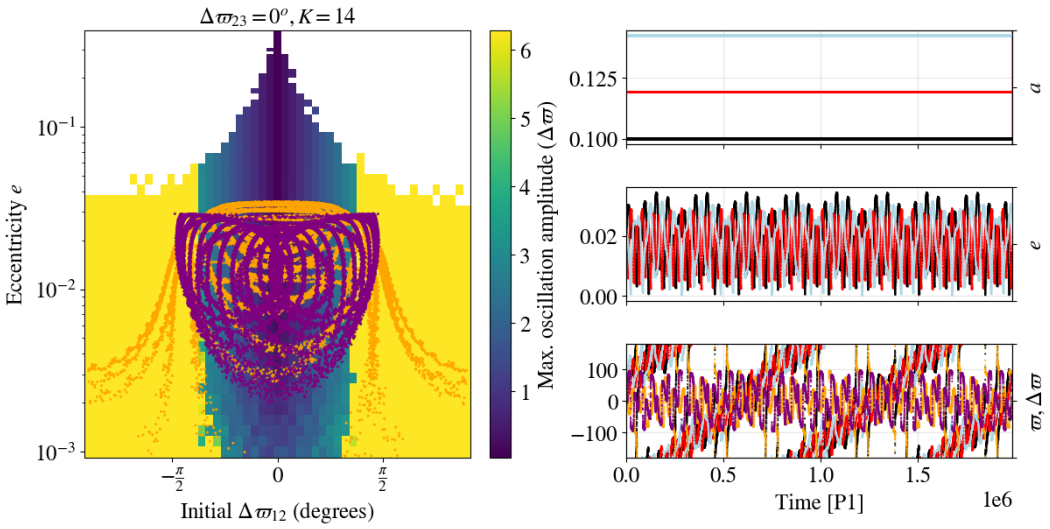
Results: Thresholds of apsidal alignment



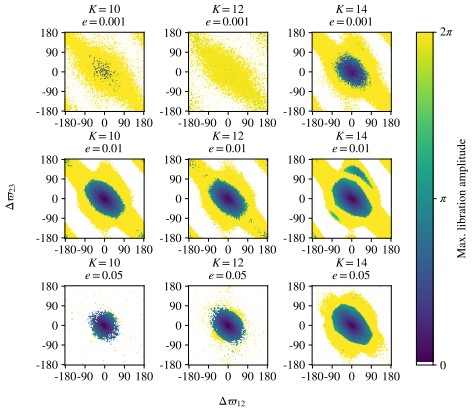
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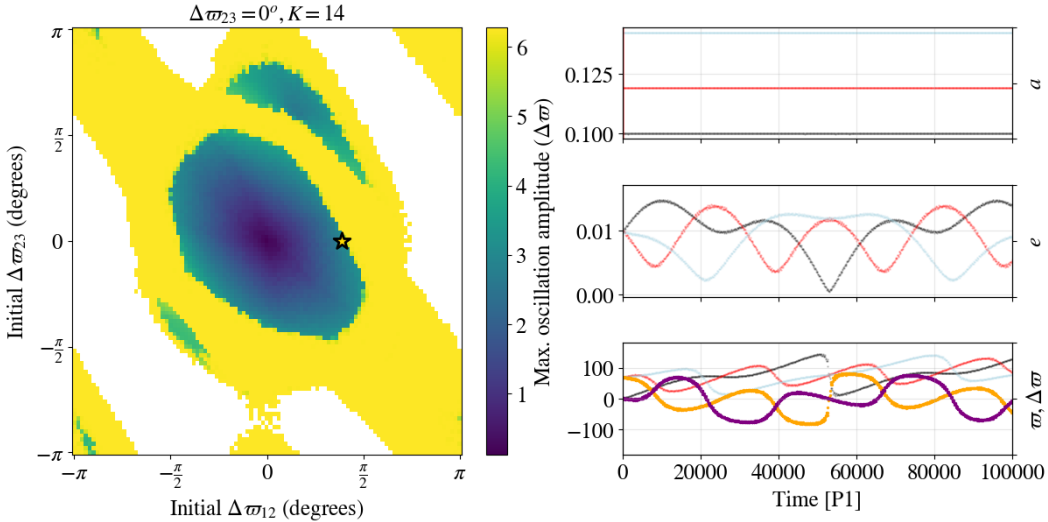


Results: Extent of apsidal corotation

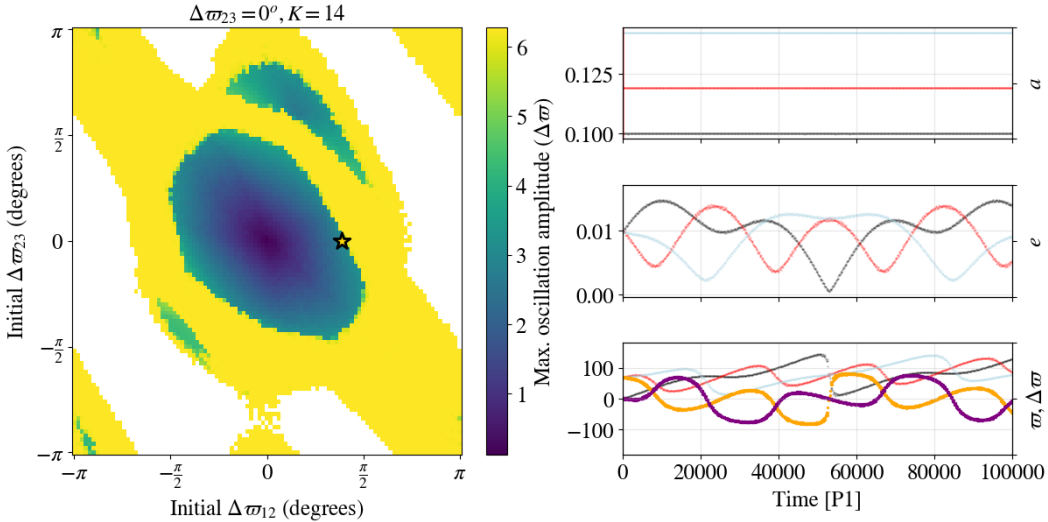


- We should see as a function of both phases.
- It becomes clearer that systems need non-negligibel eccentricities to allow for apsidal alignment
  - Region of apsidal corotation depends on K and e
  - In more unstable systems (i.e. $K = 10, e = 0.05$), most stable regions are the ones in which apsidal alignment takes place

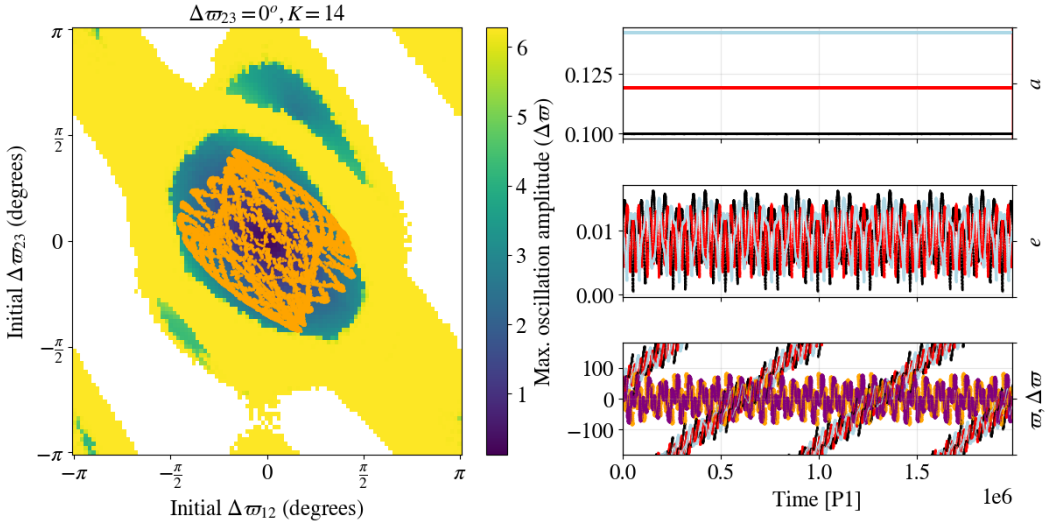
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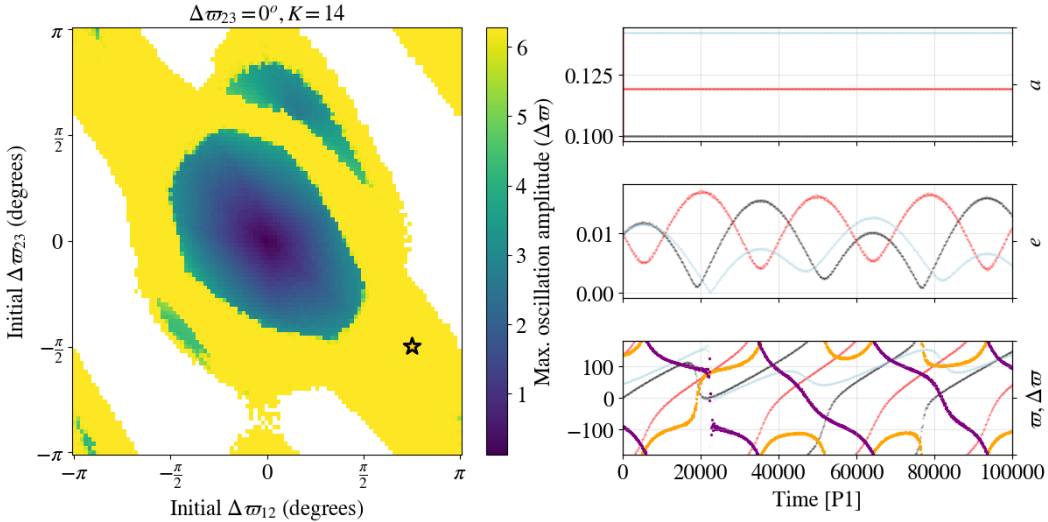
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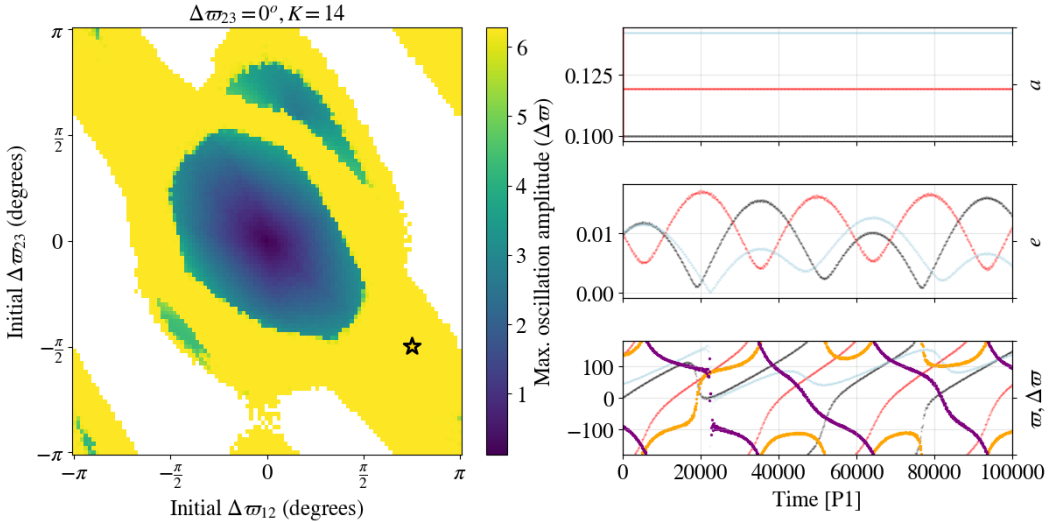
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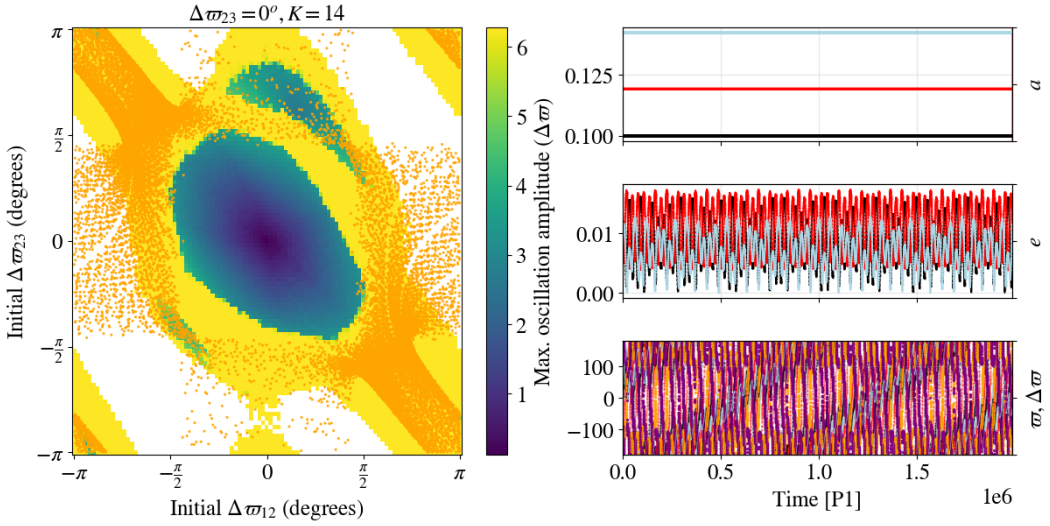
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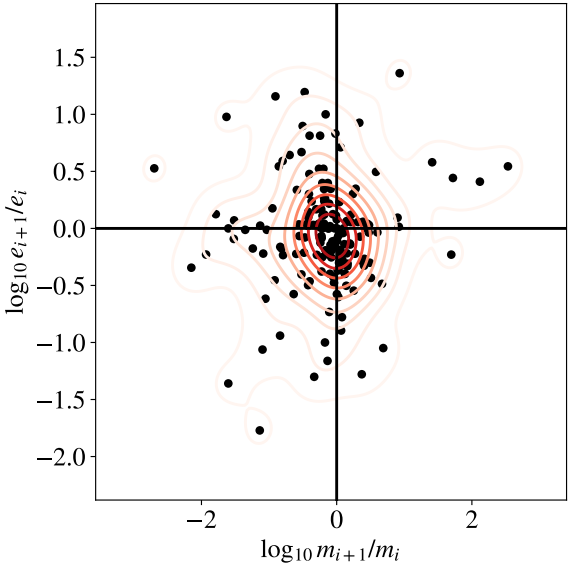
Results: Extent of apsidal corotation



Results: Extent of apsidal corotation



From observations



- Observed multi-planetary systems are highly uniform,
- Compact, similar eccentricities and masses,
- Apsidal configurations might be described by our approach,
- Simple way to characterize apsidal architecture?

Conclusions & future works

- Apsidal alignment allows for very eccentric systems to survive much longer.
  - Different instability mechanism: rupture of apsidal corotation
  - Plenty of room for apsidal corotation take place.
- We might have some candidates:
  - GJ 876, K2-72, K2-19, K2-266 etc...
- Next steps:
  - Investigate the instability mechanism in more detail.
  - Make a solid comparison with observed sample of eccentric planetary systems

P.S.: I'm applying for post docs starting from April/2027, let me know if you ~~want to help me pay my bills~~ are interested in my work and have any open positions!

Thank you very much!

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https://gabrieltxg.github.io

