The rings that should not exist

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partly based on Salo & Sicardy *Astron. Astrophys*. (2026)

A&A, 708, A9 (2026)
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Astronomy & Astrophysics

Rings around irregular bodies

II. Numerical simulations of the 1/3 spin-orbit resonance confinement and applications to Chariklo

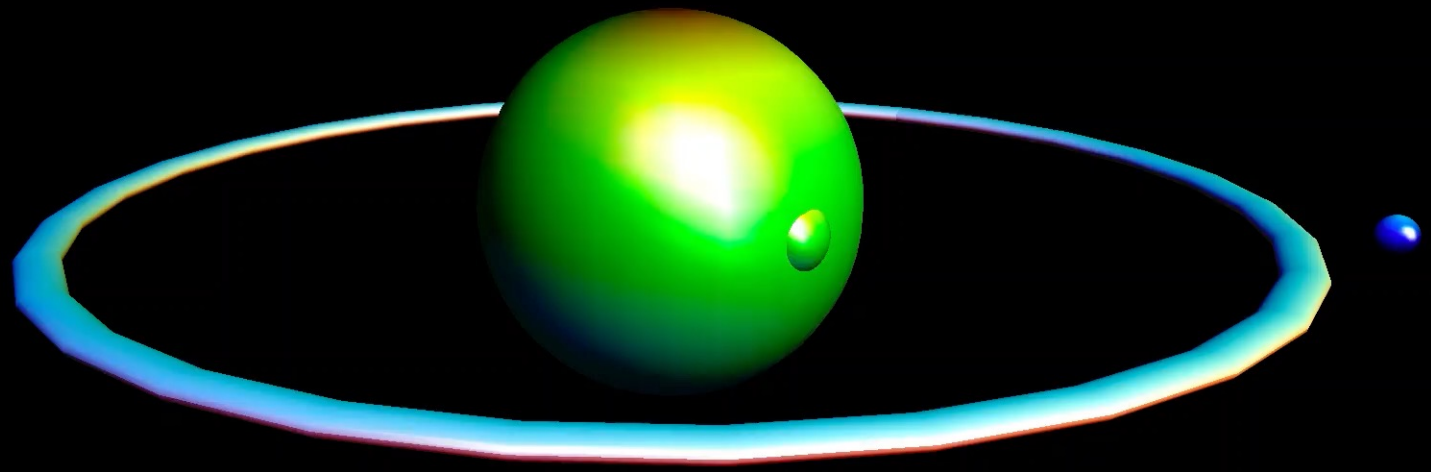
H. Salo[1,⋆], and B. Sicardy[2]

LTE anr agence nationale de la recherche

The “Roche” project

The context

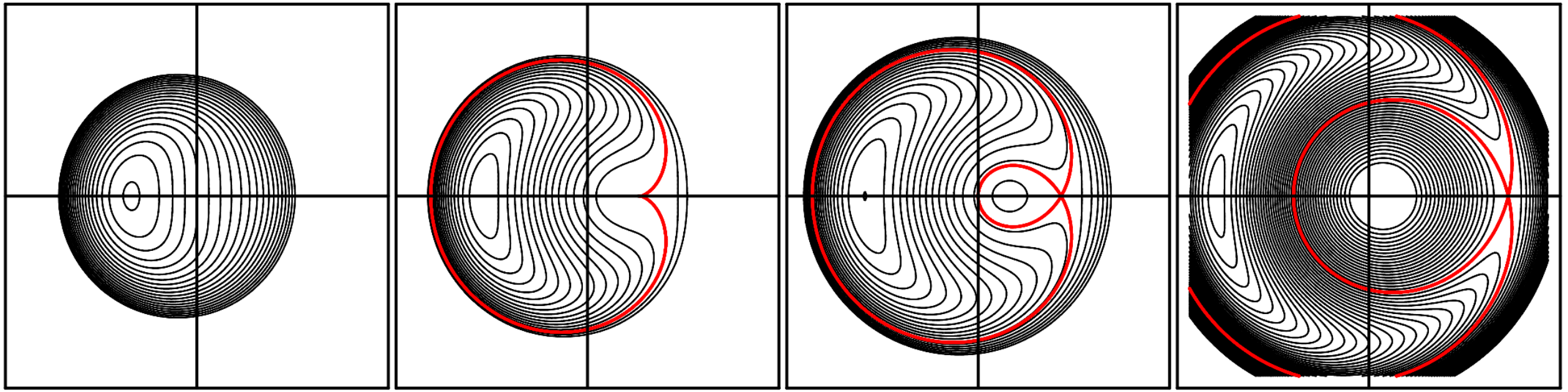
Rings around small bodies can be perturbed by Spin-Orbit Resonances (SORs) with the body or Mean-Motion Resonances (MMRs) with a satellite



The problem

more complicated than a “Celestial Mechanics“ problem:
collective effects create spiral waves at resonances

Phase portraits of MMRs in the 3BRP



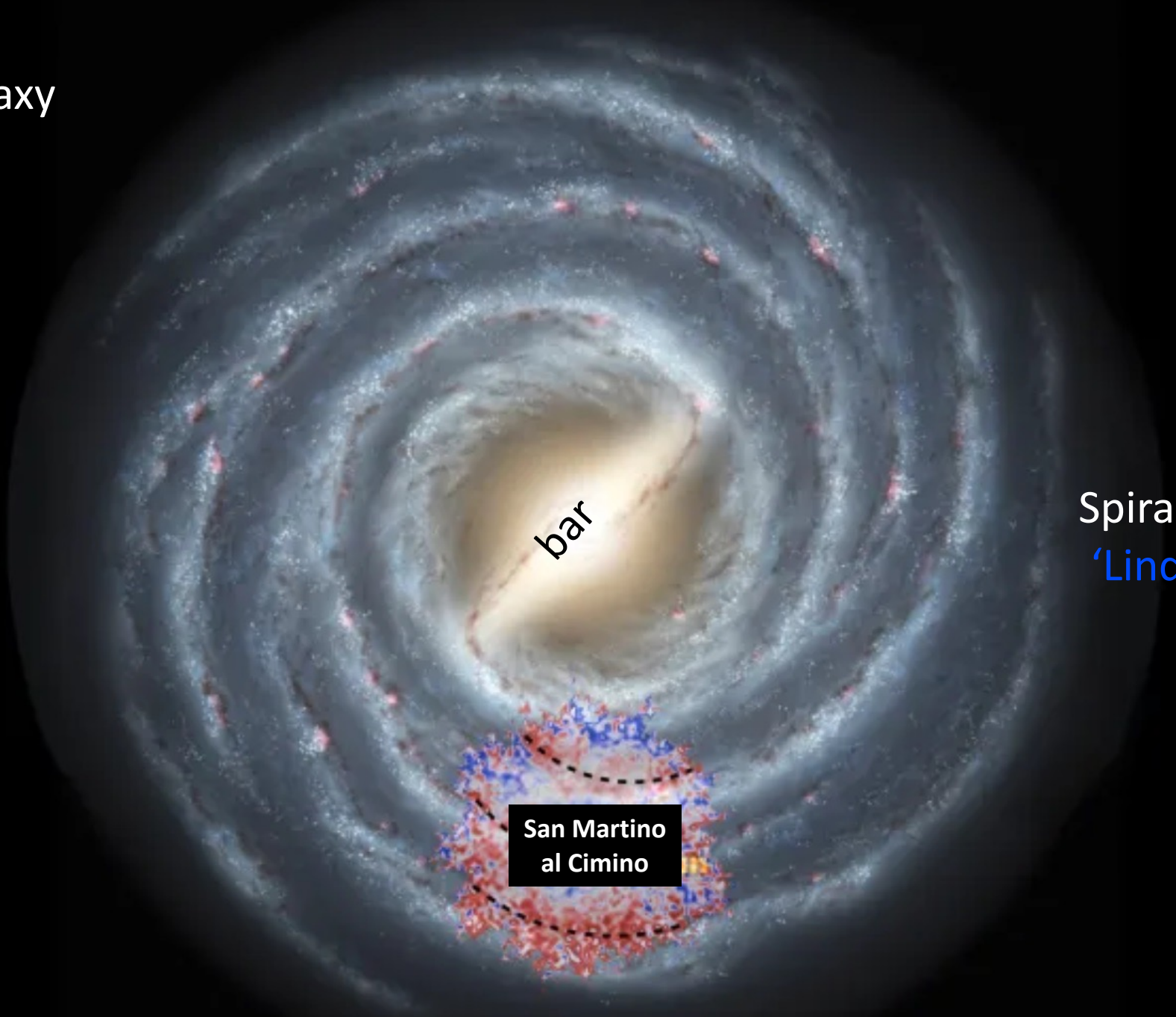
… parametrized by the Jacobi constant

$$J = a\,[1 + (m-1)e^2] \sim a$$ for small e

The question

is there **phase portrait** of spiral waves?
if yes, what does it look like?

Our Galaxy



Spiral waves excited by
'Lindblad' resonances

https://www.inkl.com/news/milky-way-galaxy-s-spiral-arms-revealed-in-stunning-detail-by-chemical-mapping

What is a **Lindblad resonance**?

$\kappa = m(n - \Omega)$

κ: epicyclic frequency of particle

n: mean motion of particle

Ω: frequency of the perturber (bar, satellite, mass anomaly, you name it…)

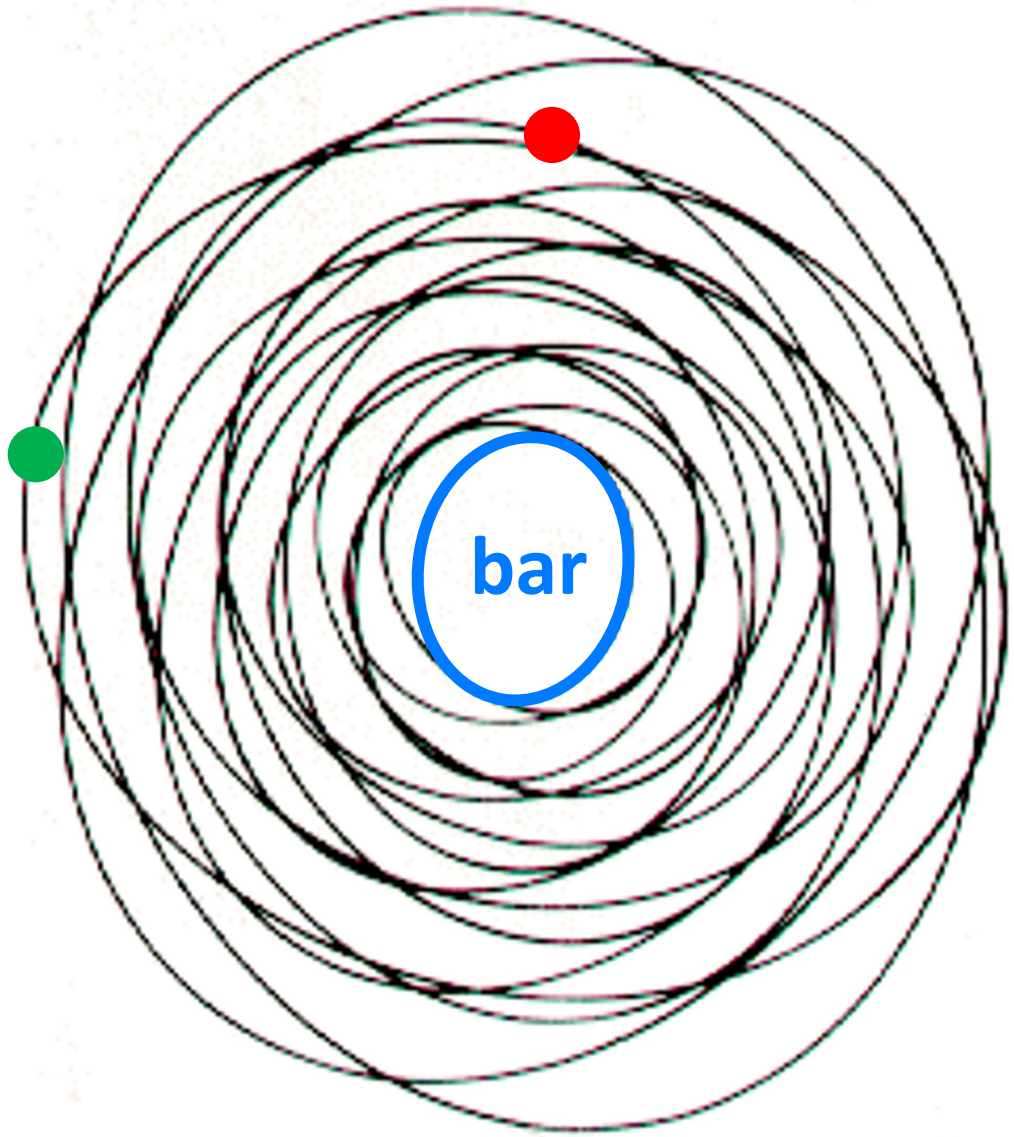
$|m|$: number of arms

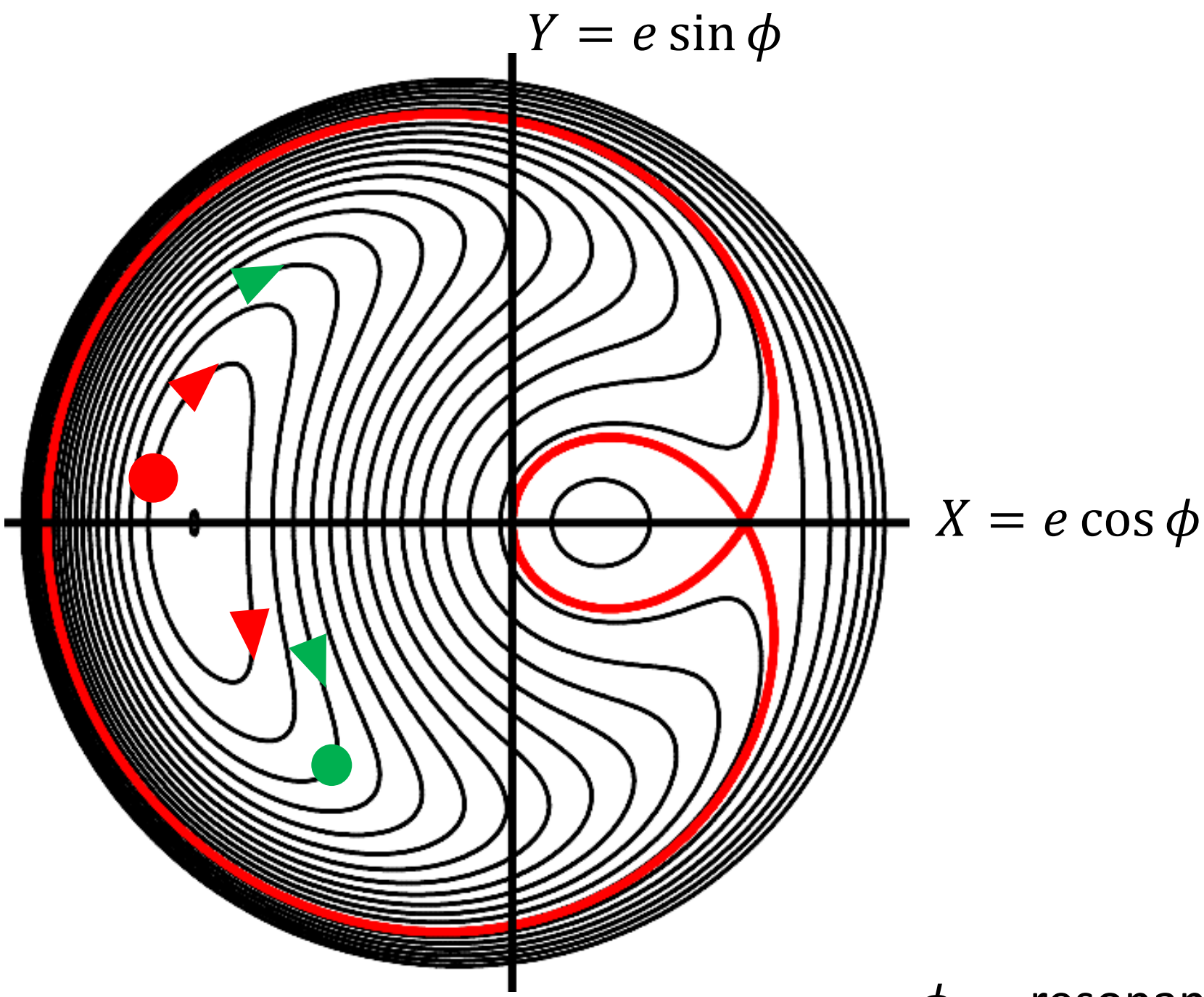
In planetary rings, the motion is ~ Keplerian : $\kappa \sim n$ →

$$\frac{n}{\Omega} \sim \frac{m}{m-1}$$

(first-order resonance)

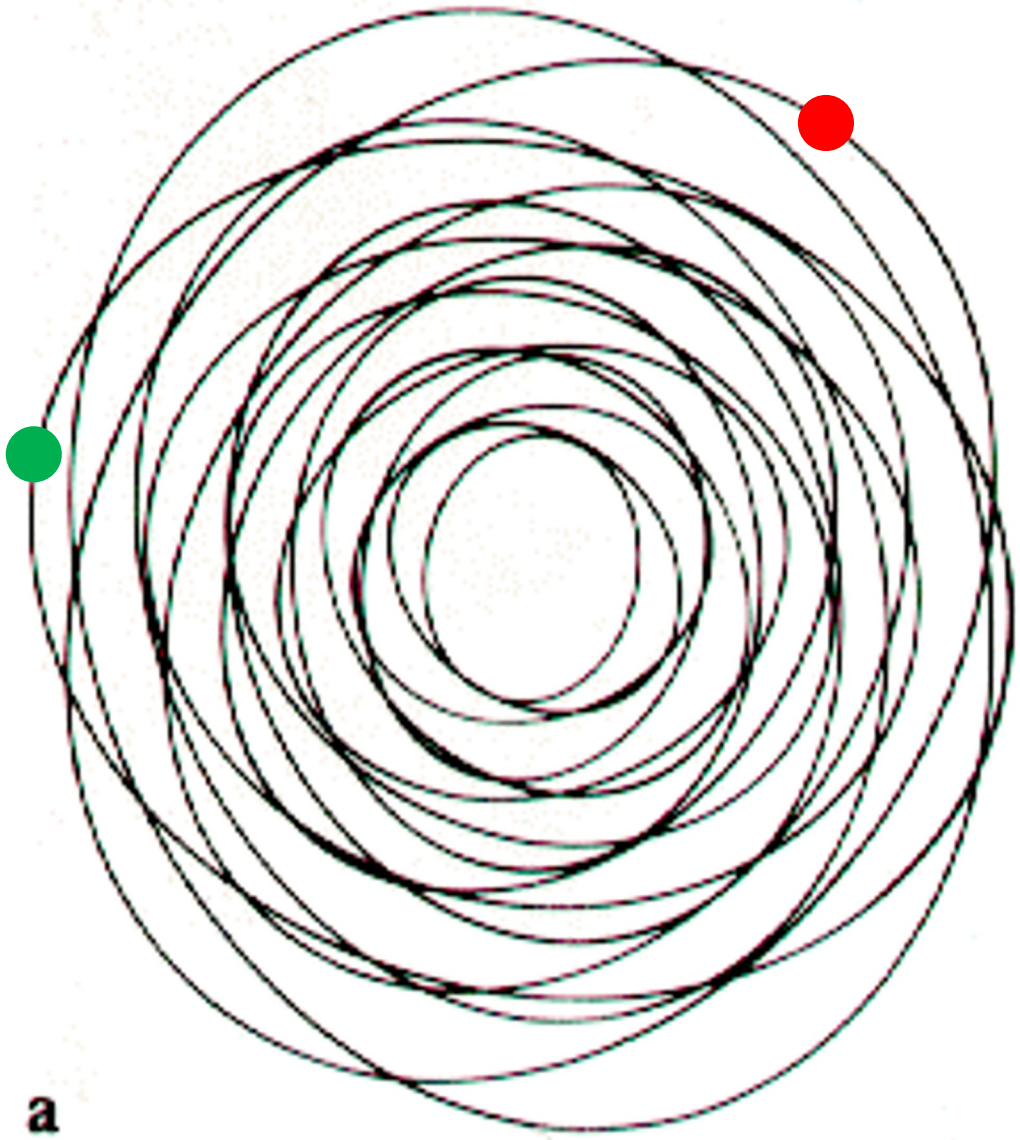
non-colliding particles →
“Celestial Mechanics“





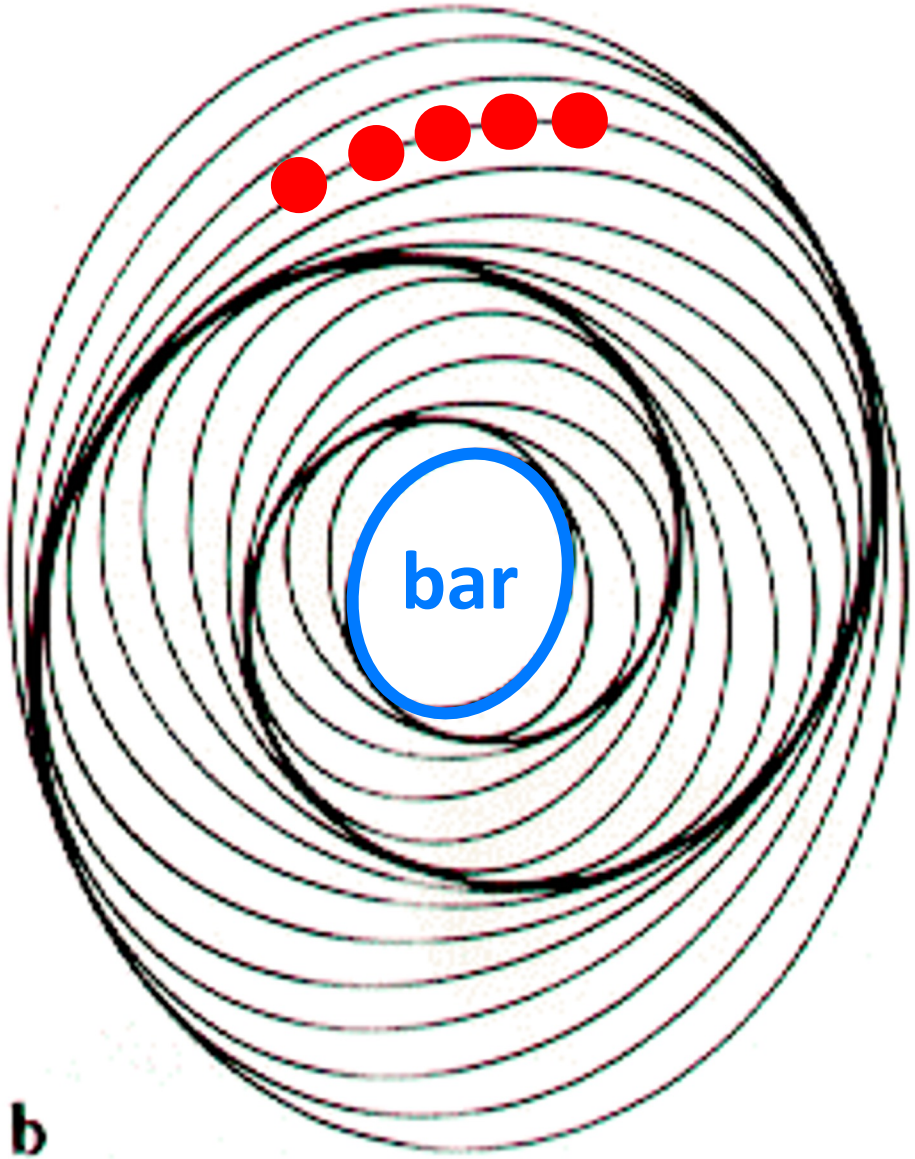
$\phi =$ resonant angle

non-colliding
particles: obey
“Celestial Mechanics“



a

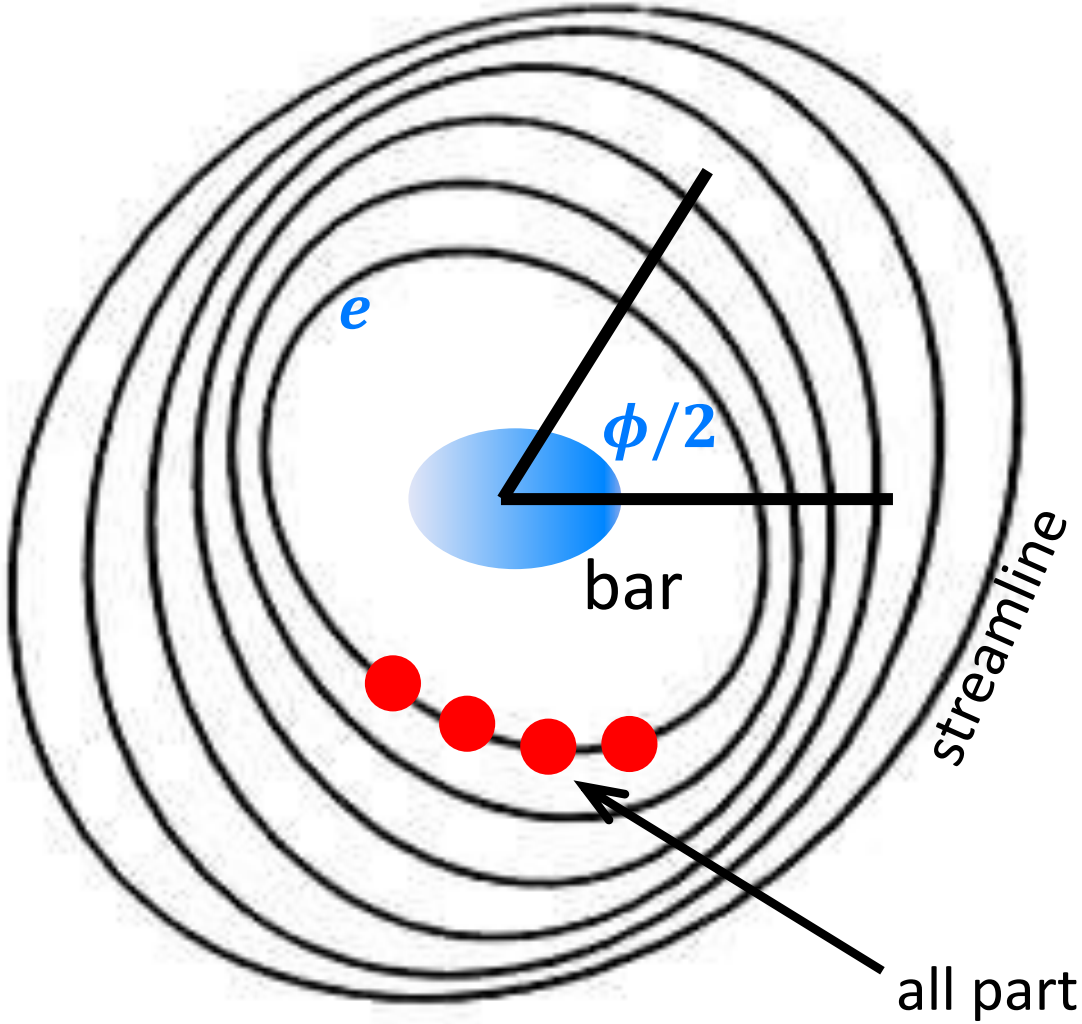
interacting particles:
a spiral galaxy is born! →
nested streamlines



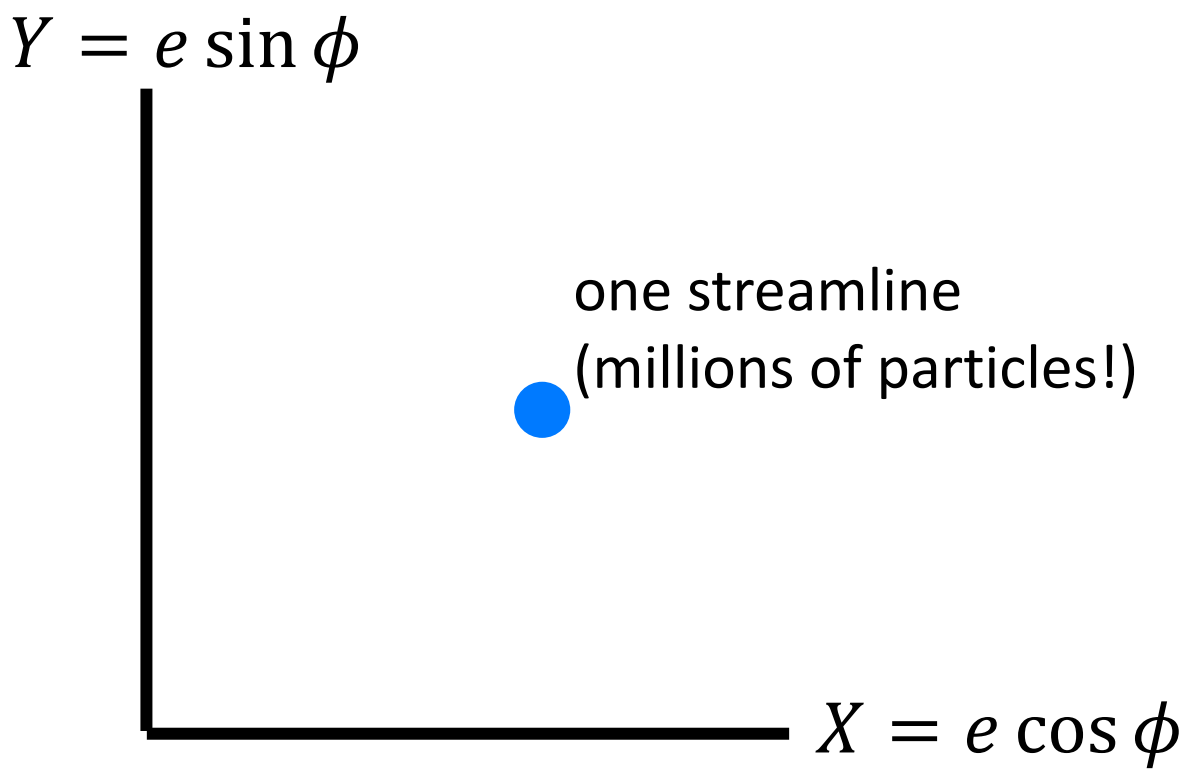
b

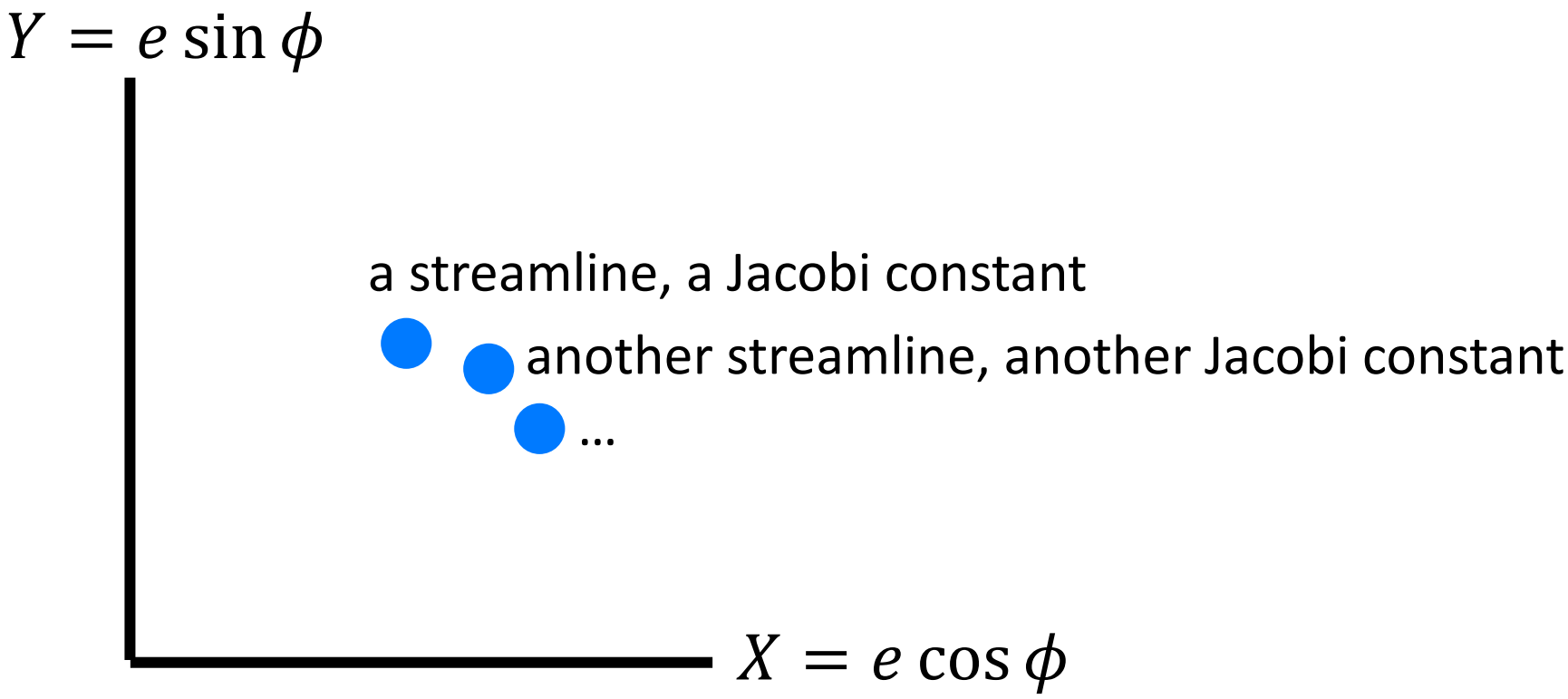
…an example: $\phi = 3\lambda - 2\lambda' - \varpi$

(2/3 outer Lindblad resonance)



all particles of streamline have the same orbital eccentricity e and the same resonant angle ϕ


$Y = e \sin \phi$
one streamline
(millions of particles!)
$X = e \cos \phi$


$Y = e \sin \phi$
a streamline, a Jacobi constant
another streamline, another Jacobi constant
…
$X = e \cos \phi$

Equations describing a disk around a planet perturbed by a satellite, accounting for self-gravity, pressure and viscosity

$$\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\boldsymbol{\nabla}\right)\mathbf{U} = -\boldsymbol{\nabla}(\phi_{\mathrm{p}} + \psi_{\mathrm{S}} + \psi_{\mathrm{D}})$$

planet satellite disk

$$-\frac{1}{\Sigma}\boldsymbol{\nabla}\cdot\mathbf{P}$$

$$\frac{\partial}{\partial t}(\Sigma) + \boldsymbol{\nabla}(\Sigma\mathbf{U}) = 0$$

$$\nabla^2\phi_{\mathrm{D}} = 4\pi\Sigma G\delta(z)$$

$$dP/d\Sigma = c^2. \qquad (27)$$

Meyer-Vernet & Sicardy (1987)

suppressing the pressure and viscosity

$$\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\boldsymbol{\nabla}\right)\mathbf{U} = -\boldsymbol{\nabla}(\phi_{\mathrm{p}} + \psi_{\mathrm{S}} + \psi_{\mathrm{D}})$$

$$-\frac{1}{\Sigma}\boldsymbol{\nabla}\cdot\mathbf{P}$$

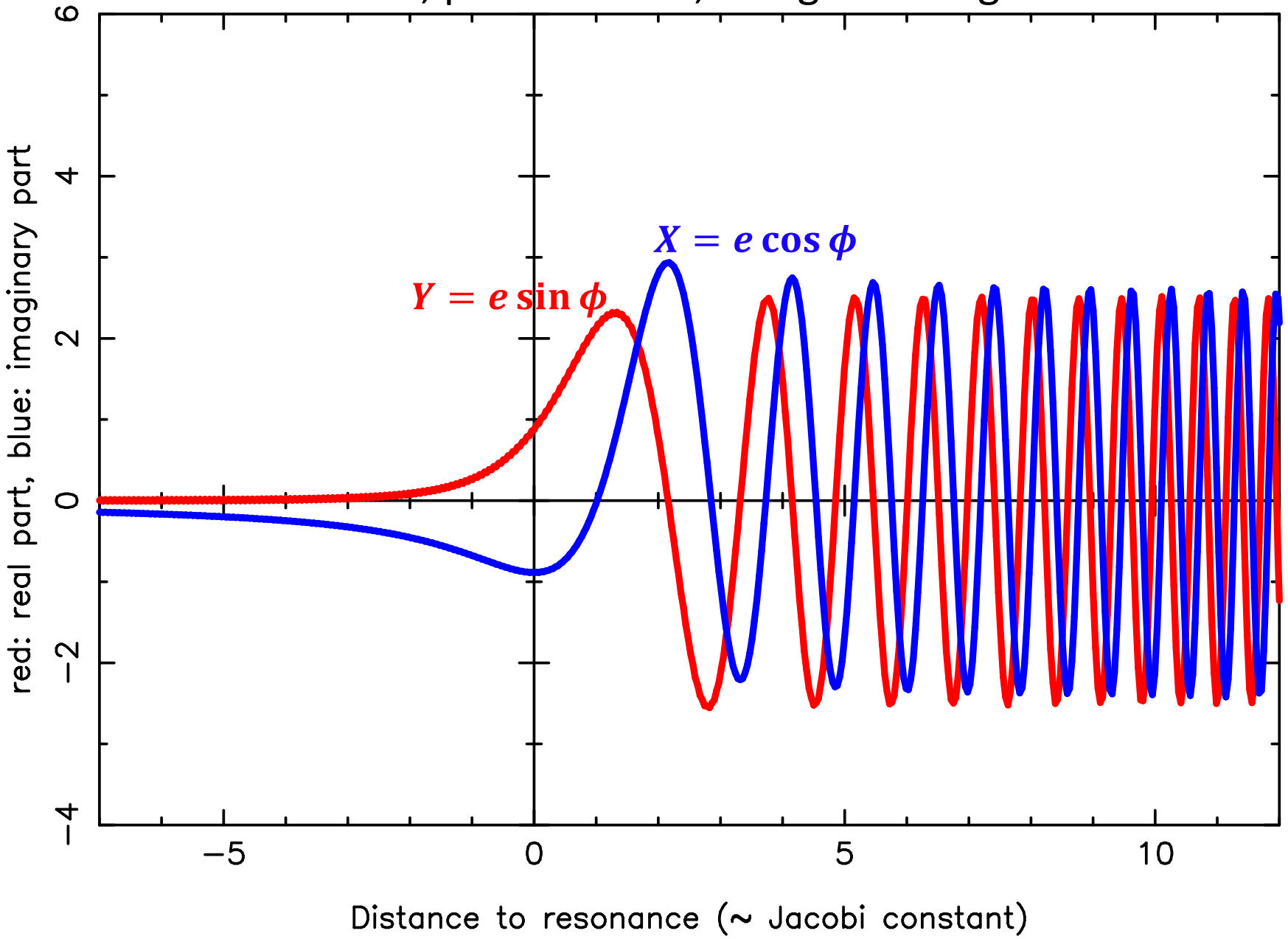
$$\frac{\partial}{\partial t}(\Sigma) + \boldsymbol{\nabla}(\Sigma\mathbf{U}) = 0$$

$$\nabla^2\phi_{\mathrm{D}} = 4\pi\Sigma G\delta(z)$$

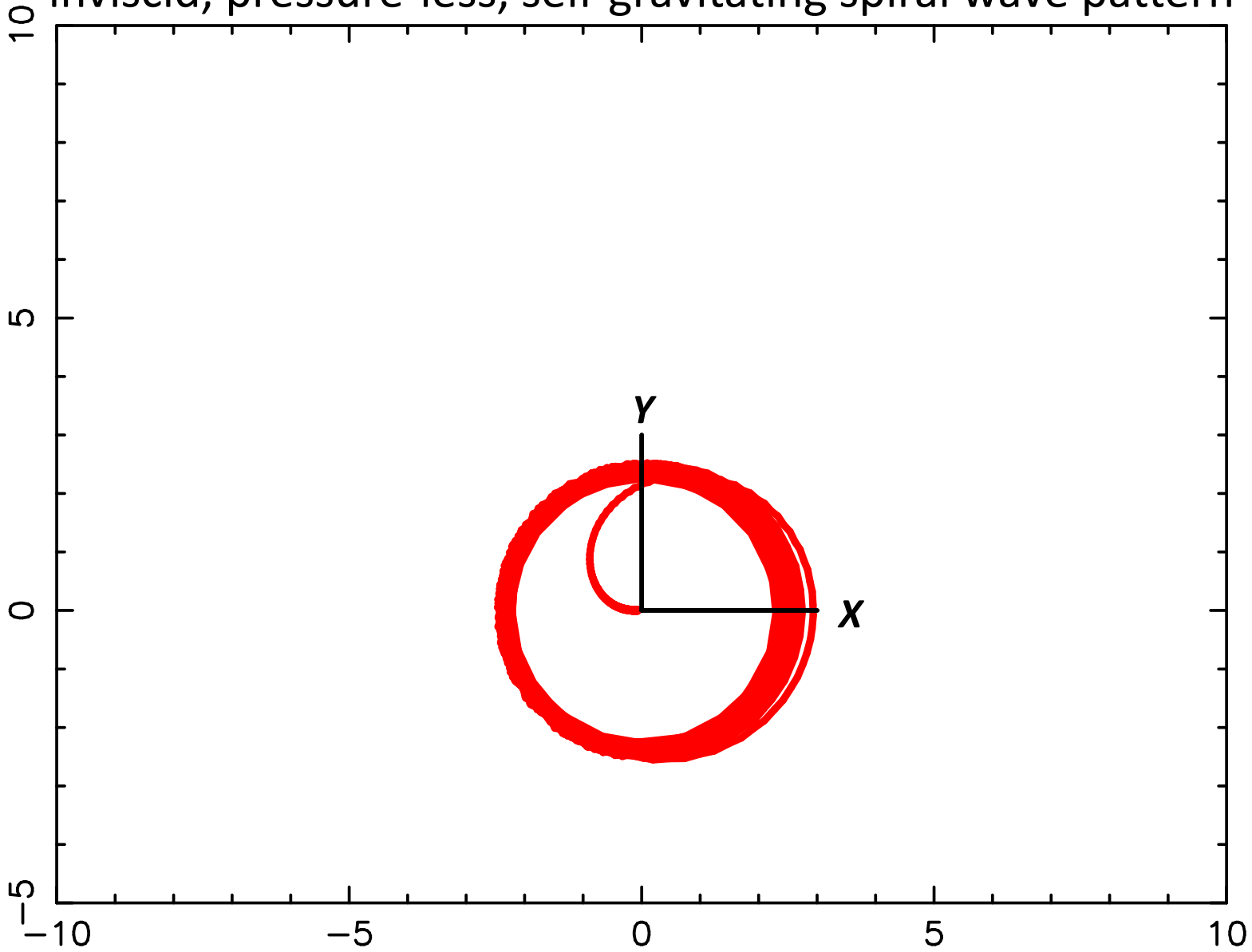
$$dP/d\Sigma = c^2. \qquad (27)$$

Meyer-Vernet & Sicardy (1987)

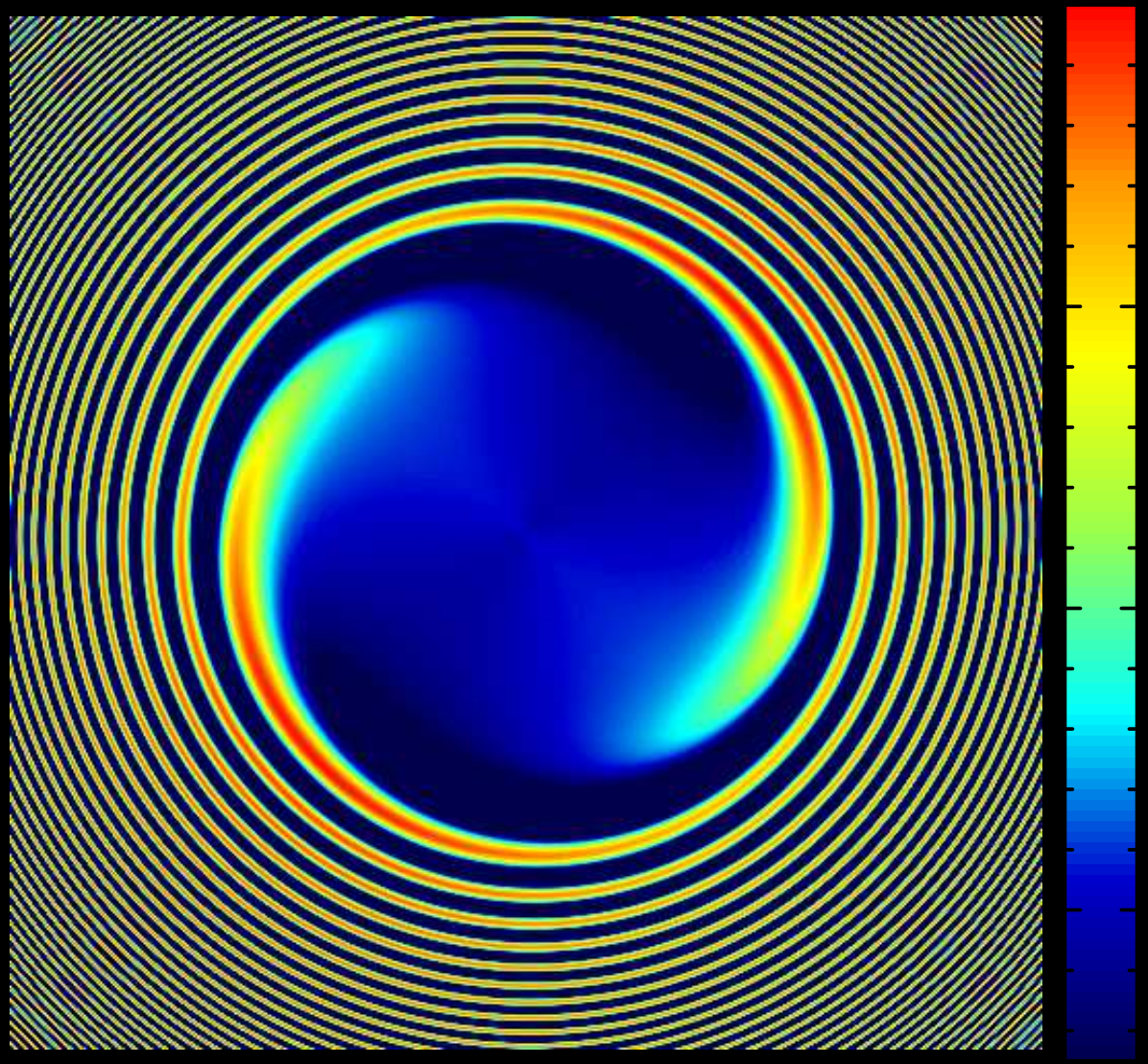
… after (long 😢) calculations: the response of an inviscid, pressure-less, self-gravitating disk:



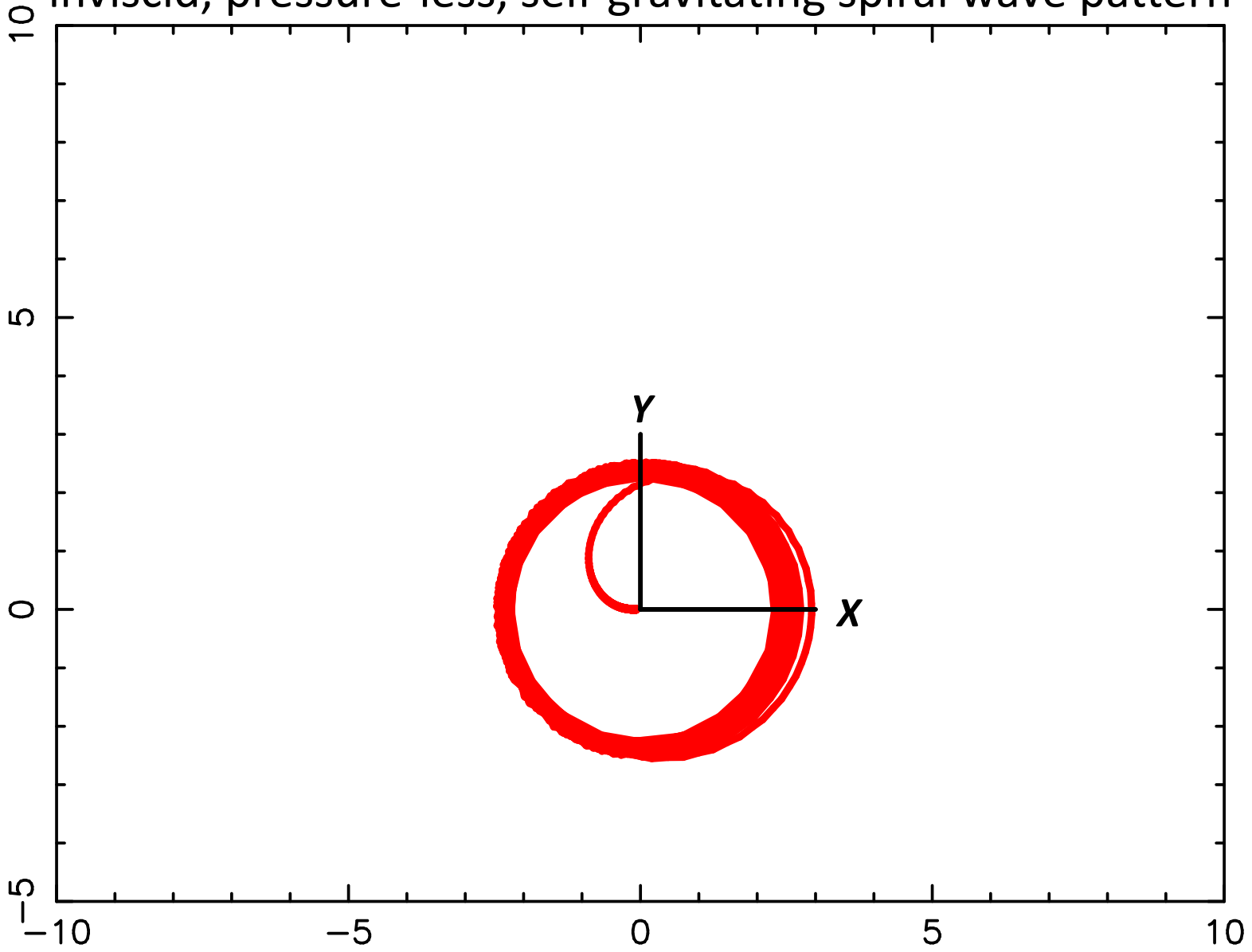
the phase portrait of an
inviscid, pressure-less, self-gravitating spiral wave pattern

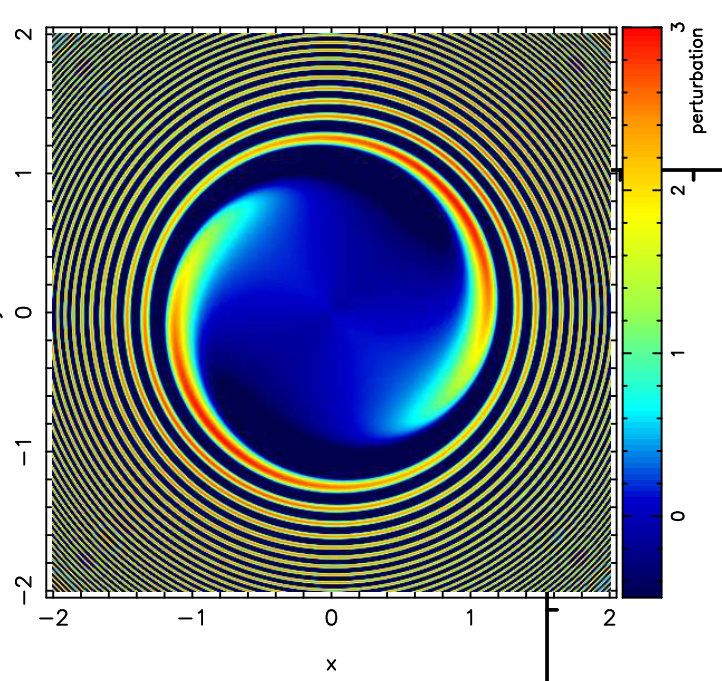


an inviscid, pressure-less self-gravitating disk
(for instance at 2/3 Lindblad resonance)



the phase portrait of an
inviscid, pressure-less, self-gravitating spiral wave pattern





This is the “DNA” helix of the 2/3 spiral pattern shown at left

one streamline

Jacobi constant

Y

X

This “DNA encodes all the information
of the spiral wave pattern







https://www.magnific.com/es/vector-premium/doble-helice-adn-diferentes-posiciones-parte-genoma-codigo-genetico-humano_136885496.htm

suppressing the pressure, keeping self-gravity and viscosity

$$\left(\frac{\partial}{\partial t} + \mathbf{U}\cdot\boldsymbol{\nabla}\right)\mathbf{U} = -\boldsymbol{\nabla}(\phi_{\mathrm{p}} + \psi_{\mathrm{S}} + \psi_{\mathrm{D}}) - \frac{1}{\Sigma}\boldsymbol{\nabla}\cdot\mathbf{P}$$

viscosity only

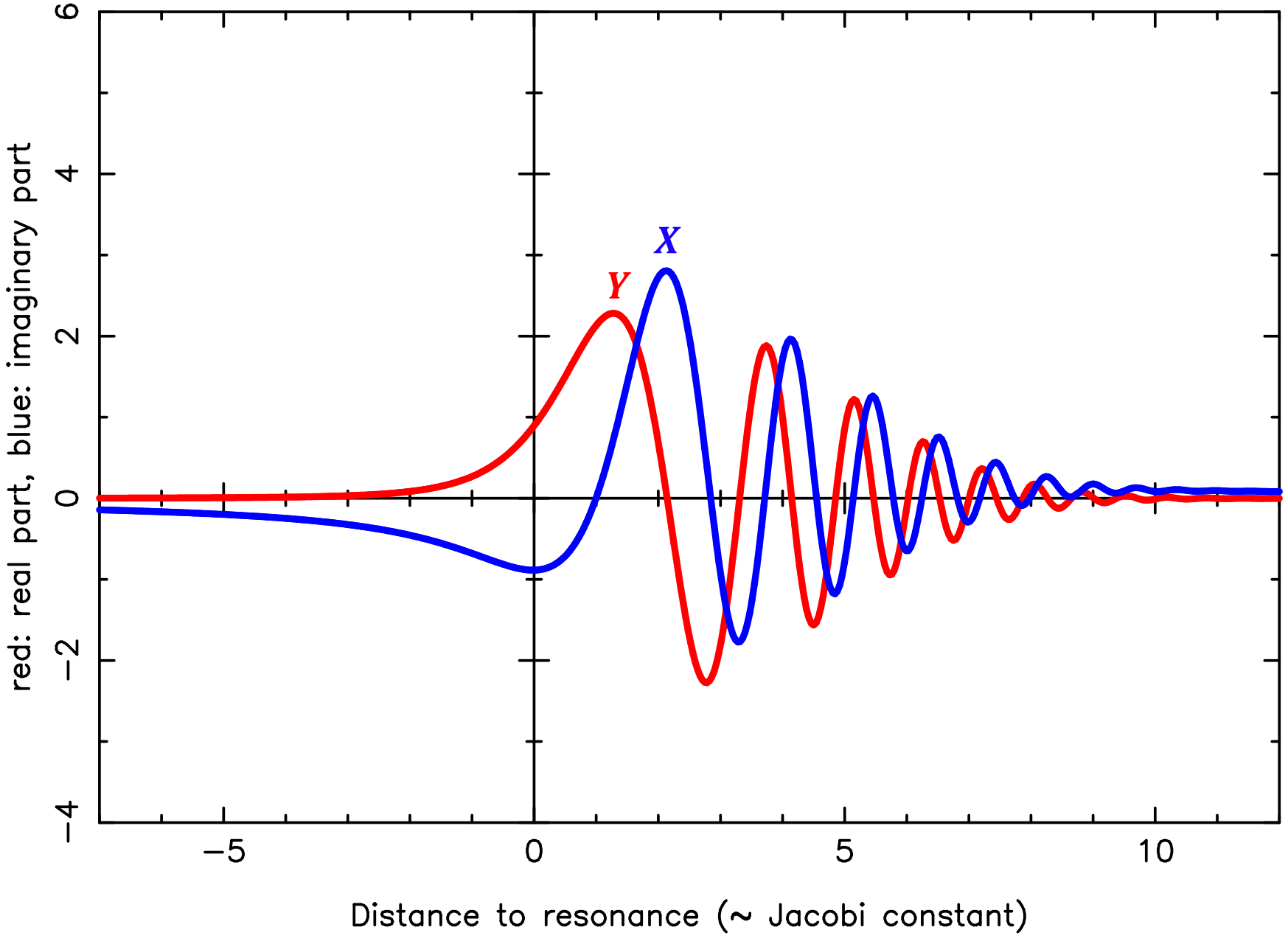
$$\frac{\partial}{\partial t}(\Sigma) + \boldsymbol{\nabla}(\Sigma\mathbf{U}) = 0$$

$$\nabla^2\phi_{\mathrm{D}} = 4\pi\Sigma G\delta(z)$$

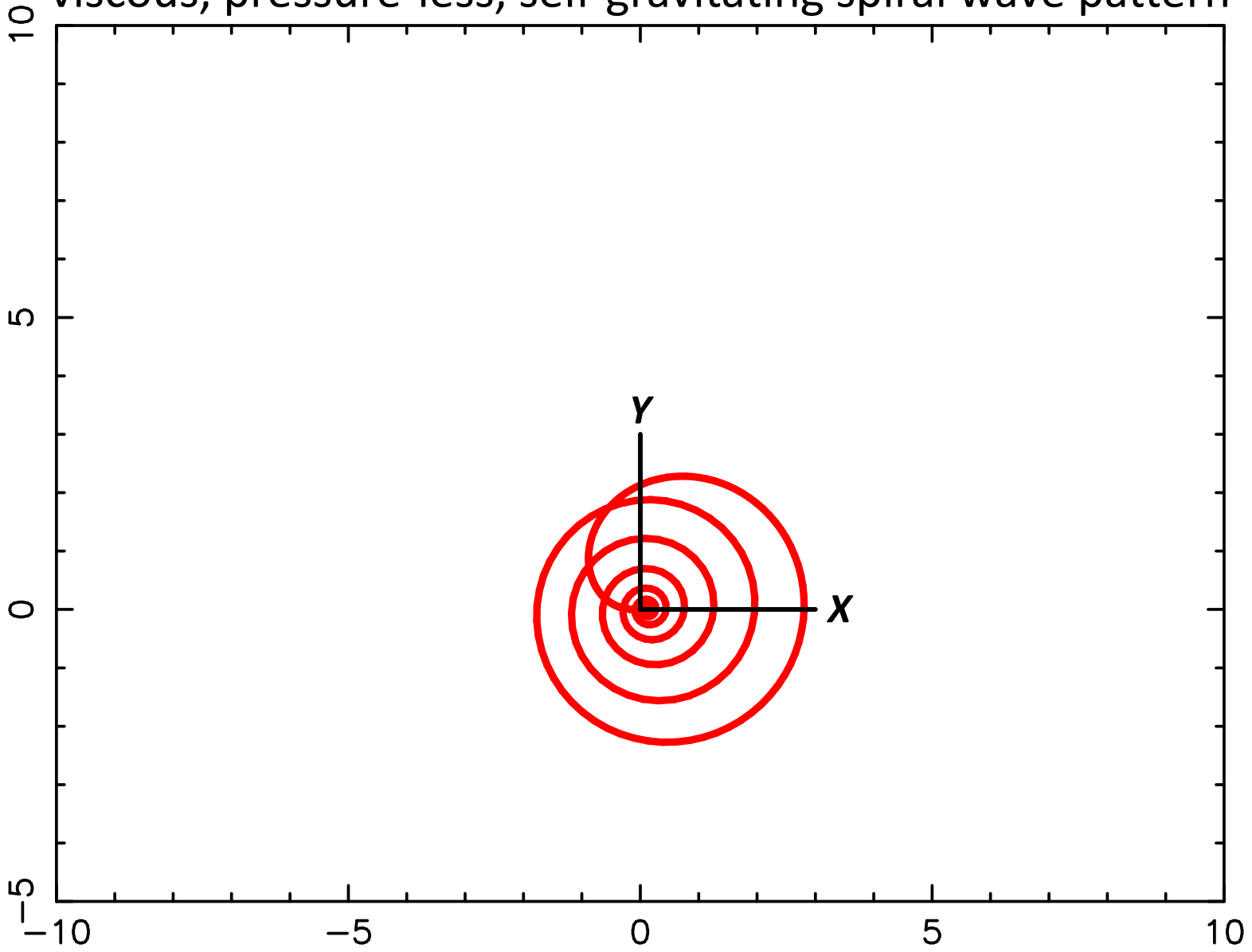
$$dP/d\Sigma = c^2. \qquad (27)$$

Meyer-Vernet & Sicardy (1987)

the response of a viscous, pressure-less, self-gravity disk

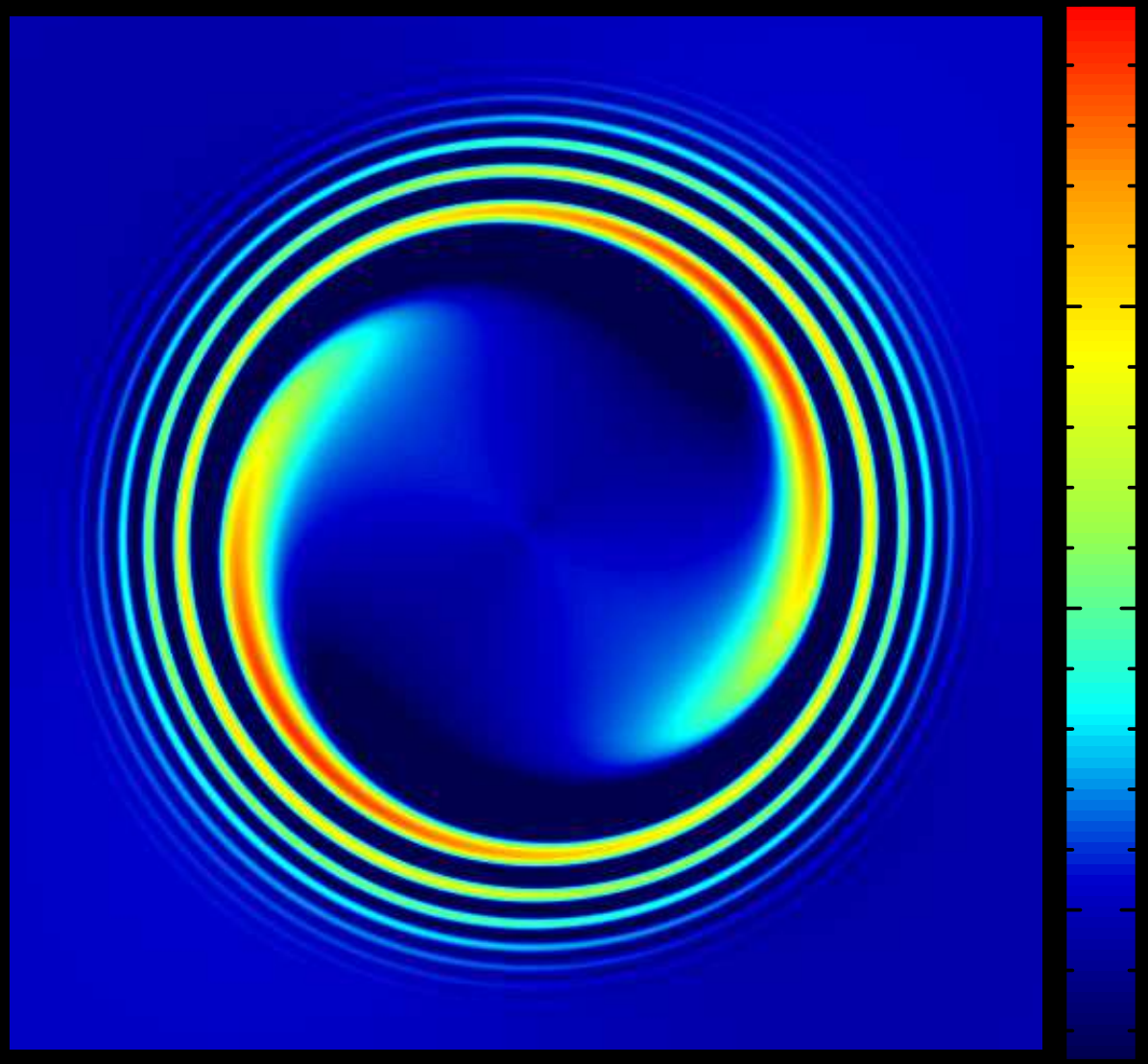


the phase portrait of a
viscous, pressure-less, self-gravitating spiral wave pattern

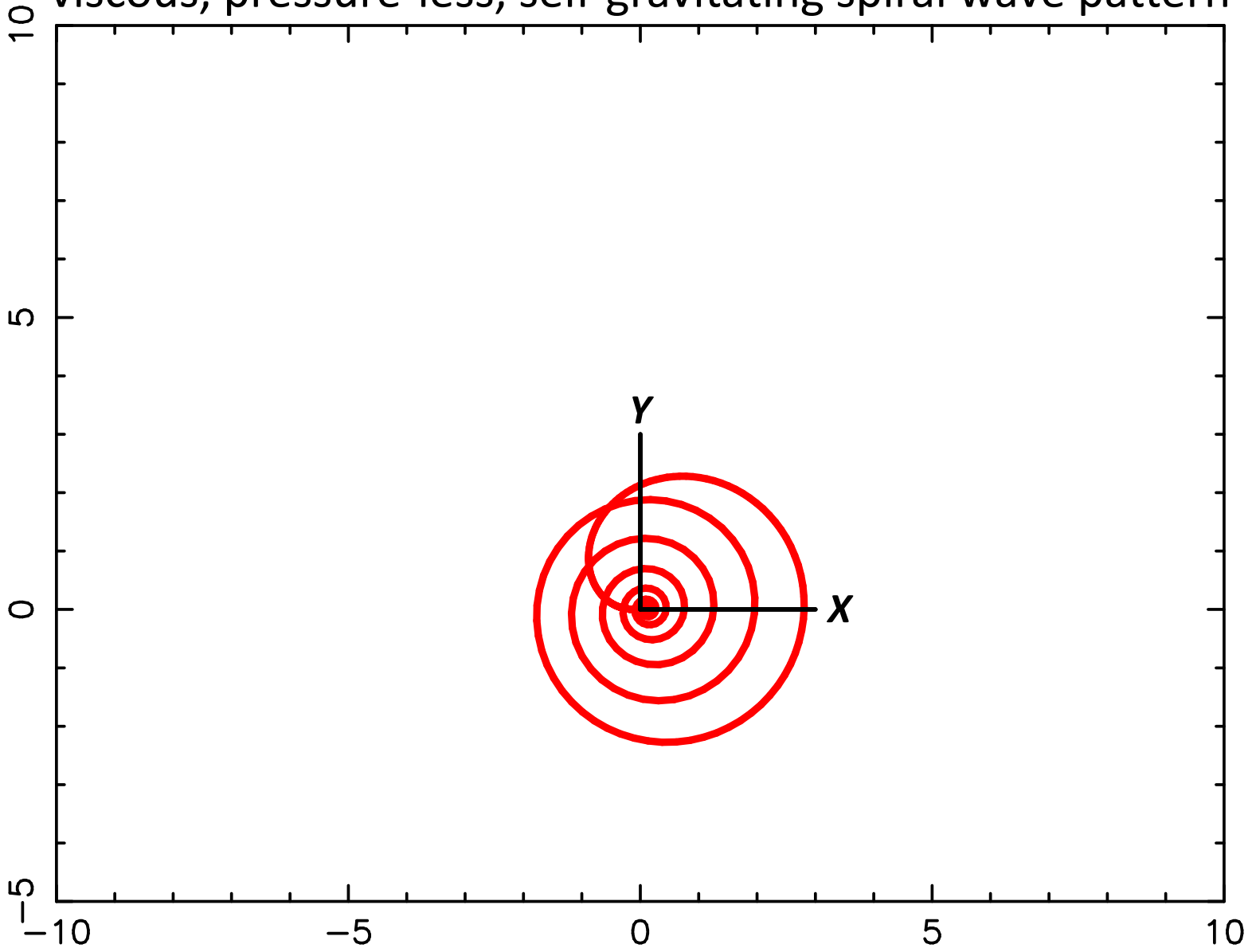


⊙⊙im_re_x_G_1_P_0_nu_0p25_perspective_B_90_P_0_phi_m90.ps/cps

a viscous, self-gravitating, no pressure disk
(for instance at 2/3 Lindblad resonance)

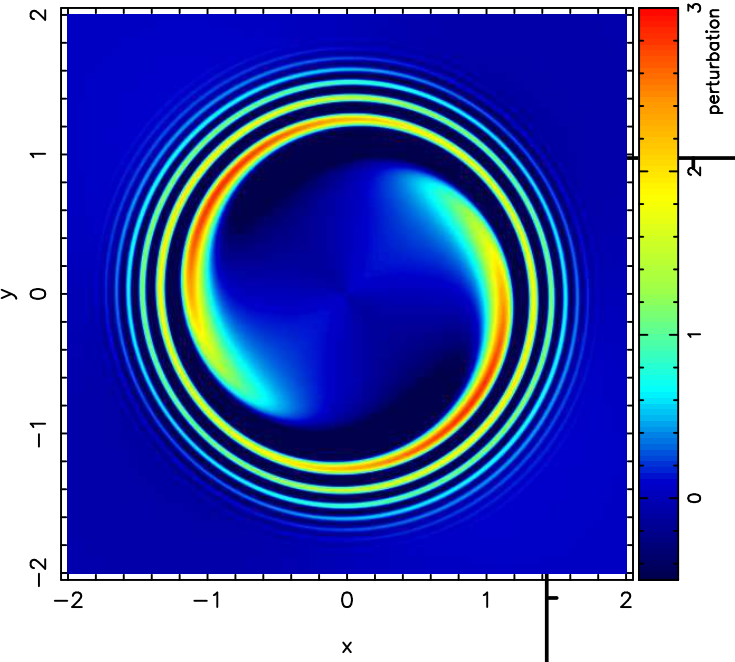


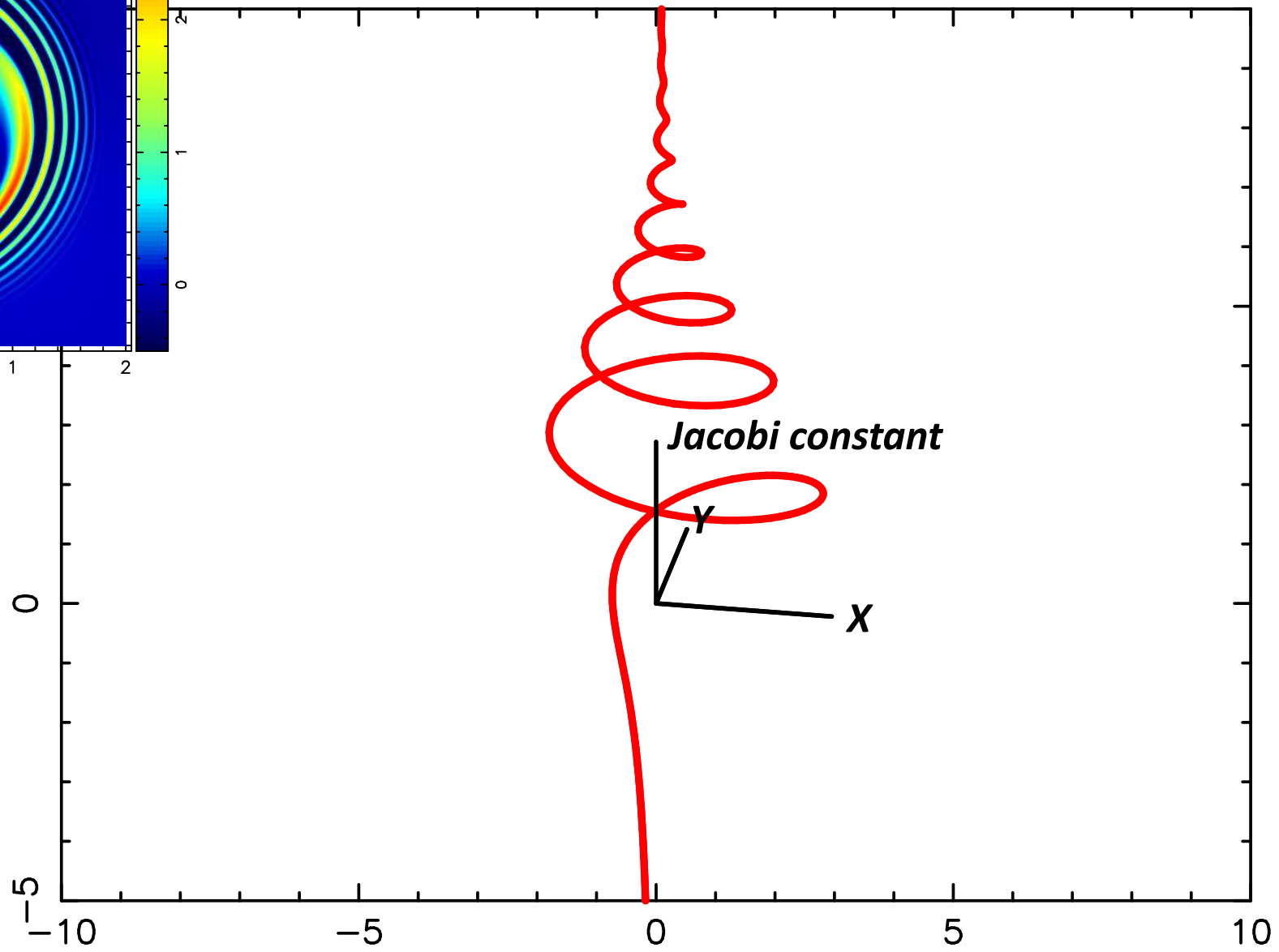
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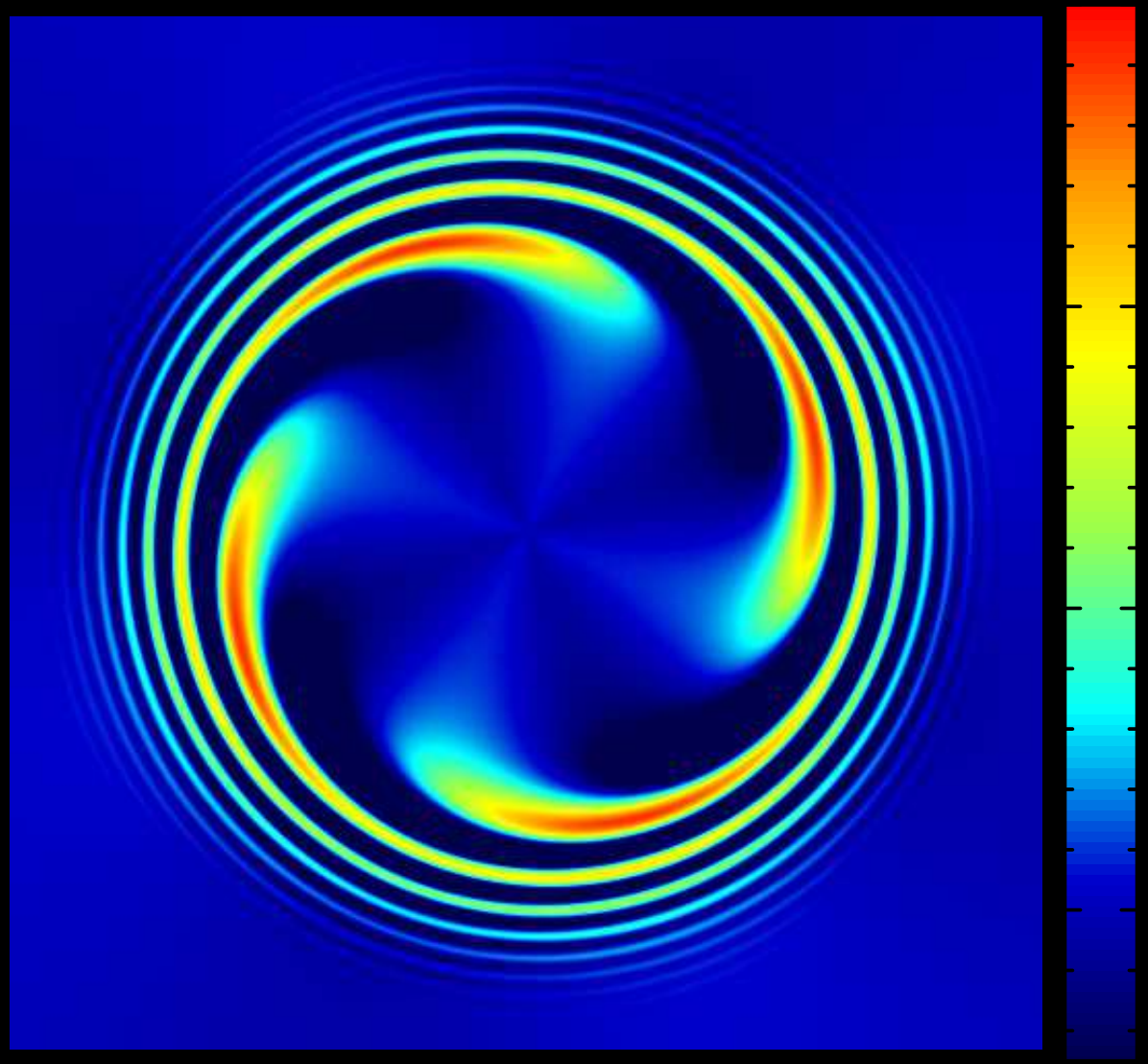
⊙⊙im_re_x_G_1_P_0_nu_0p25_perspective_B_90_P_0_phi_m90.ps/cps

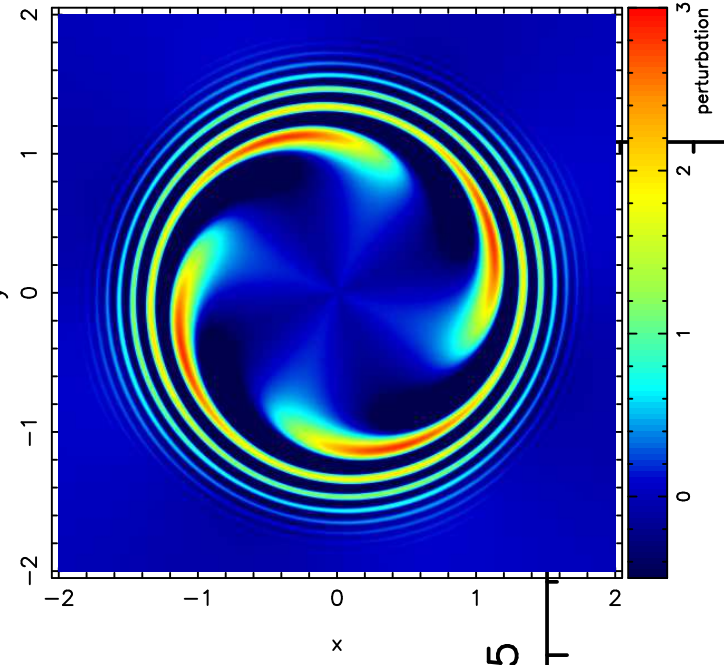
self-gravity, viscosity only (no pressure)





a viscous, self-gravitating, no pressure disk
(for instance at 4/5 Lindblad resonance)





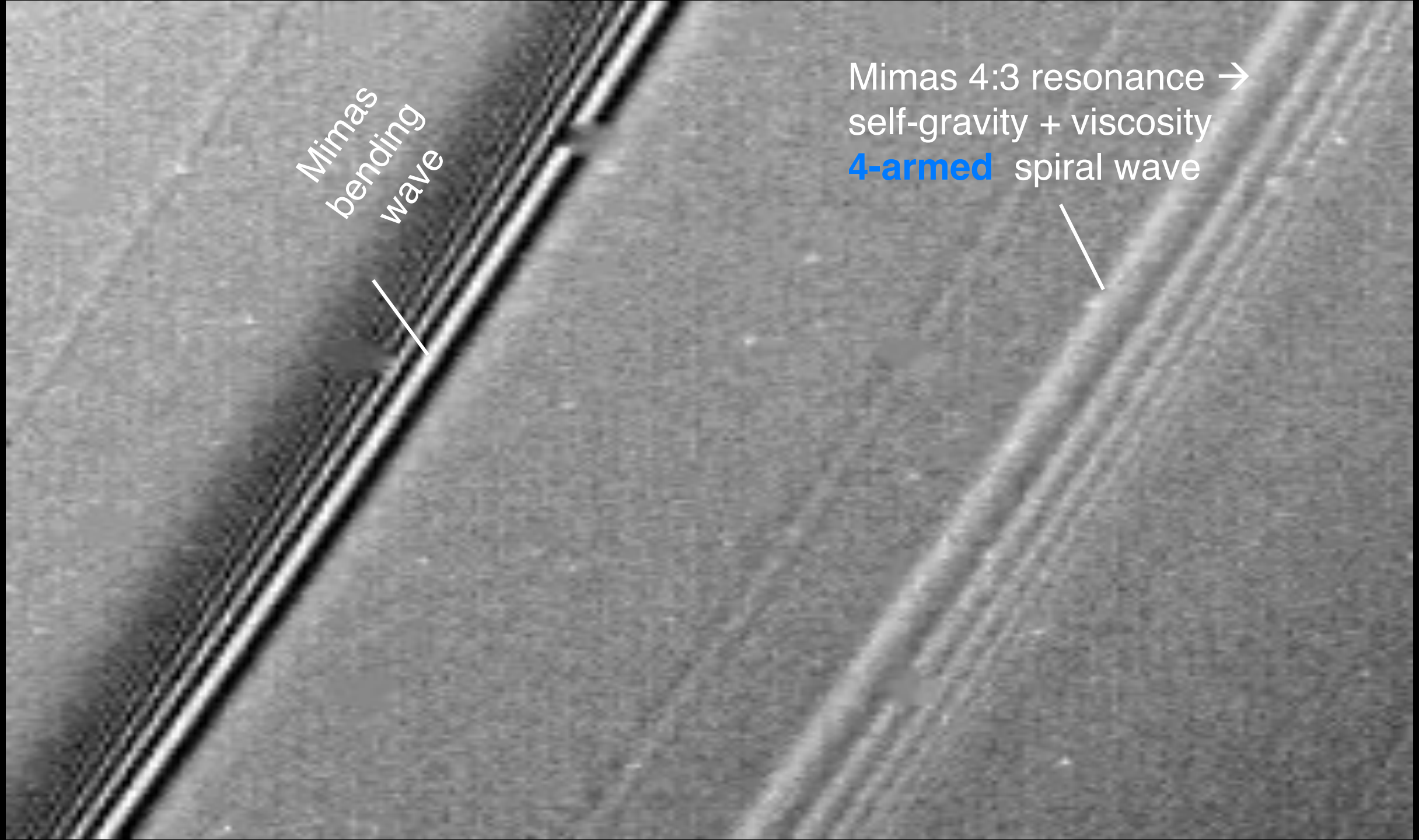
The 4/5 spiral pattern has the **same DNA** as the 2/3 spiral wave pattern s

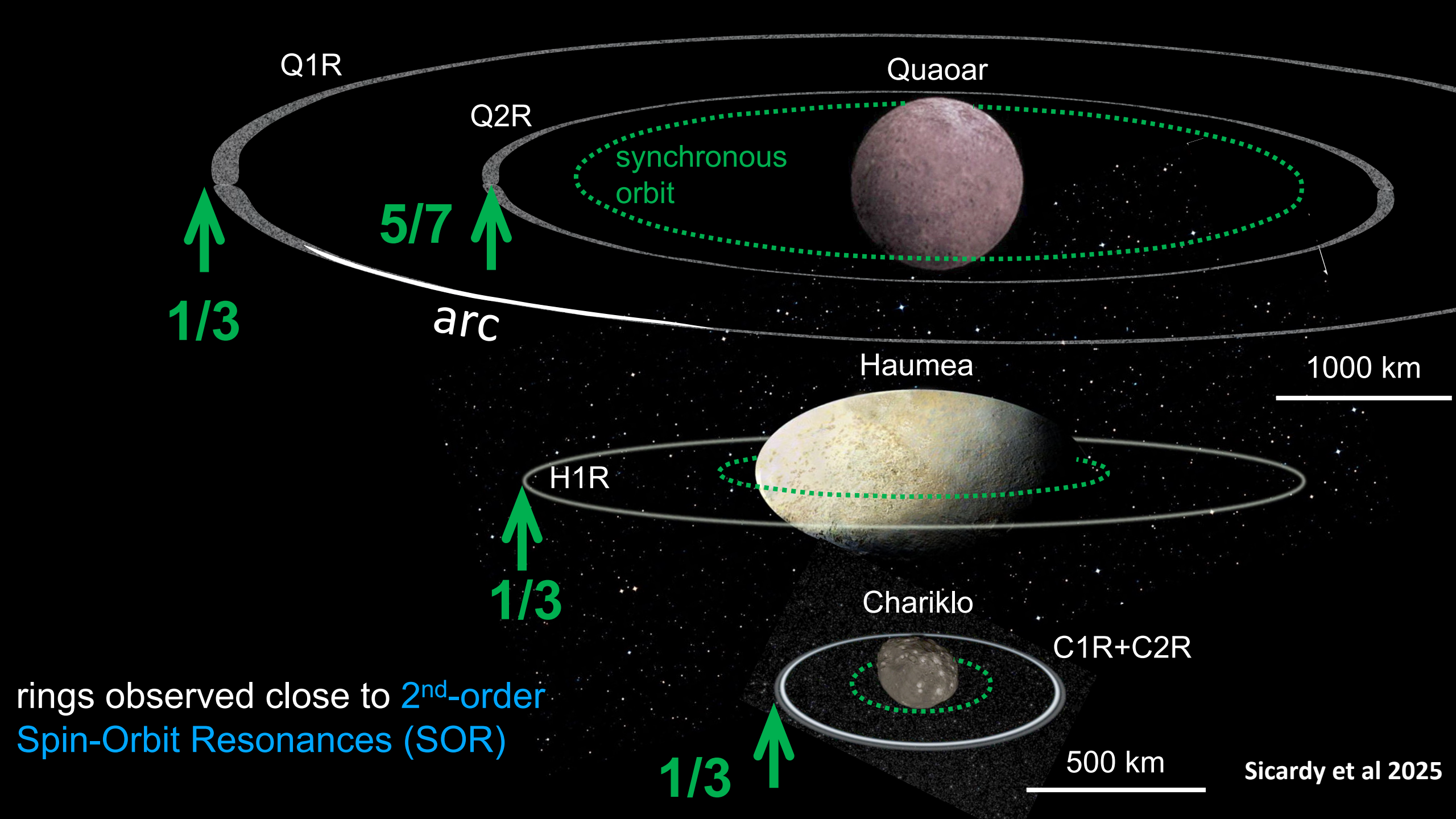
Jacobi constant

Y

X

What does all this have to do
with rings around small bodies?


Mimas bending wave
Mimas 4:3 resonance →
self-gravity + viscosity
4-armed spiral wave


Q1R
Quaoar
Q2R
synchronous
orbit
5/7
1/3
arc
1000 km
Haumea
H1R
1/3
Chariklo
C1R+C2R
1/3
500 km
rings observed close to 2nd-order
Spin-Orbit Resonances (SOR)
Sicardy et al 2025

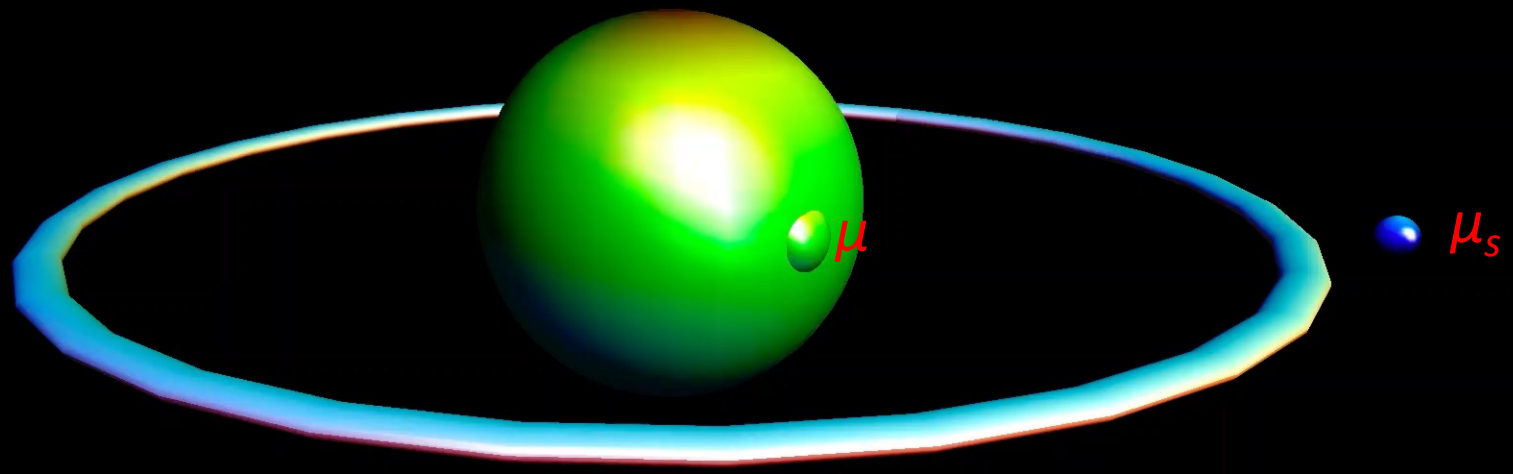
Mass anomaly μ

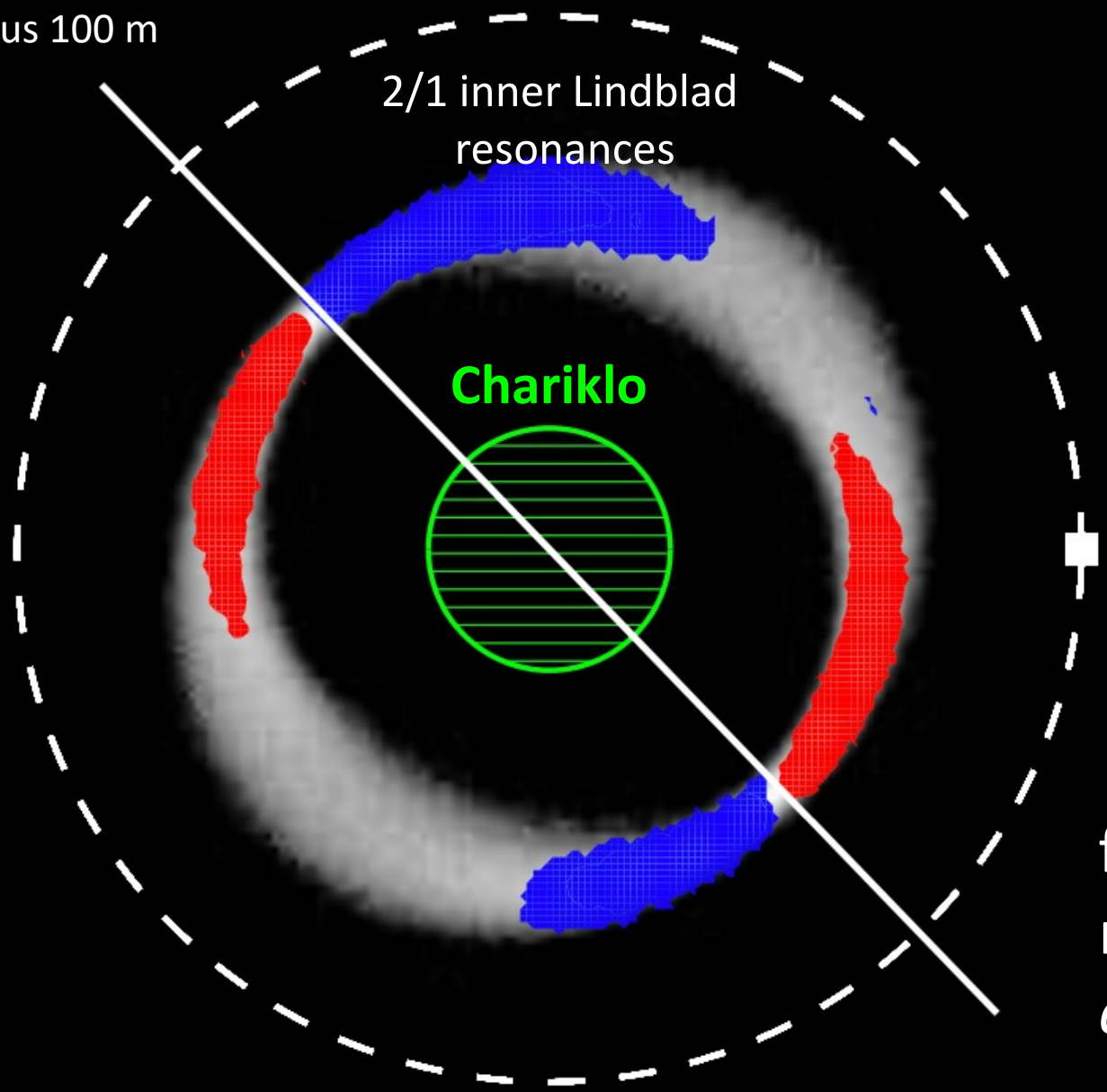
N-body collisional simulation

7,500 particles, radius 100 m

complete (3D, 2π), *no* self-gravity

Salo & Sicardy (2026)





positive angular momentum flux → spreading

negative angular momentum flux → confinement of the ring! (Hänninen & Salo 1994, Goldreich, Rappaport & Sicardy 1995)

satellite $\mu_s = 2 \times 10^{-4}$

for Lindblad resonances, the angular momentum flux reversal *can be treated analytically*

Salo & Sicardy (2026)

next…

what about second-order resonances (e.g. 1/3) ?

$2\kappa = m(n - \Omega)$ (e.g. for 1/3 res: m= -1)

not considered by Lindblad because…

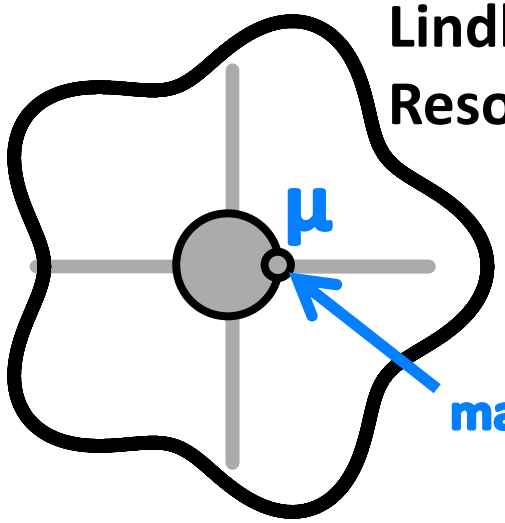
Houston, we've had a problem here…



Jack Swigert

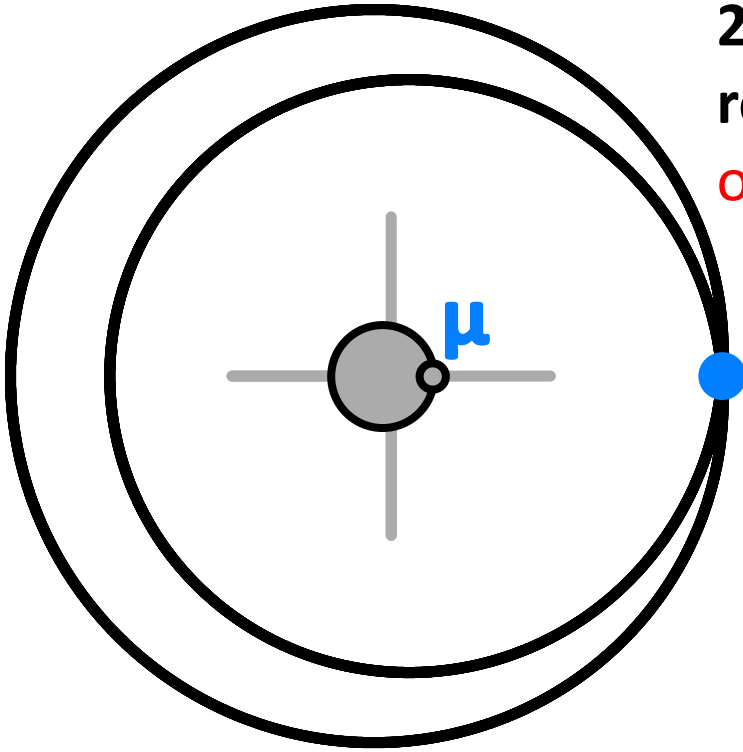
m/m-j resonance →

|m|(j-1) intersection points



Lindblad
Resonance: *j=1* → no intersection points

mass anomaly



2nd-order 1/3
resonance: *m= -1, j= 2* →
one intersection point

the problem: streamline self-intersection!


the 1/3 resonance:
the recipee for a disaster…
😱

self-intersecting streamlines
are in principle a route to chaos

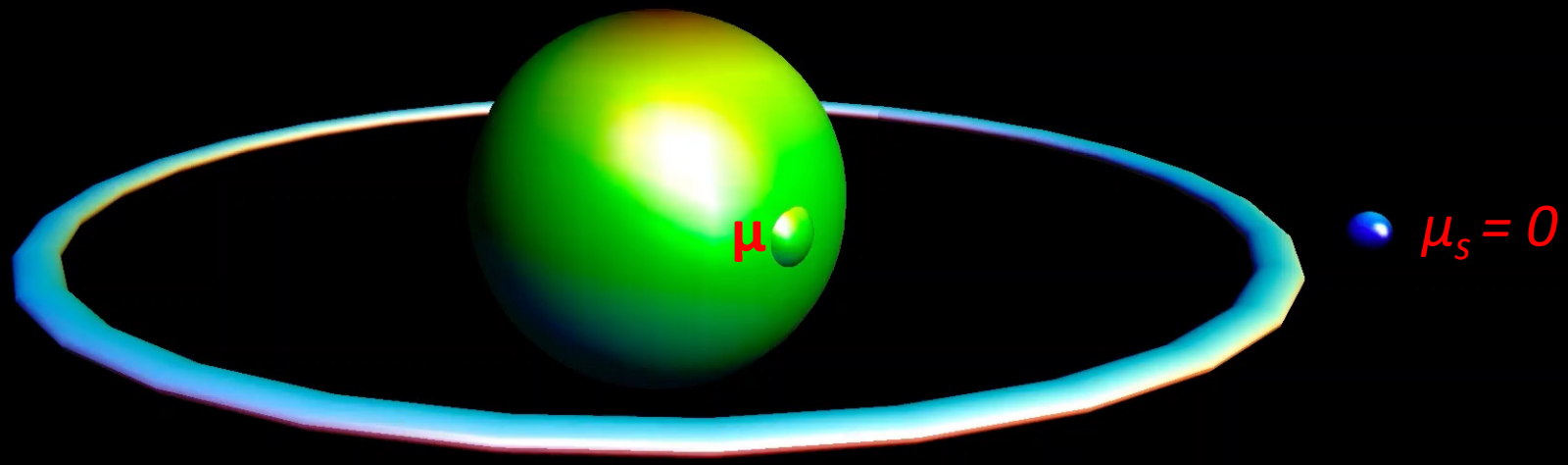
Mass anomaly μ= 0.003

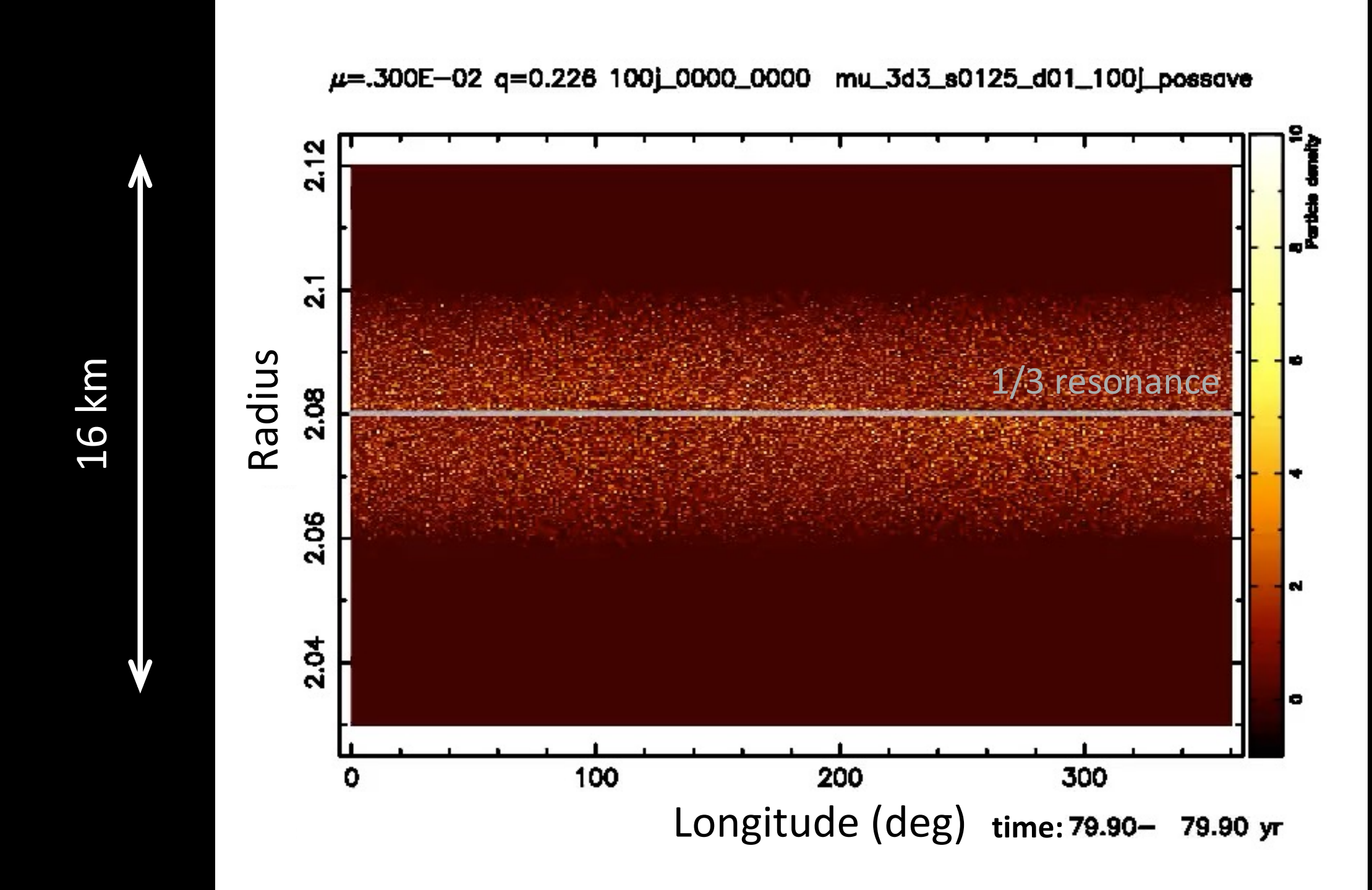
N-body collisional simulation

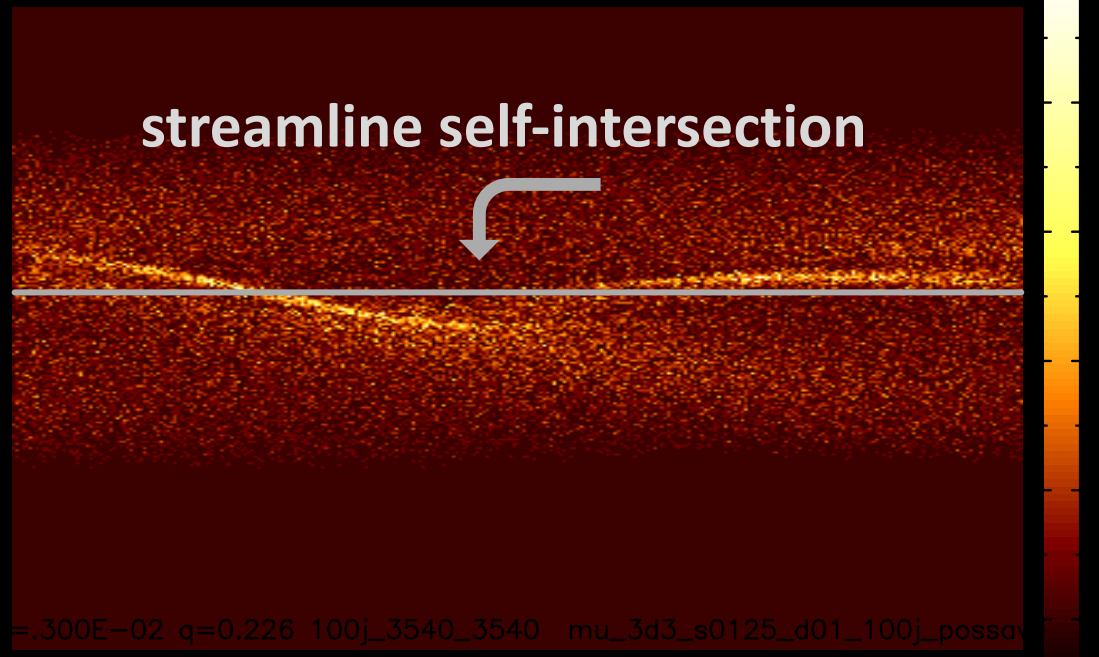
40,000 particles, radius 25 m

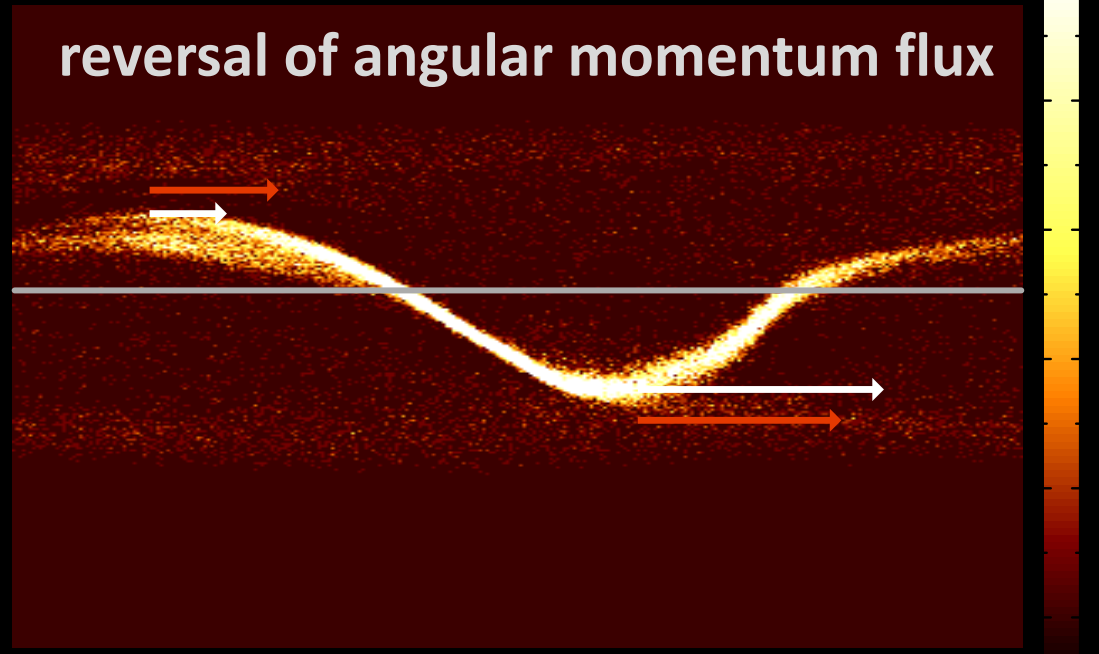
complete (3D, 2π), *no* self-gravity

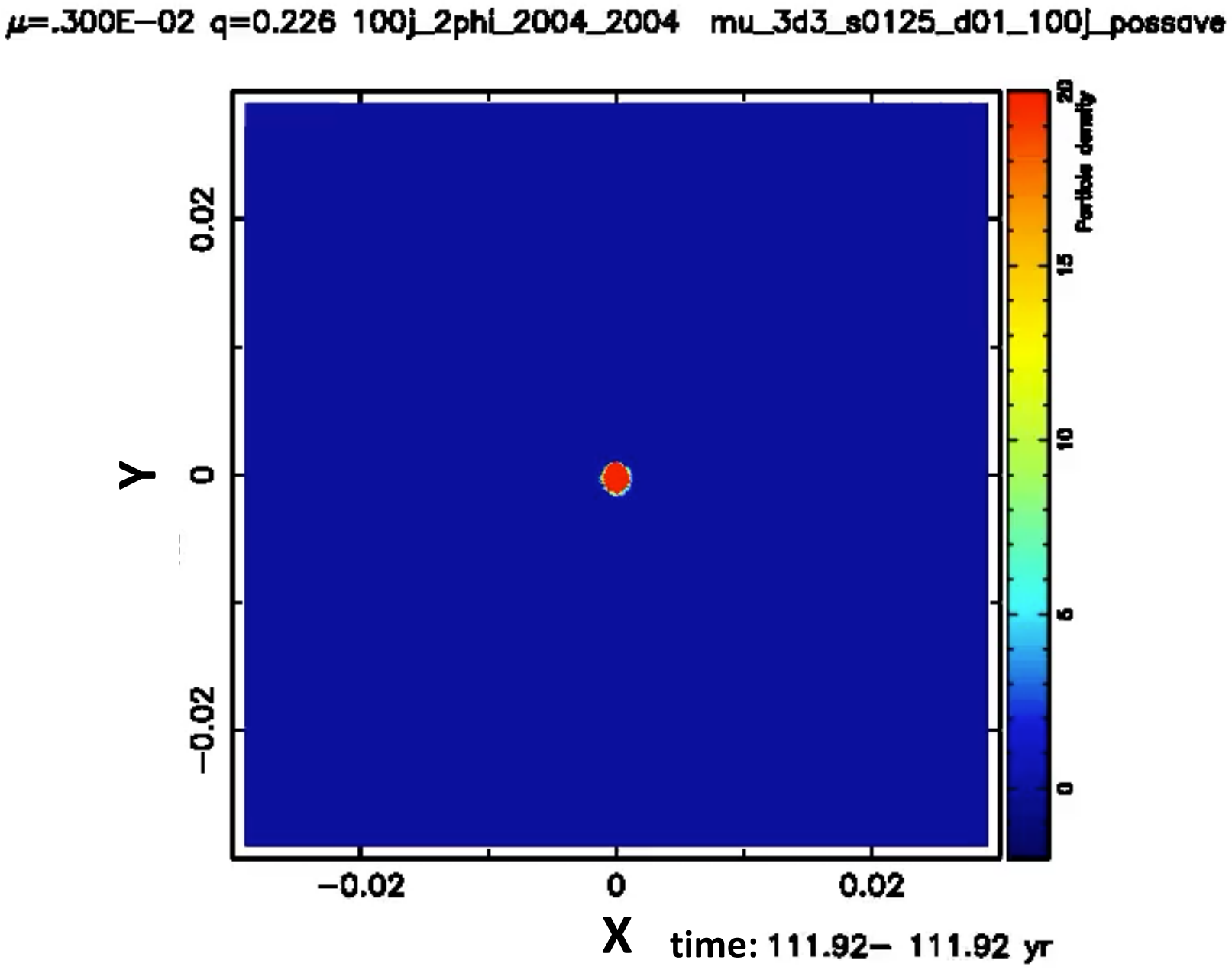
Salo & Sicardy (2026)



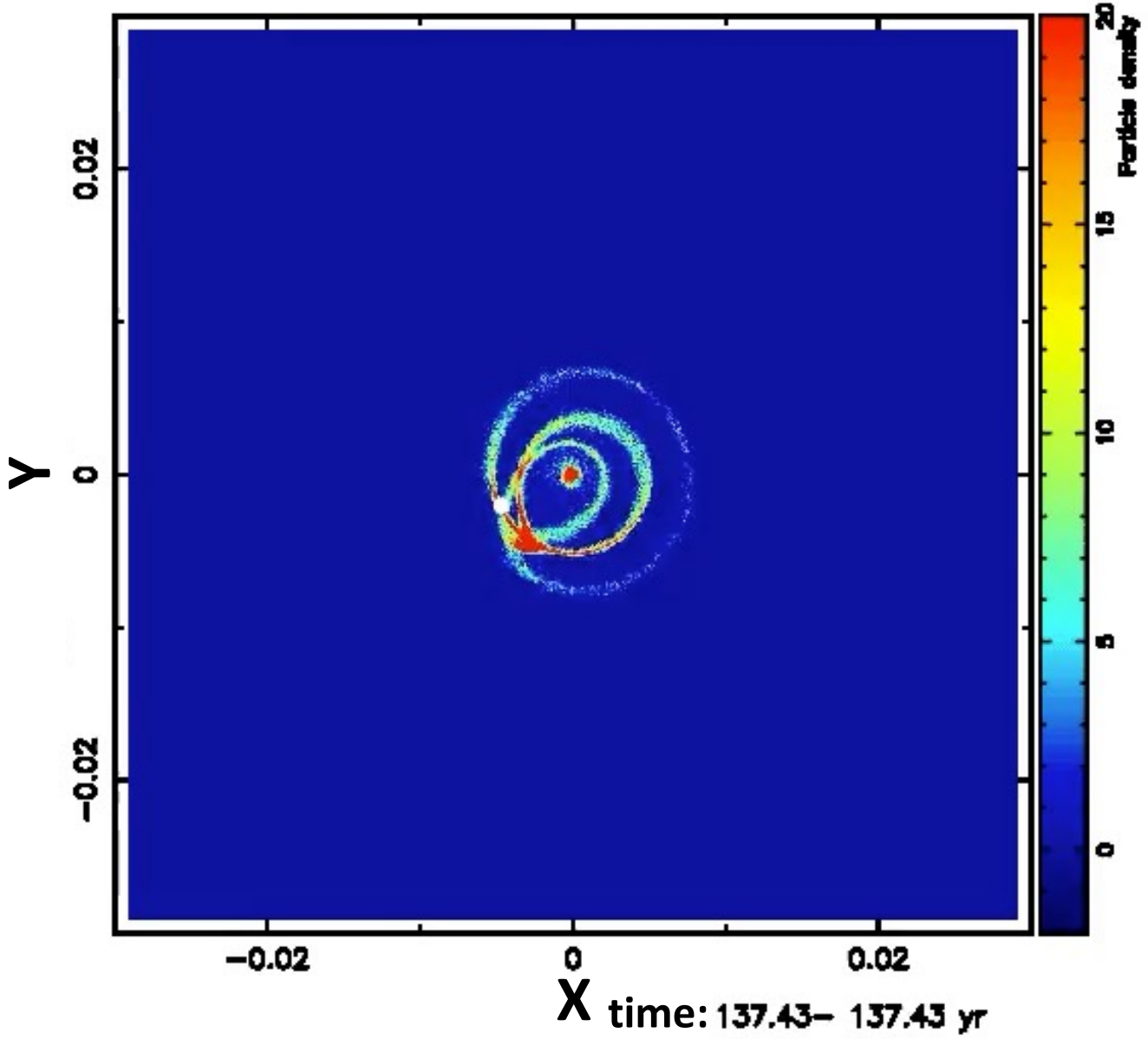

μ=.300E−02 q=0.226 100j_0000_0000 mu_3d3_s0125_d01_100j_possave
16 km
Radius
2.12
2.1
2.08
2.06
2.04
1/3 resonance
0
100
200
300
Longitude (deg)
time: 79.90– 79.90 yr
Particle density
0
2
4
6
8
10


streamline self-intersection
=.300E−02 q=0.226 100j_3540_3540 mu_3d3_s0125_d01_100j_possav


reversal of angular momentum flux

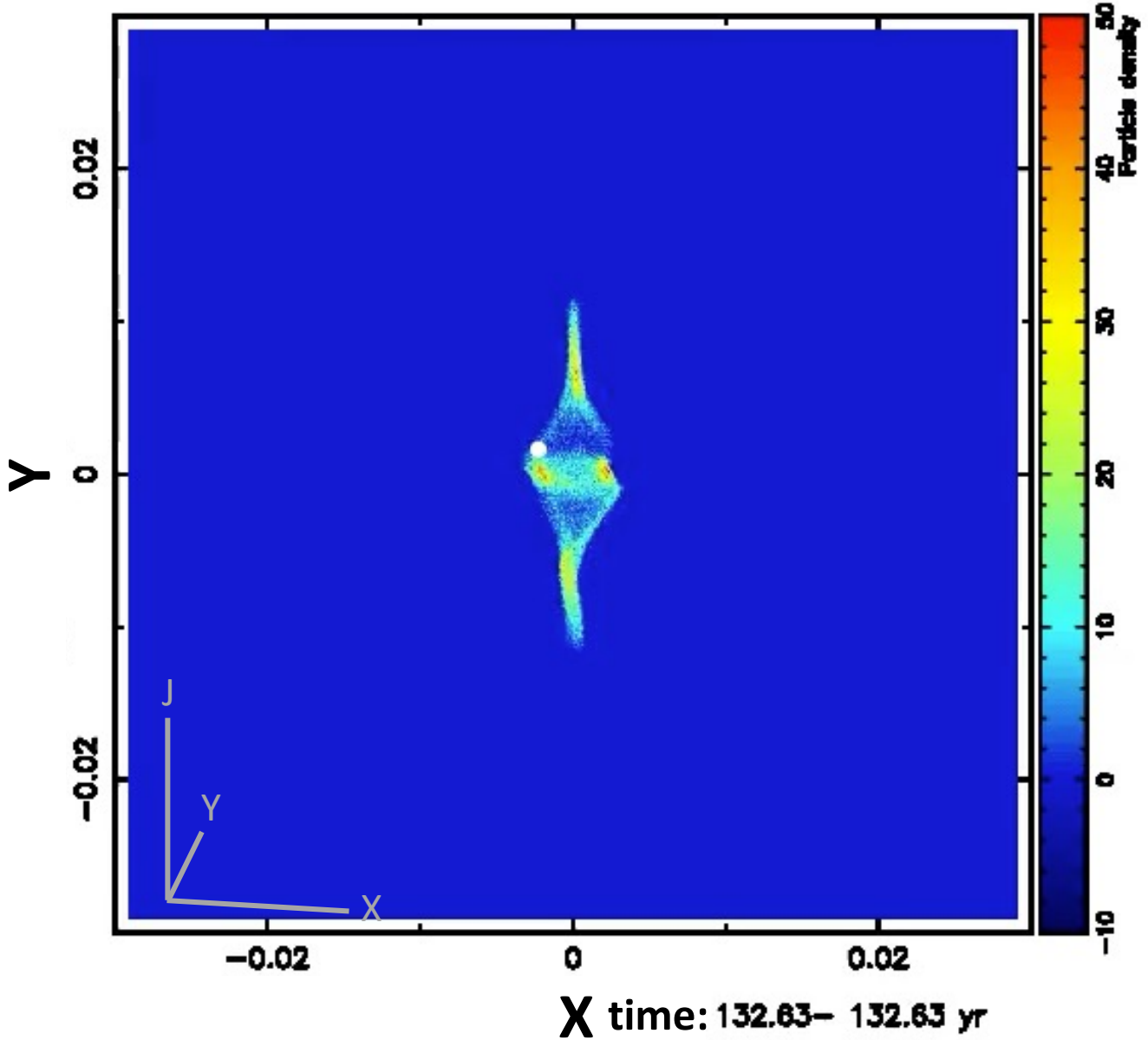

μ=.300E−02 q=0.226 100j_2phi_2004_2004 mu_3d3_s0125_d01_100j_possave
0.02
0
−0.02
Y
−0.02
0
0.02
X
time: 111.92− 111.92 yr
Particle density
20
15
10
5
0

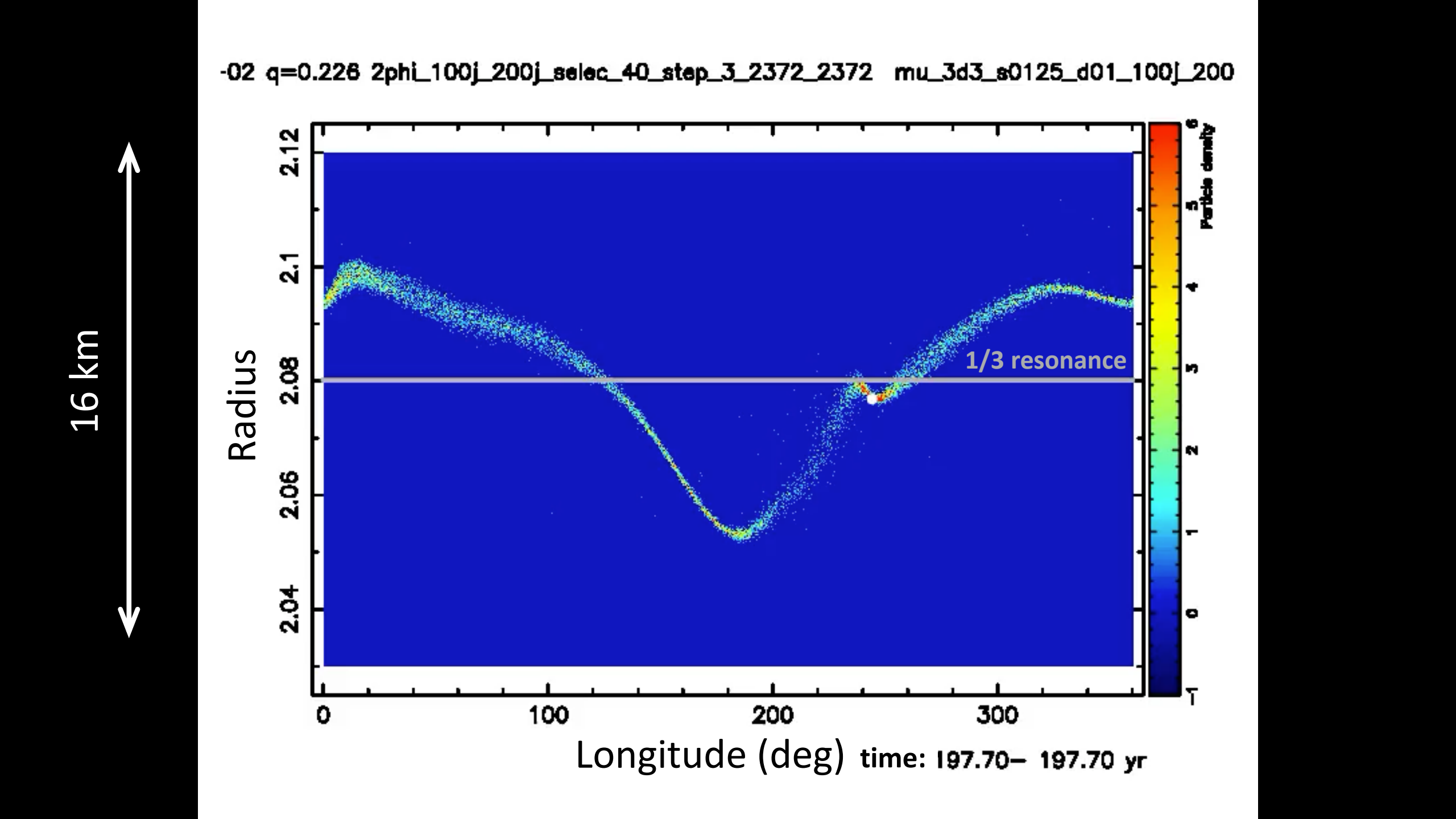
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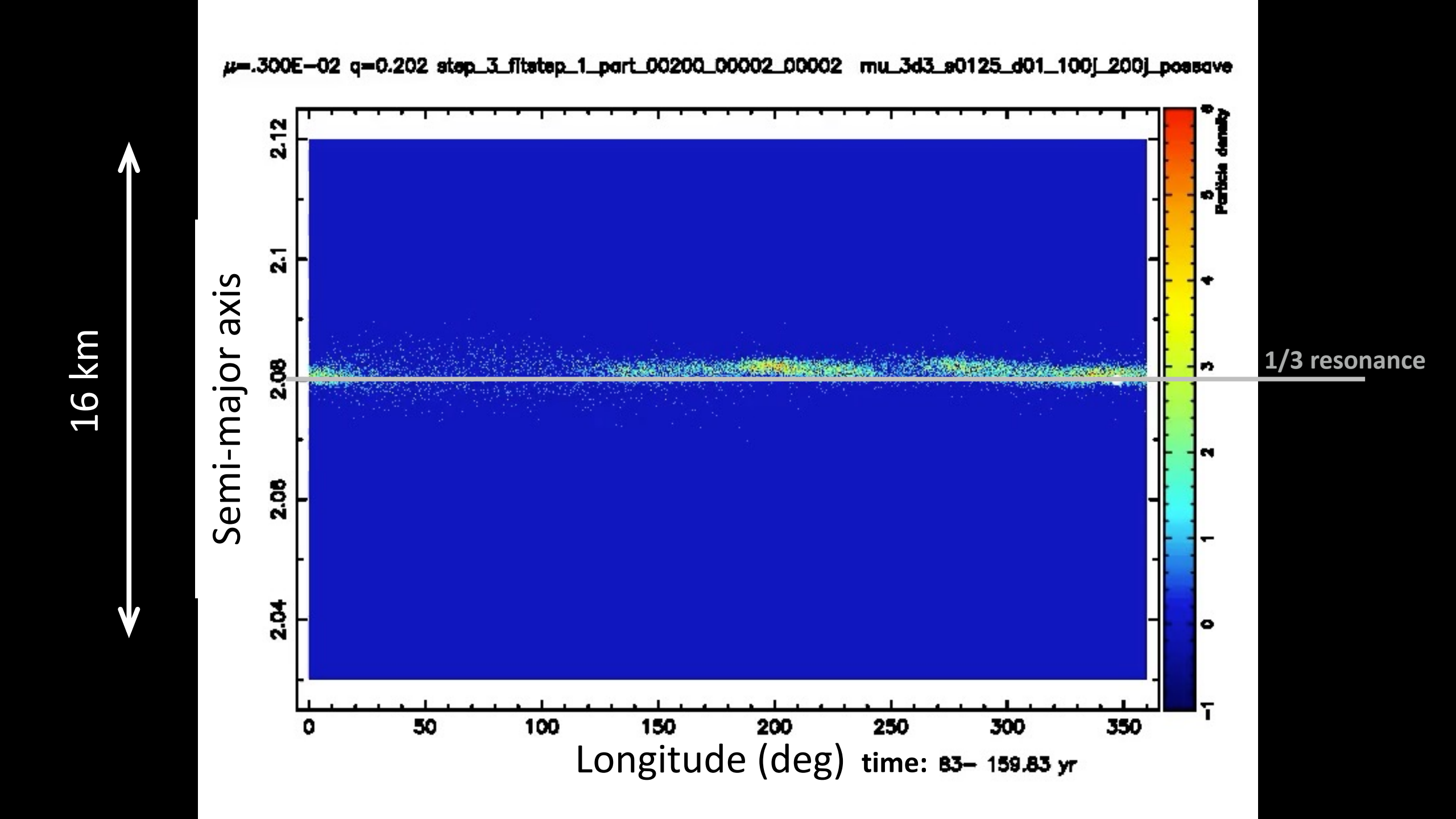


time: 137.43– 137.43 yr

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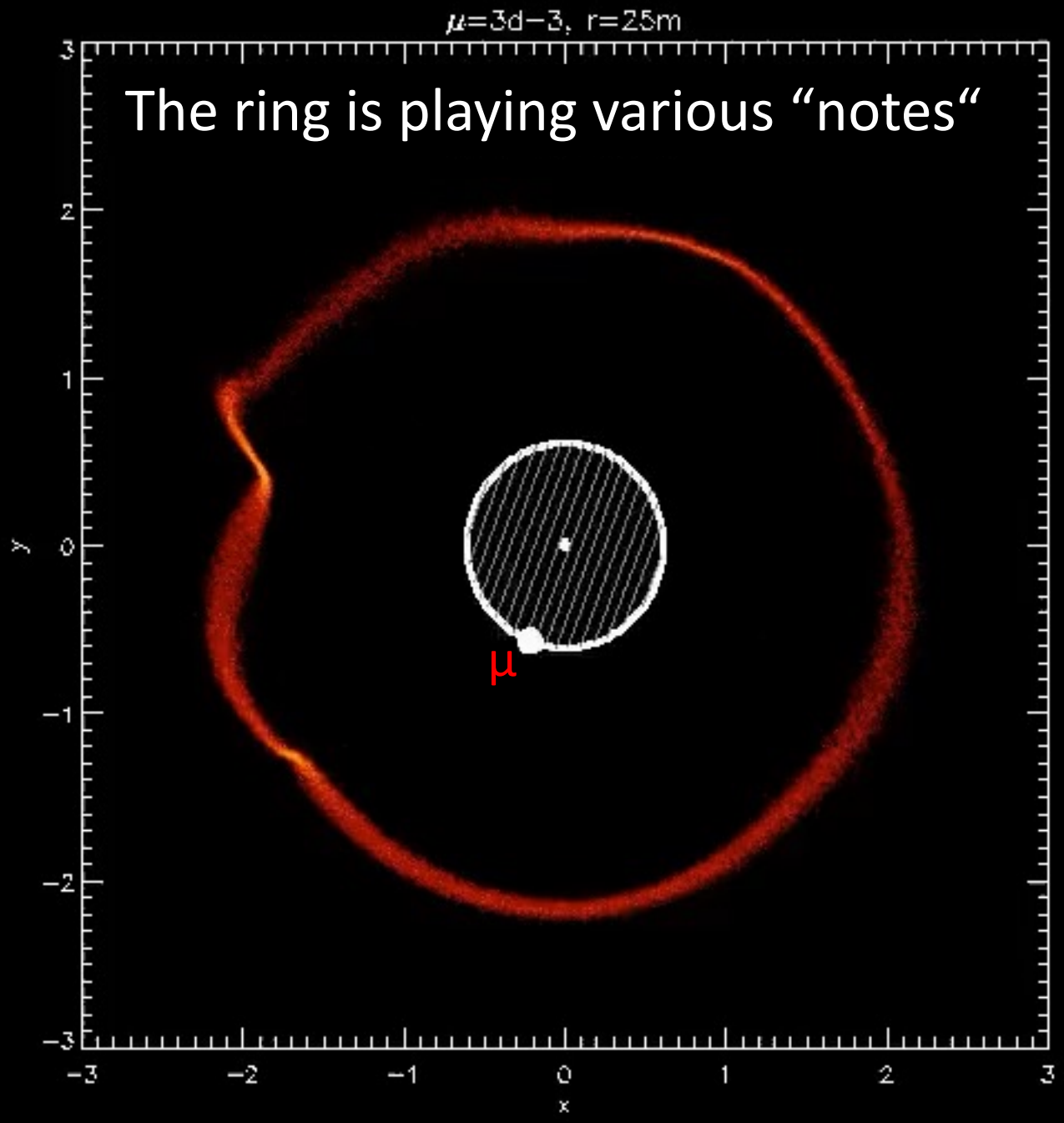


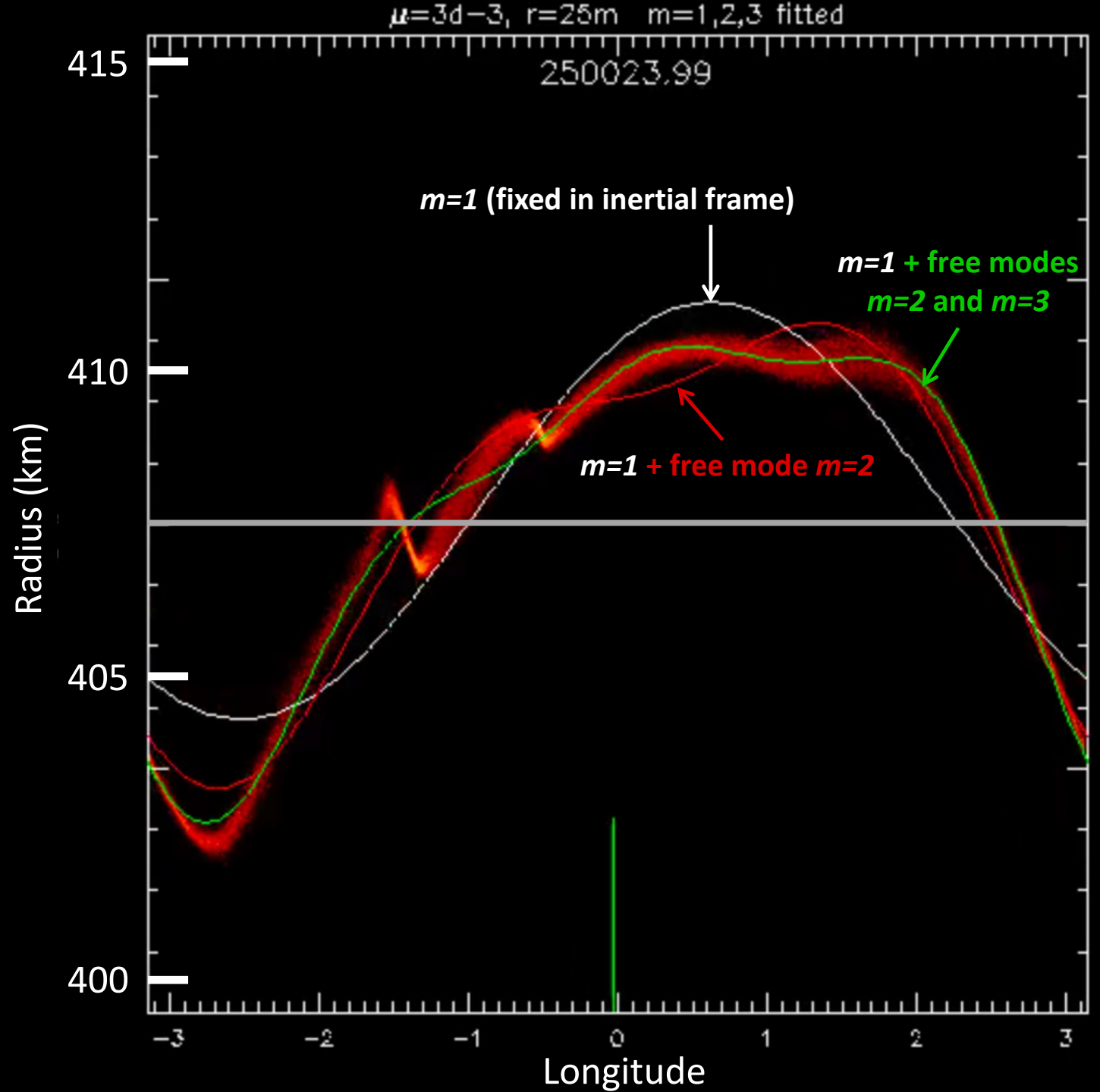

-02 q=0.226 2phi_100j_200j_selec_40_step_3_2372_2372 mu_3d3_s0125_d01_100j_200
16 km
Radius
2.12
2.1
2.08
2.06
2.04
1/3 resonance
0
100
200
300
Longitude (deg)
time: 197.70– 197.70 yr
Particle density
6
5
4
3
2
1
0
-1


μ=.300E−02 q=0.202 step_3_f\tstep_1_part_00200_00002_00002 mu_3d3_s0125_d01_100J_200J_posave
16 km
Semi-major axis
2.12
2.1
2.08
2.06
2.04
1/3 resonance
0
50
100
150
200
250
300
350
Longitude (deg)
time: 83– 159.83 yr
Particle density
5
4
3
2
1
0
-1

The ring is playing various “notes“

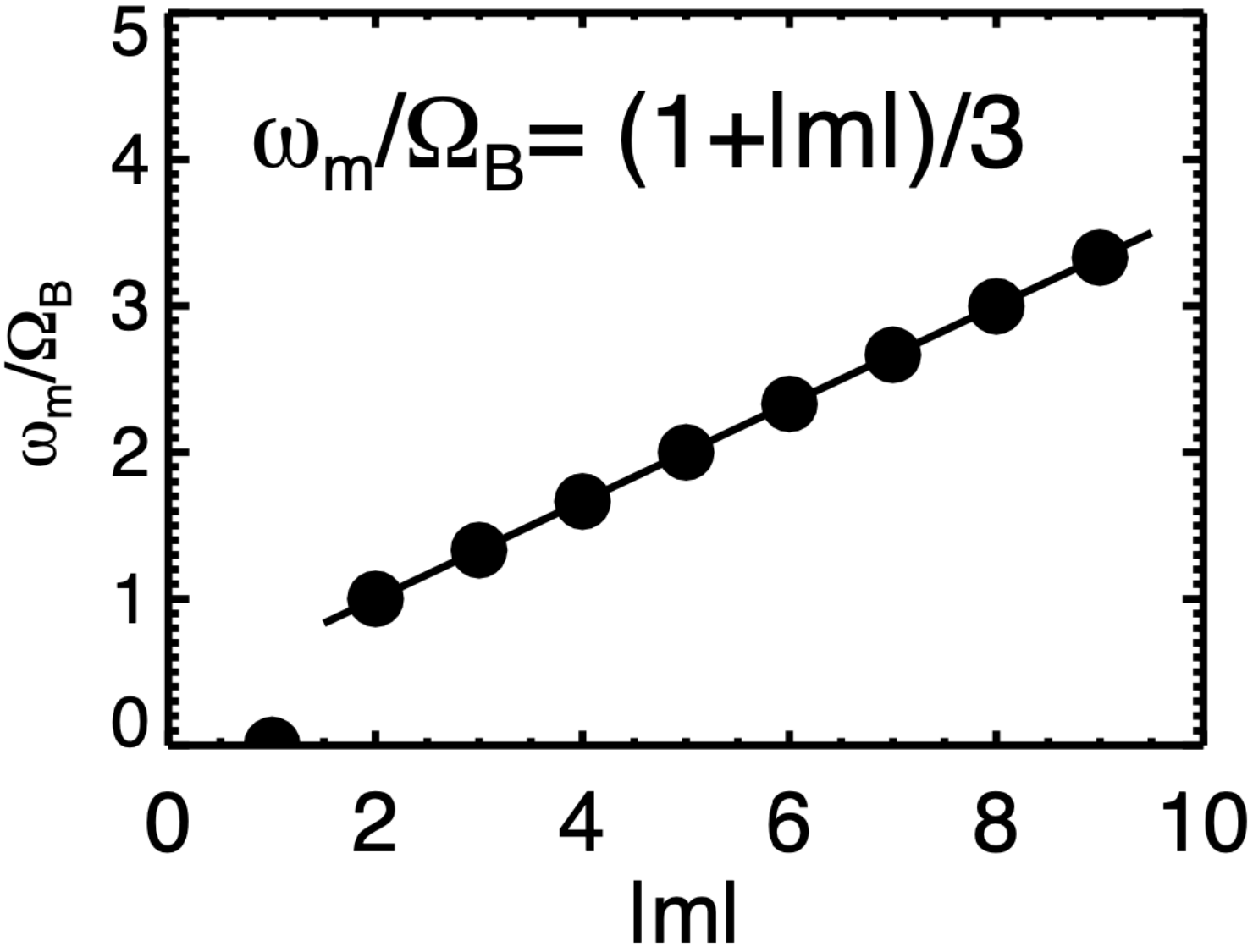
The ring seen pole on
in Cartesian coordinates
Radial excursion x5
Width x50





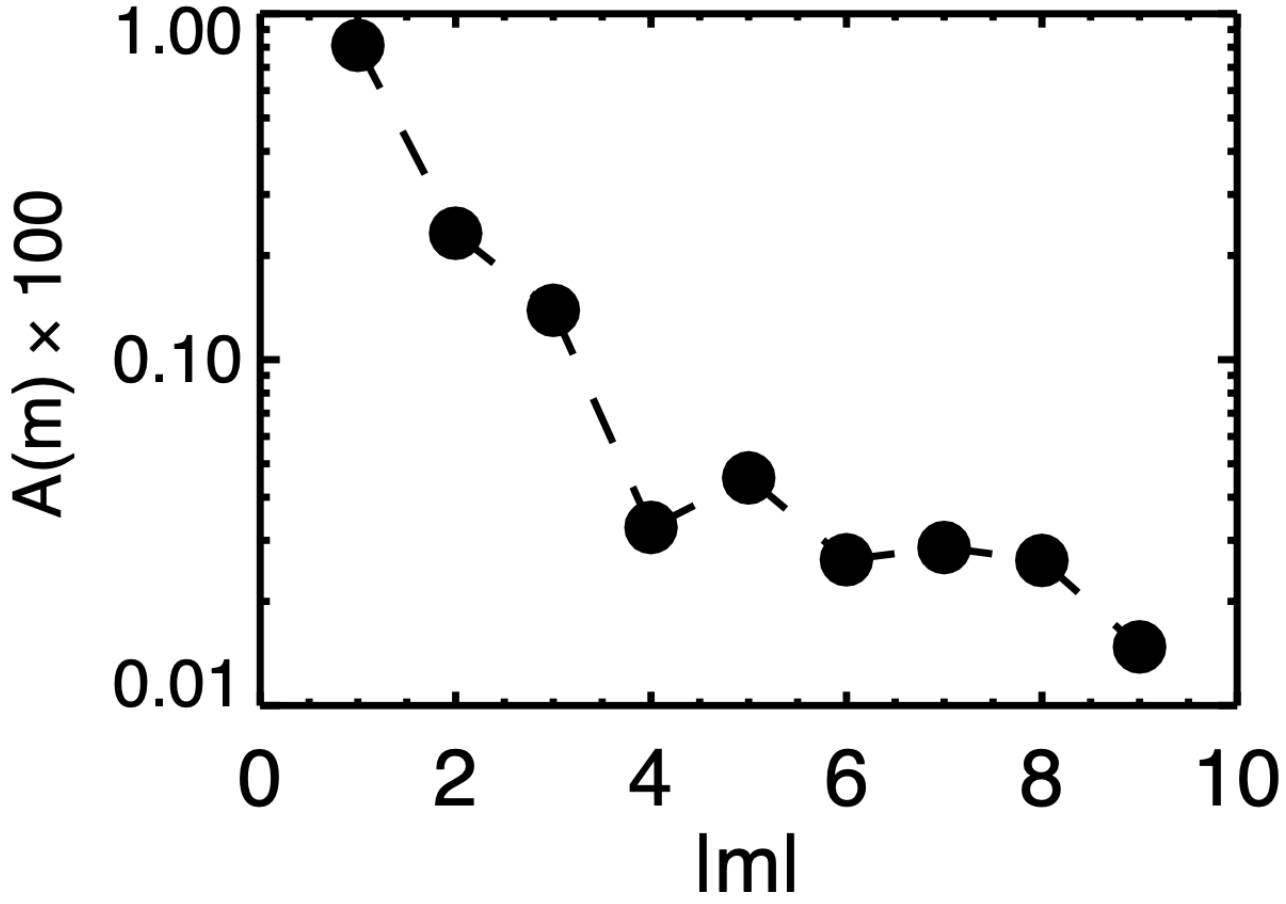
Free modes, Lindblad-type
m-lobed oscillations

The ring is playing various “notes“

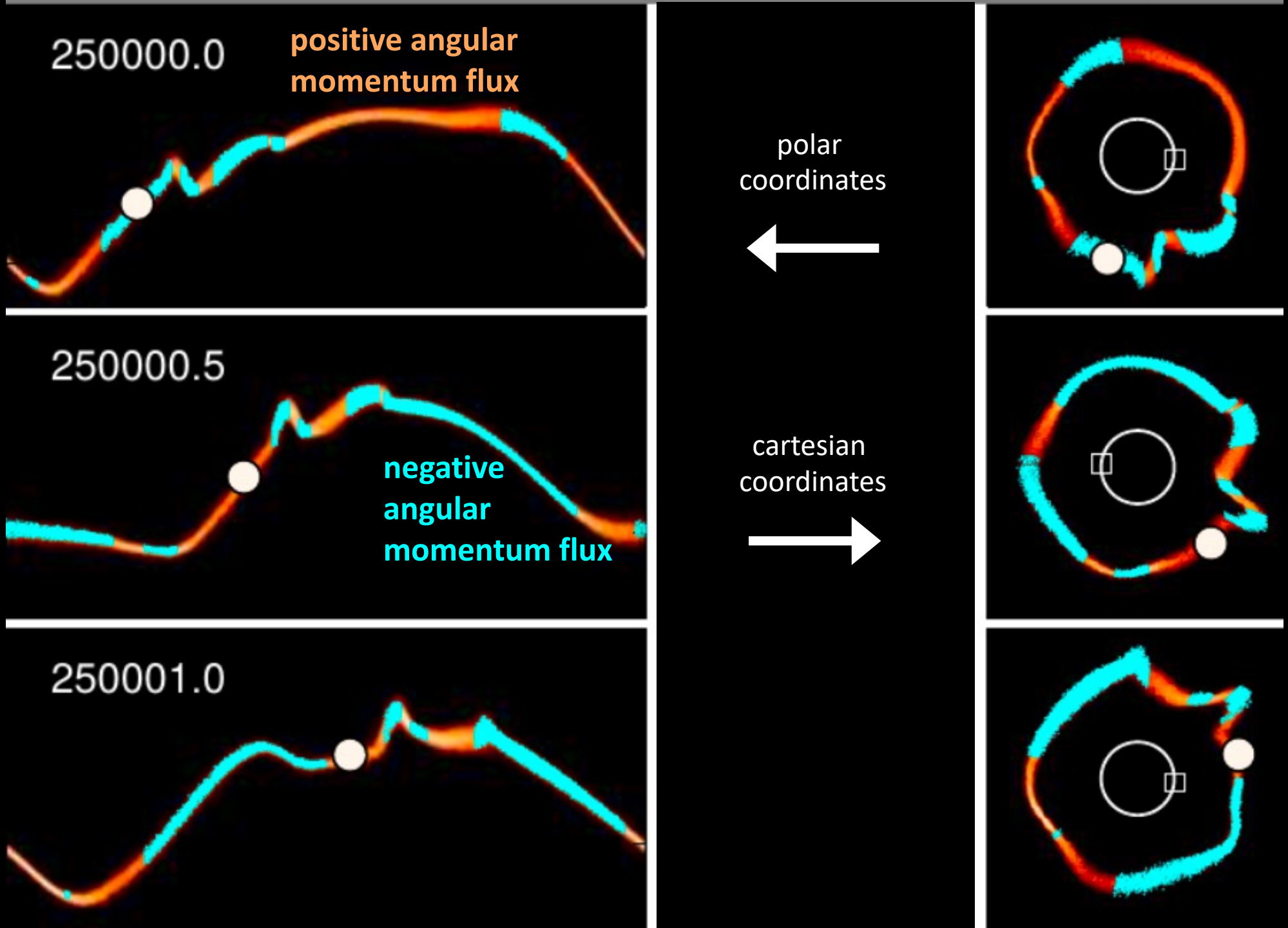


$$r(L,t) = A_m \cos[m(L - \Omega_m t) + \phi_m]$$

where $\Omega_m = \dfrac{m-1}{m} n$ → free Lindblad modes



Salo & Sicardy
A&A 2026


250000.0
positive angular momentum flux
250000.5
negative angular momentum flux
250001.0
polar coordinates
cartesian coordinates

Conclusions I

1st-order (Lindblad) resonances create viable spiral waves in disks (no self-intersection problem)

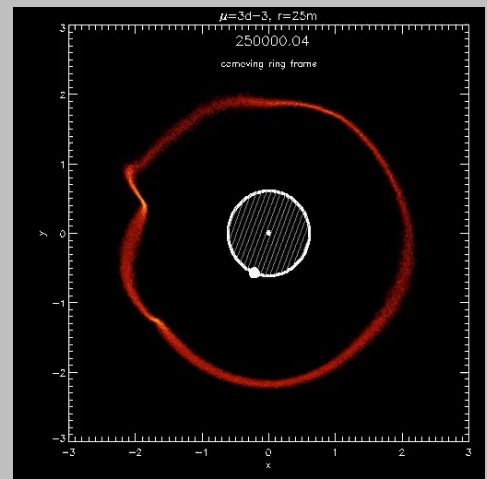
They are analytically tractable, with ‘DNA‘ phase portraits

They lead to ring confinement

Higher-order resonances all have self-intersection problems

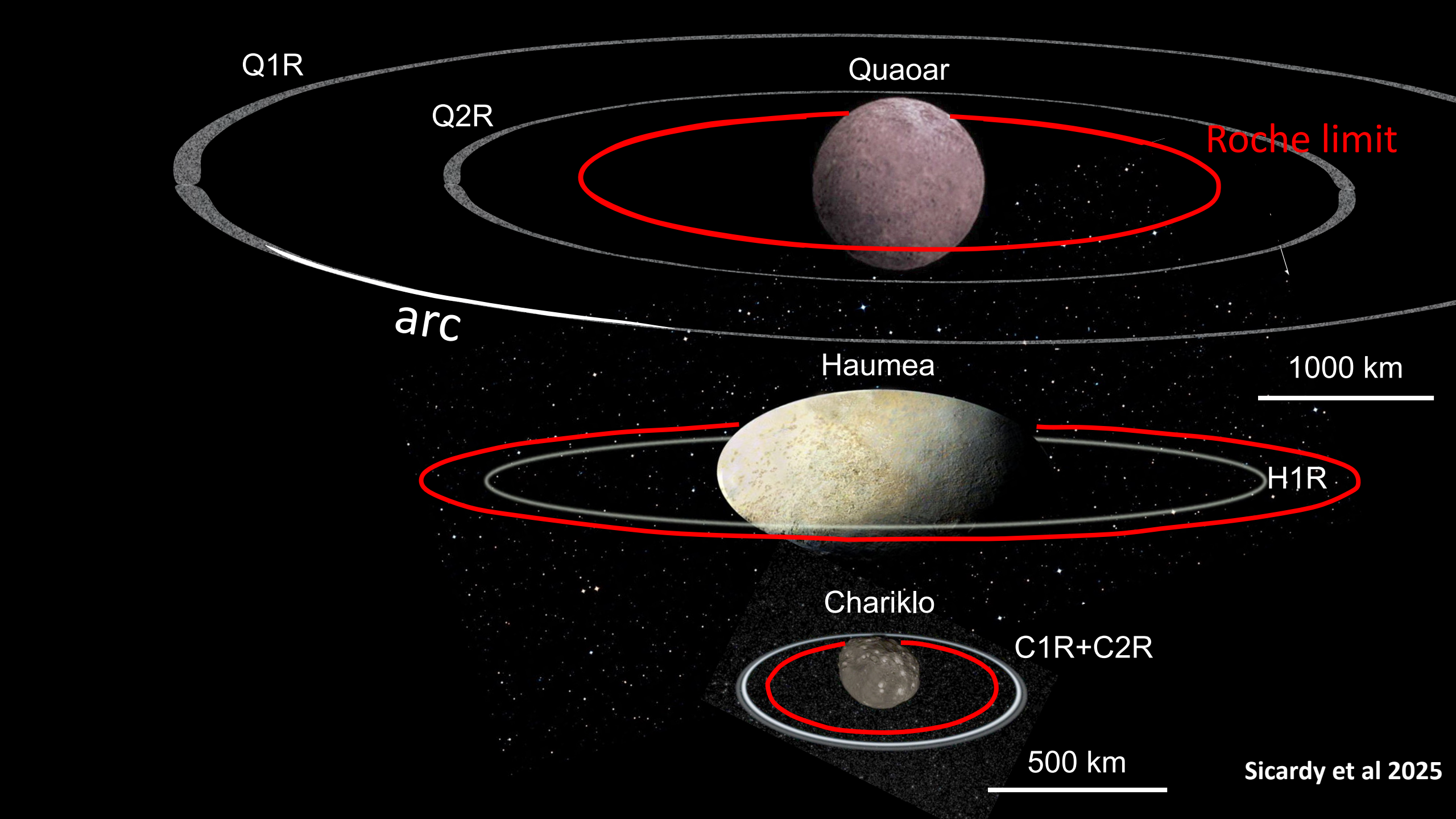
Yet, they also lead to ring confinement through the excitation of free Lindblad modes: order instead of chaos

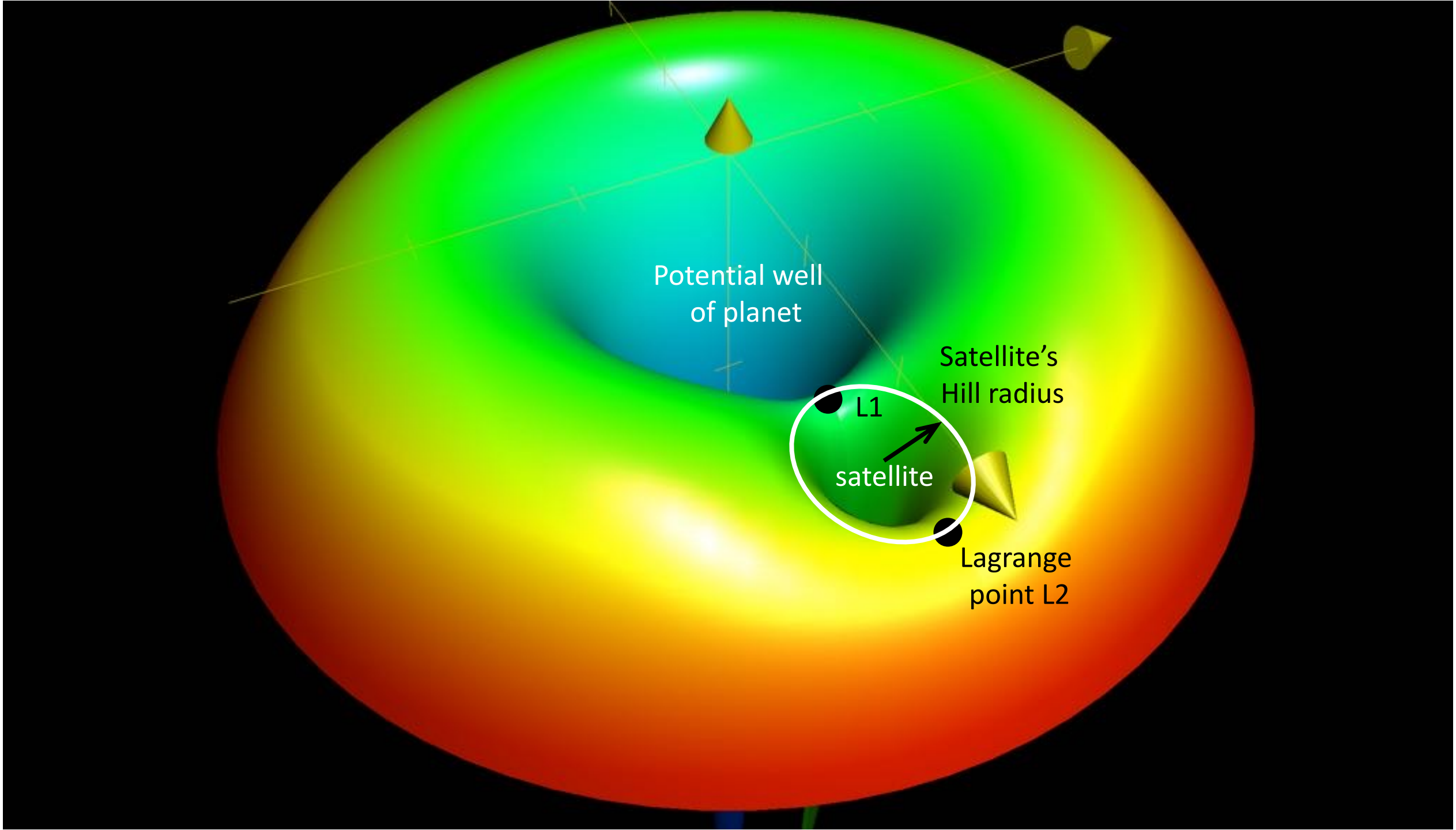
A problem yet to be treated analytically

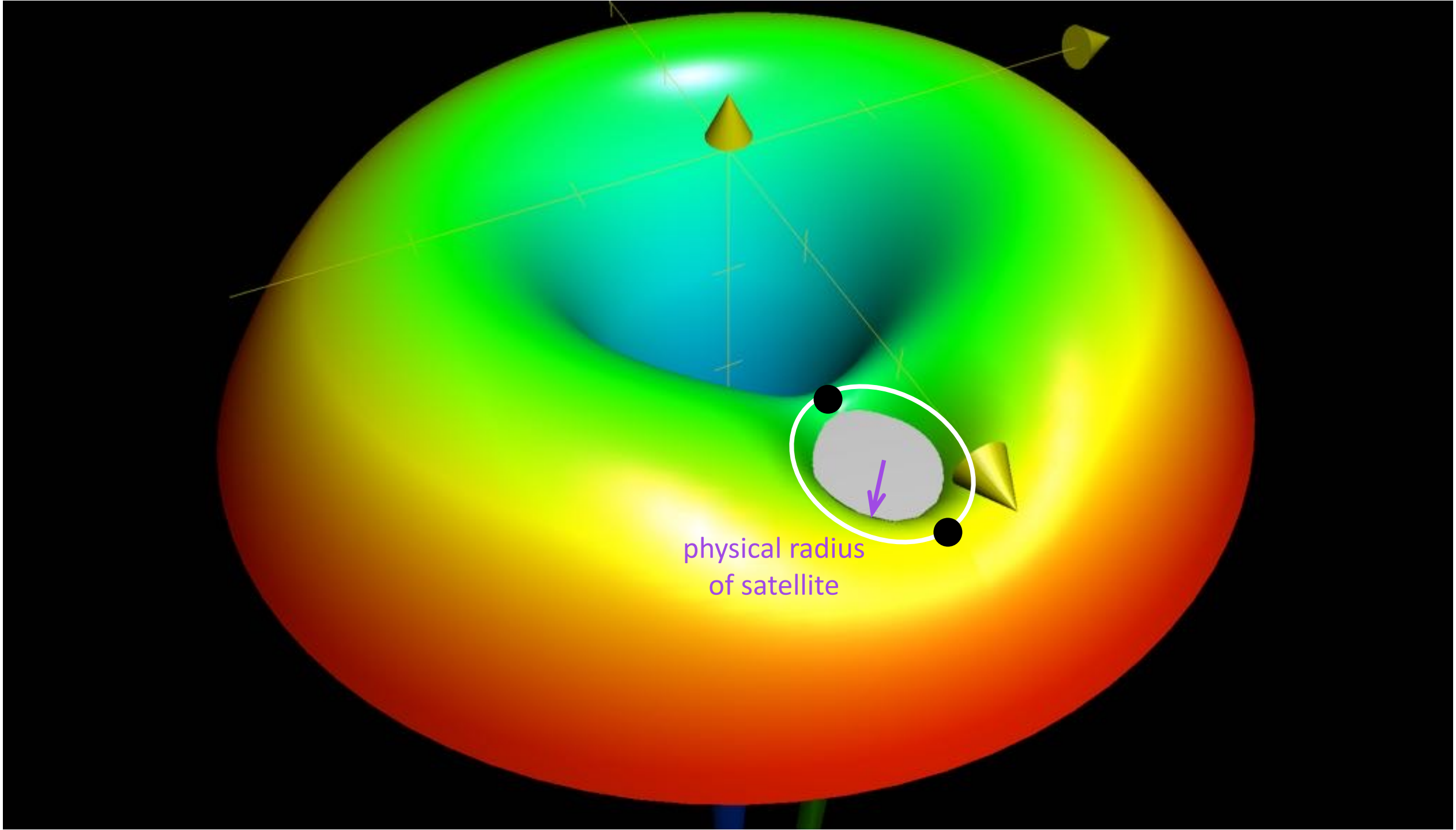


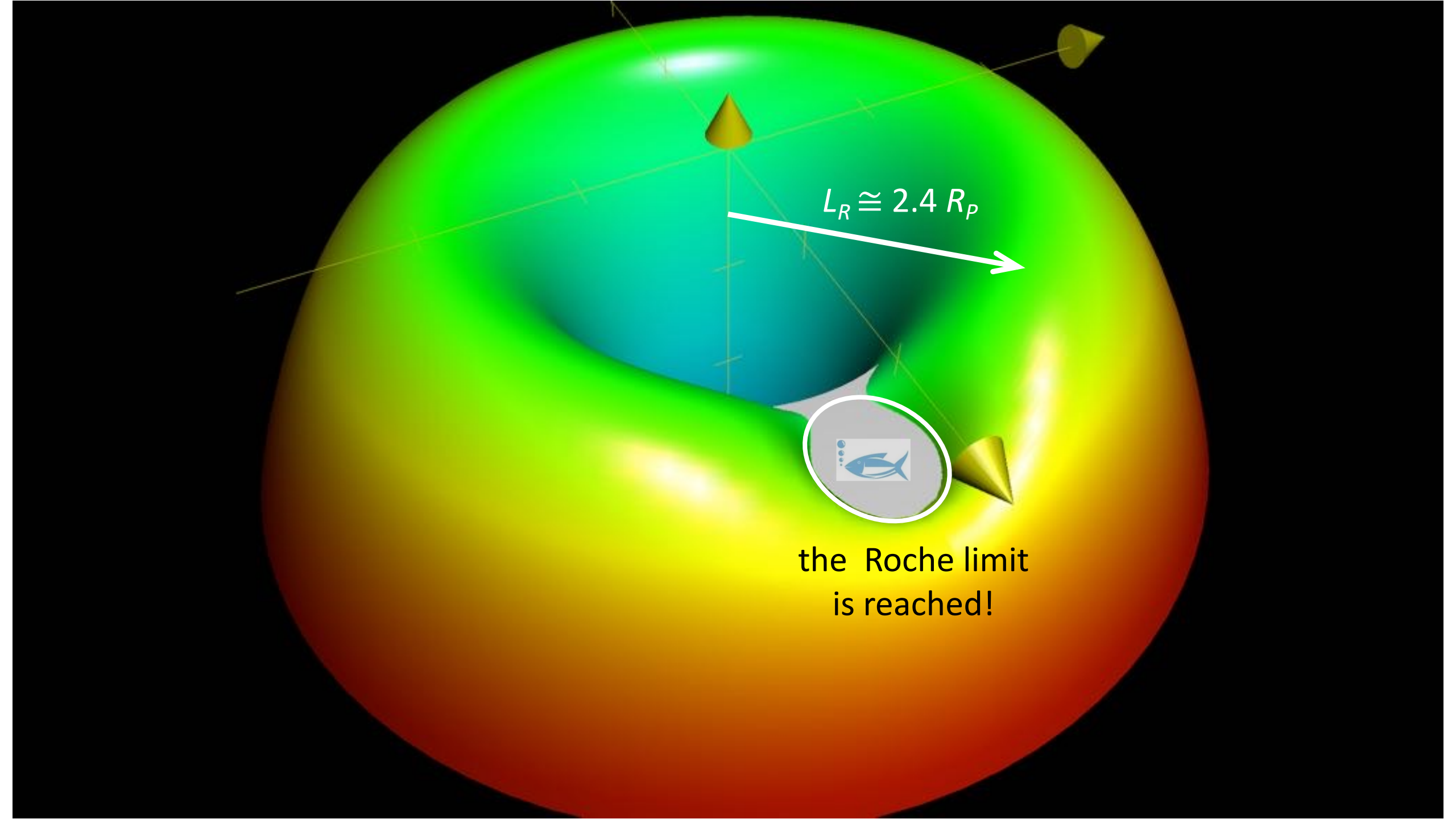
A new definition of
the Roche limit

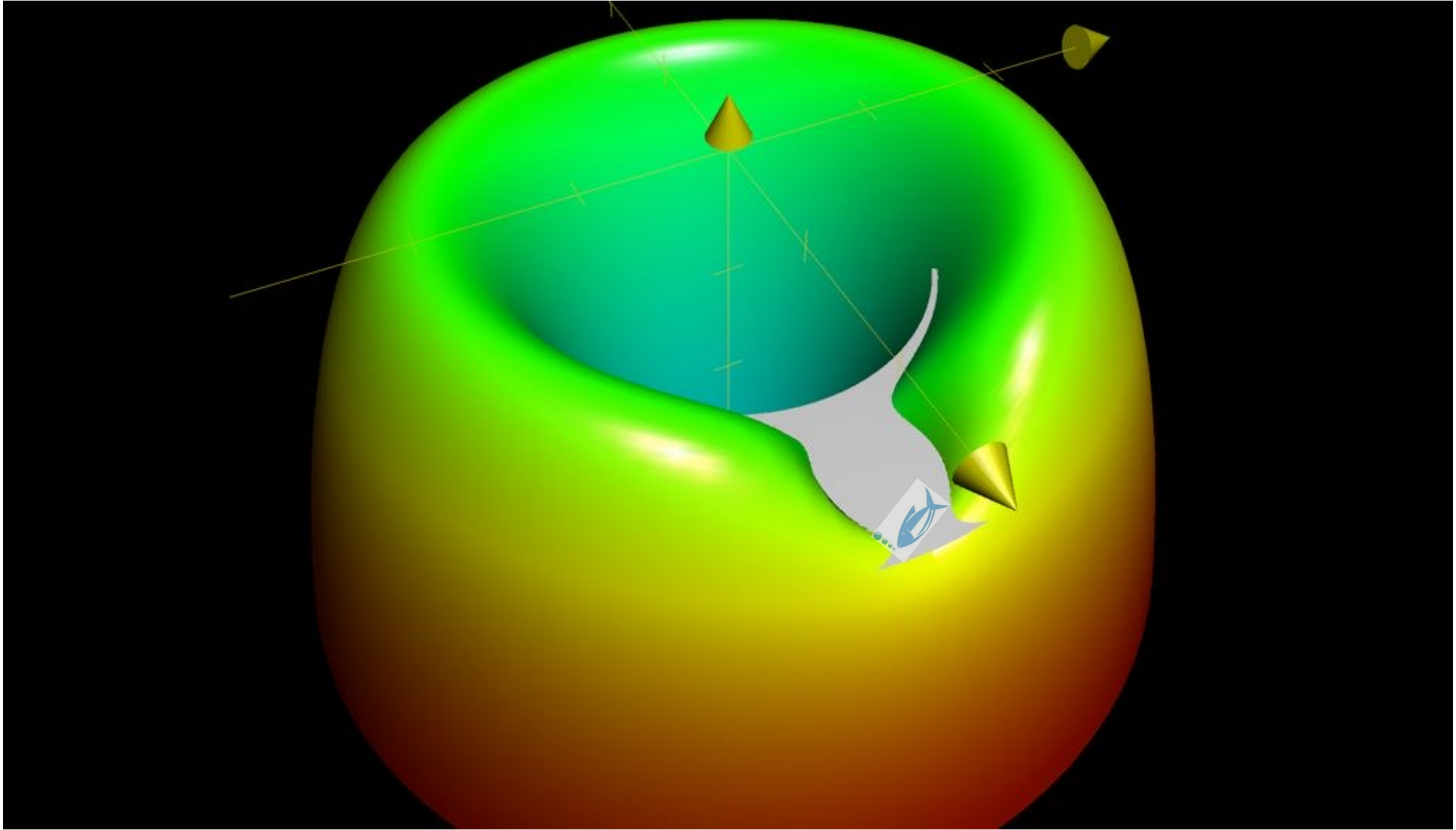


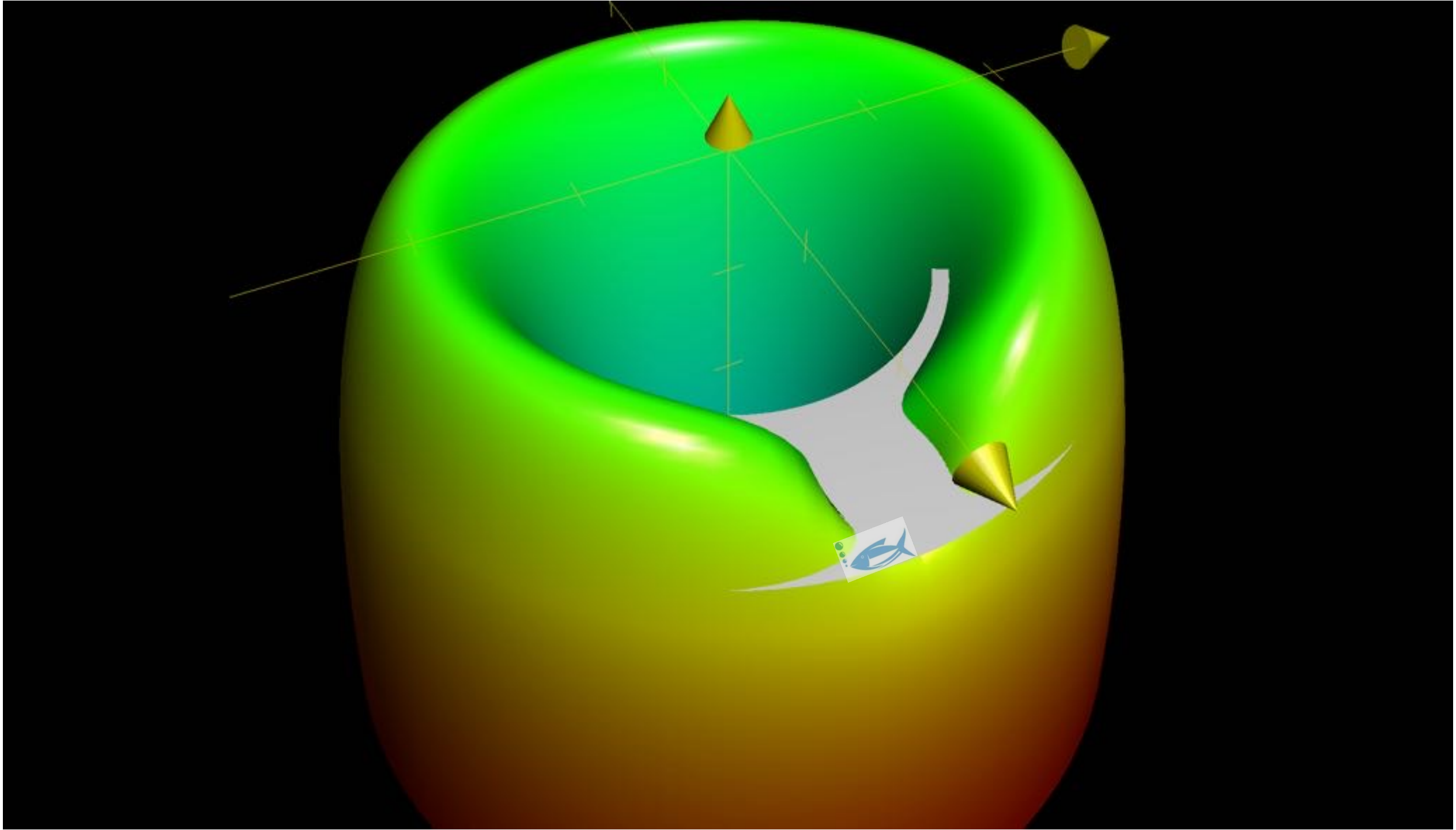

Q1R
Q2R
Quaoar
Roche limit
arc
Haumea
1000 km
H1R
Chariklo
C1R+C2R
500 km
Sicardy et al 2025

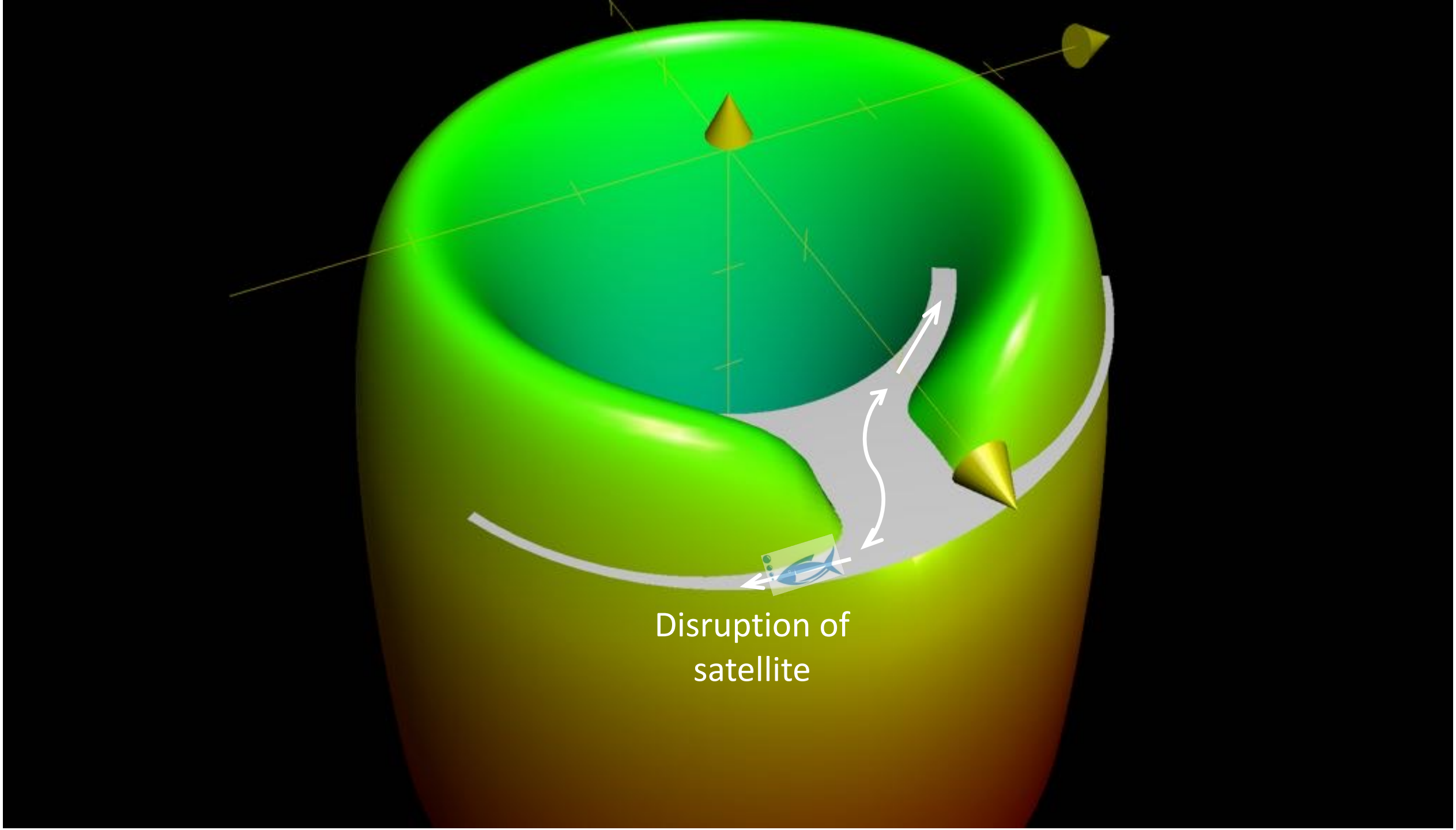

Potential well
of planet
Satellite's
Hill radius
L1
satellite
Lagrange
point L2

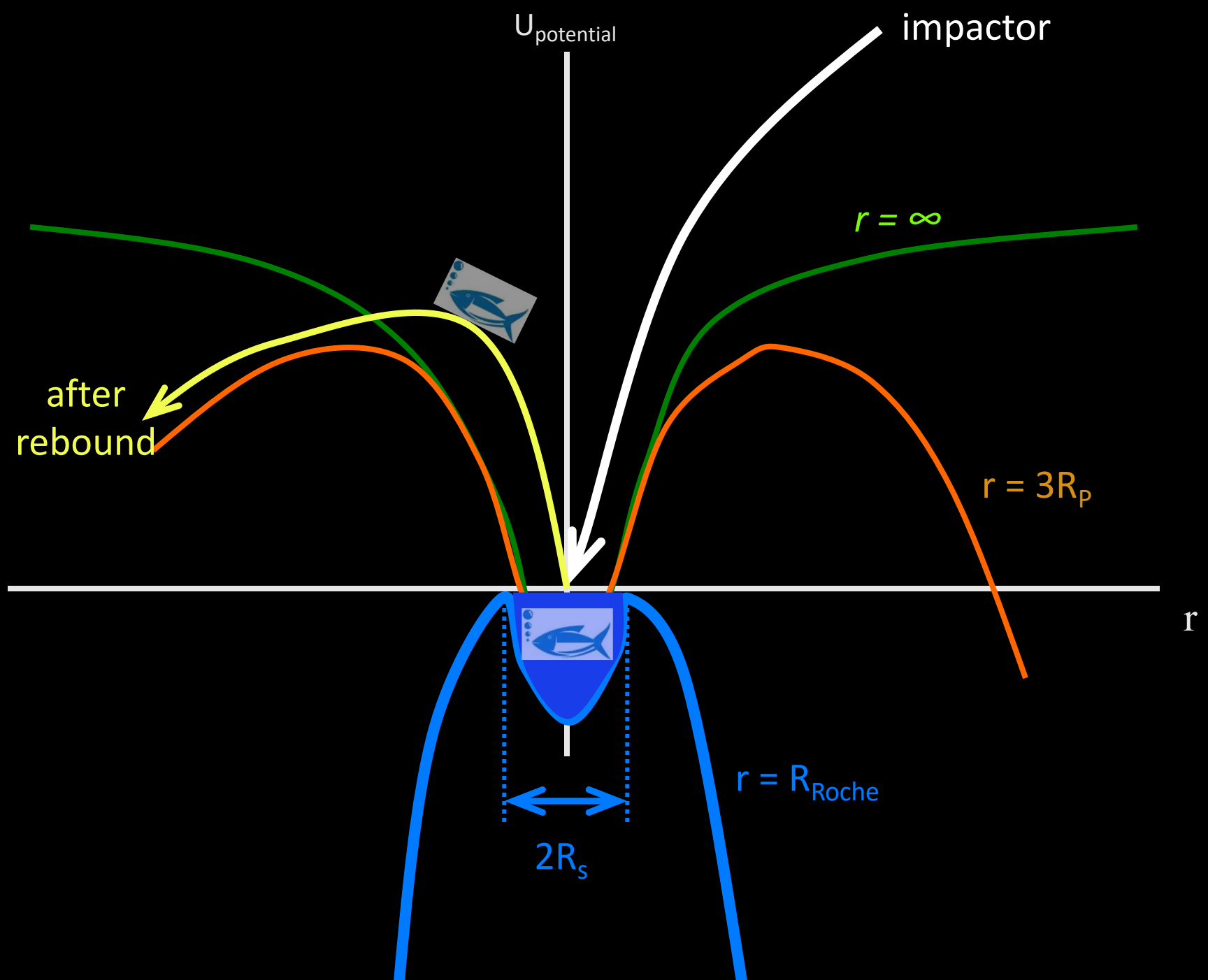

physical radius
of satellite


$L_R \cong 2.4\ R_P$
the Roche limit
is reached!



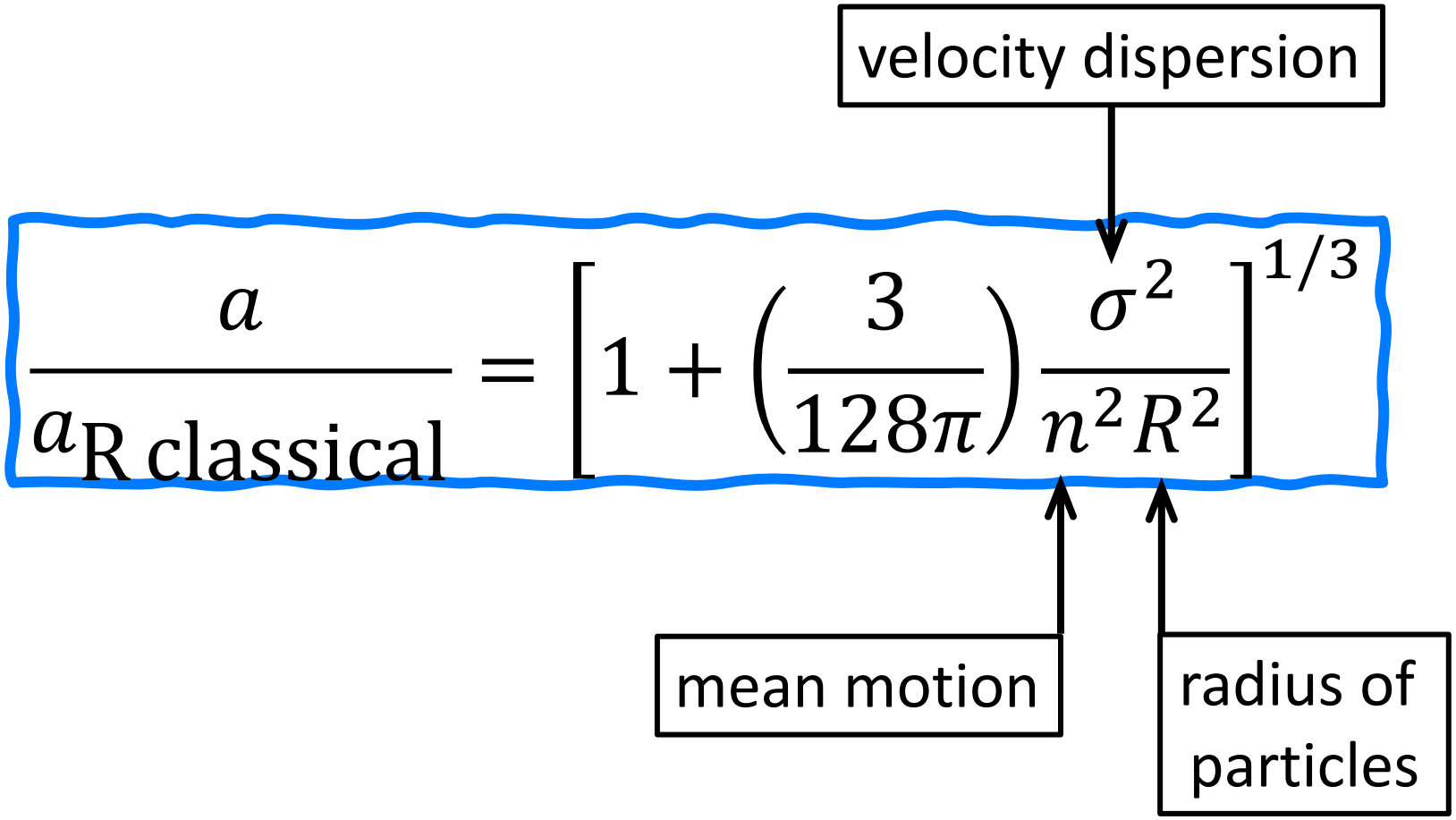


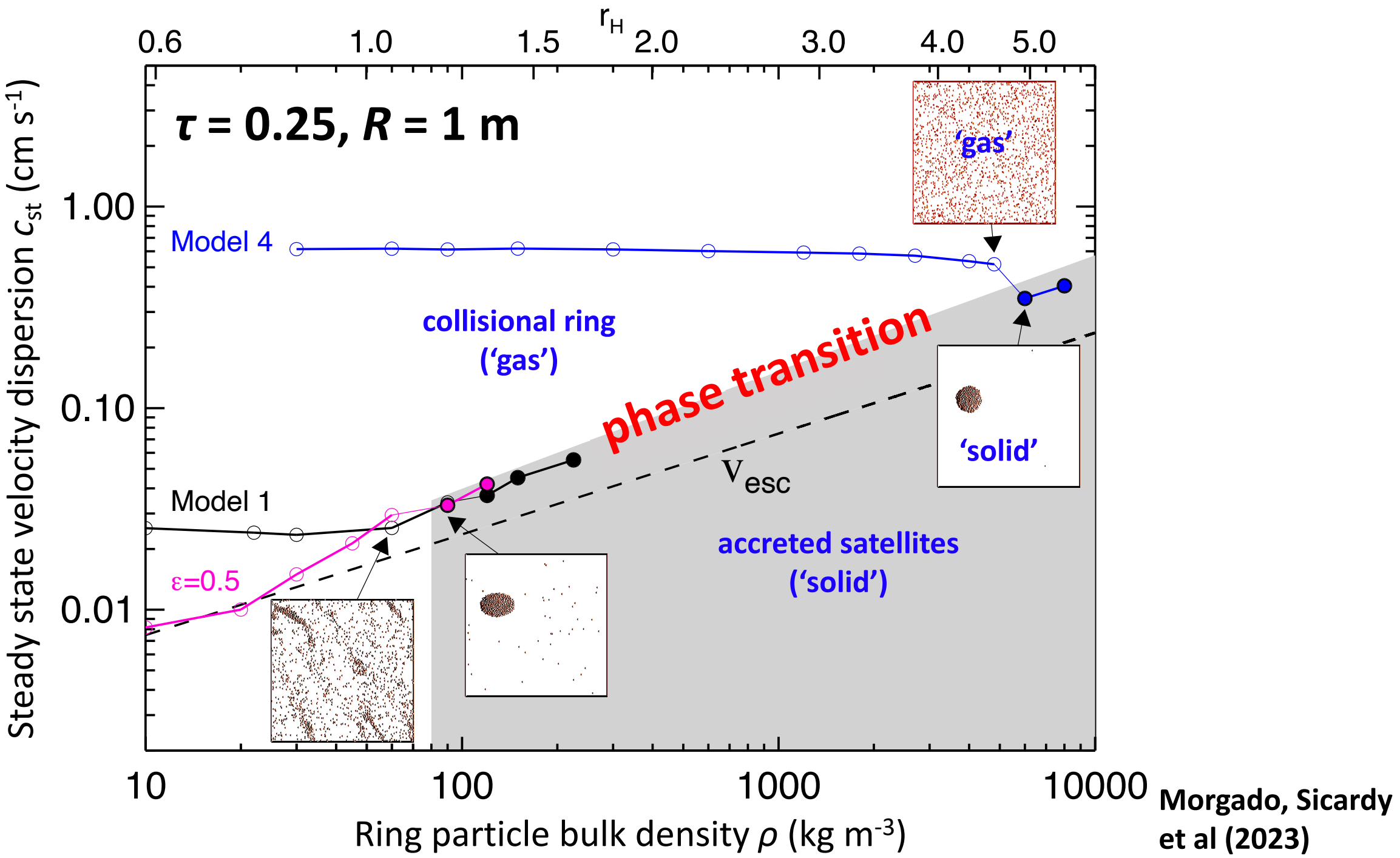

Disruption of
satellite


$U_{potential}$
impactor
$r = \infty$
after
rebound
$r = 3R_P$
r
$r = R_{Roche}$
$2R_s$

A Statistical Model Model for Stable Rings Outside the Roche Limit

J. T. Dinsmore arXiv:2608.17004v1





Morgado, Sicardy et al (2023)

Conclusions II

The classical definition of the Roche limit is static

For a ring, the kinematic of the particles must be accounted for.
The allows rings to be maintained well beyond the classical Roche limit

Take away conclusions →

“Streamlines can start to intersect, but don’t worry“

“A fish can escape the bowl if the water spills out, or if the fish jumps up”