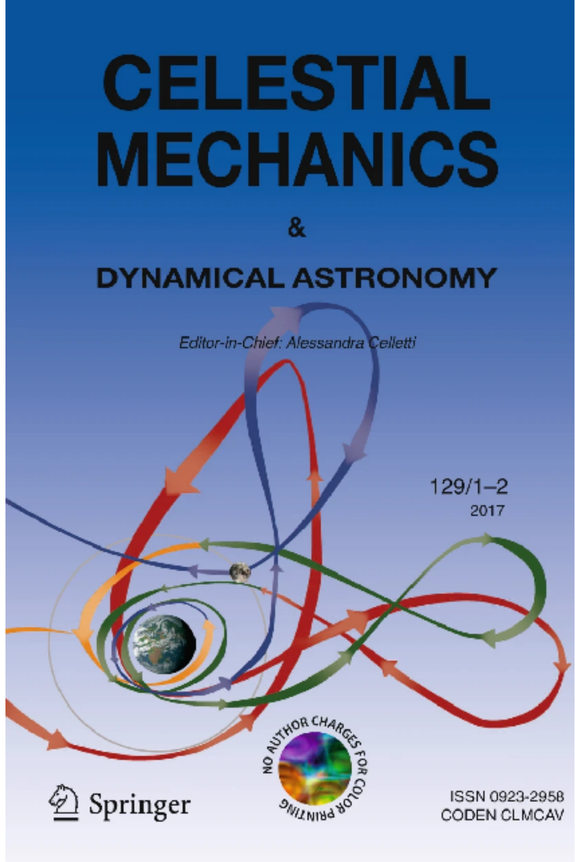
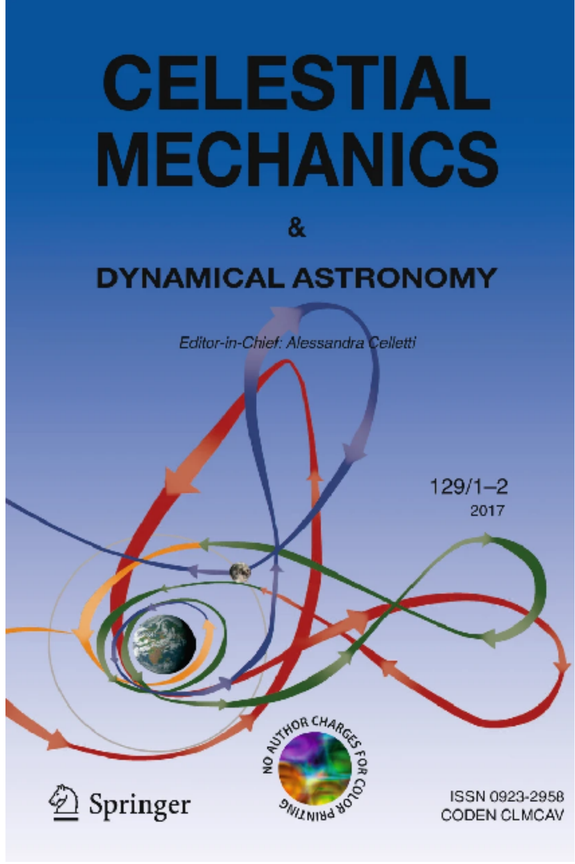
Celestial Mechanics & Dynamical Astronomy

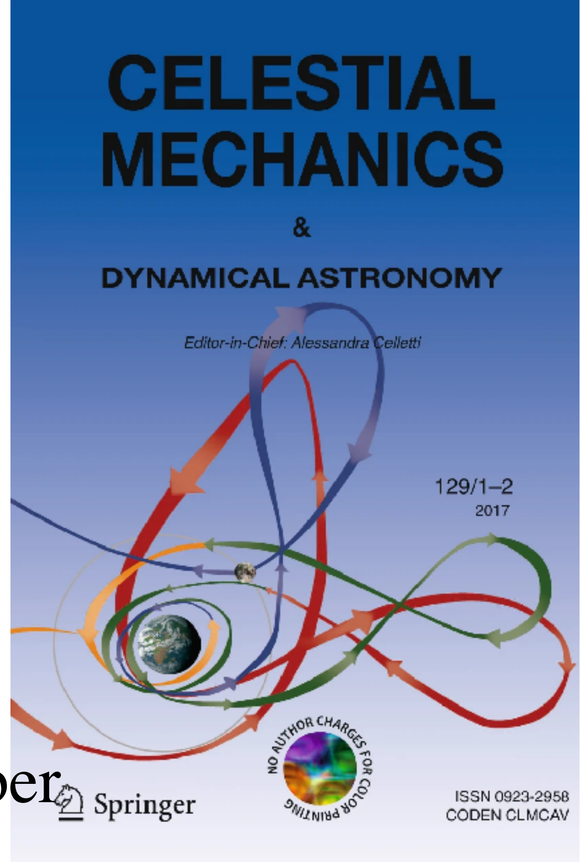


Celestial Mechanics & Dynamical Astronomy

- The CMDA Journal has been excellently run over the last 10 years by Alessandra Celletti as Editor-in-Chief!
  - She has expertly guided the journal through covid and many other difficult issues
  - Due to Springer-Nature rules on the tenure of Editors-in-Chief, they requested that Alessandra step down and that a new EiC be selected
  - She obliged, and the Celestial Mechanics Institute started a search for a new EiC

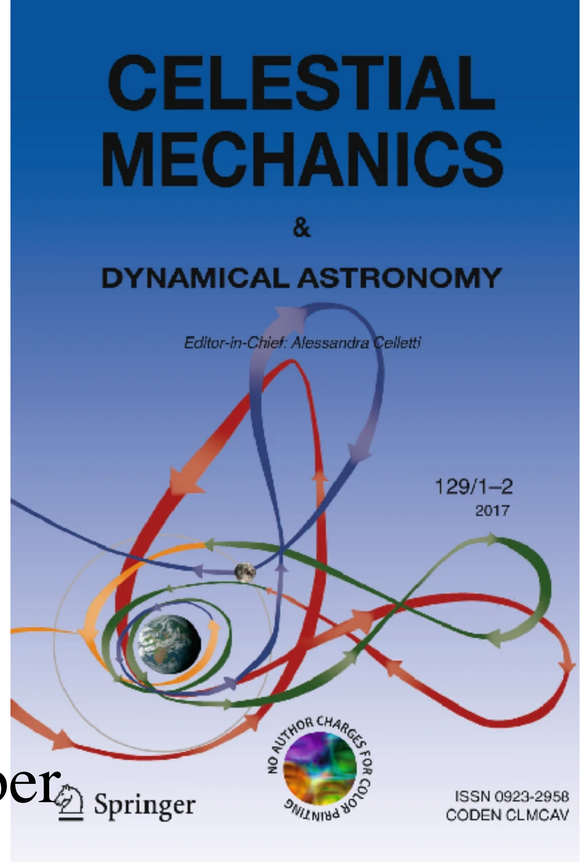


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  - Extensive experience in editorial roles, and has many contributions to dynamical astronomy
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- Alessandra will stay on as co-EiC for the remainder of the year and will transition to Senior Editor in the new year
  - We thank her for her exceptional service to the Journal!!!
  - We look forward to a new era with Dimitri Veras!!!

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Celestial and Spaceflight Mechanics Laboratory

Formation of Contact Binaries through the Creation of Asteroid Pairs

D.J. Scheeres & E. White

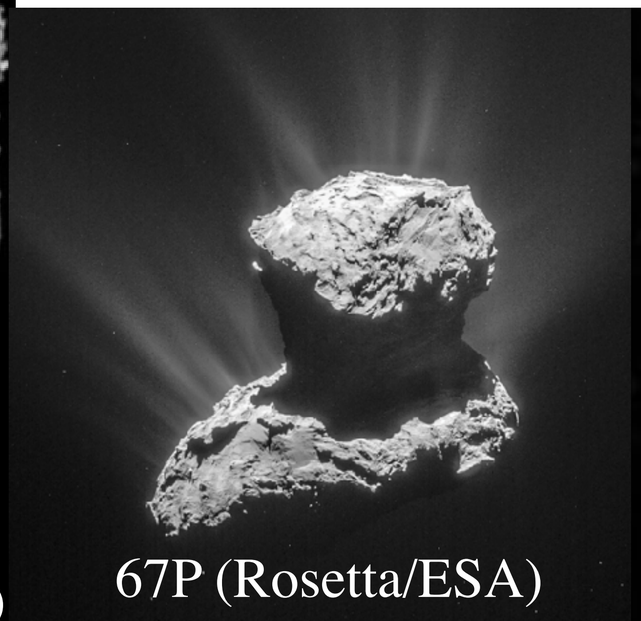
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CELMEC 2026

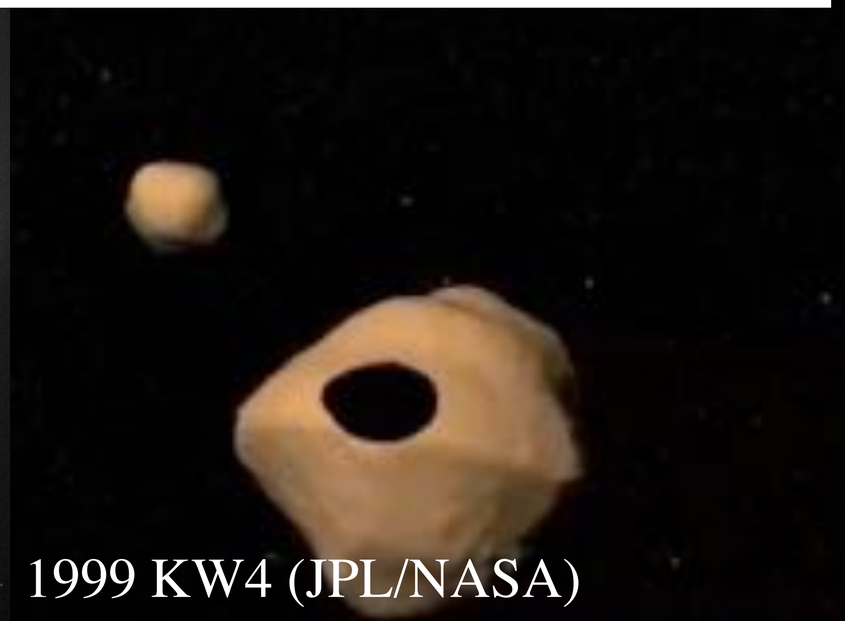
Monday, September 14



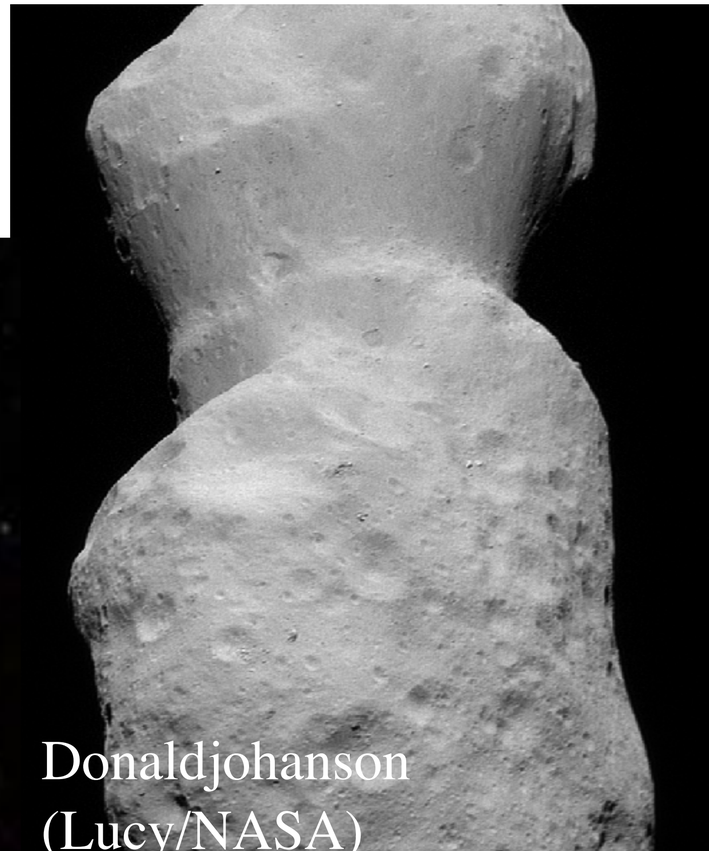
Itokawa (Hayabusa/JAXA)



67P (Rosetta/ESA)



1999 KW4 (JPL/NASA)



Donaldjohanson (Lucy/NASA)

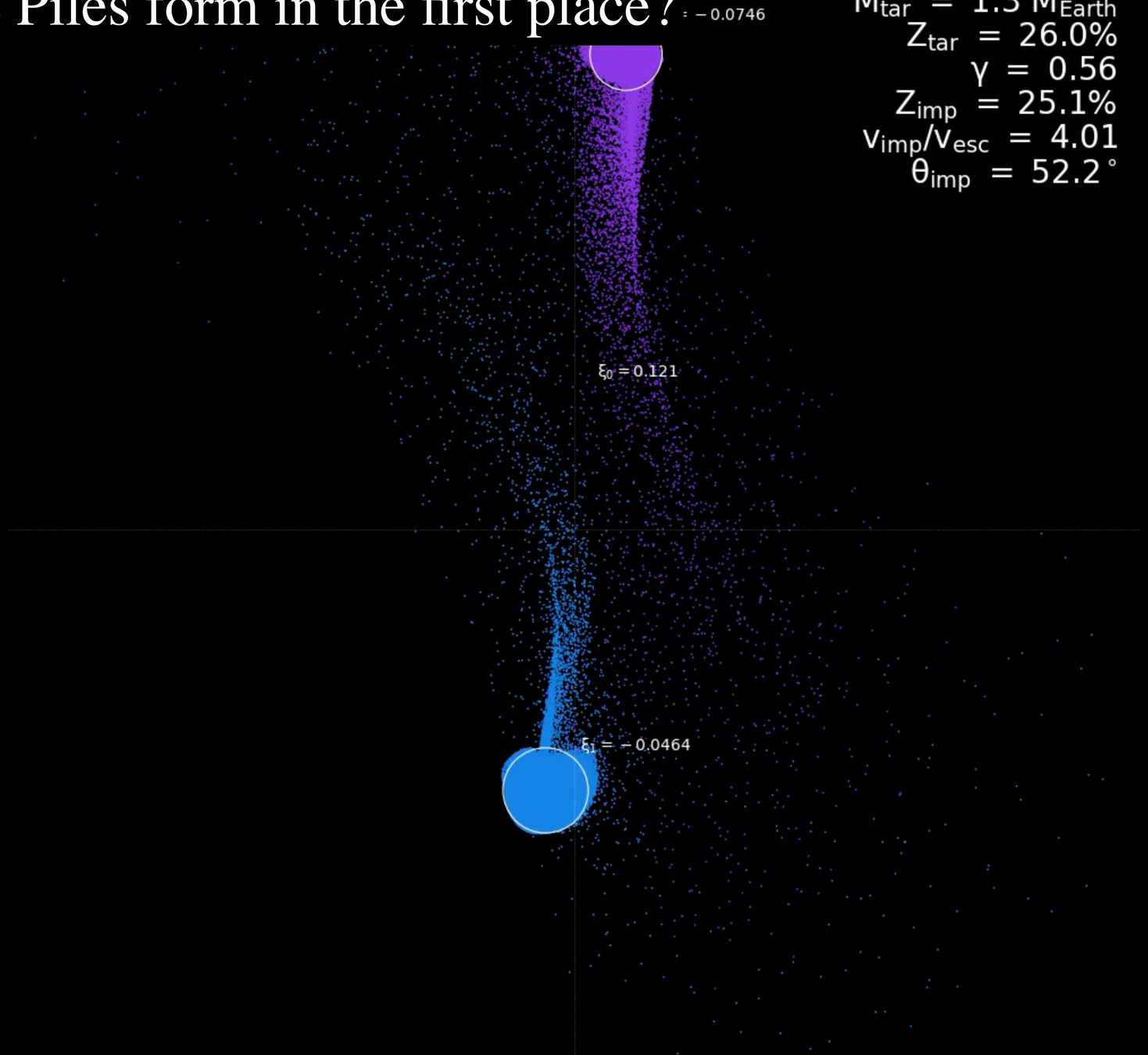
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Multi-Component Bodies…



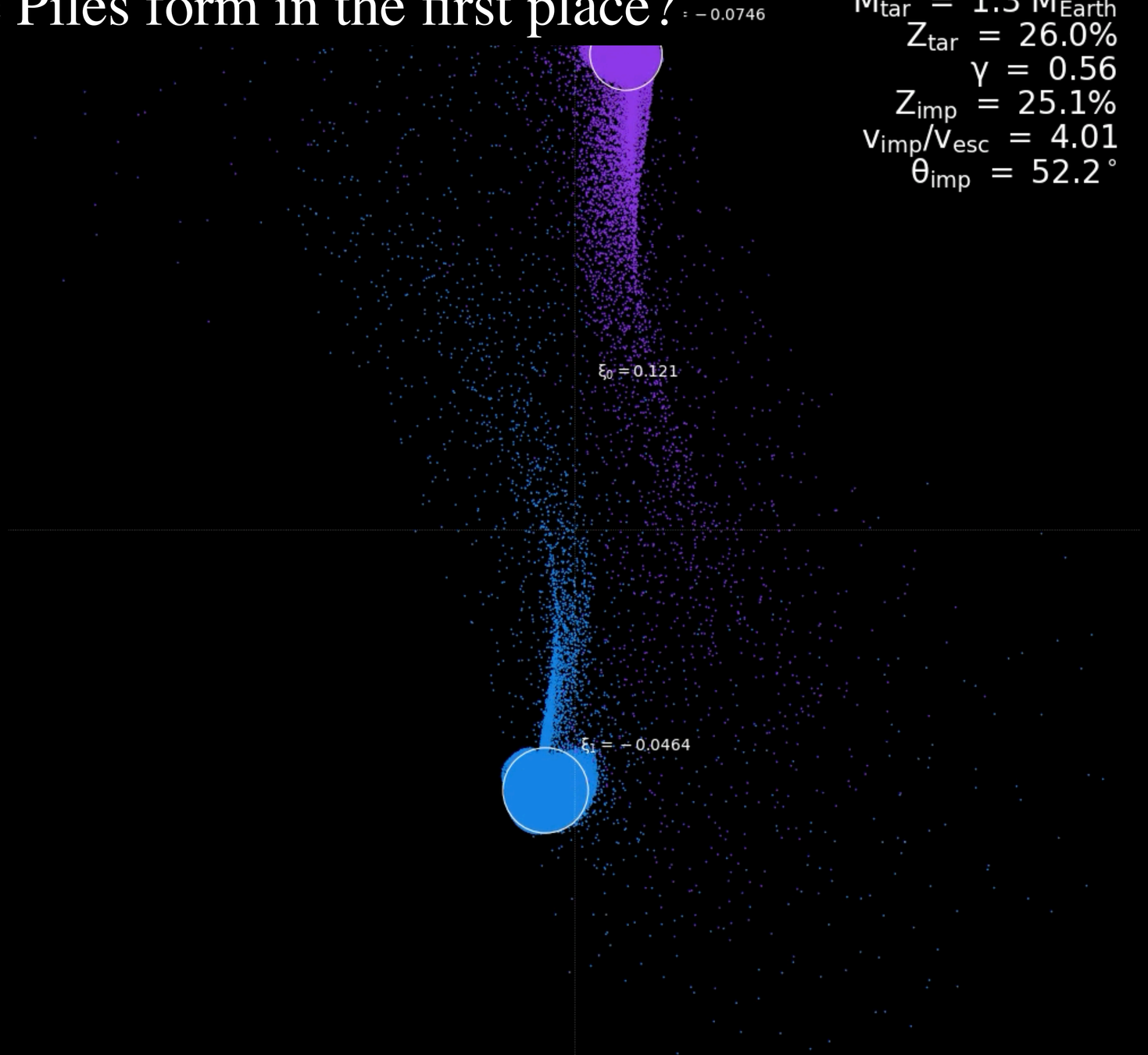
- … are ubiquitous in the asteroid population
  - Binary asteroids: ~15%
  - Contact binary bodies: ~30%
  - Asteroid Pairs / Clusters: ~?
- At least ~50% of small bodies have binarity in their morphology, and have evidence of being parts of primordial bodies with multiple components
- "Components" themselves can be rubble piles — the amount of strength that two rubble piles need to rest on each other without collapse is minimal… (Meyer & Scheeres 2024).
- Here we probe dynamical questions regarding the formation and evolution of these bodies:
  - When and where does the multi-component nature of these bodies arise?
  - Are asteroid pairs and contact binary structures related?
  - How efficient is this creation process?

How do Rubble Piles form in the first place?



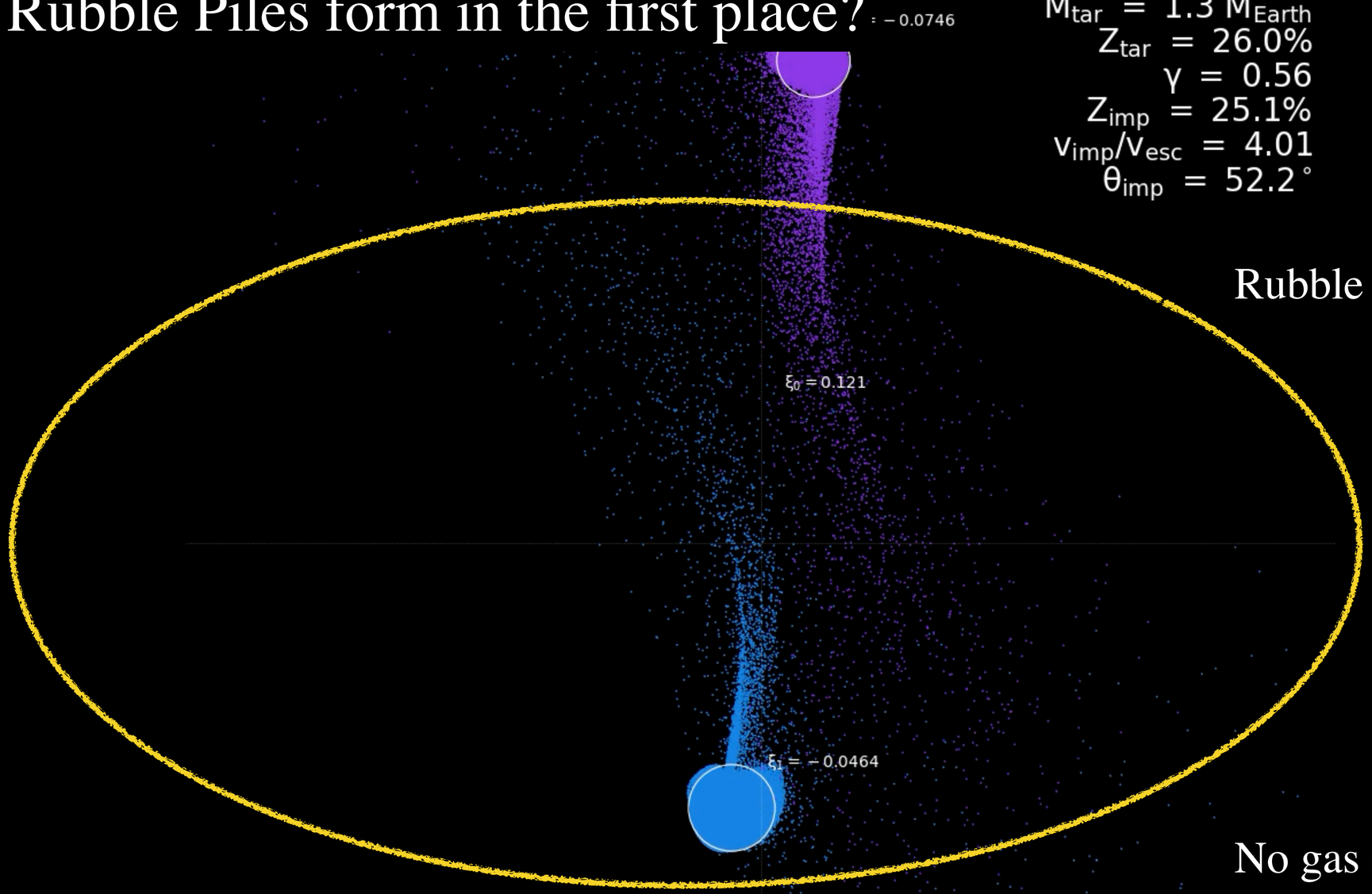
Movie from Alexandre Emsenhuber at: https://emsenhuber.ch/coll/lhs_1000/0#graph

How do Rubble Piles form in the first place?



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How do Rubble Piles form in the first place?



$M_{tar} = 1.3\ M_{Earth}$
$Z_{tar} = 26.0\%$
$\gamma = 0.56$
$Z_{imp} = 25.1\%$
$v_{imp}/v_{esc} = 4.01$
$\theta_{imp} = 52.2^\circ$

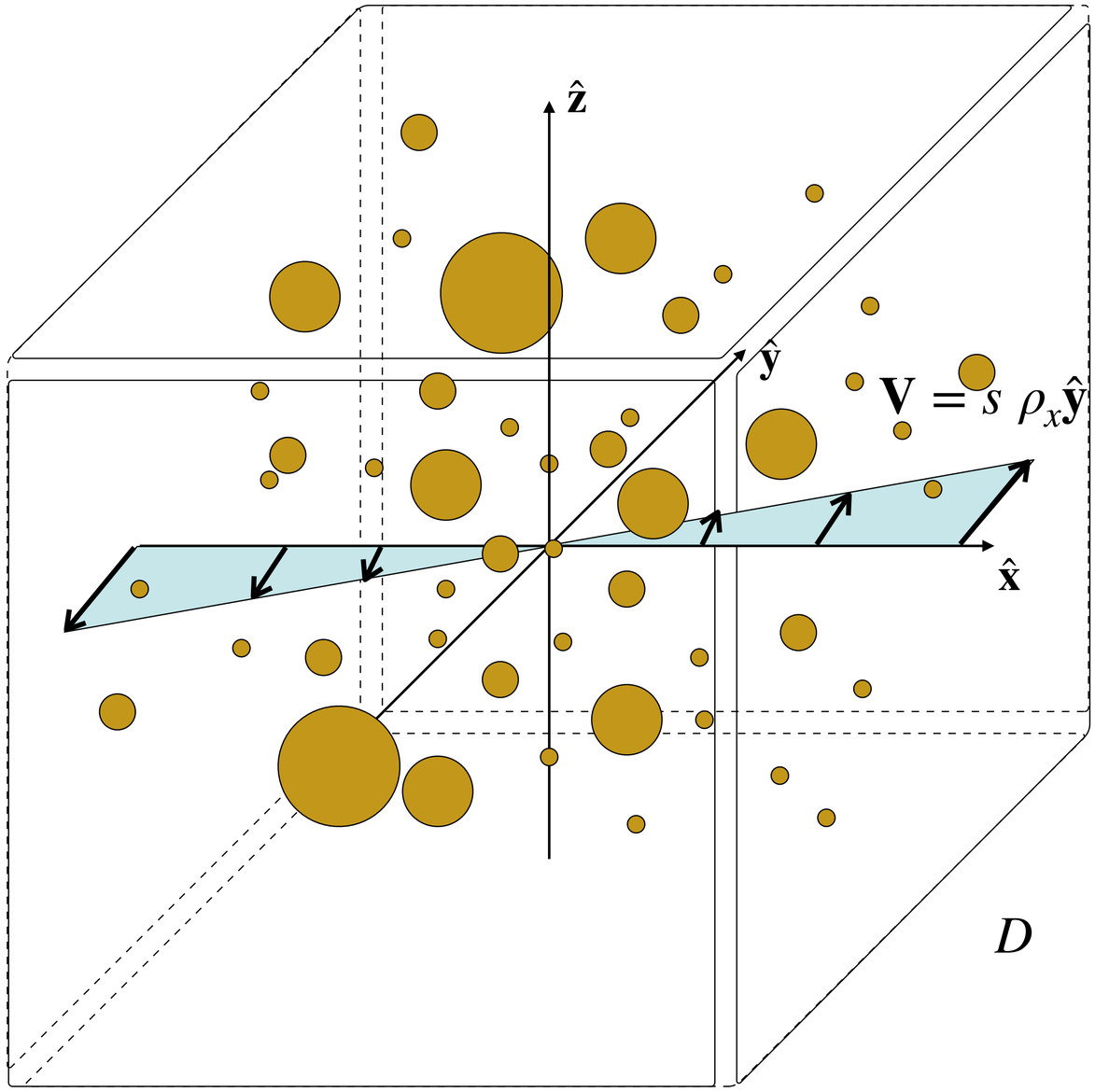
Rubble Pile Factory?

No gas drag expected
Low-speed dynamics
Rigid body rotations
Reimpact dynamics

Movie from Alexandre Emsenhuber at: https://emsenhuber.ch/coll/lhs_1000/0#graph

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Simple post-impact environment



$$\mathbf{H} = \frac{1}{2M}\int_{\mathscr{B}}\int_{\mathscr{B}} (\mathbf{r}-\mathbf{r}')\times(\mathbf{v}-\mathbf{v}')\,dm\;dm'$$

$$E = T + \mathscr{U}$$

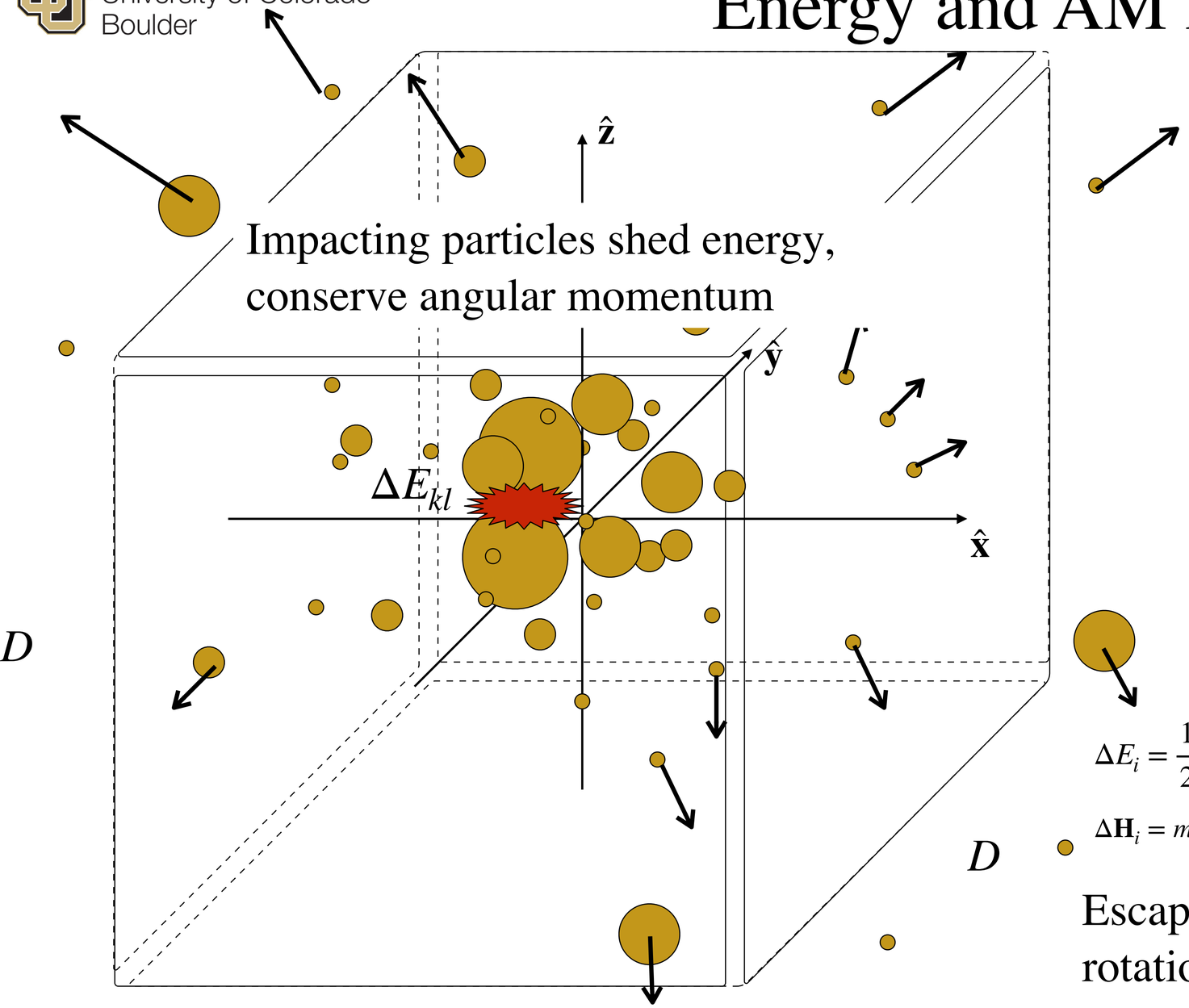
$$T = \frac{1}{4M}\int_{\mathscr{B}}\int_{\mathscr{B}} (\mathbf{v}-\mathbf{v}')\cdot(\mathbf{v}-\mathbf{v}')\,dm\;dm'$$

$$\mathscr{U} = -\frac{\mathscr{G}}{2}\int_{\mathscr{B}}\int_{\mathscr{B}} \frac{dm\;dm'}{\left|\mathbf{r}-\mathbf{r}'\right|}$$

$$I_H = \frac{1}{2M}\int_{\mathscr{B}}\int_{\mathscr{B}} \left[(\mathbf{r}-\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}') - \left(\hat{\mathbf{H}}\cdot(\mathbf{r}-\mathbf{r}')\right)^2\right] dm\;dm'$$

System-level quantities of Angular Momentum and Energy should define the outcome, independent of the particulars of the distribution.

Energy and AM Loss



Impacting particles shed energy, conserve angular momentum

$$\mathbf{H}' = \mathbf{H} - \sum_i \Delta \mathbf{H}_i$$

$$E' = E - \sum_i \Delta E_i - \sum_{kl} \Delta E_{kl}$$

The remaining rigid components will seek out minimum energy configurations

$$\Delta E_i = \frac{1}{2} m_i V_{\infty,i}^2 + \frac{1}{2} I_i \Omega_i^2 + \sum_{j \neq i} \mathcal{U}_{ij} + \mathcal{U}_{ii}$$

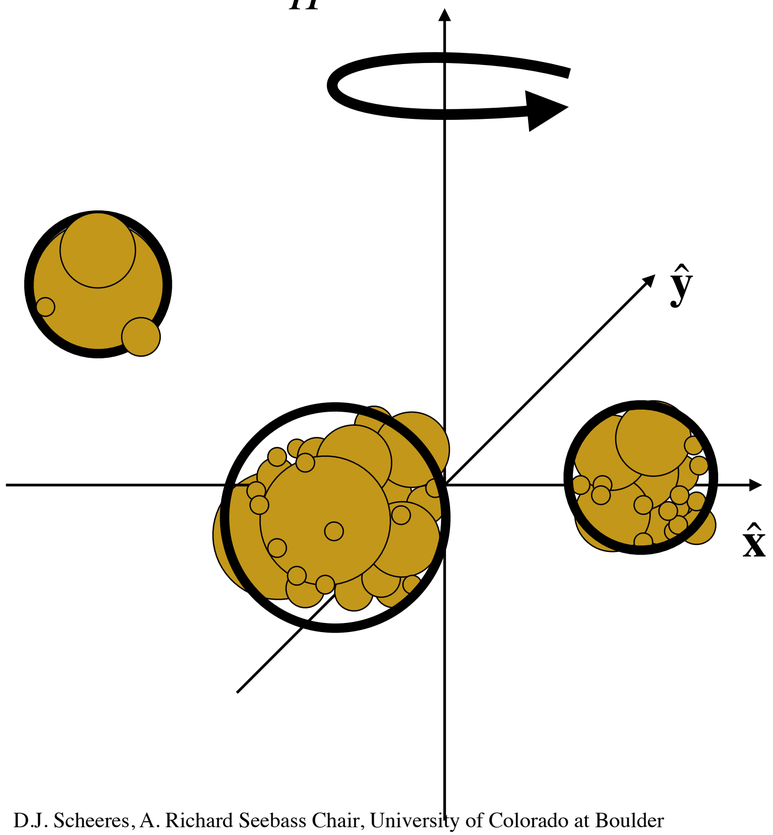
$$\Delta \mathbf{H}_i = m_i V_{\infty,i} b_{\infty,i} \hat{\mathbf{H}}_i + I_i \Omega_i$$

Escaping particles carry away translational and rotational energy and angular momentum

Leading to multi-component systems…

The remaining mass distributions are coupled and continue to evolve, the possible final outcomes can be defined as a function of the total angular momentum, energy and mass distribution through the amended potential:

$$\mathscr{E} = \frac{H^2}{2I_H} + \mathscr{U}$$

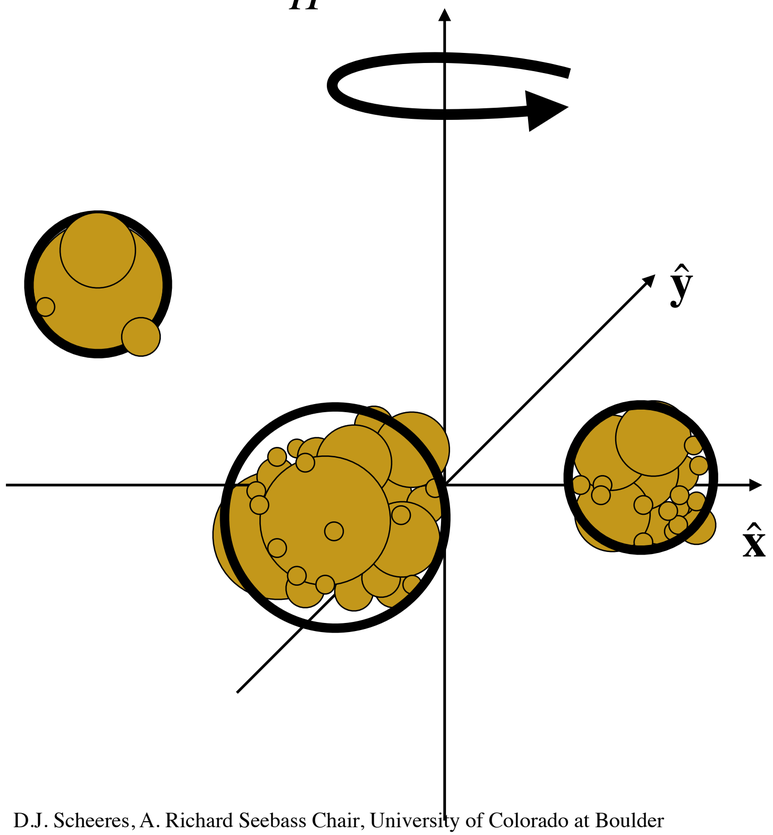


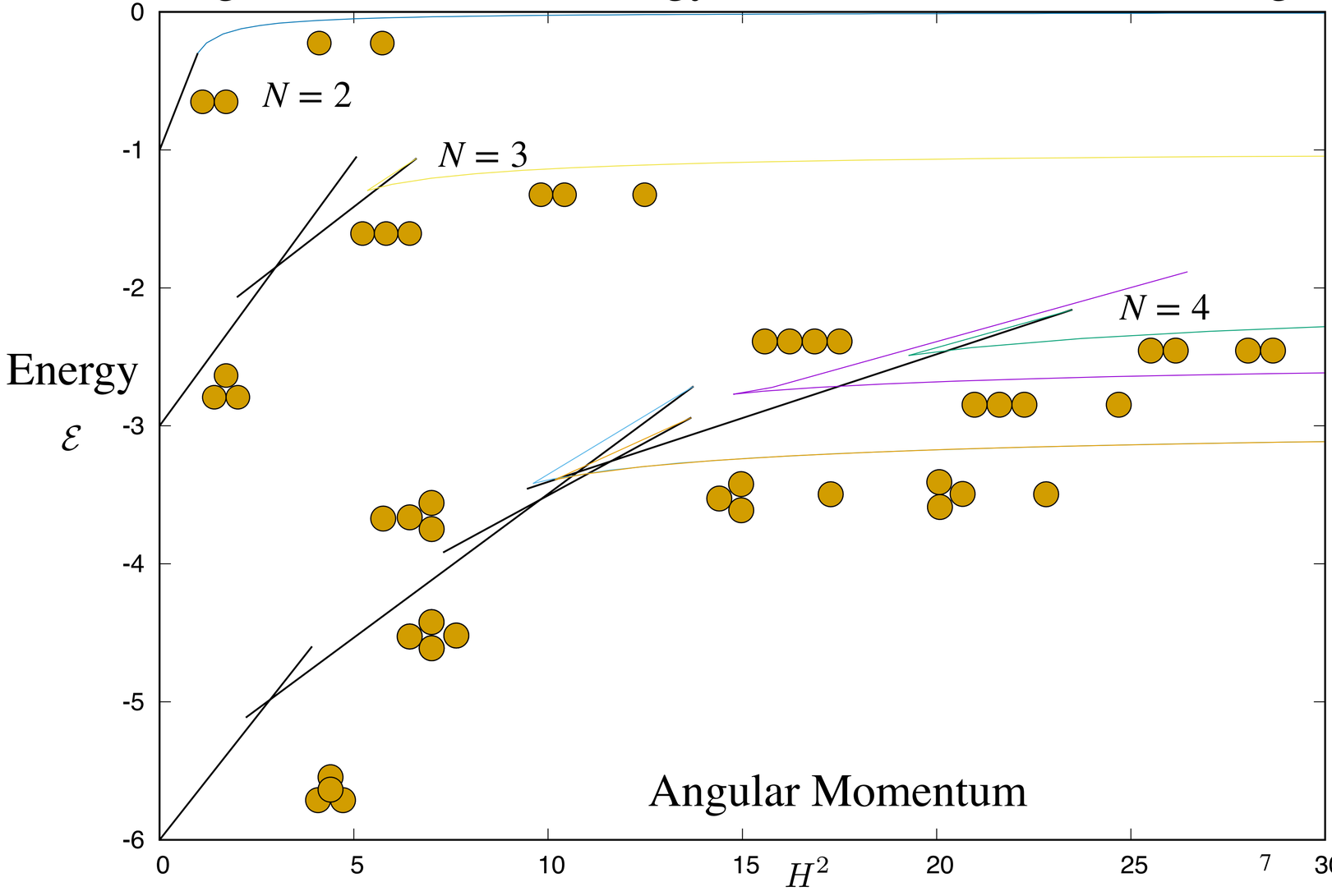
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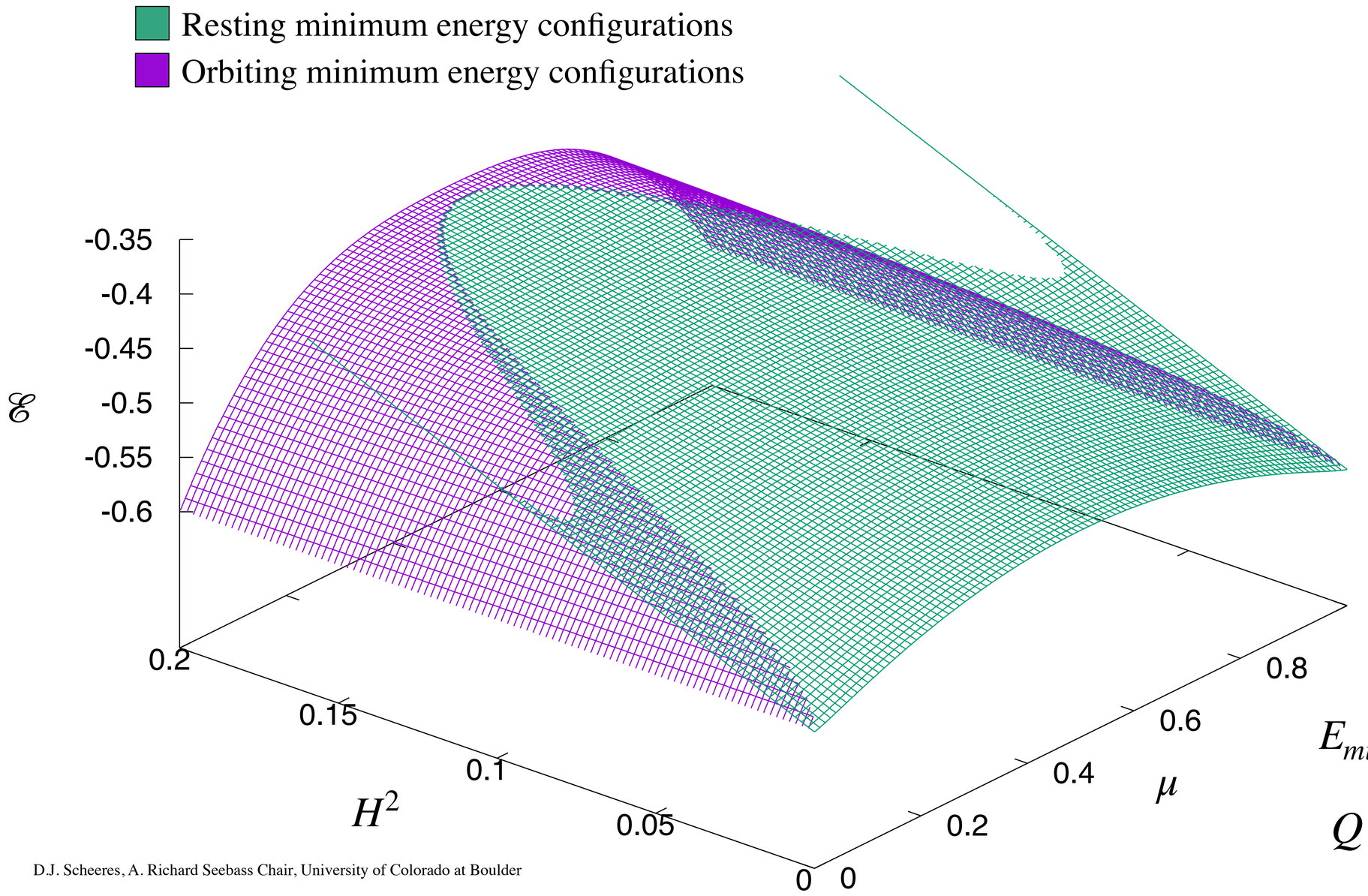
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What are the energetics of collapse?



The final shape and morphology of these condensates is a function between angular momentum, final energy and mass distributions. Under a "spherical approximation" is defined as the minimum of the amended potential $\mathscr{E}$ for 2 spherical bodies defined by their mass fraction

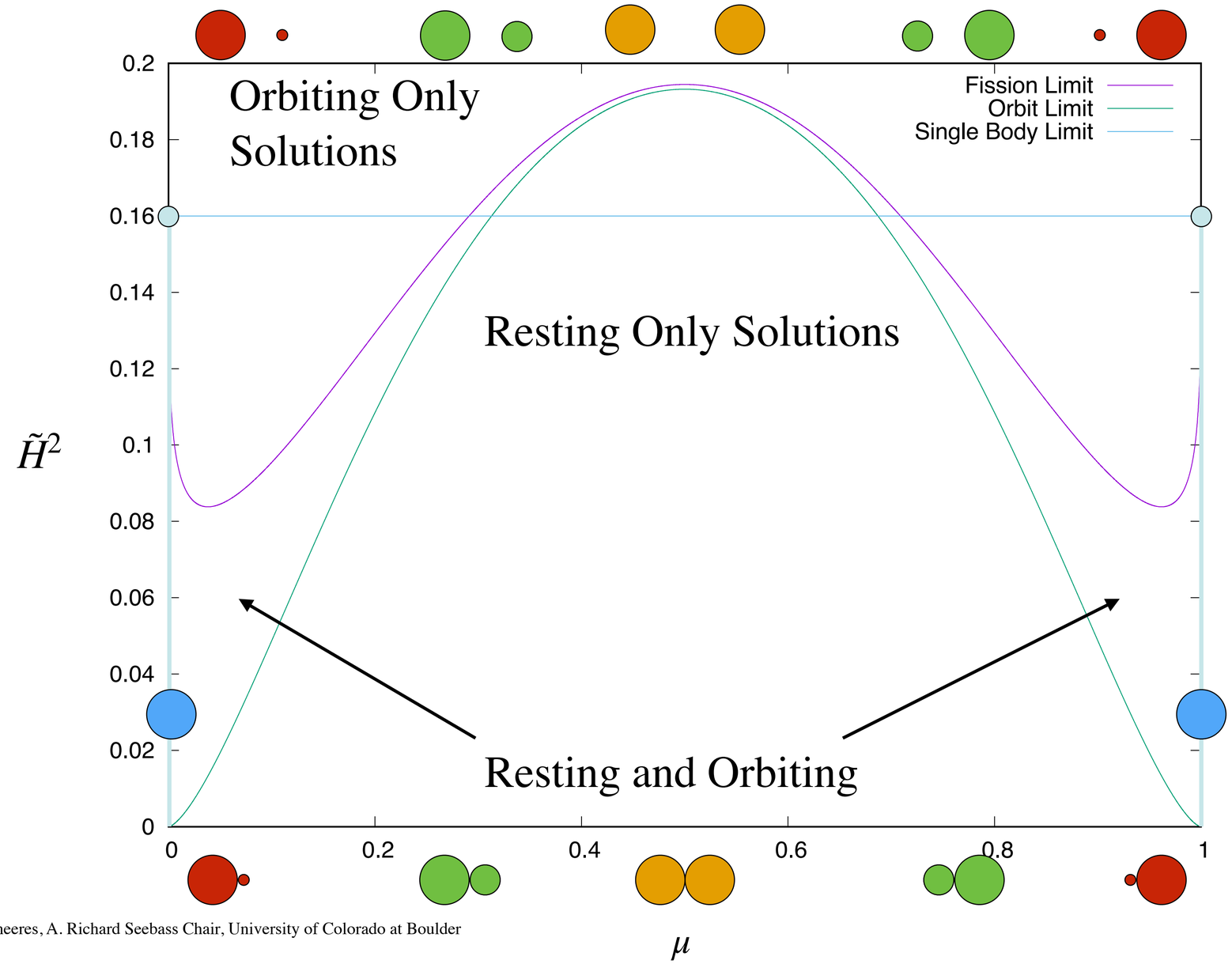
$$\mu = \frac{m_1}{m_1 + m_2}$$

$$E_{min}(H^2, \mu) = \min_Q(\mathscr{E}(H^2, \mu, Q))$$

Q = internal degrees of freedom

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Angular Momentum vs Morphology



Phase diagram showing the minimum energy configurations as a function of angular momentum and "mass distribution".

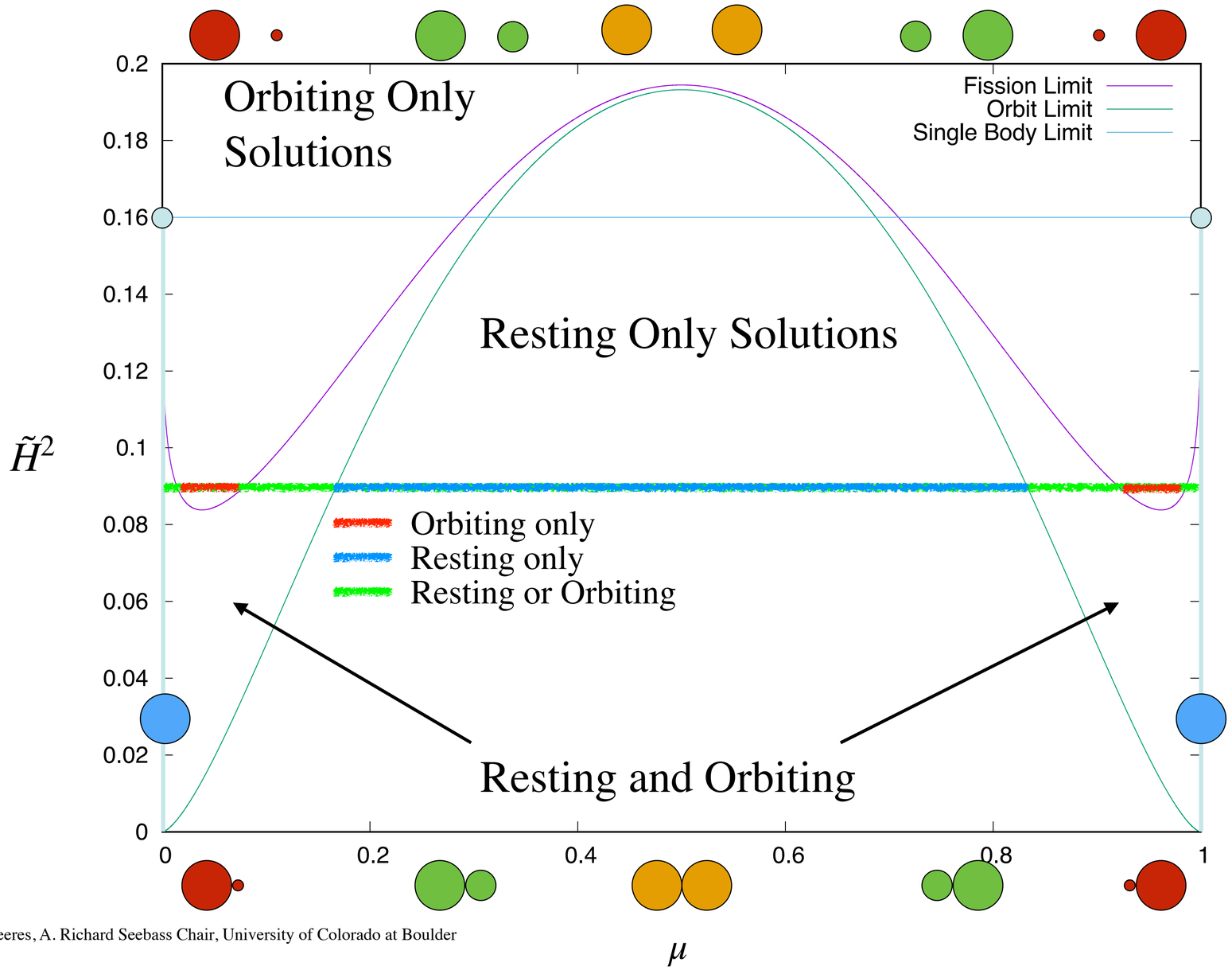
Approximations made by restricting final shapes to be spherical.

Relative probability of formation should scale with "area".

For small rubble pile asteroids, bodies can retain their shapes even when resting on each other (Meyer & Scheeres, ApJL 2024)

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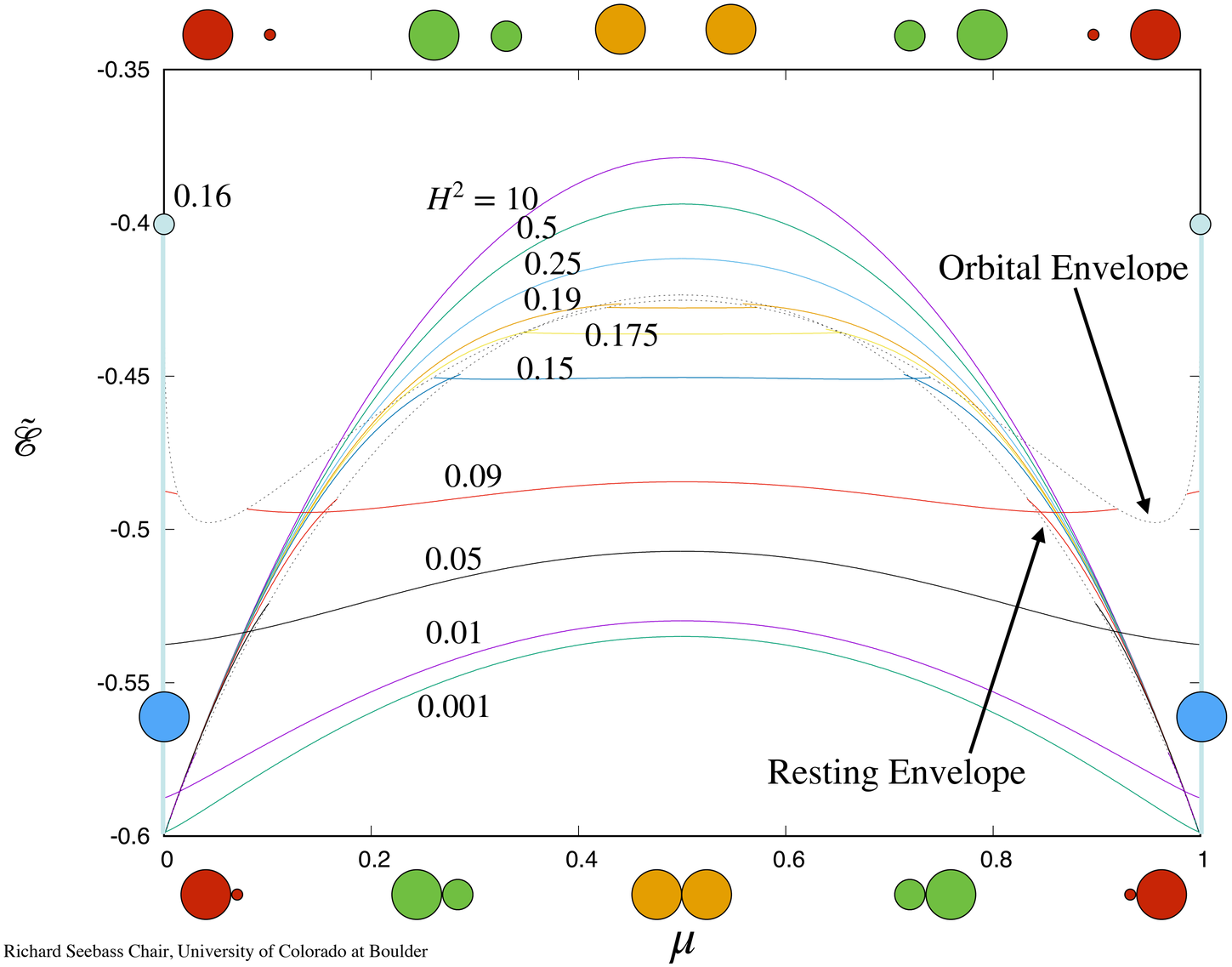
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Energy vs Morphology

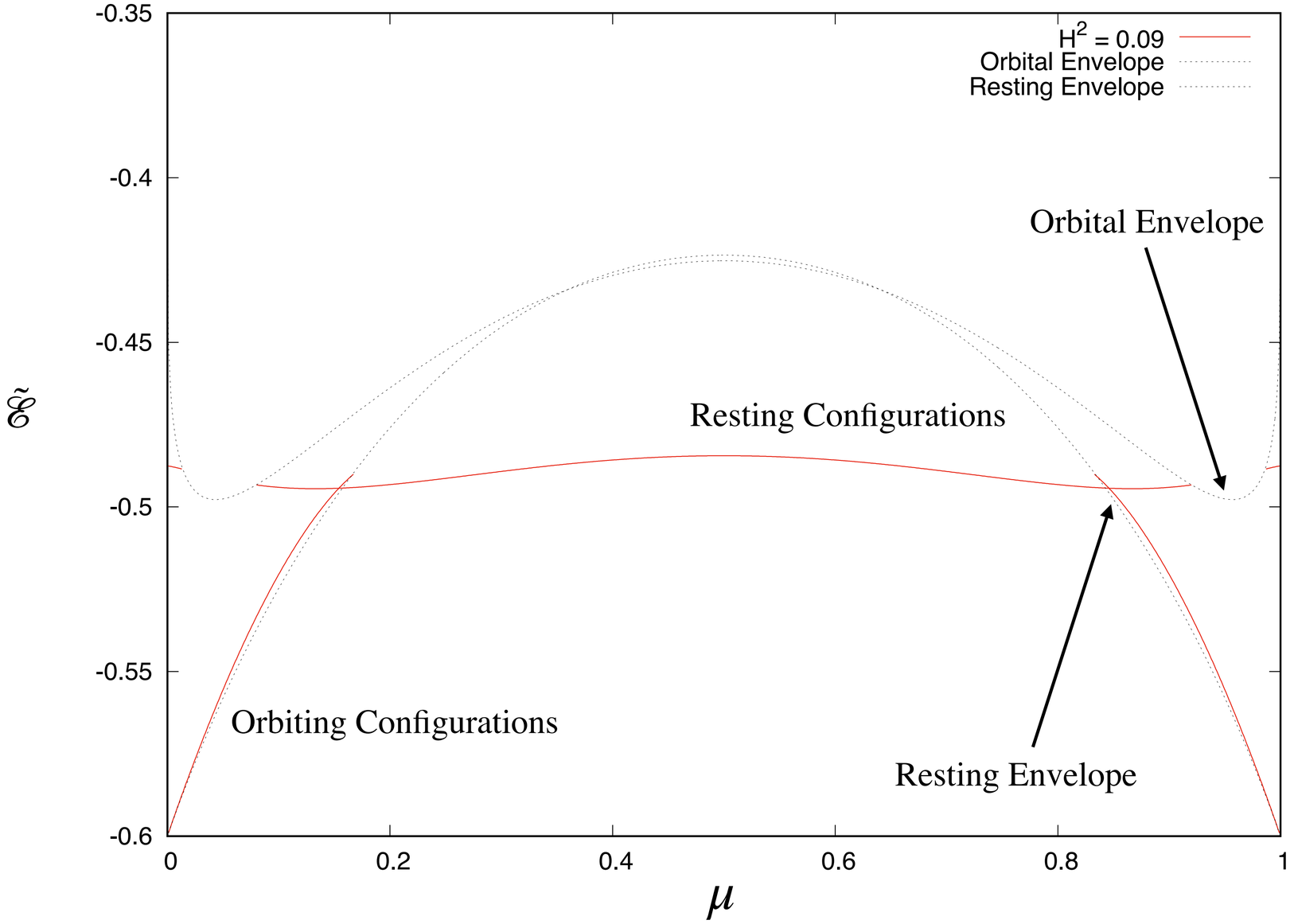


Phase diagram showing the minimum energy configurations as a function of final energy, angular momentum and “mass distribution” with different angular momentum values indicated.

Theory can accommodate non-minimum energy configurations, such as singly-synchronous binaries and tumbling single asteroids.

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Detail showing multiple solutions

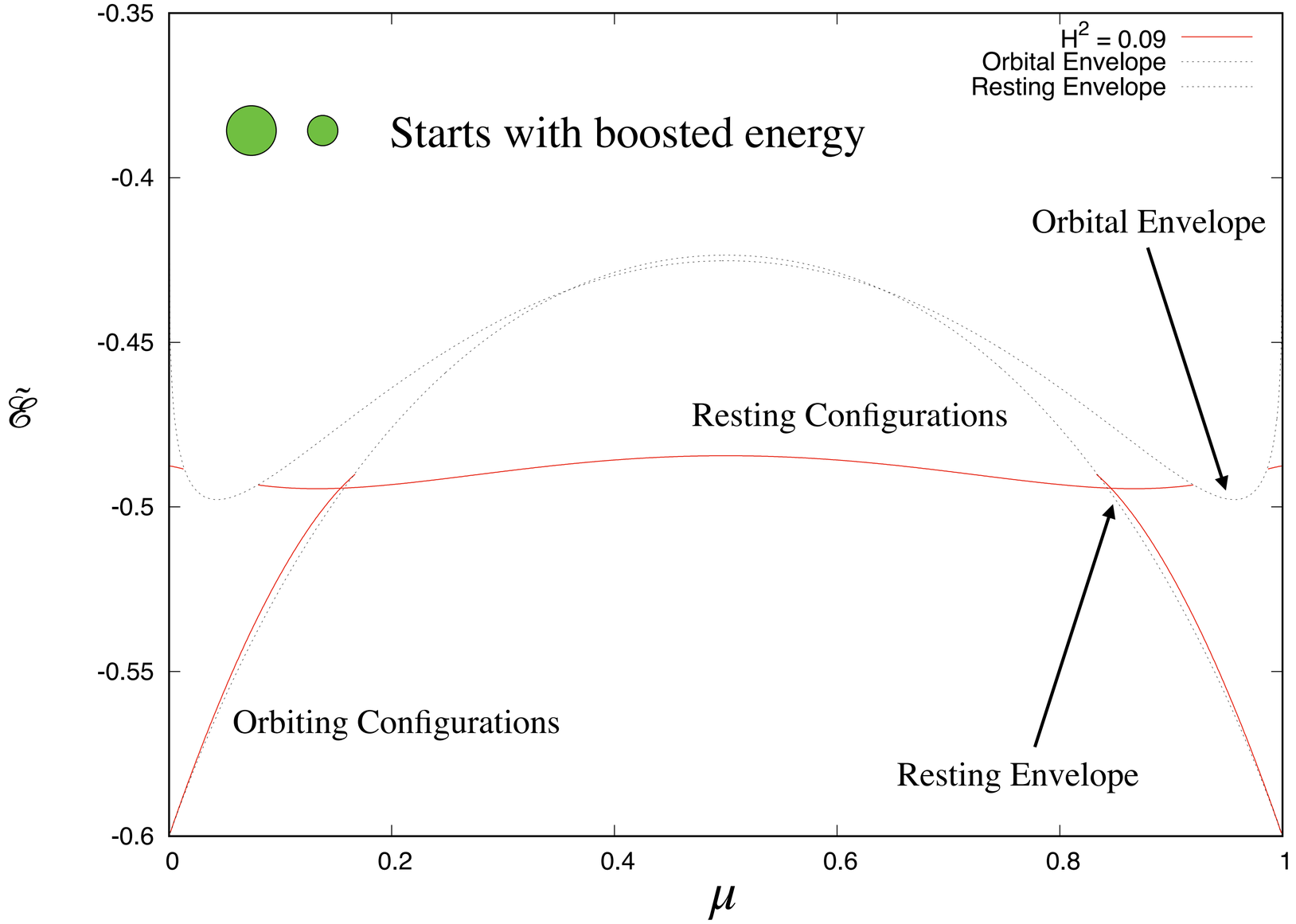


Specific case showing that multiple final states can exist, i.e., orbiting minimum energy states or contact binary minimum energy states.

This implies that the stable end states of a rubble pile collapse are not unique, and will have a stochastic component.

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Detail showing multiple solutions

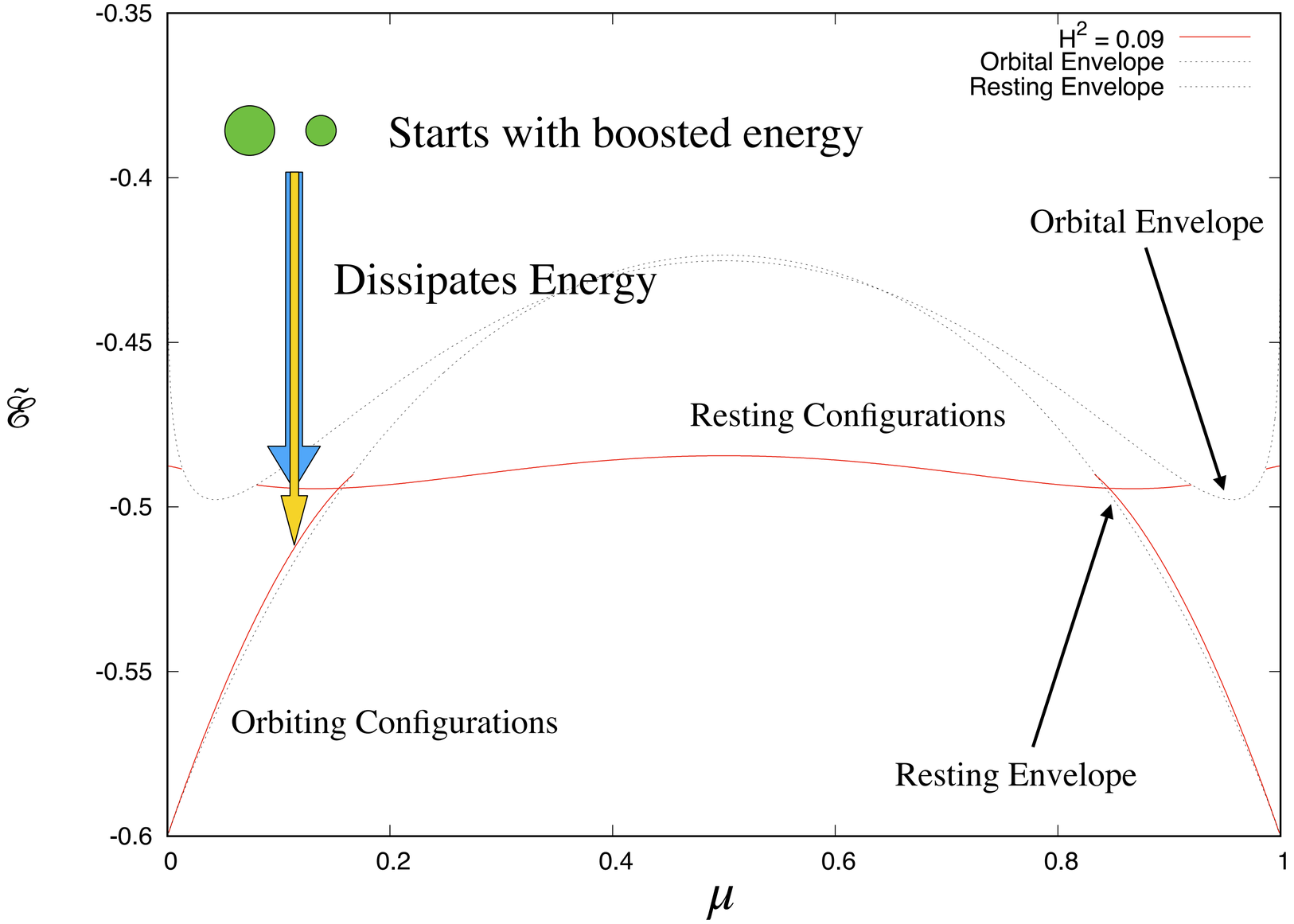


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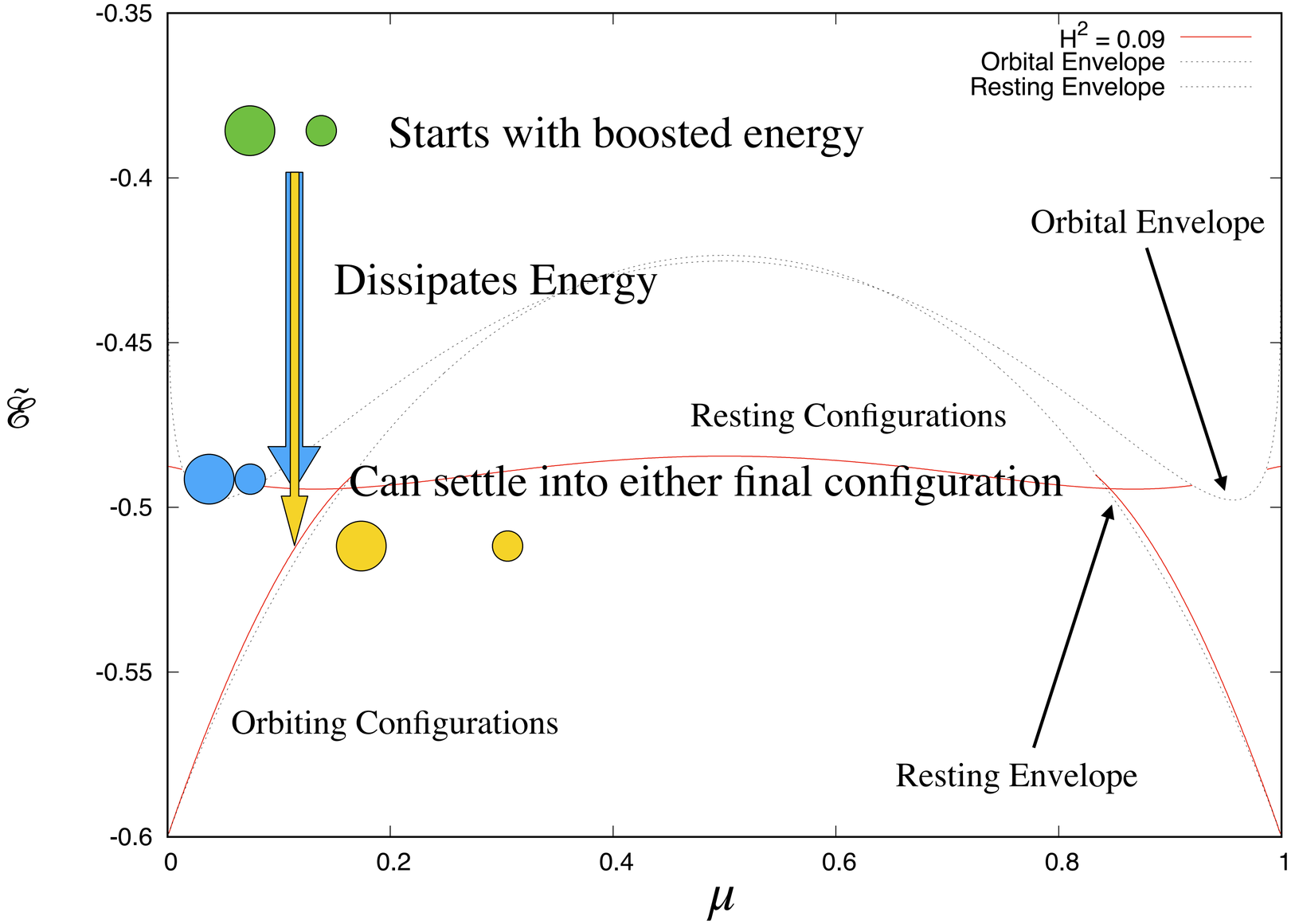


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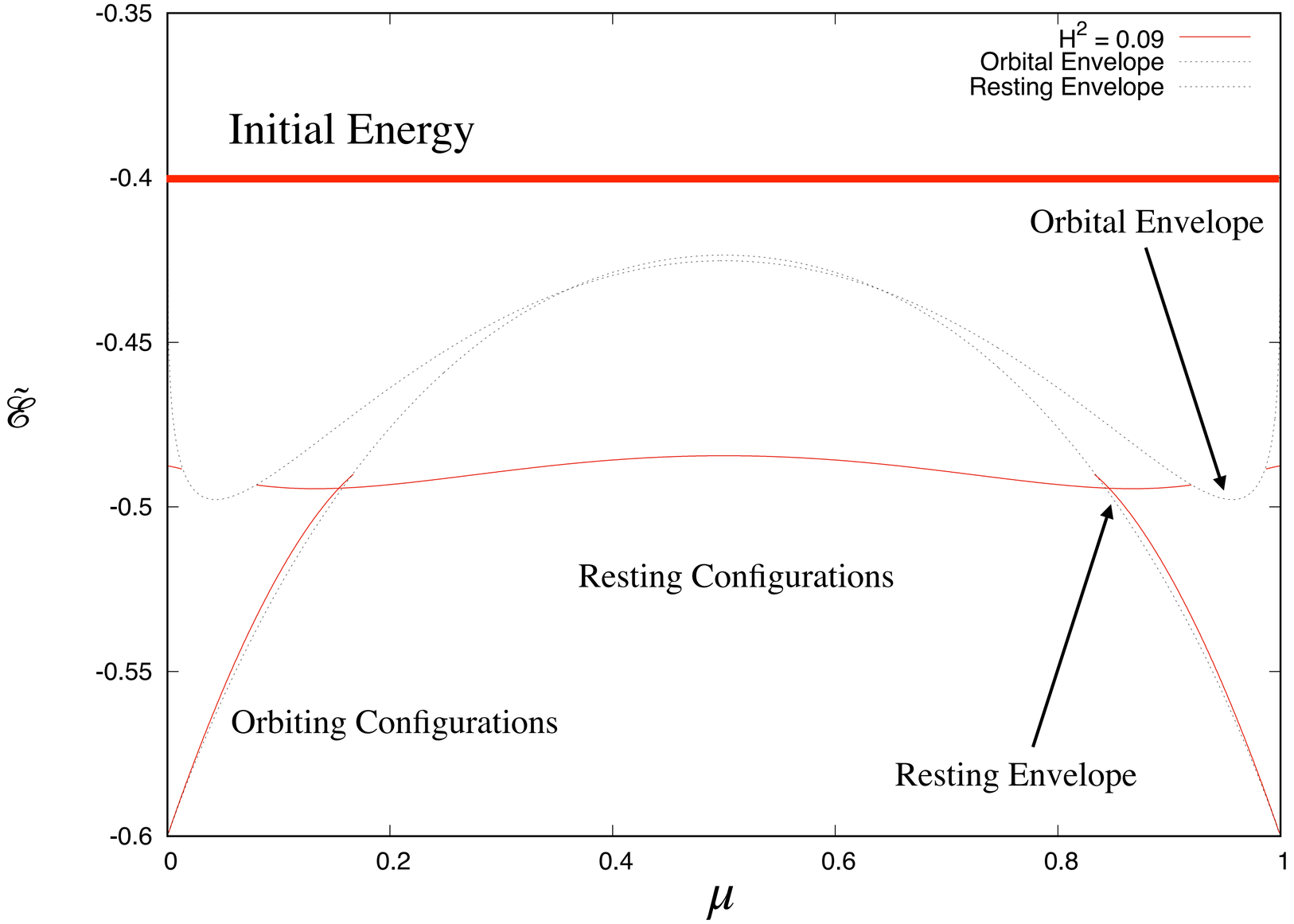


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Energy Dissipation Across all Shapes



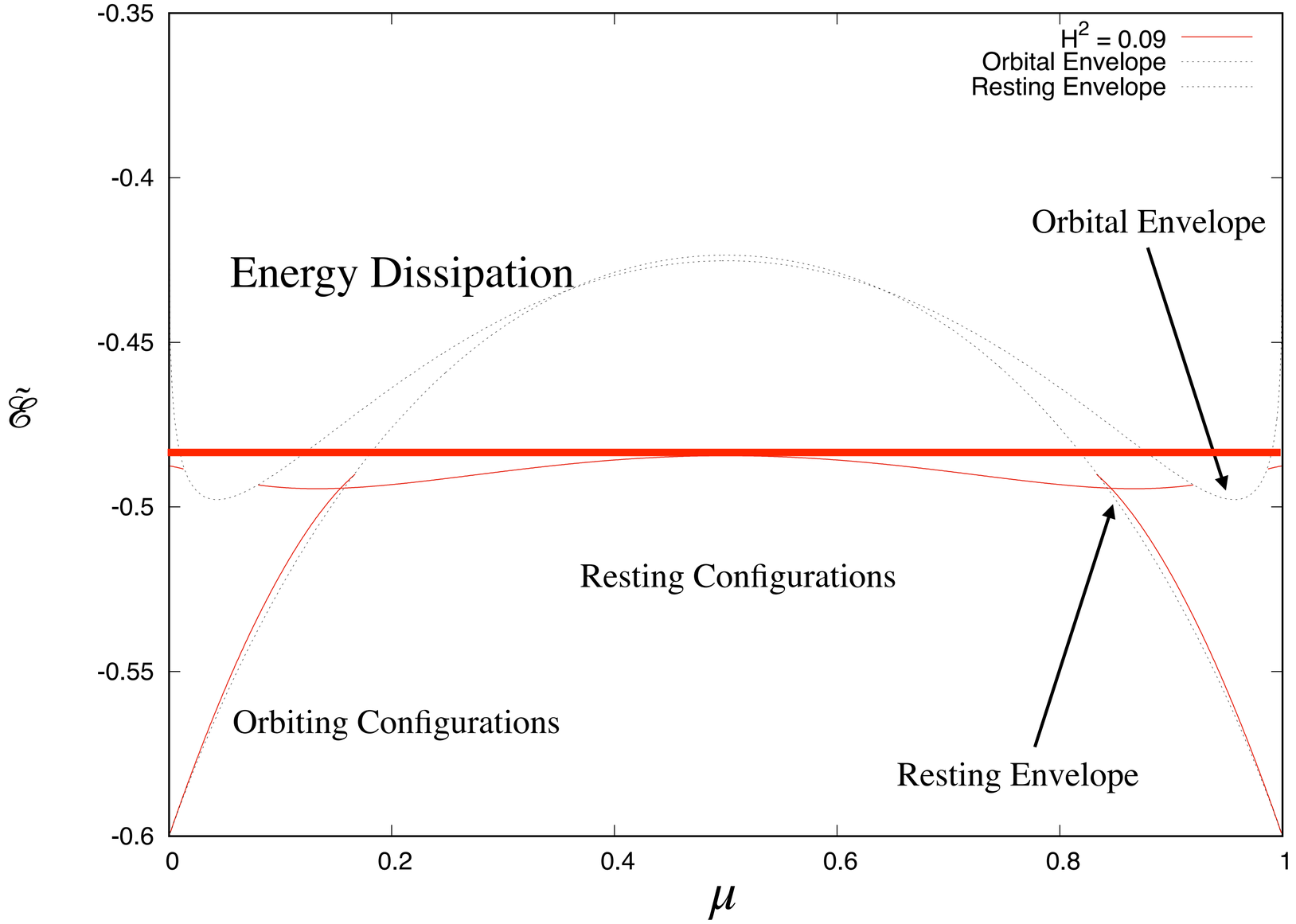
The different “distillates” have extremely different system morphologies.

Although they may be close in energy they are in general far from each other in system geometry.

Additional interaction dynamics and probabilities needed to fully convert into probabilities.

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Energy Dissipation Across all Shapes



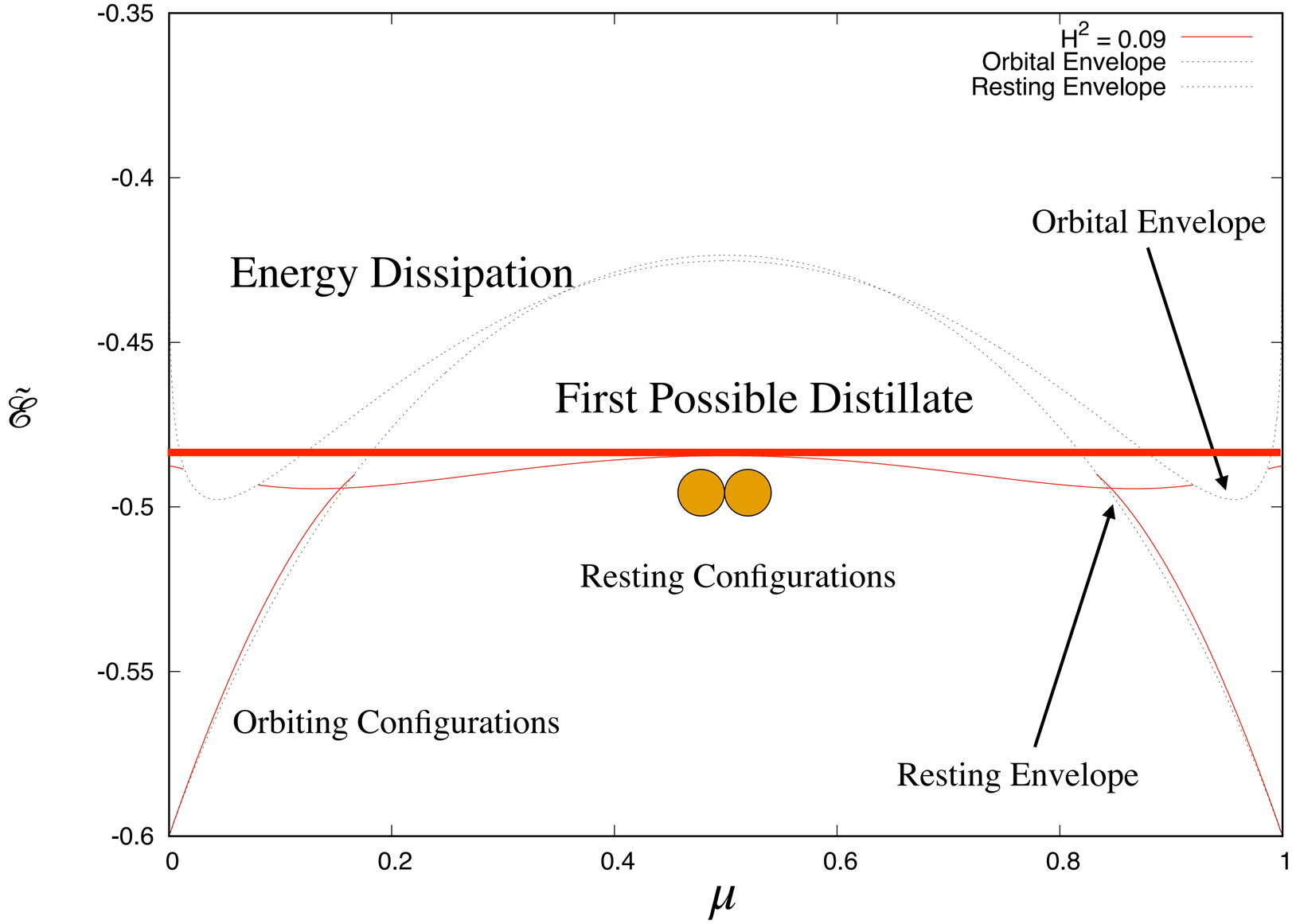
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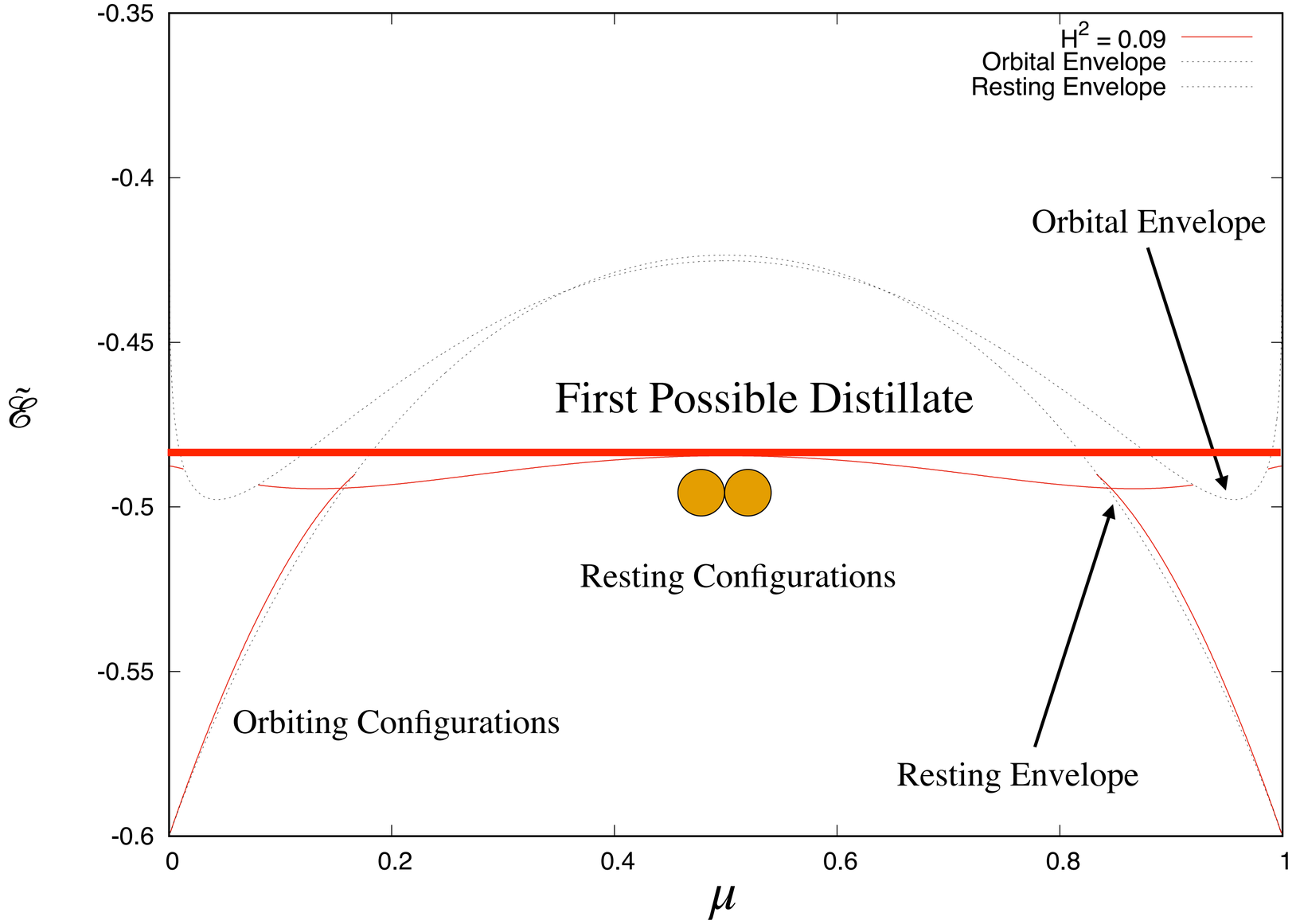
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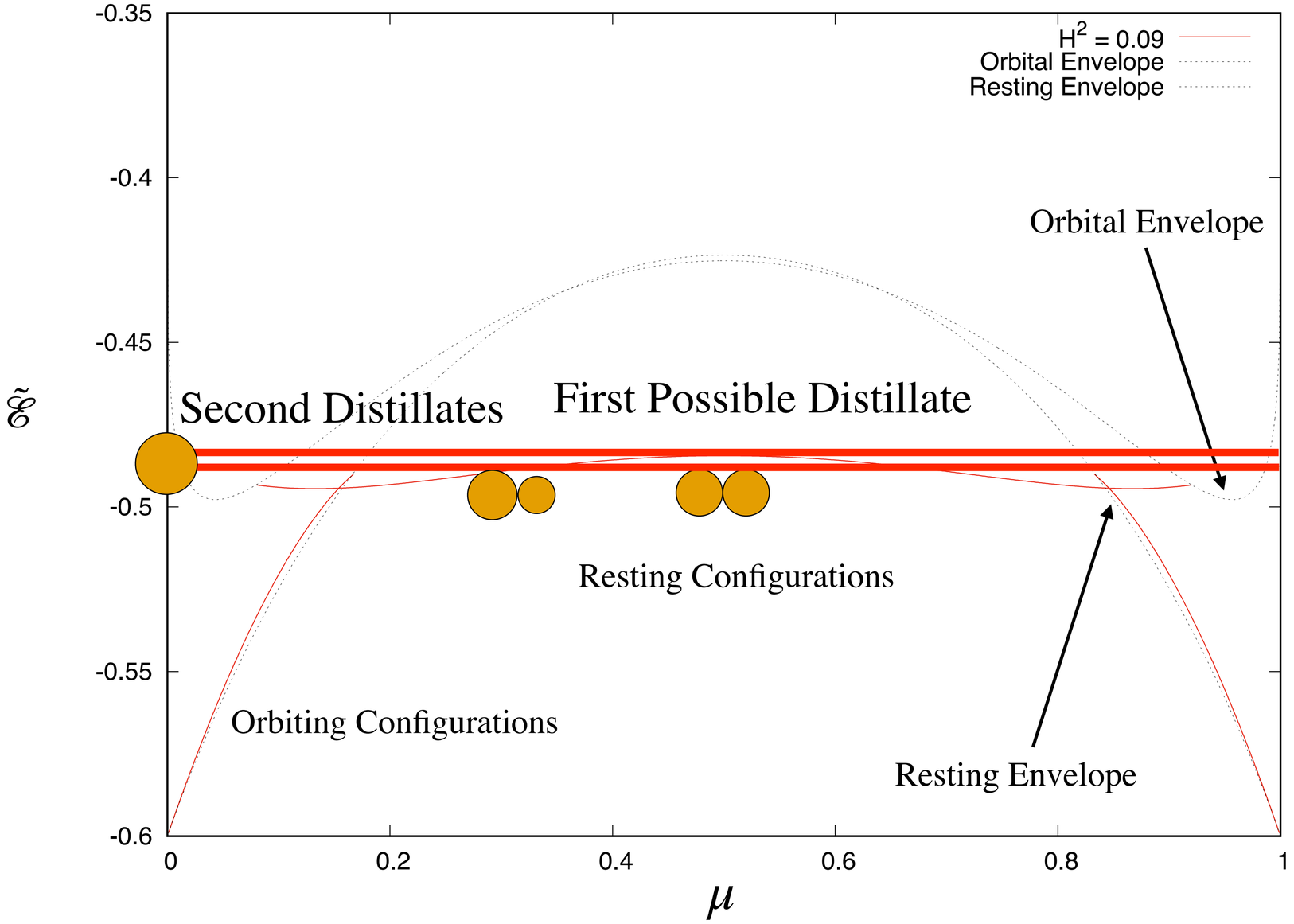
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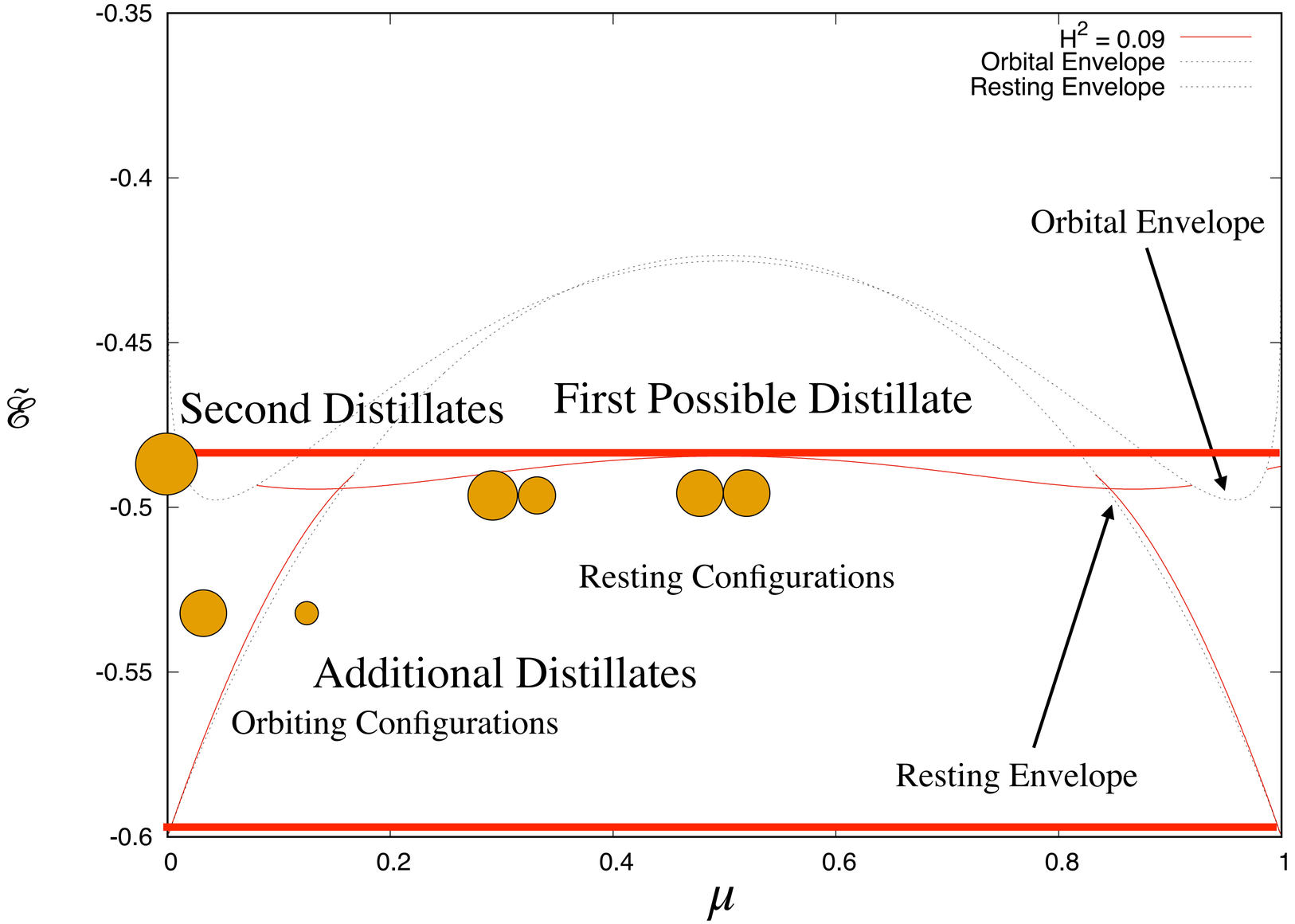
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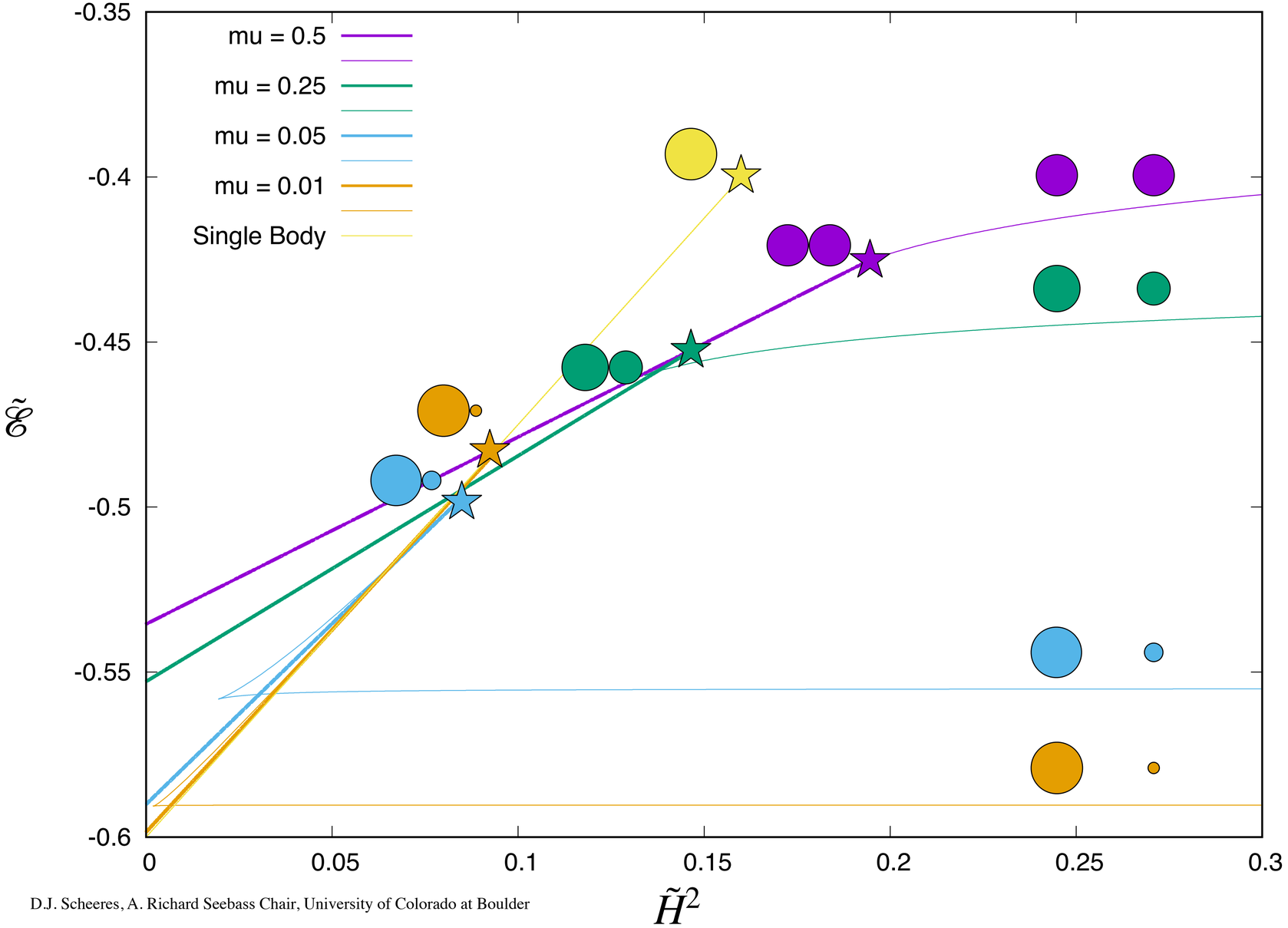
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Energy vs Angular Momentum

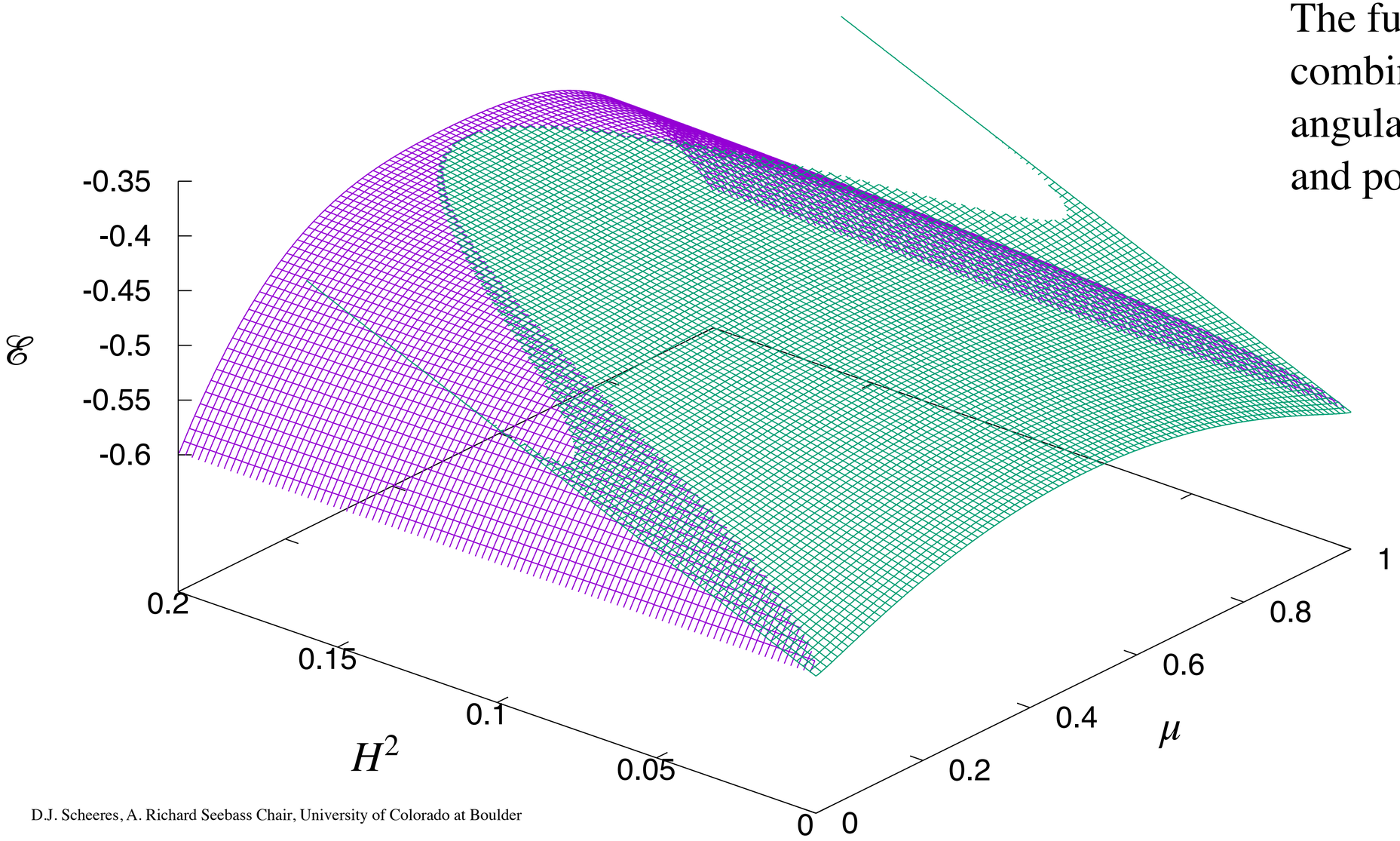


Phase diagram showing the minimum energy configurations as a function of final energy and angular momentum for $N = 2$.

Earlier plots showed that this basic morphology persisted for larger values of N.

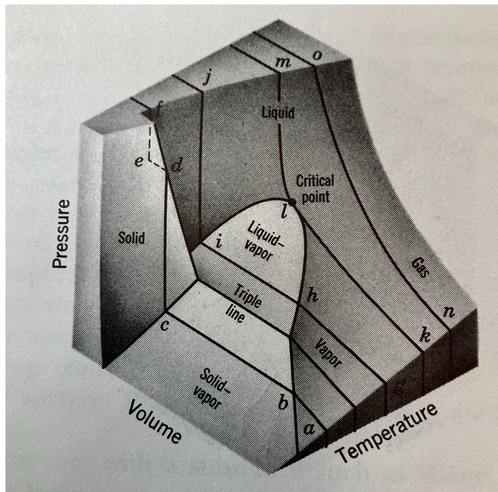
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Rubble Pile Formation Phase Diagram



The full phase diagram shows the combined relationship between angular momentum, final energy and possible mass distributions.

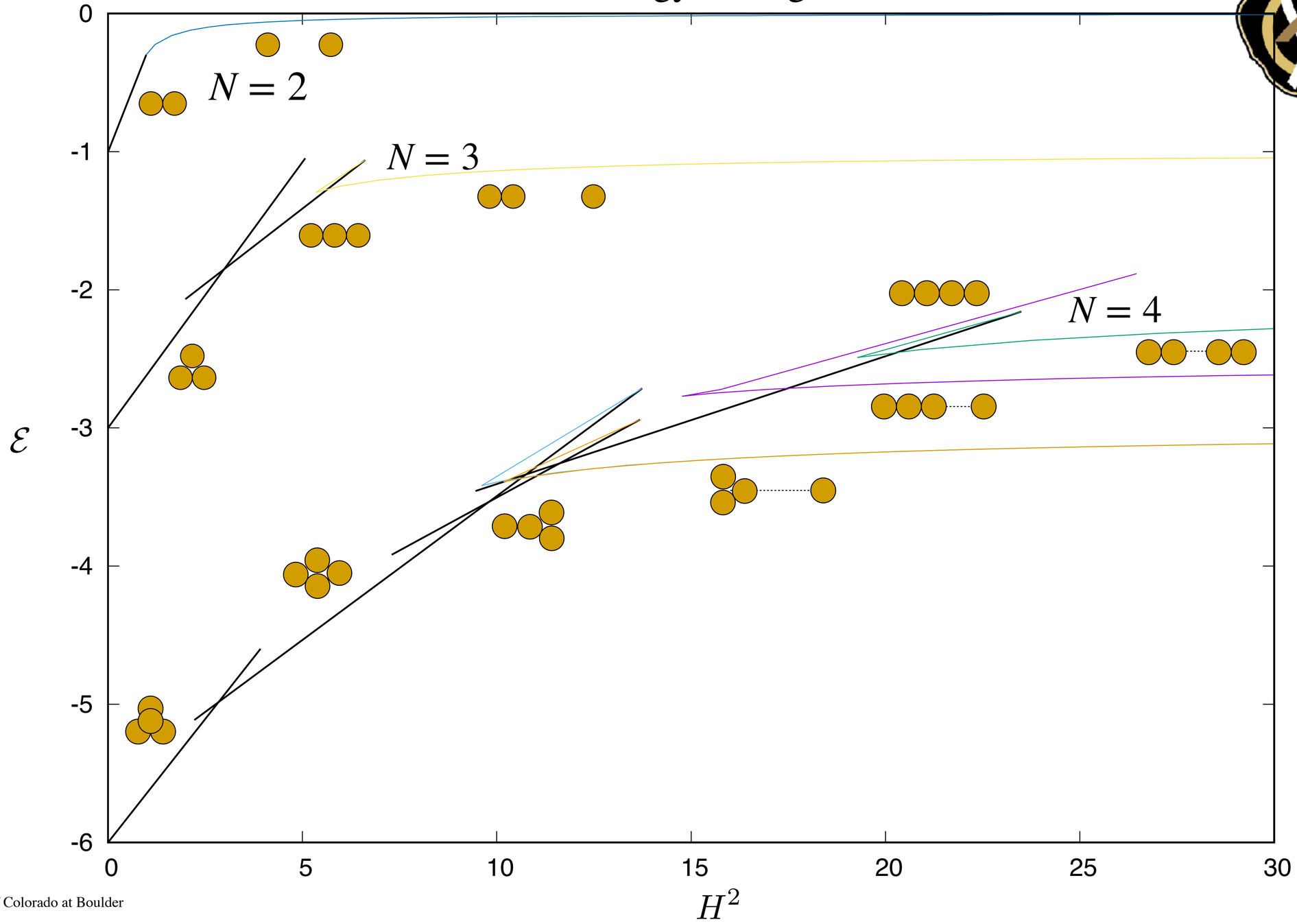
Reminiscent of thermodynamic phase diagrams



Minimum Energy Configurations

We study the dynamics of these multi-body systems when started at energies above their minimum configurations.

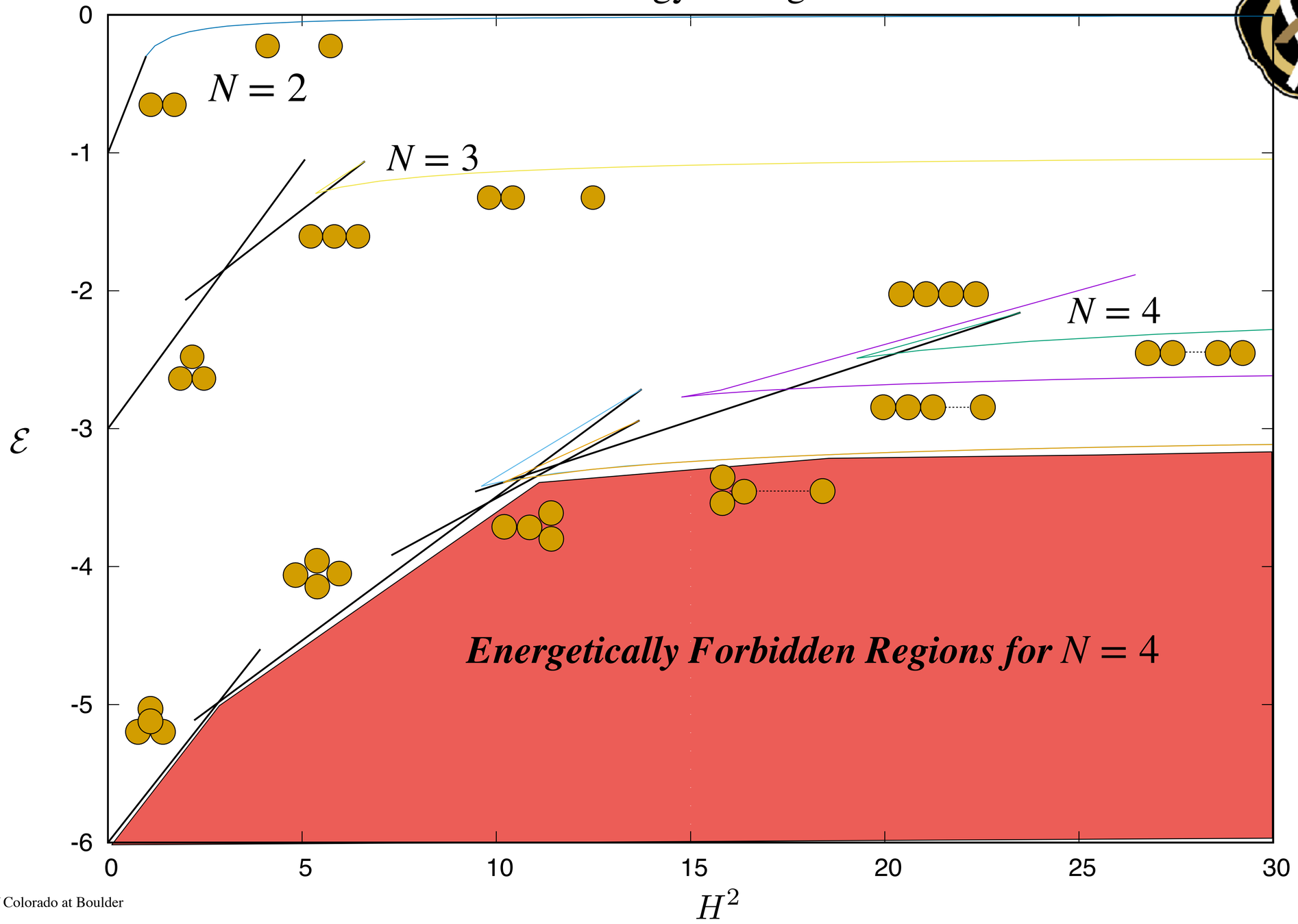
Depending on the energy of the system, particles can be ejected, while conserving system energy and angular momentum



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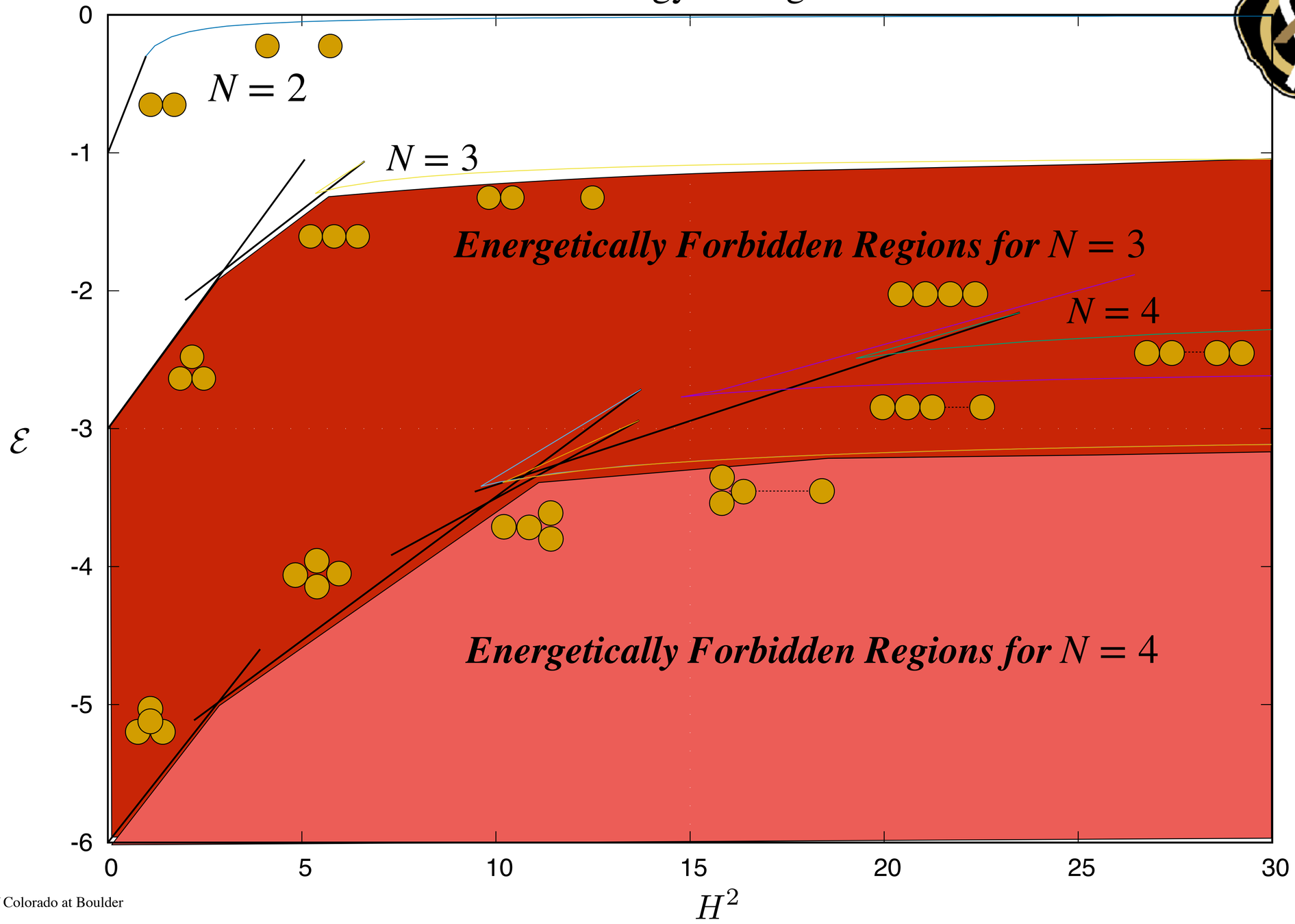
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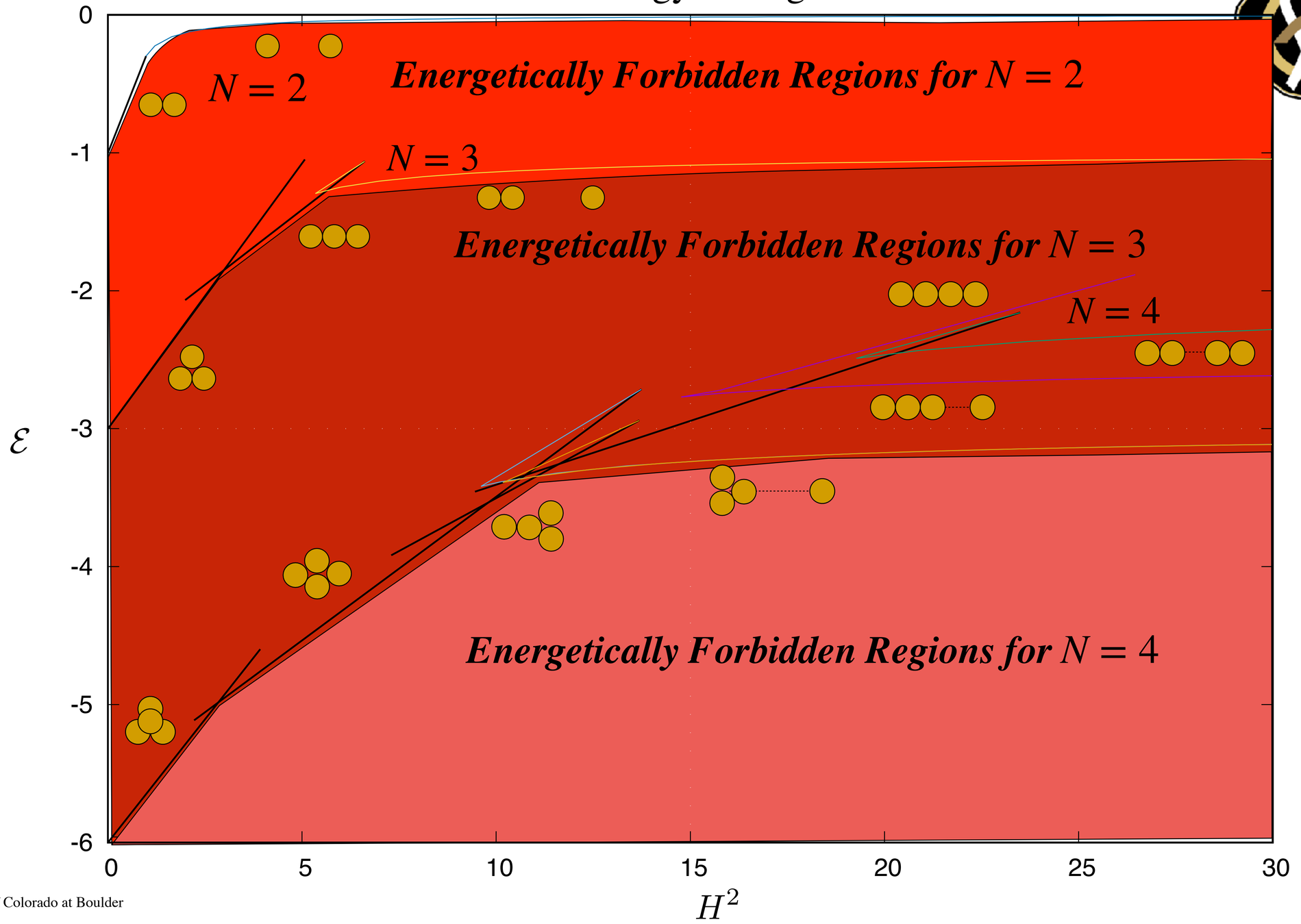
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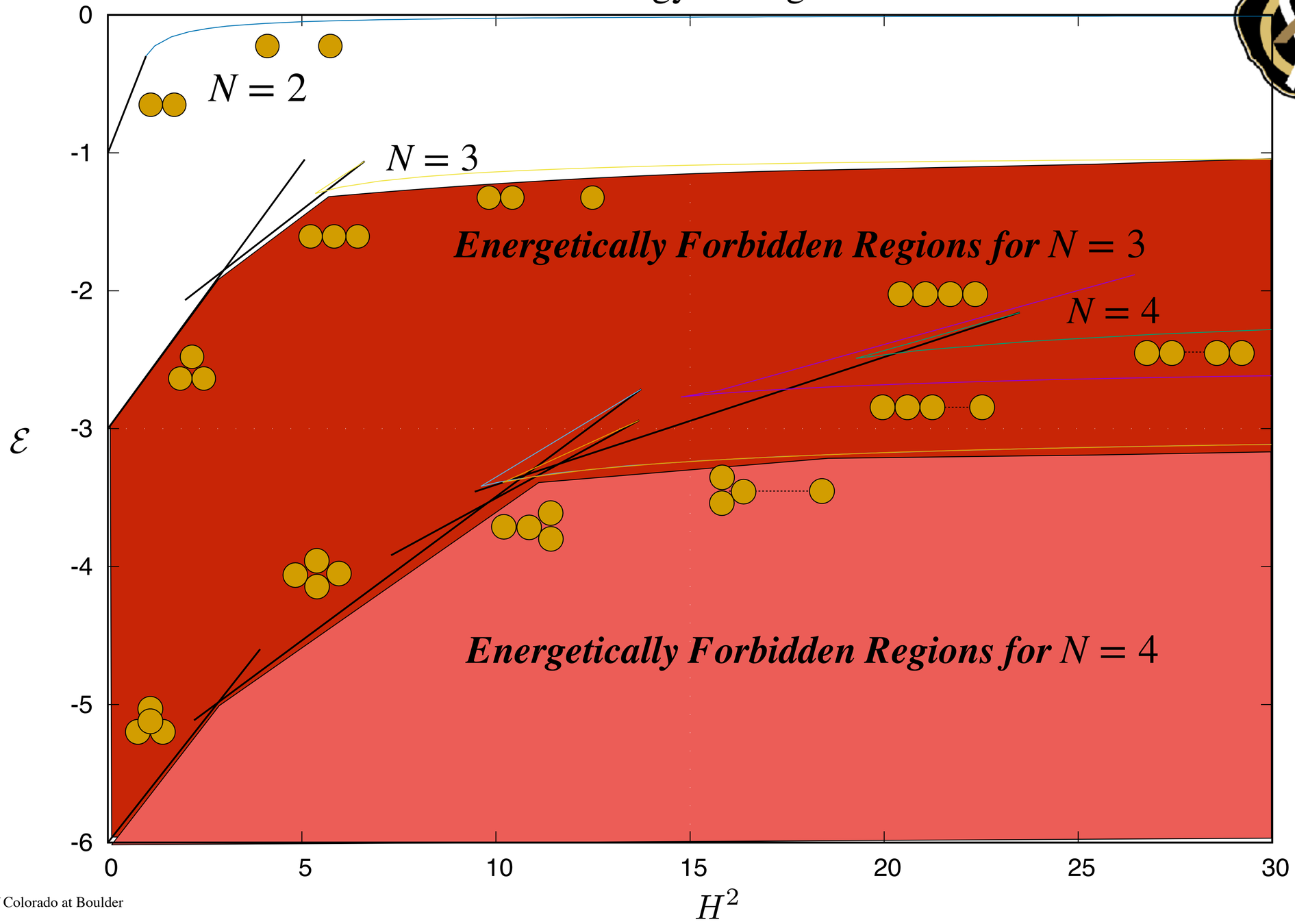
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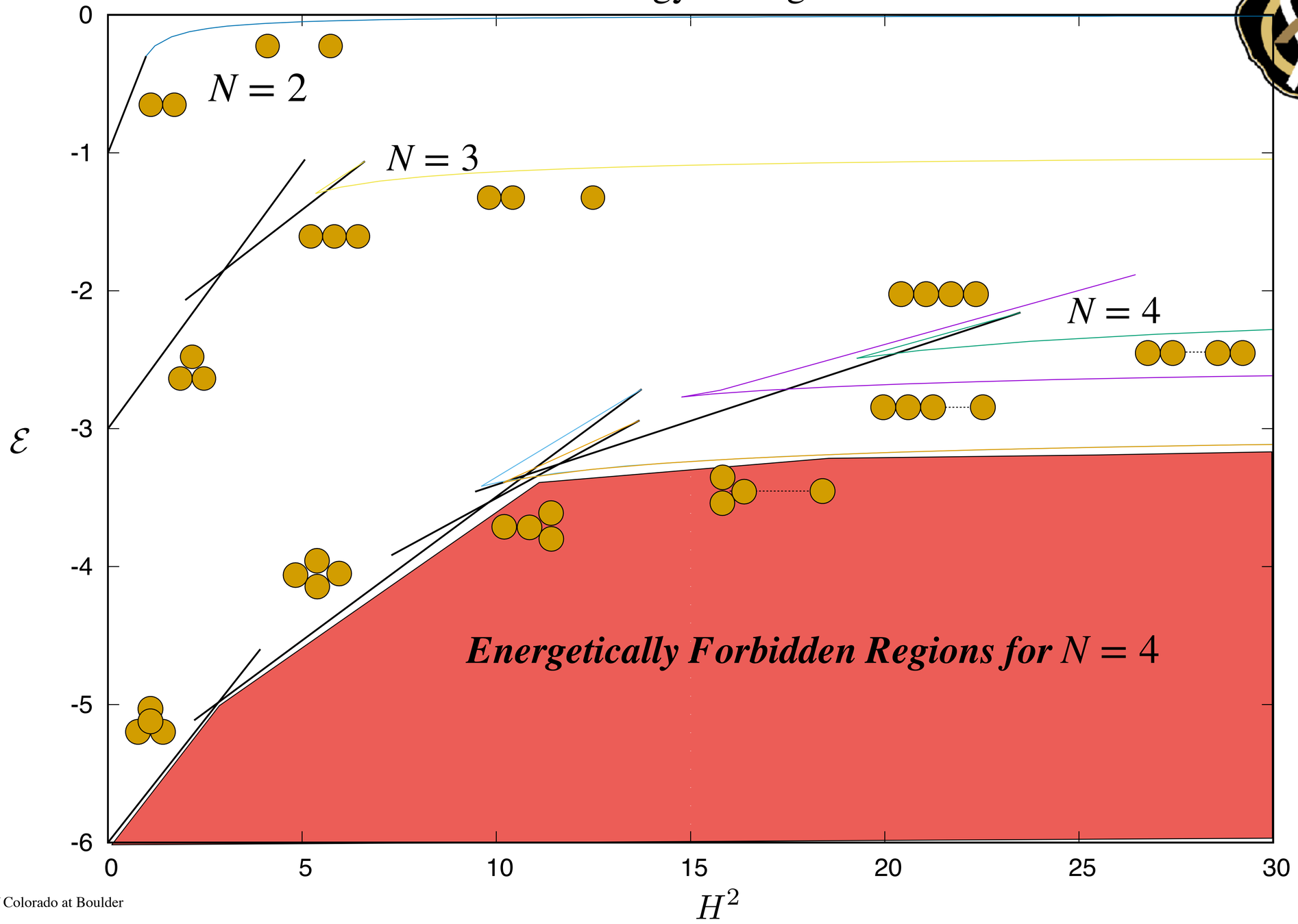
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Minimum Energy Configurations

We study the dynamics of these multi-body systems when started at energies above their minimum configurations.

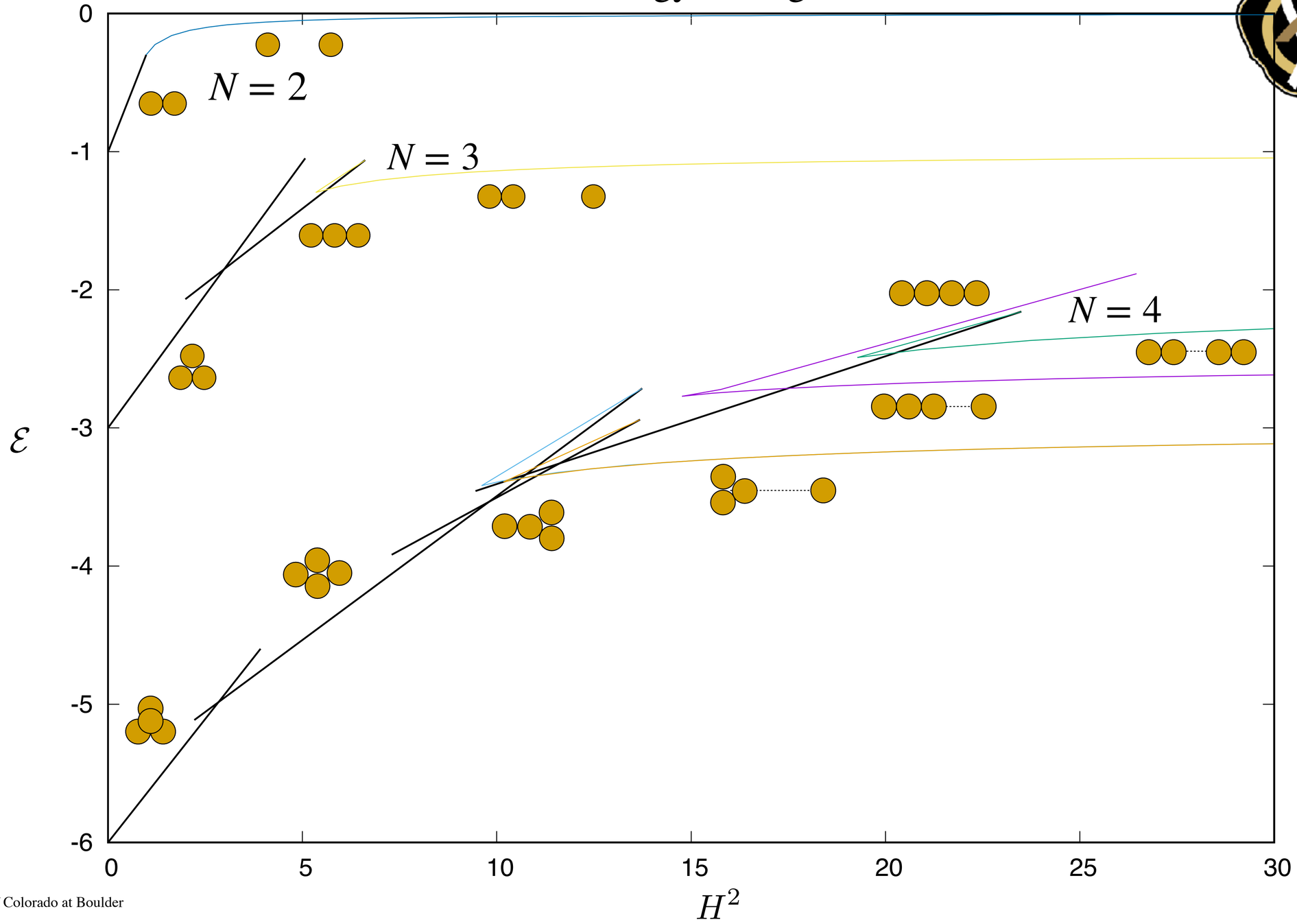
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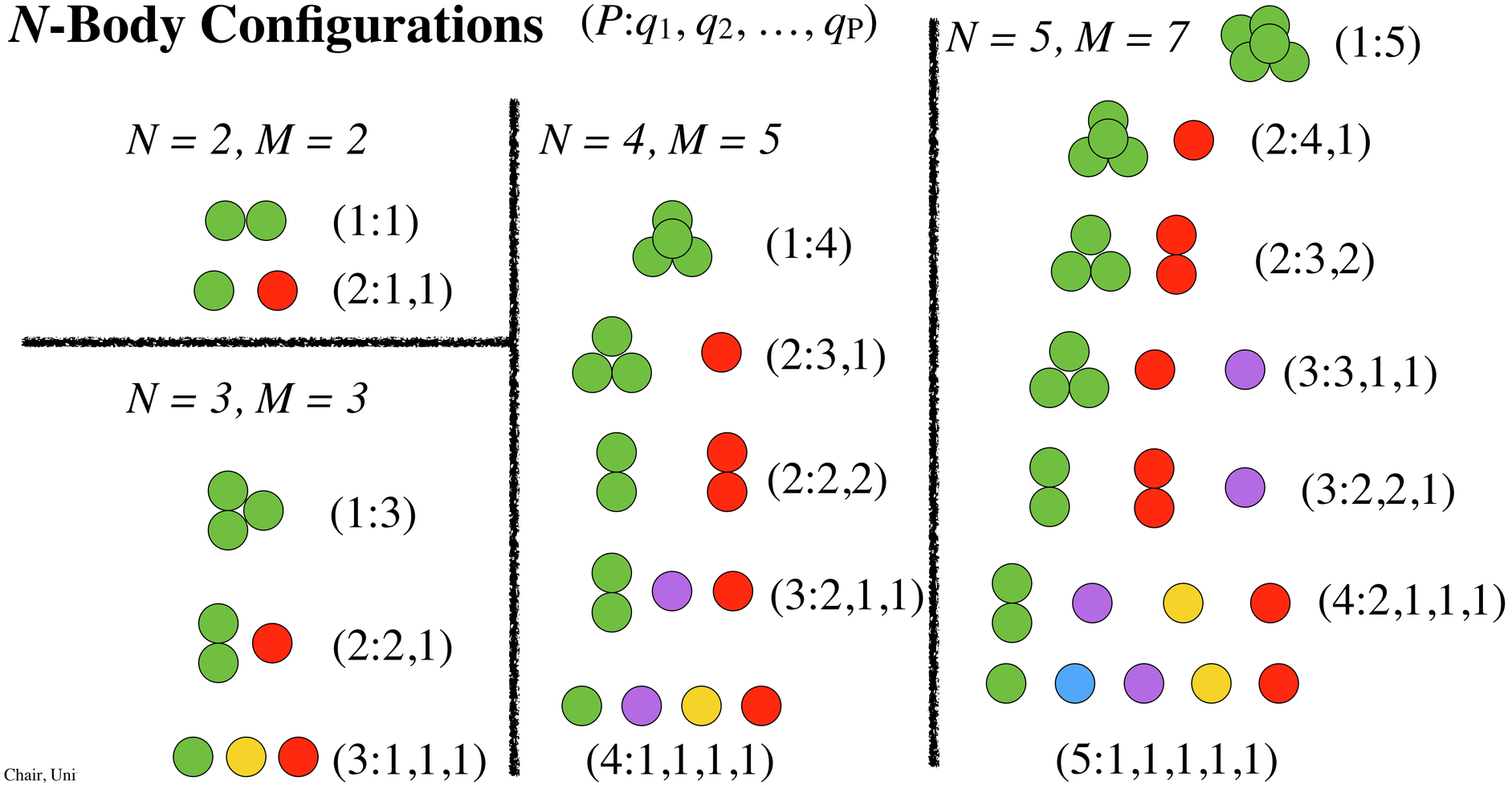
Depending on the energy of the system, particles can be ejected, while conserving system energy and angular momentum



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Fission and Escape for *N* Bodies

- Given *N* bodies, a *Configuration* is a collection of these bodies into *P* groups, with each group having a set number of bodies q_i. Does not consider the state of the configurations



Hill Stability: Definitions & Theorems

System Energy of an *N*-Body Problem

$$E = T + \mathcal{U}$$

Gravitational Potential Energy for Finite Density Bodies

$$\mathcal{U} = -\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{1}{r_{ij}}$$

Minimum Potential

$$\mathcal{U}_m(N) = \min_{r_{ij}\geq 1}\mathcal{U}$$

- A *Configuration* of *P* components with q_i; *i = 1, 2, … P* bodies in each component is *Configuration Hill Stable* if none of these components can mutually escape to infinity.
- Theorem: *A given configuration of P components with q_i bodies in each component is Configuration Hill Stable if*

$$E < \sum_{i}^{P}\mathcal{U}_m(q_i)$$

$$\mathcal{U}_m(2) = -1 \qquad \mathcal{U}_m(4) = -6$$

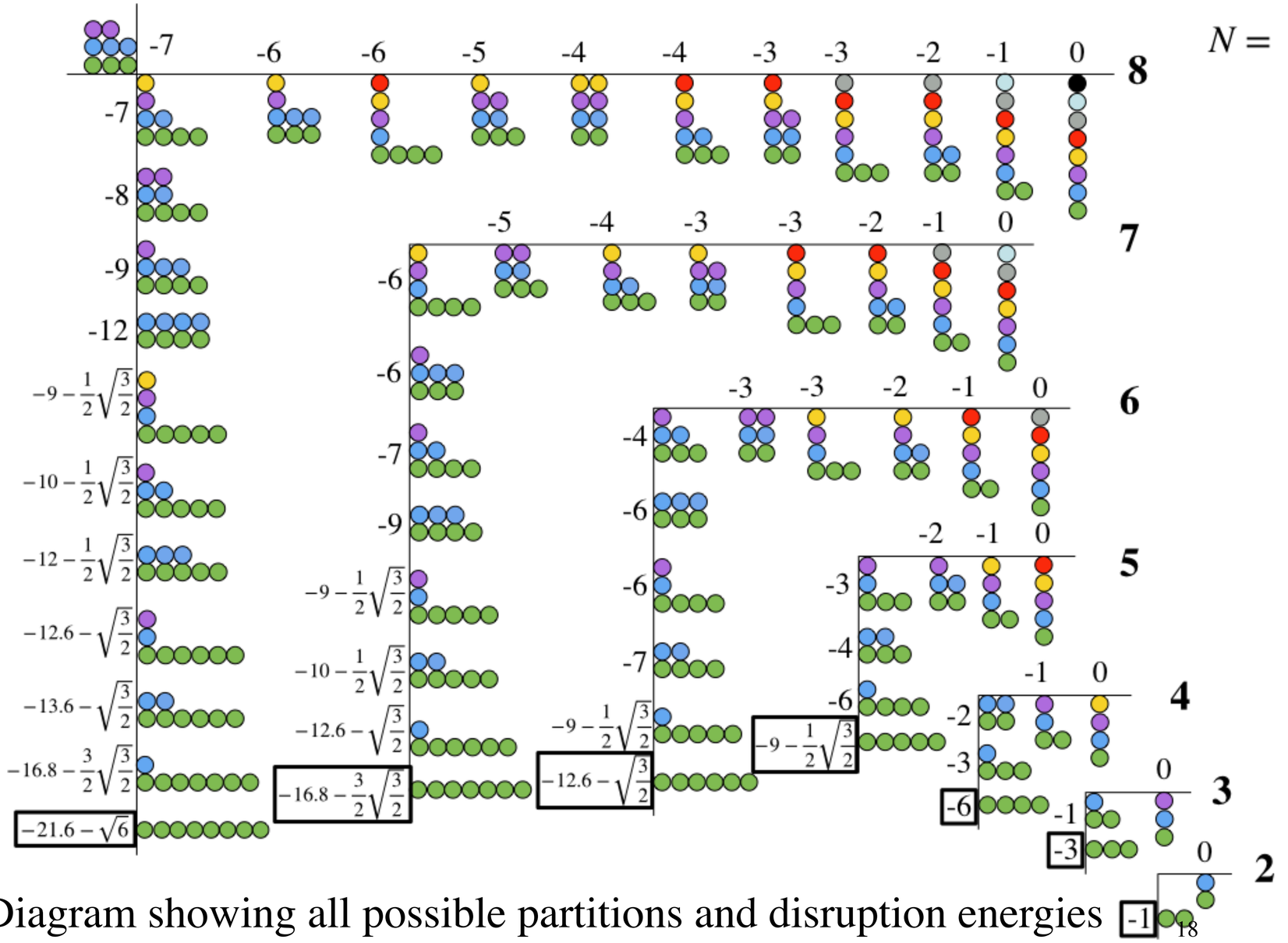
$$\mathcal{U}_m(3) = -3 \qquad \mathcal{U}(5) \sim -10$$

- Proof: *Since the distance between bodies is bounded, there is a minimum gravitational potential energy, which restricts escape.*

How many ways can a Rubble Pile Disaggregate?

- For a rubble pile with *N* components… the number of ways it can disaggregate equals the number of "integer partitions" of *N*

N	Number of Partitions
2	2
3	3
4	5
5	7
6	11
7	15
8	22



Young Diagram showing all possible partitions and disruption energies

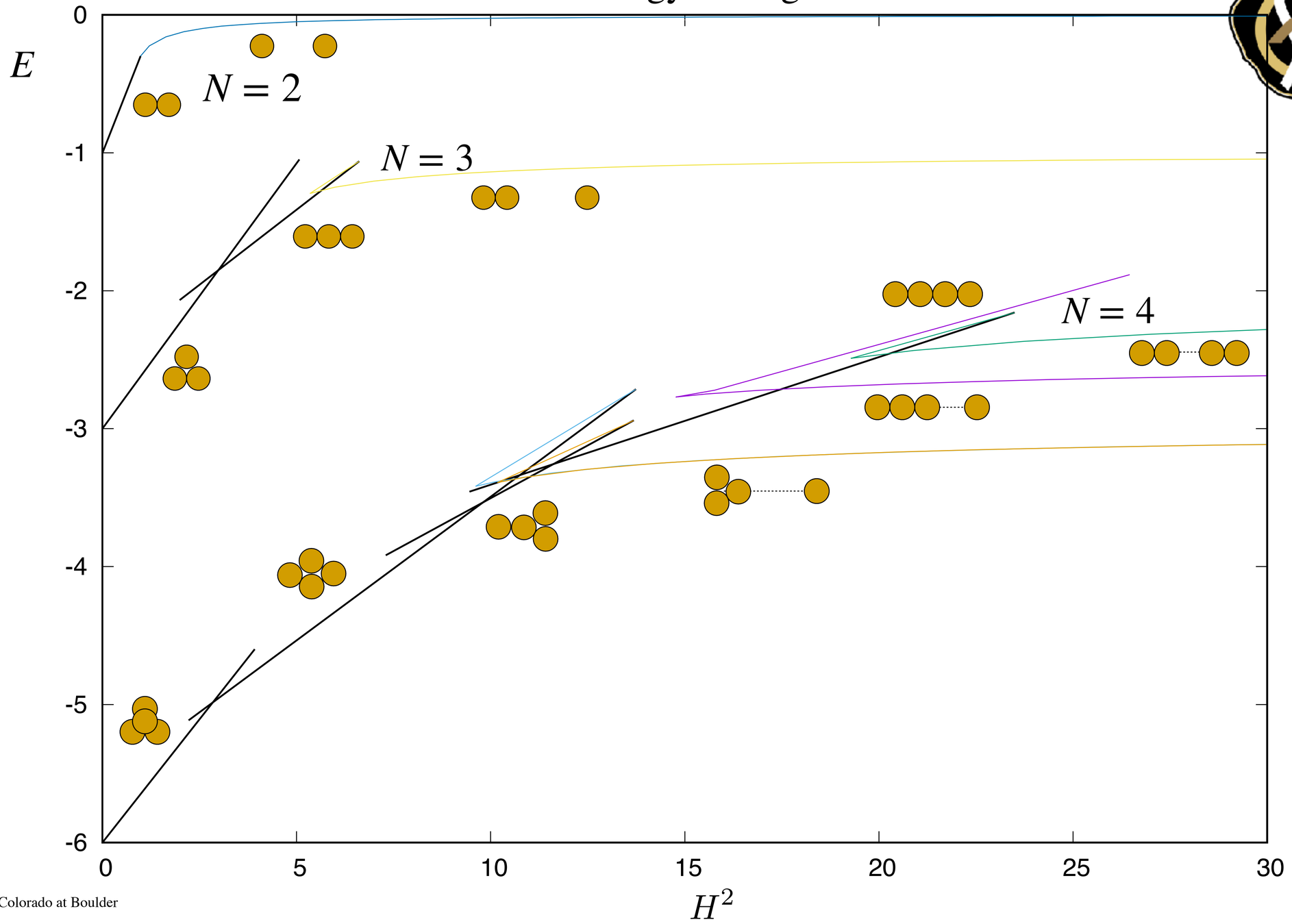
Minimum Energy Configurations

Ejections for $N = 2$ require an energy greater than $E = 0$

Ejections for $N = 3$ require an energy greater than $E = -1$

Ejections for $N = 4$ require an energy greater than $E = -3$

Ejections for $N = 5$ require an energy greater than $E = -6$



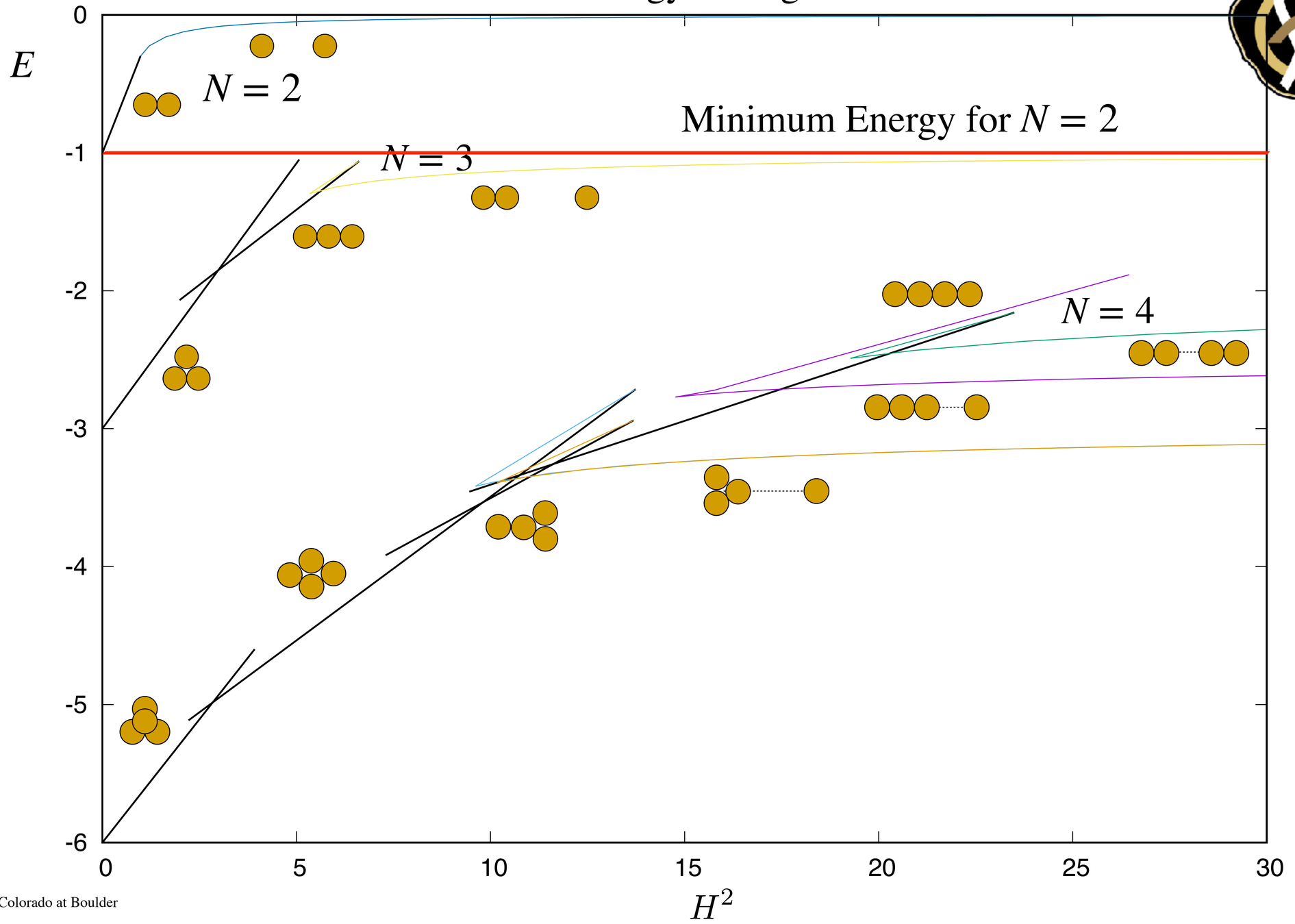
Minimum Energy Configurations

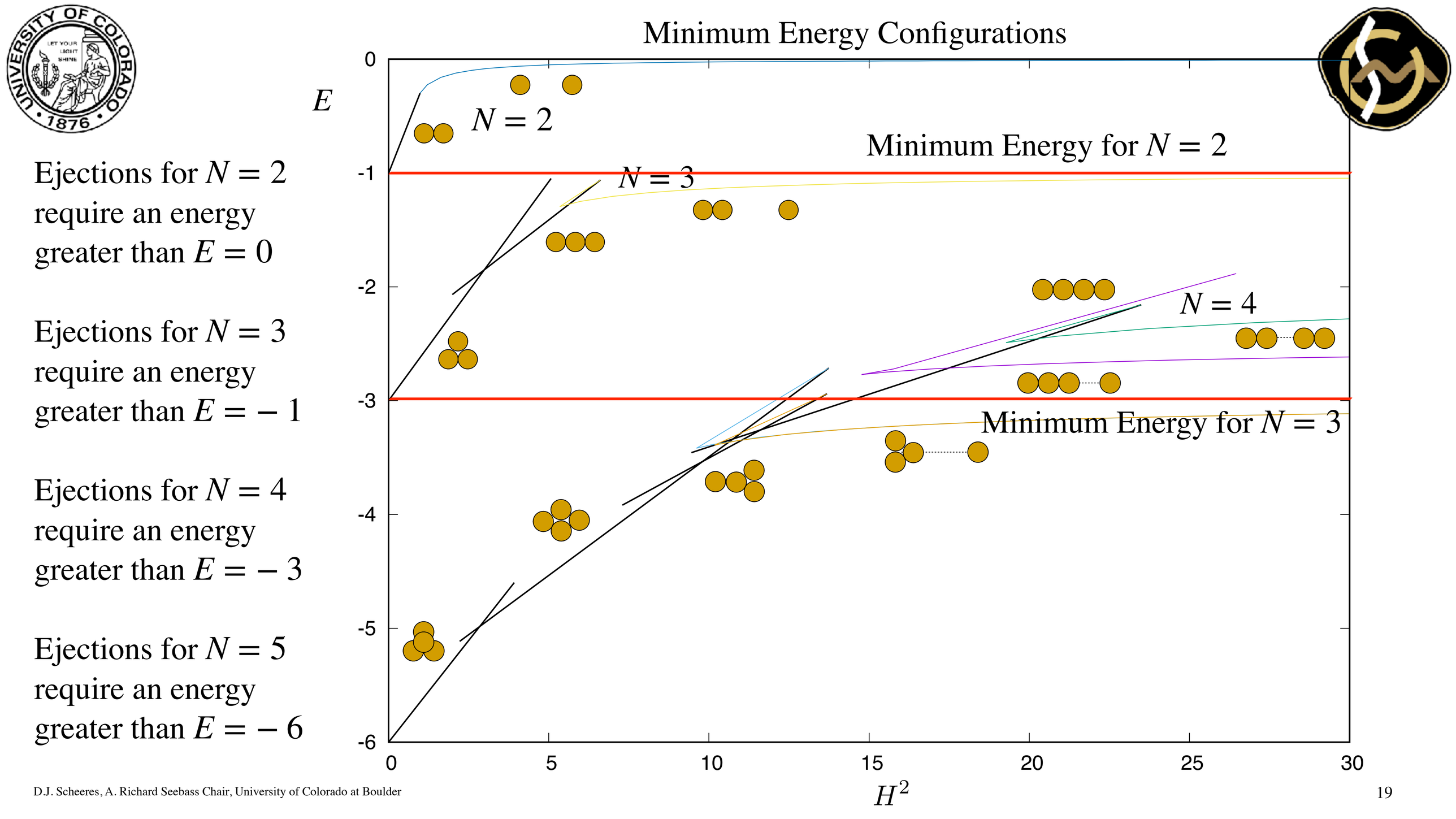
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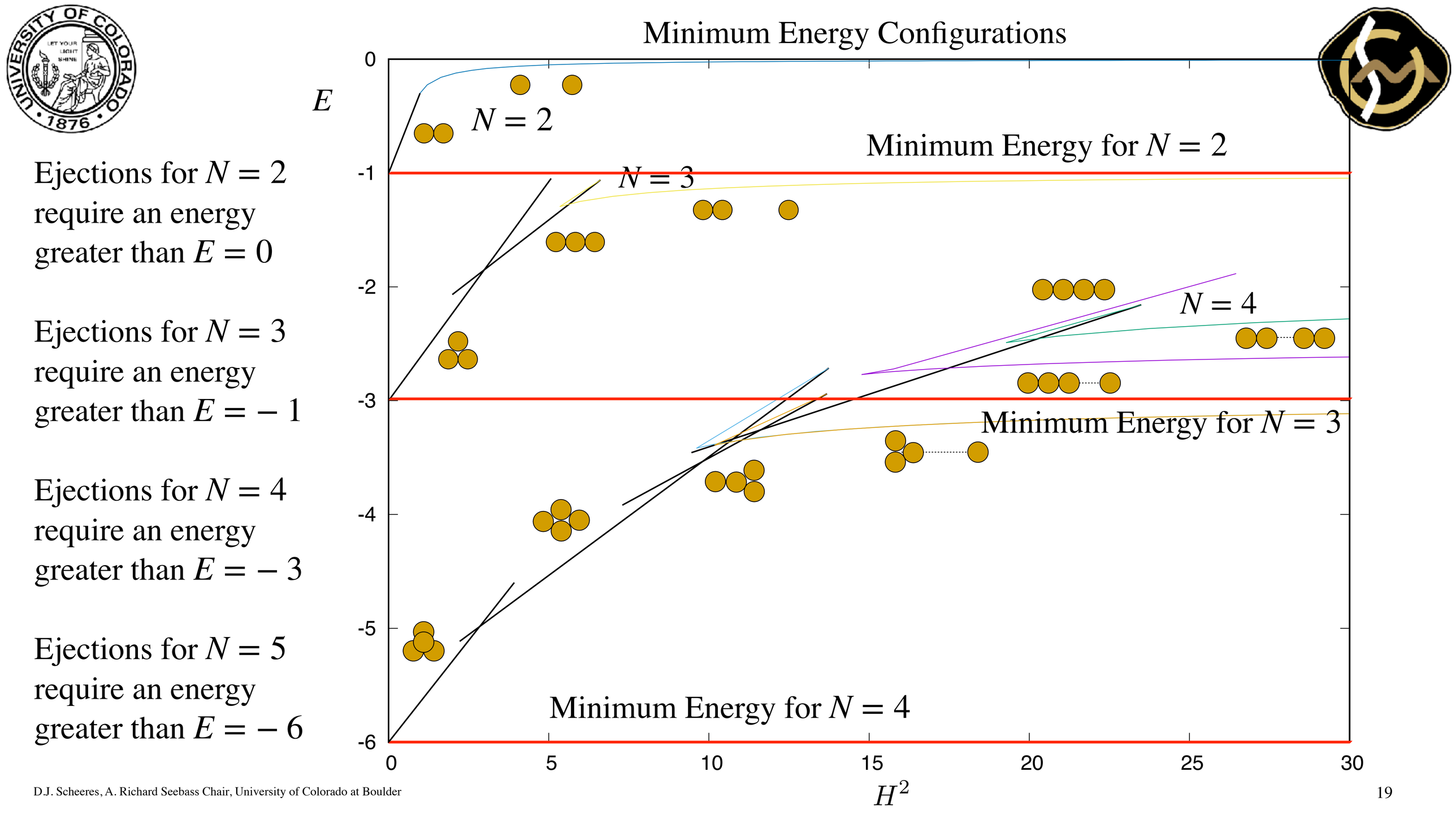


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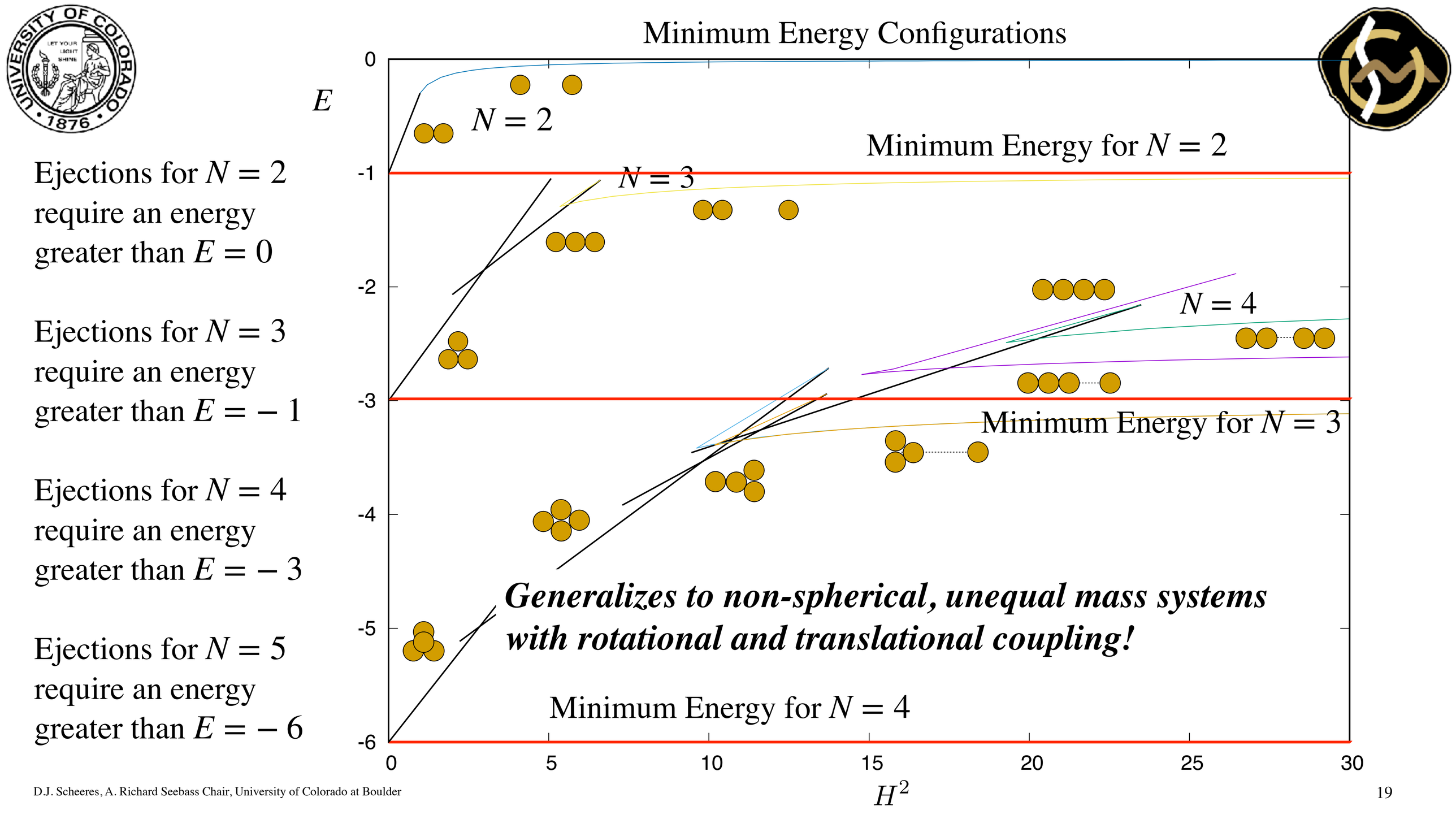


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Ejections for $N = 2$ require an energy greater than $E = 0$

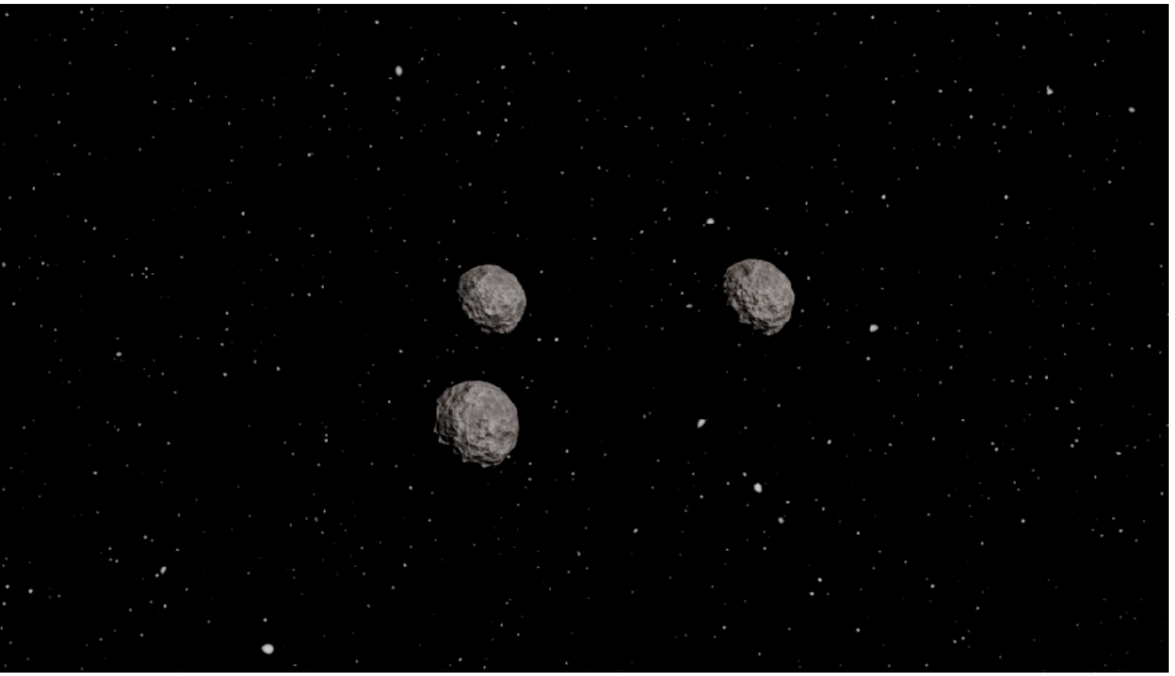
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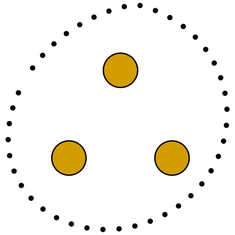
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Possible Outcomes for a 3 component system



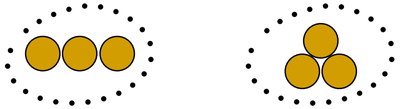
We simulate 3-body systems, accounting for rigid body rotation, surface friction, COR and gravity

Simulations are made using ReboundX, updated to account for the total system angular momentum and energy including the rigid body components and their interactions

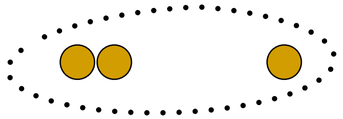


Initial condition: 3 spinning bodies in an initially bound orbit with a given E and H^2

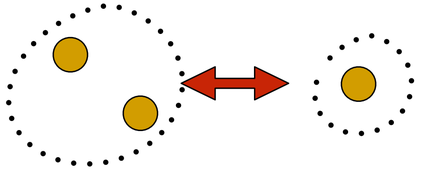
Possible Outcomes after a given time:



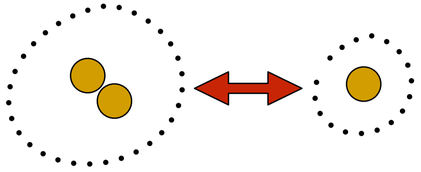
Total collapse, contact binary/ternary



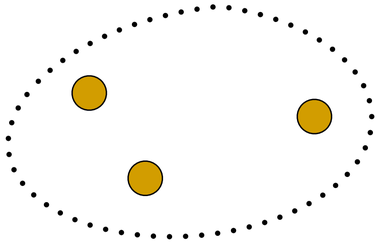
Settle into a stable binary asteroid



Escape:
Asteroid pair, leaving a binary



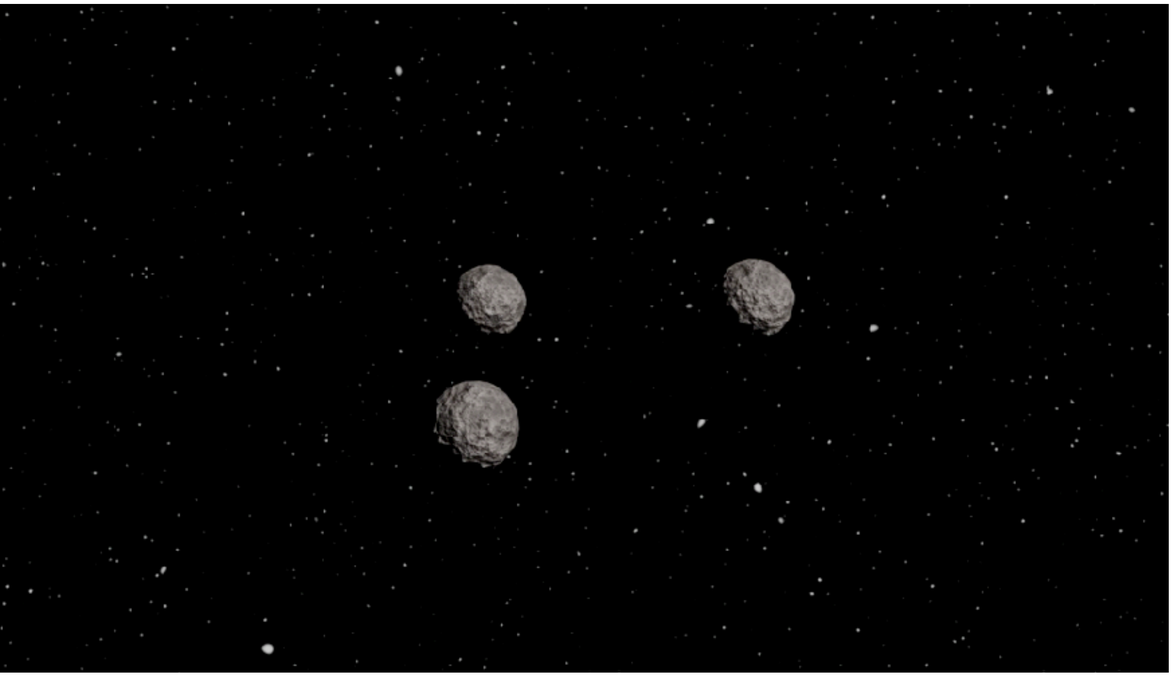
Escape:
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Continued interactions as a 3-body system…

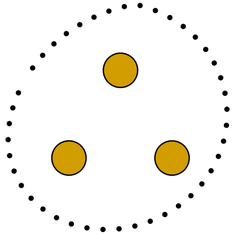
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Possible Outcomes for a 3 component system



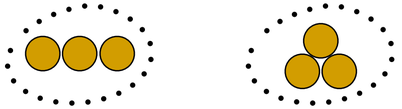
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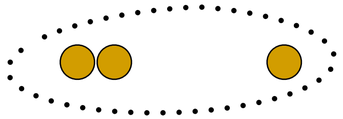


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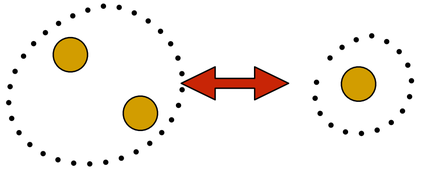
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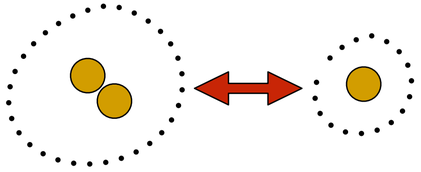
Total collapse, contact binary/ternary



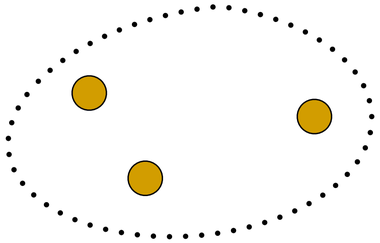
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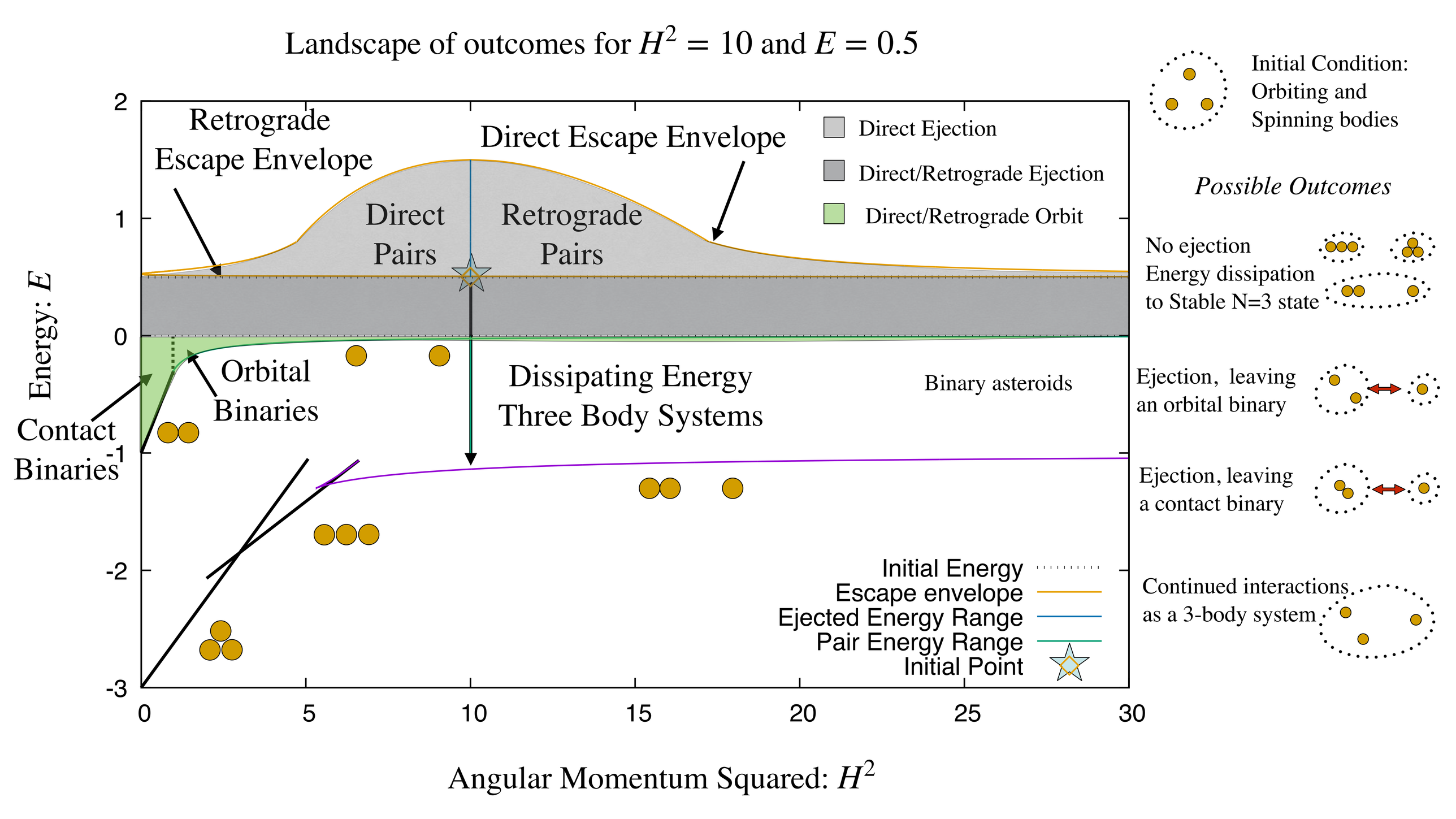
Escape:
Asteroid pair, leaving a binary

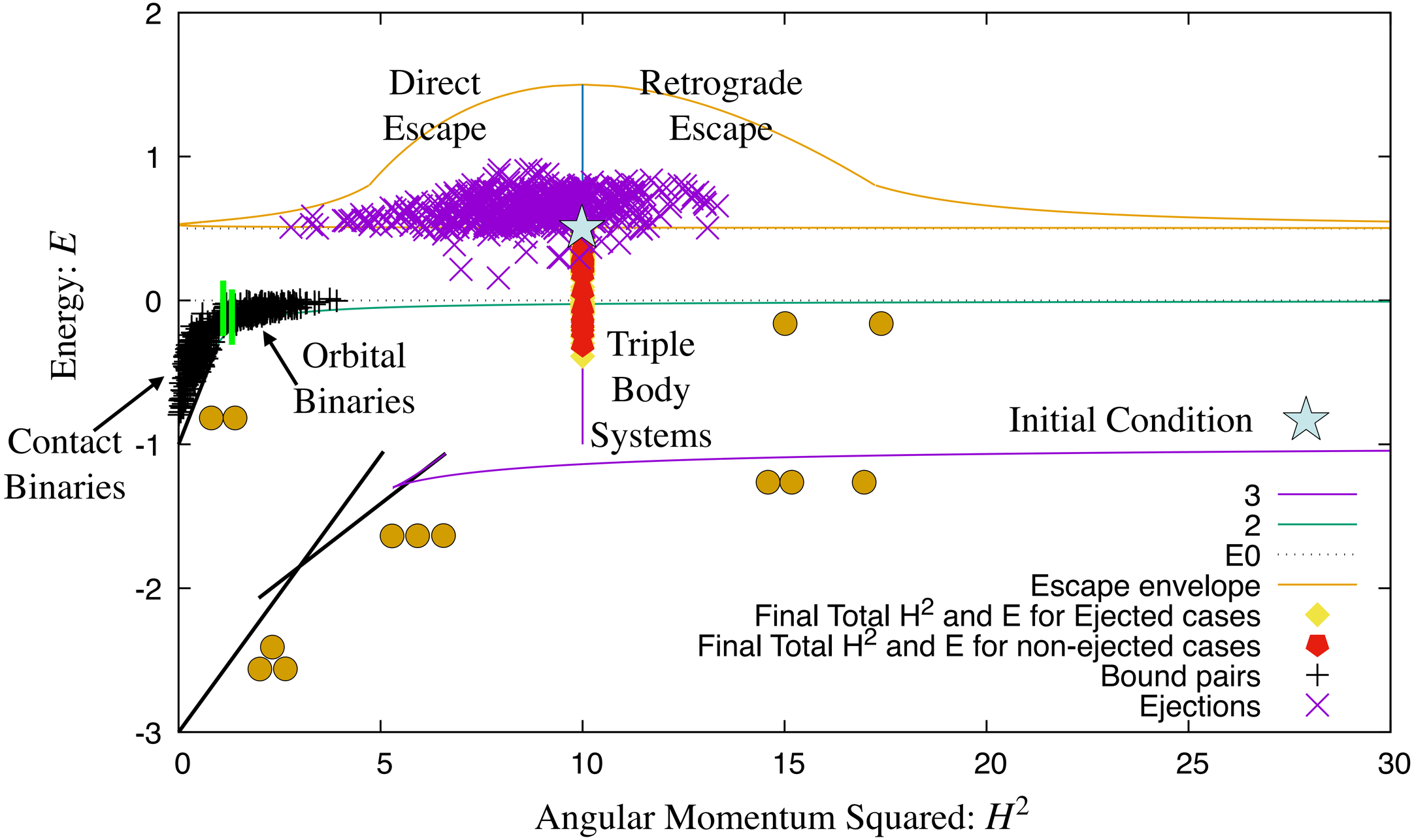


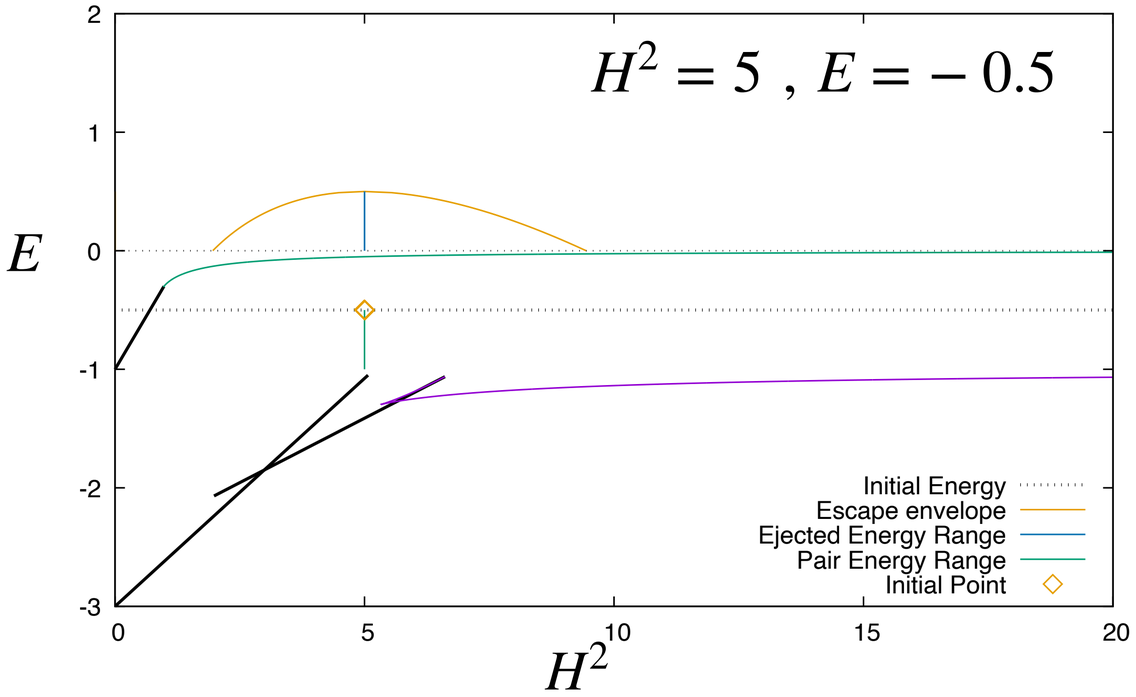
Escape:
Asteroid pair, leaving a contact binary

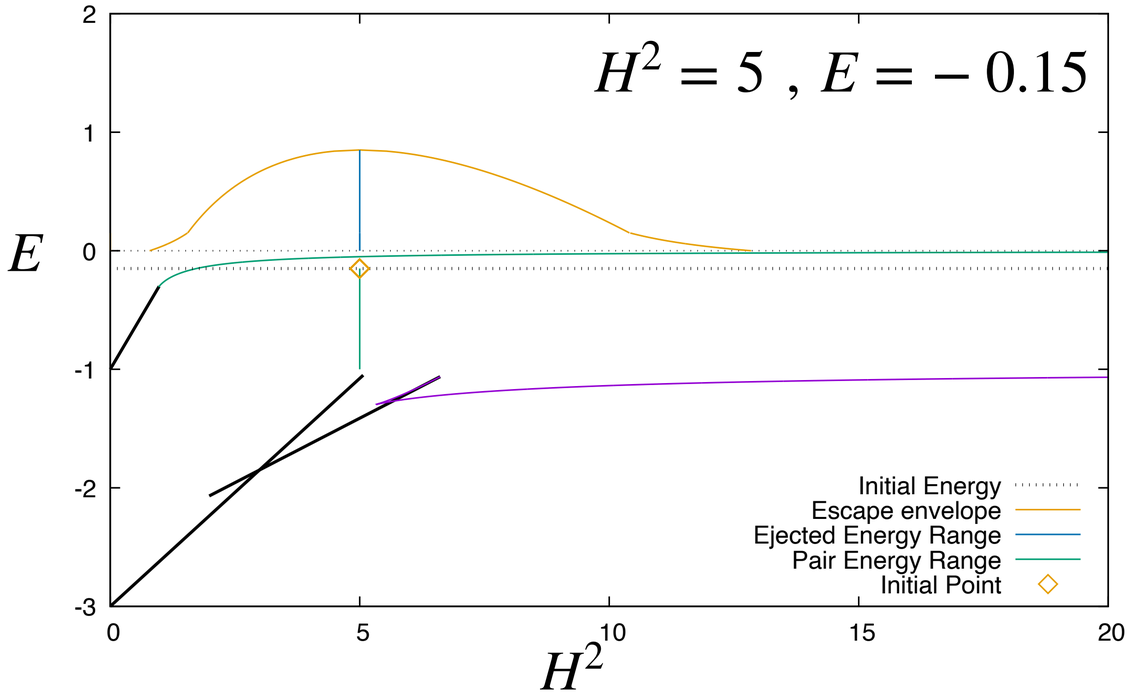


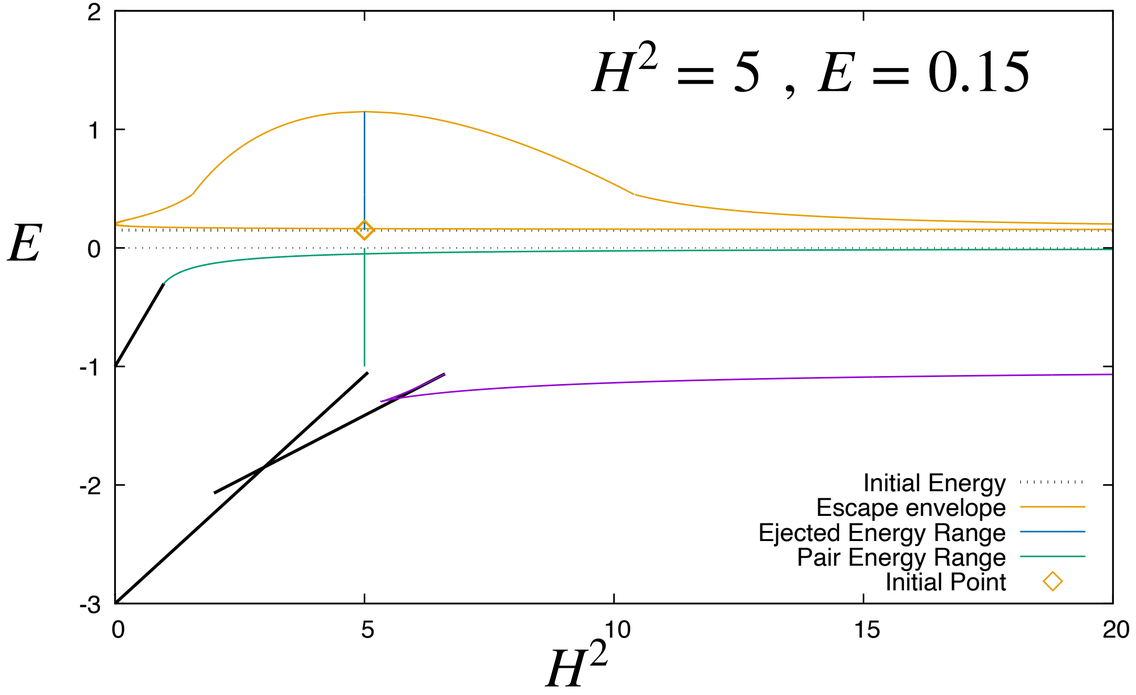
Continued interactions as a 3-body system…

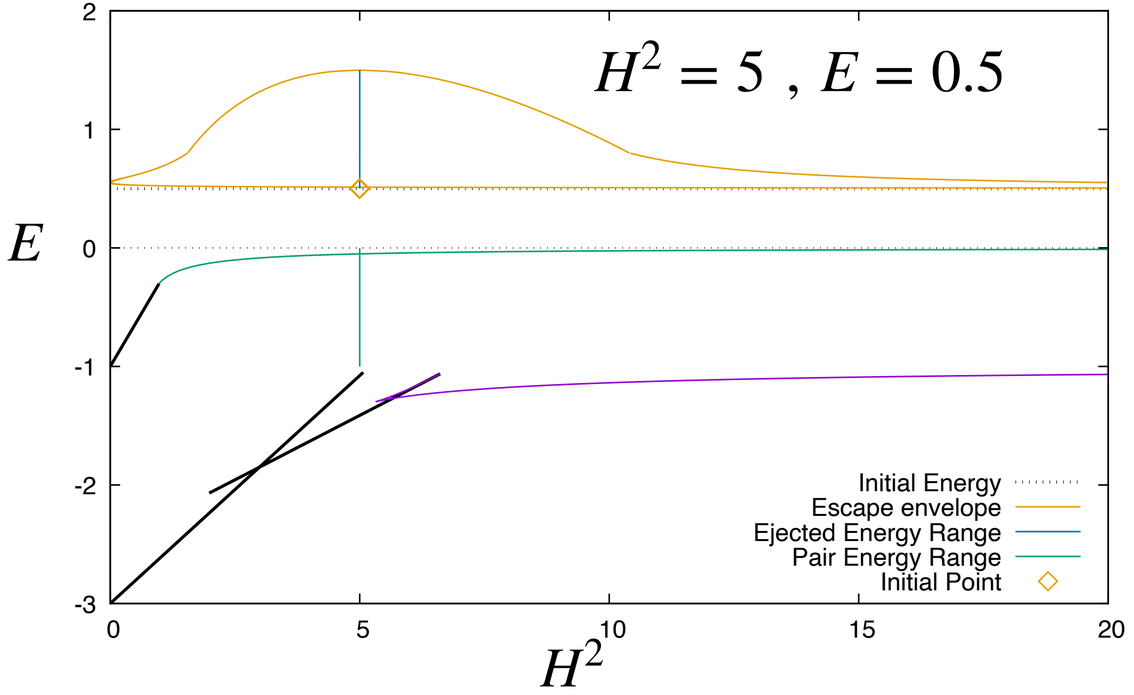

Landscape of outcomes for $H^2 = 10$ and $E = 0.5$
Retrograde Escape Envelope
Direct Escape Envelope
Direct Pairs
Retrograde Pairs
Direct Ejection
Direct/Retrograde Ejection
Direct/Retrograde Orbit
Orbital Binaries
Contact Binaries
Dissipating Energy Three Body Systems
Binary asteroids
Initial Energy
Escape envelope
Ejected Energy Range
Pair Energy Range
Initial Point
Energy: E
Angular Momentum Squared: H^2
2
1
0
-1
-2
-3
0
5
10
15
20
25
30
Initial Condition: Orbiting and Spinning bodies
Possible Outcomes
No ejection Energy dissipation to Stable N=3 state
Ejection, leaving an orbital binary
Ejection, leaving a contact binary
Continued interactions as a 3-body system


Outcome from hundreds of runs at $H^2 = 10$ and $E = 0.5$
Direct Escape
Retrograde Escape
Orbital Binaries
Contact Binaries
Triple Body Systems
Initial Condition
3
2
E0
Escape envelope
Final Total H2 and E for Ejected cases
Final Total H2 and E for non-ejected cases
Bound pairs
Ejections
Energy: E
Angular Momentum Squared: H^2
2
1
0
-1
-2
-3
0
5
10
15
20
25
30


$H^2 = 5\ ,\ E = -0.5$
E
H^2
Initial Energy
Escape envelope
Ejected Energy Range
Pair Energy Range
Initial Point


$H^2 = 5\ ,\ E = -0.15$
E
H^2
Initial Energy
Escape envelope
Ejected Energy Range
Pair Energy Range
Initial Point


$H^2 = 5\ ,\ E = 0.15$
E
H^2
Initial Energy
Escape envelope
Ejected Energy Range
Pair Energy Range
Initial Point


$H^2 = 5\ ,\ E = 0.5$
E
H^2
Initial Energy
Escape envelope
Ejected Energy Range
Pair Energy Range
Initial Point

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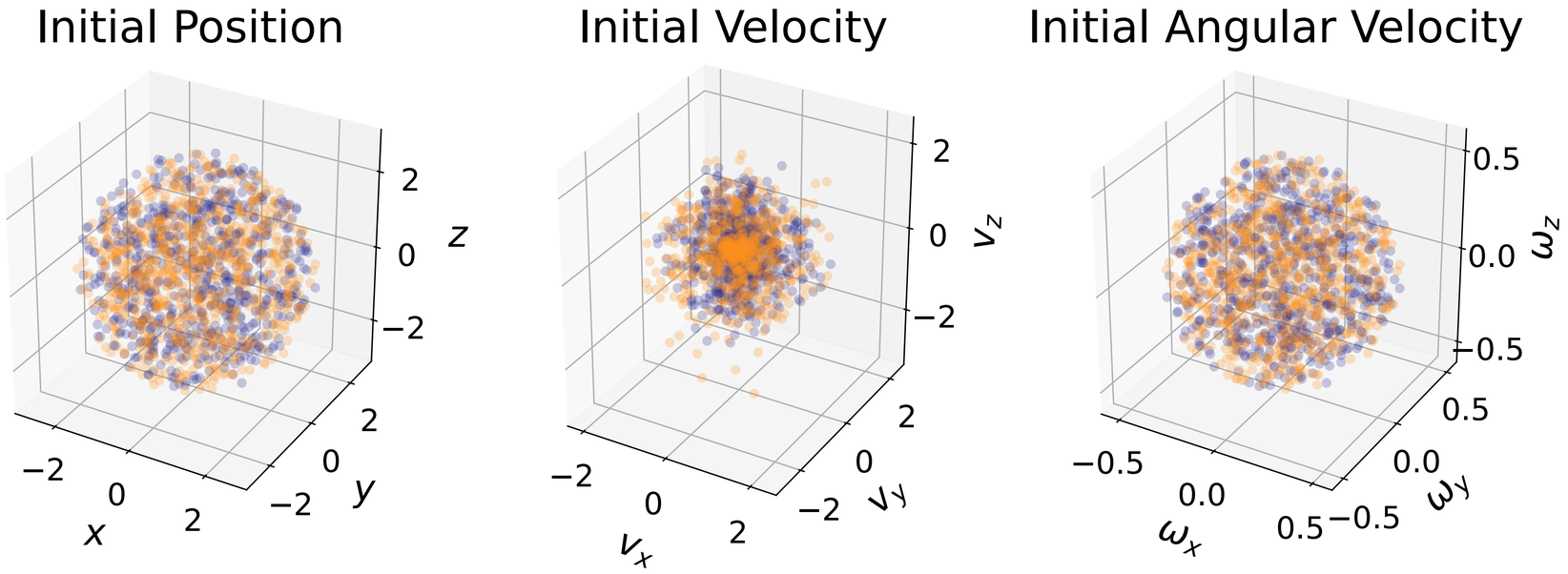
Survey of Final Outcomes

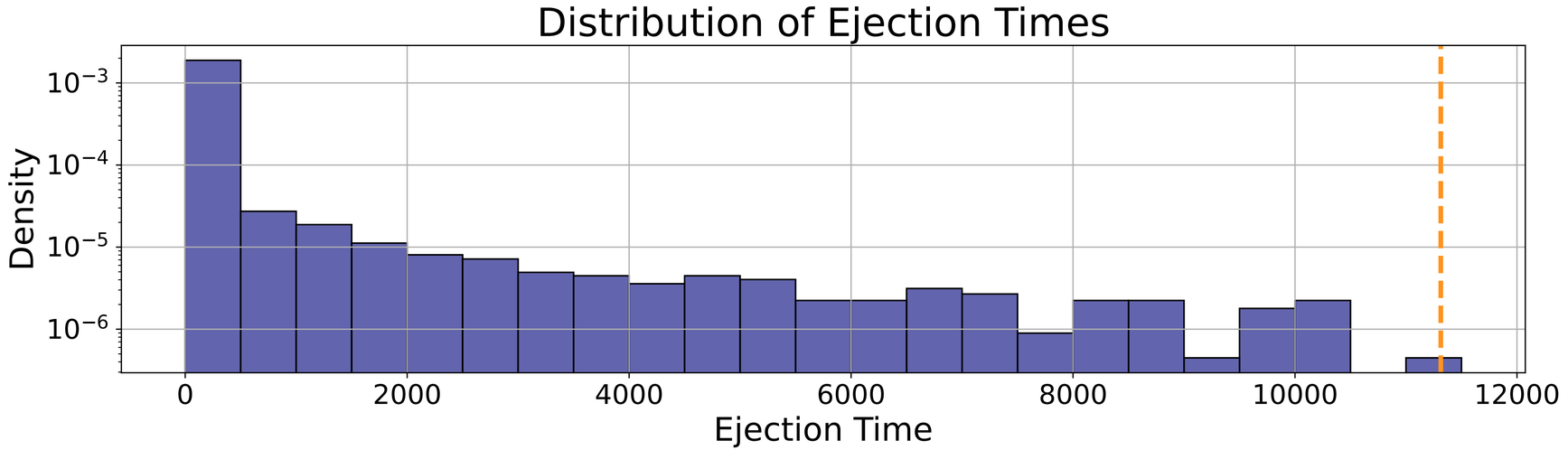


- For the equal mass Full 3-Body problem we ran thousands of cases for different values of angular momentum H^2 and initial energy E
- Evolution is tracked until:
  - Ejection of a body
  - Sufficient dissipation of energy to form a stable 3-body system
  - Pre-specified time span
- Computations resolve impacts, which transfer angular and translational momentum and energy
  - Strict numerical constraints maintained on total angular momentum conservation
  - Energy decreases at impacts due to surface forces and rebound energy loss
- Goal is to identify statistical outcomes as a function of H^2 and E
  - Since N-body interactions are chaotic, more appropriate to develop statistical approaches
  - Motivates a "Thermodynamics" of rubble pile evolution

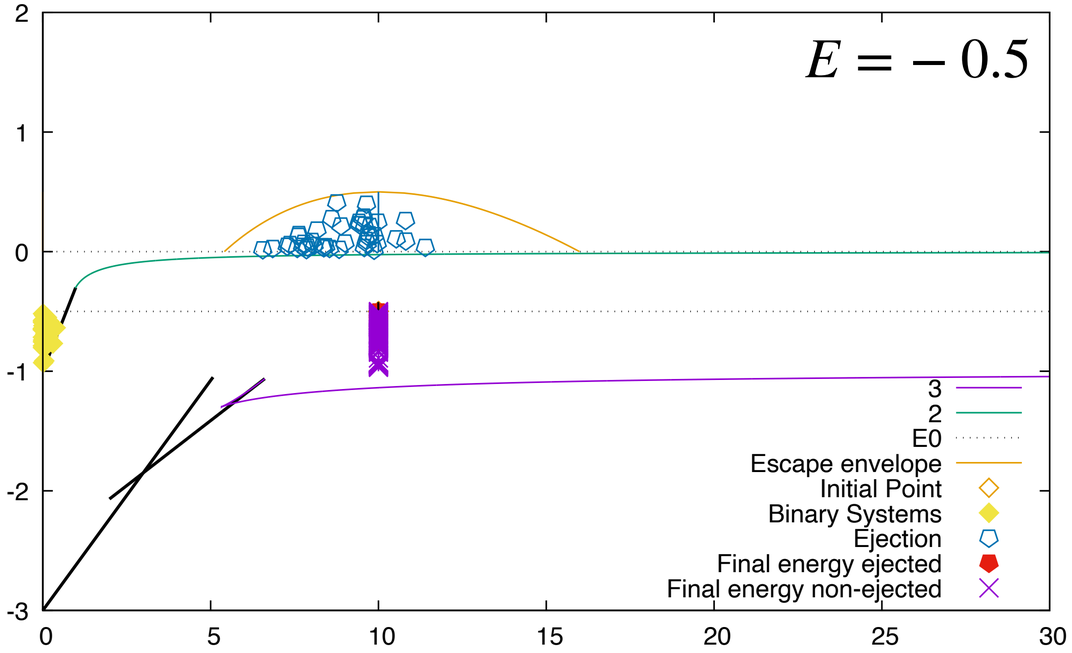
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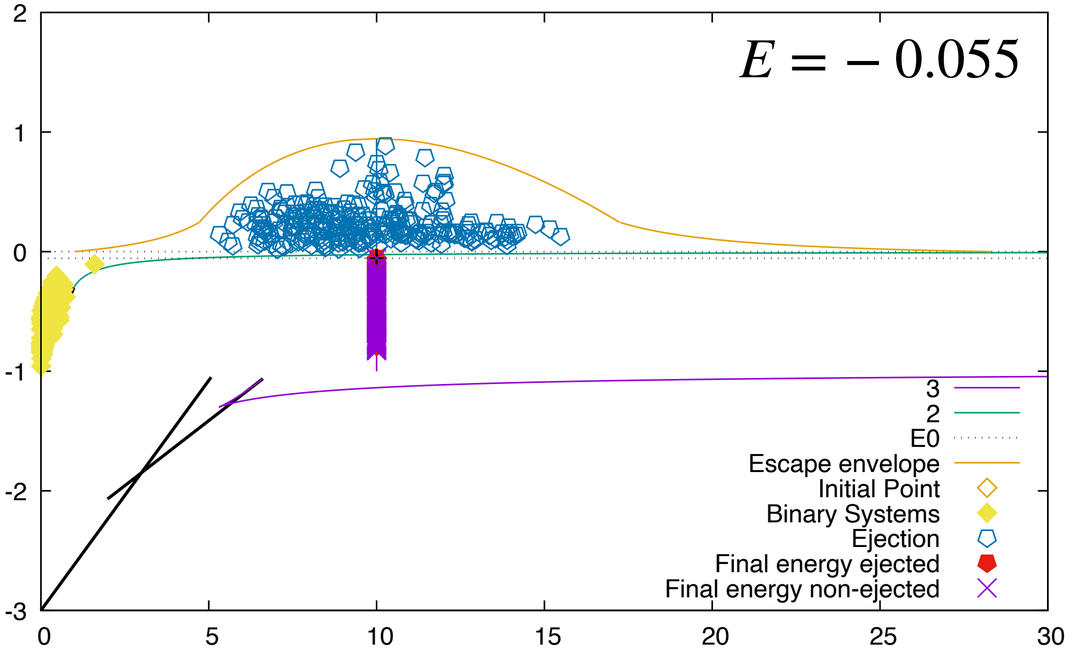
Typical initial values for a fixed H^2 and E

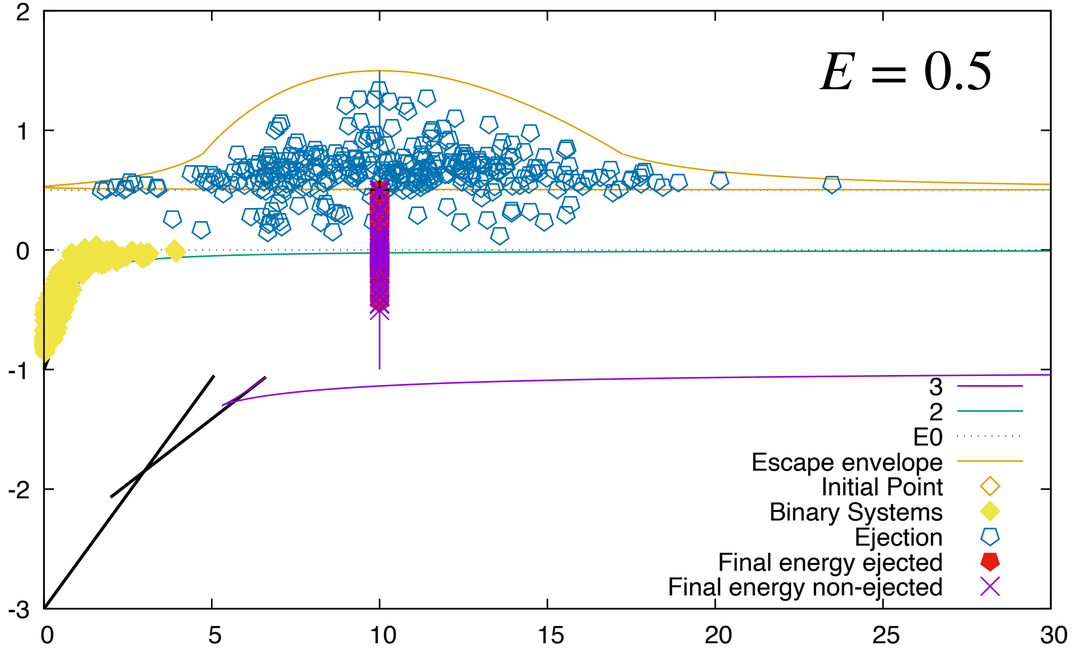




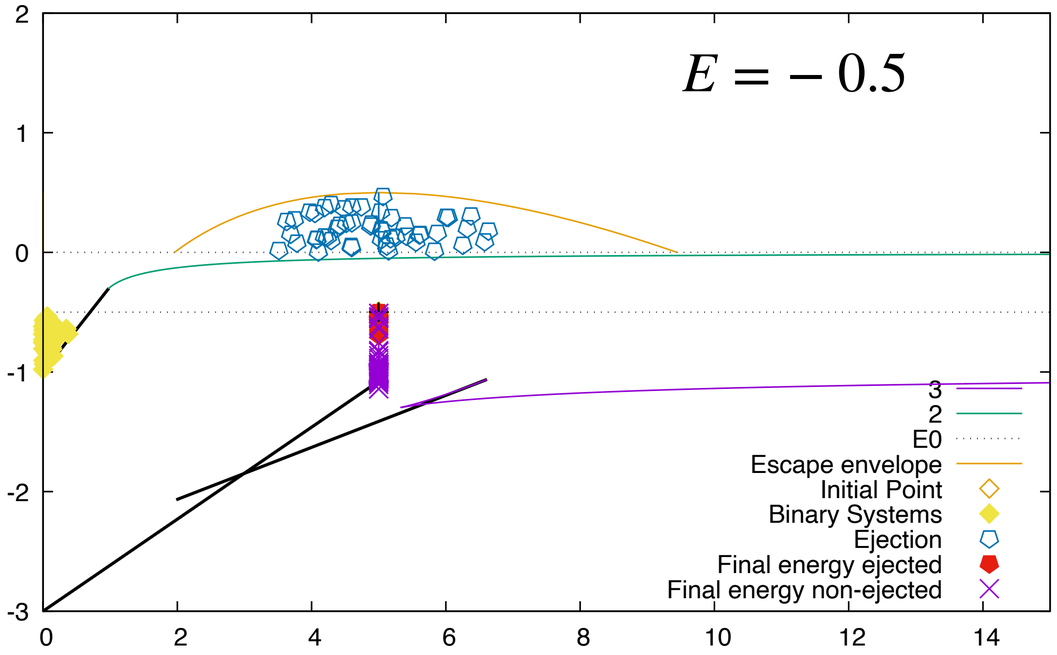
$H^2 = 10$

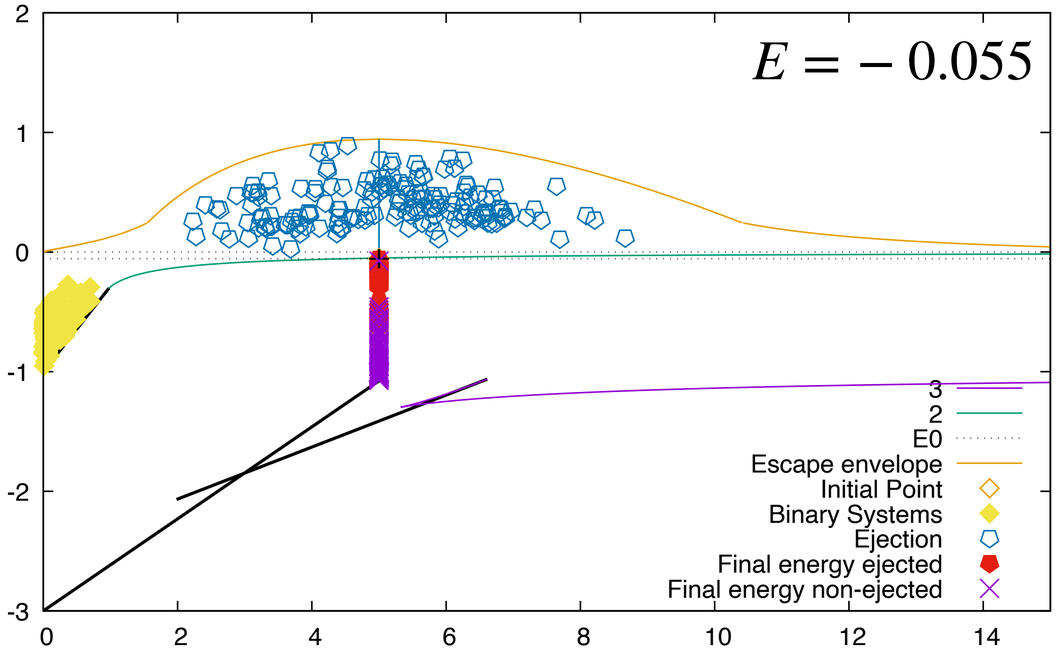

$E = -0.5$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected

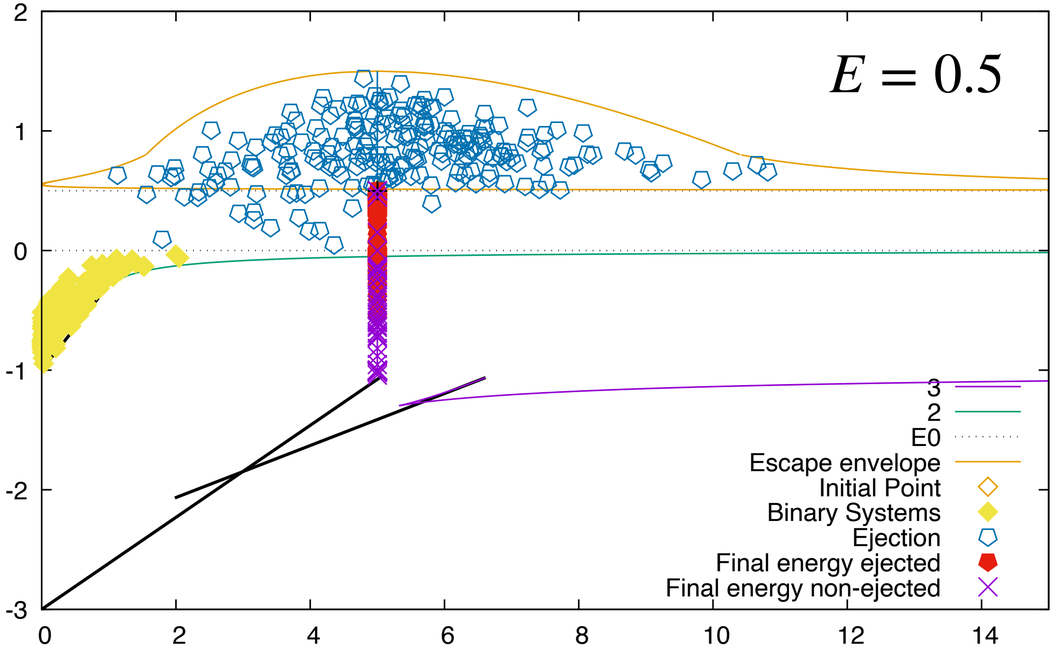

$E = -0.055$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected


$E = 0.5$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected

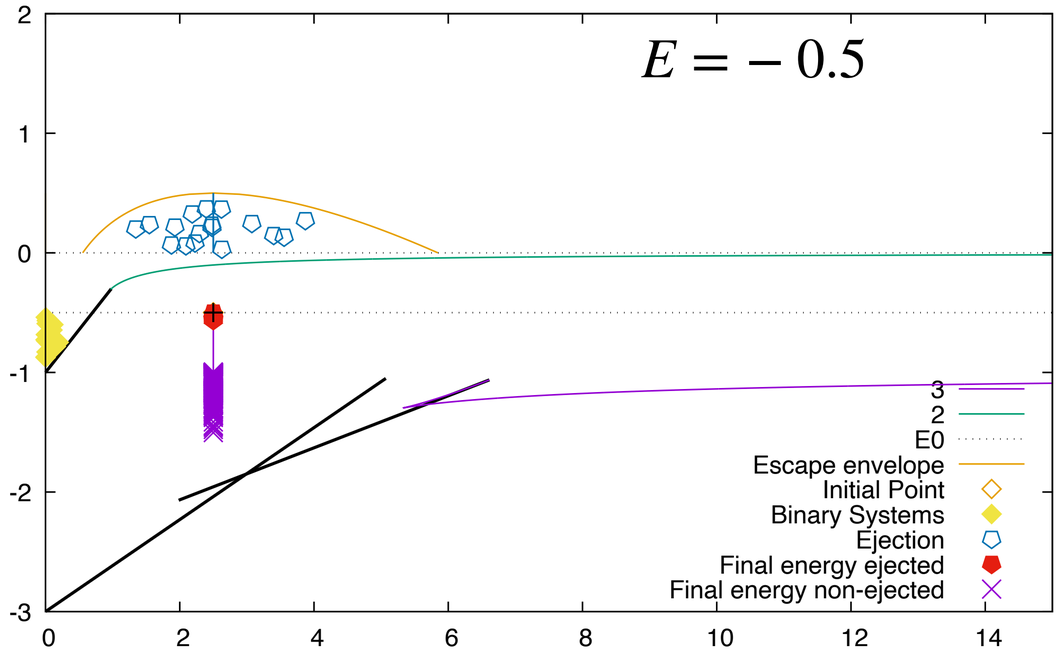
$H^2 = 5$

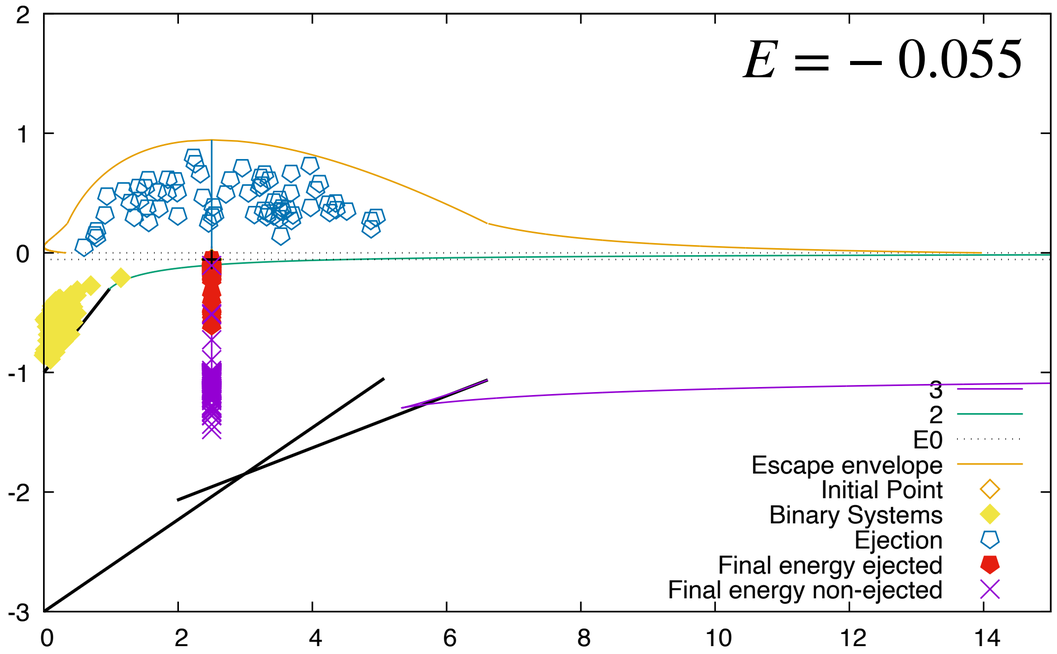

$E = -0.5$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected

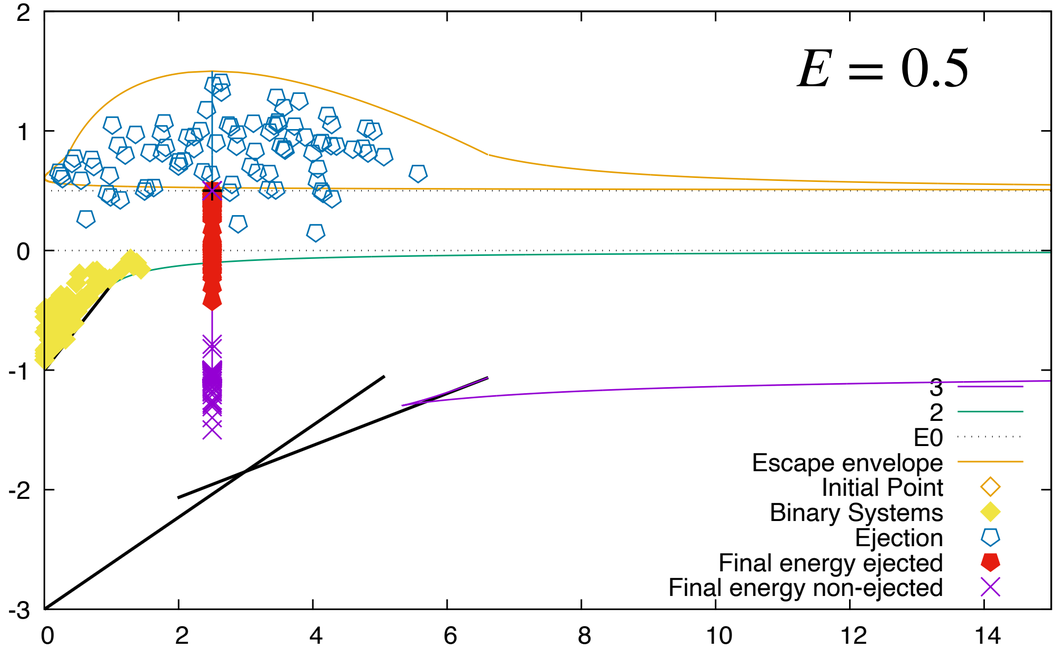

$E = -0.055$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected


$E = 0.5$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected

$H^2 = 2.5$


$E = -0.5$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected

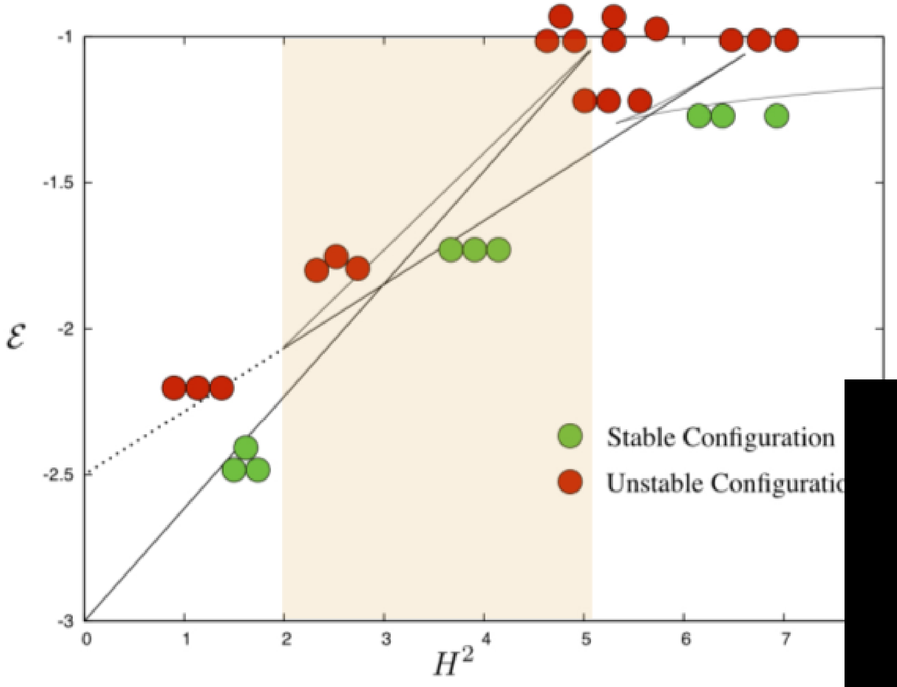

$E = -0.055$
3
2
E0
Escape envelope
Initial Point
Binary Systems
Ejection
Final energy ejected
Final energy non-ejected


$E = 0.5$
3
2
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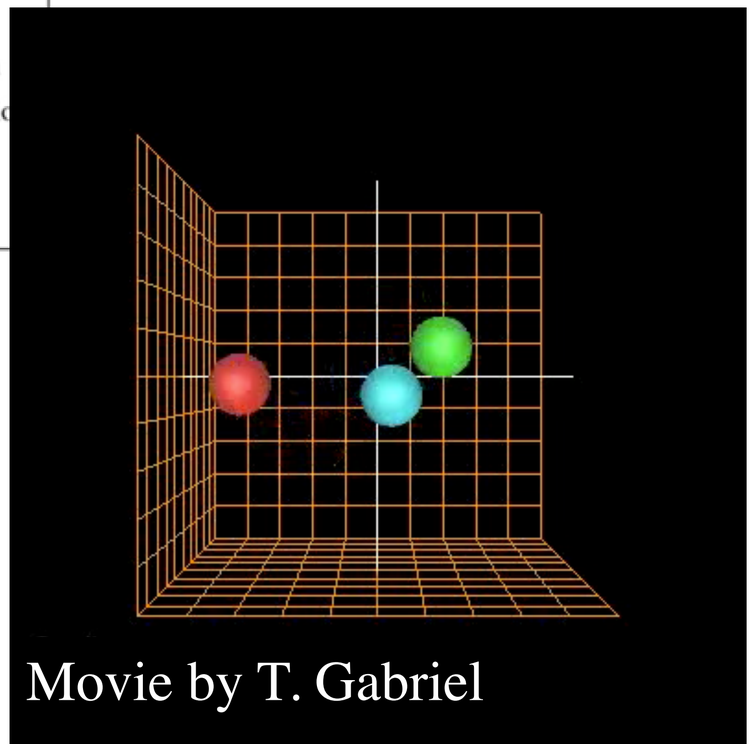
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Endgame of Energy Collapse

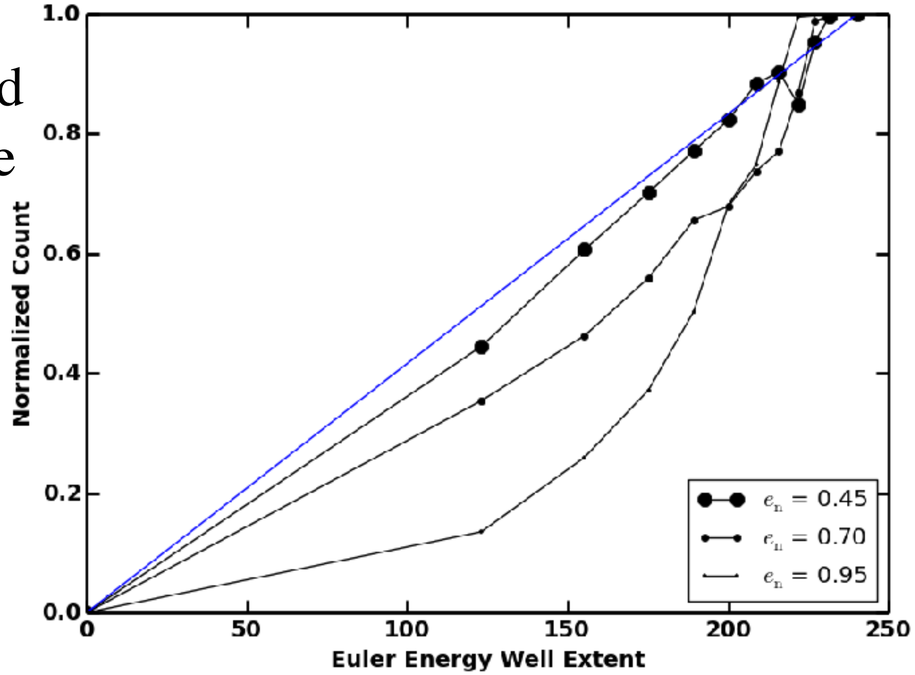
- Results are stochastic but predictable, from a thermodynamics perspective



Outcome of a collapse where either Lagrange or Euler resting configurations can exist



Fraction that end in an Euler State

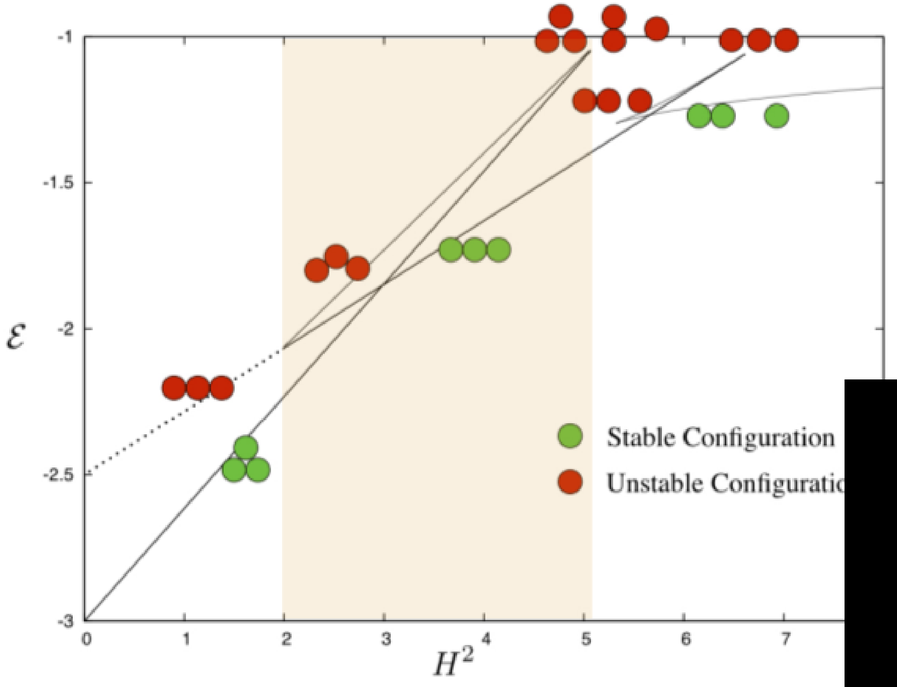


Total Angular Momentum Measure

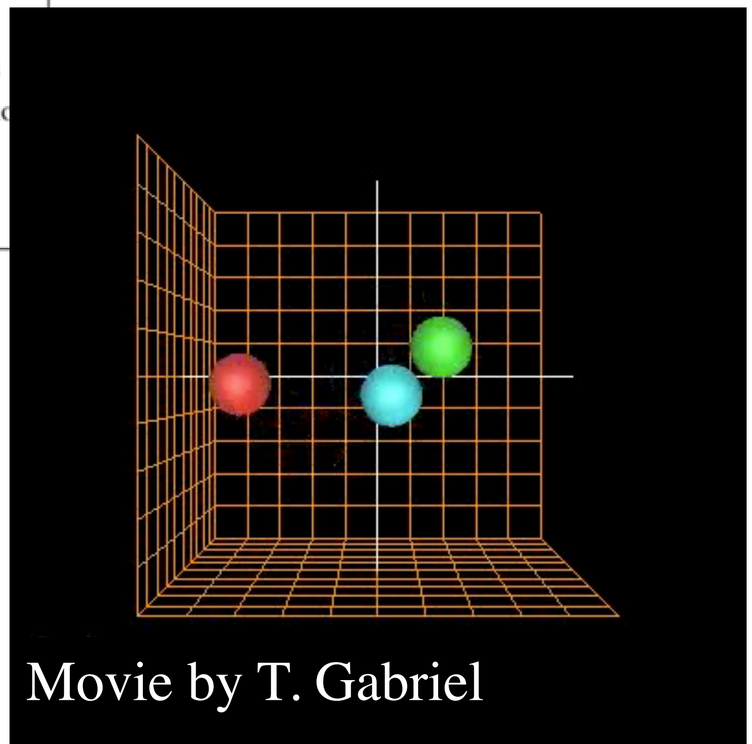
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Endgame of Energy Collapse

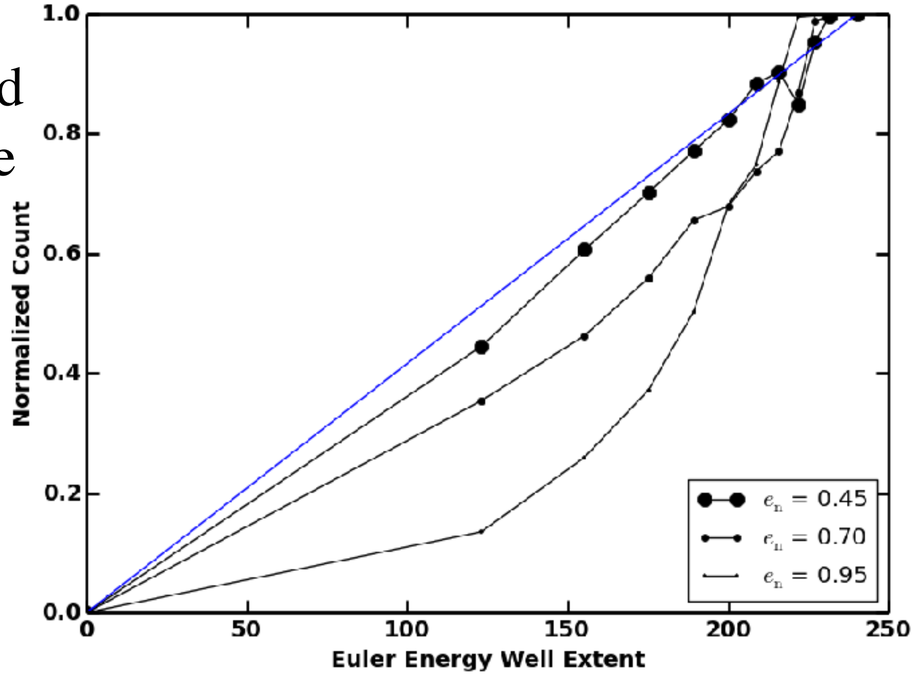
- Results are stochastic but predictable, from a thermodynamics perspective



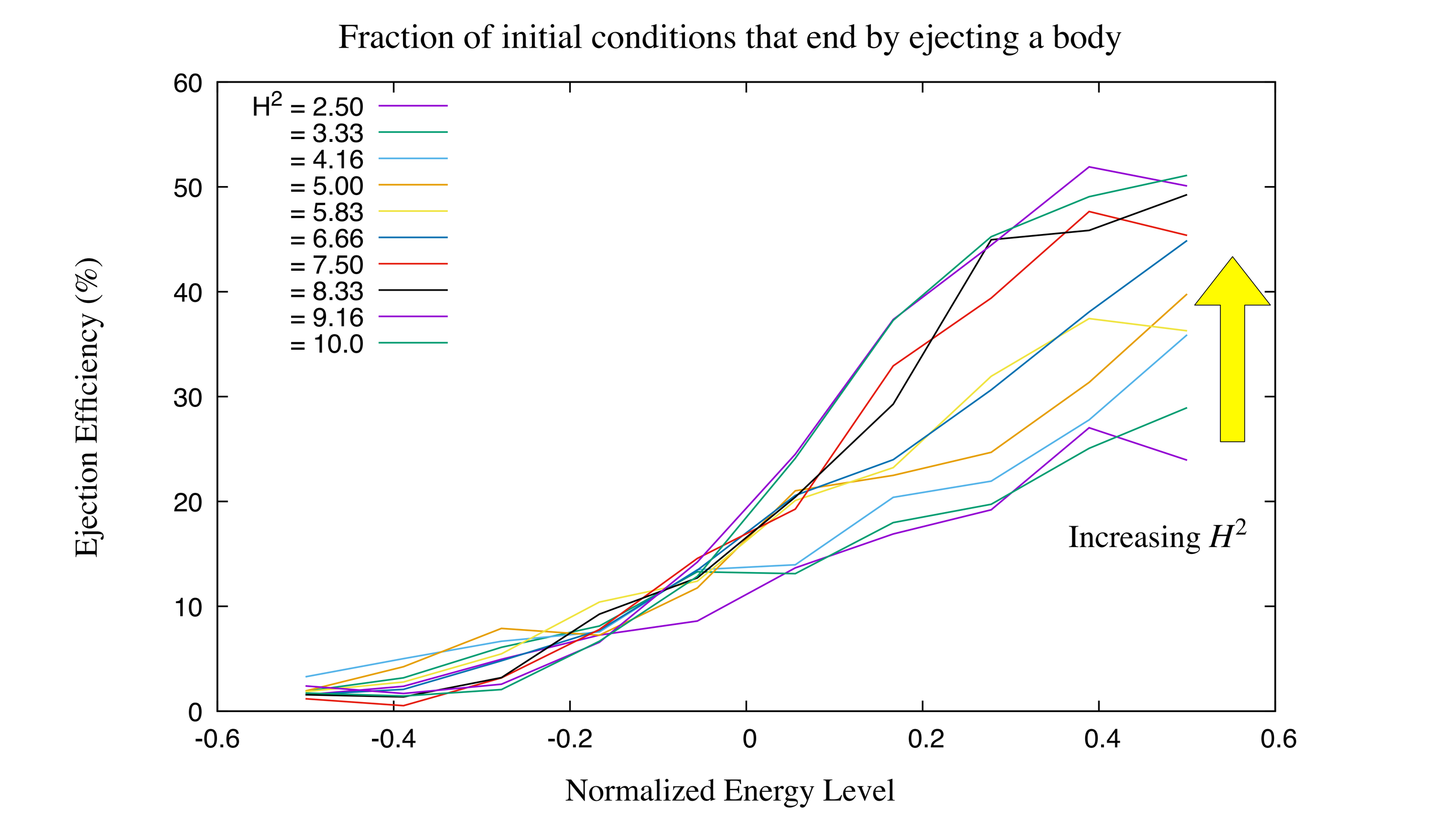
Outcome of a collapse where either Lagrange or Euler resting configurations can exist



Fraction that end in an Euler State

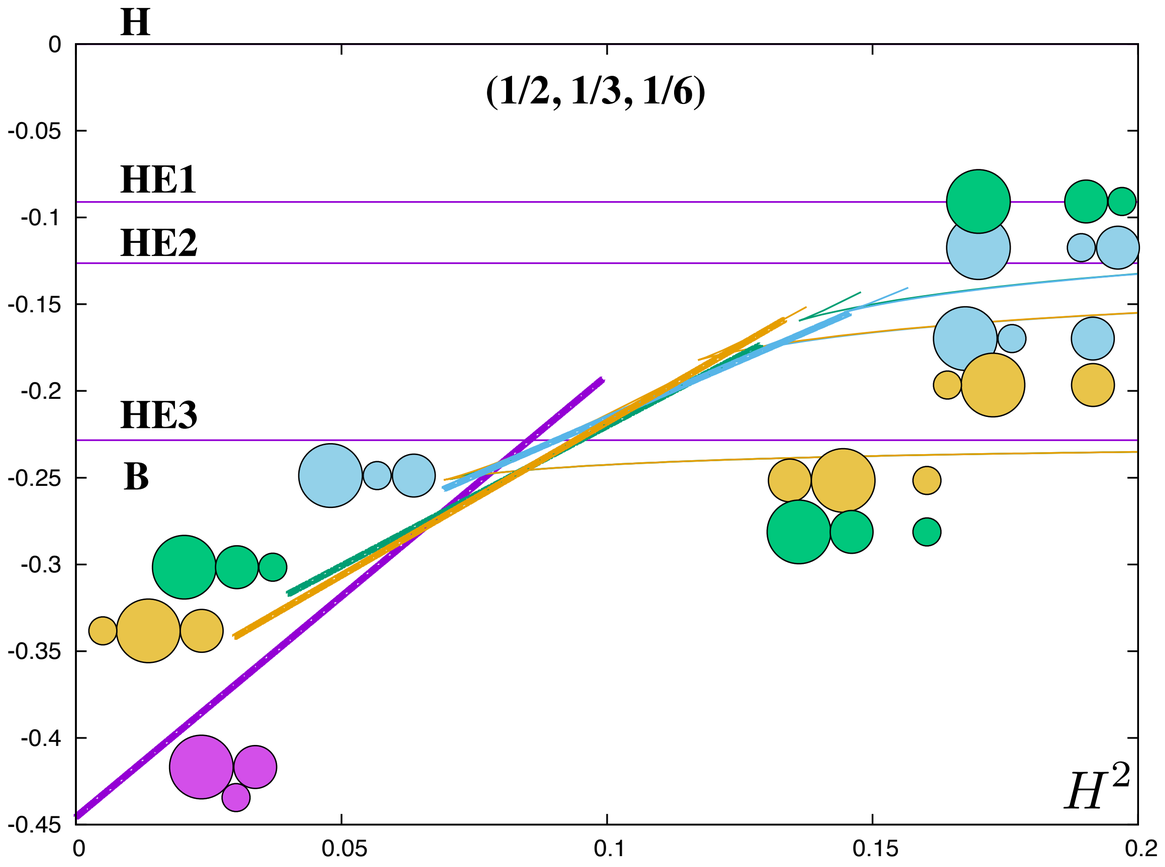


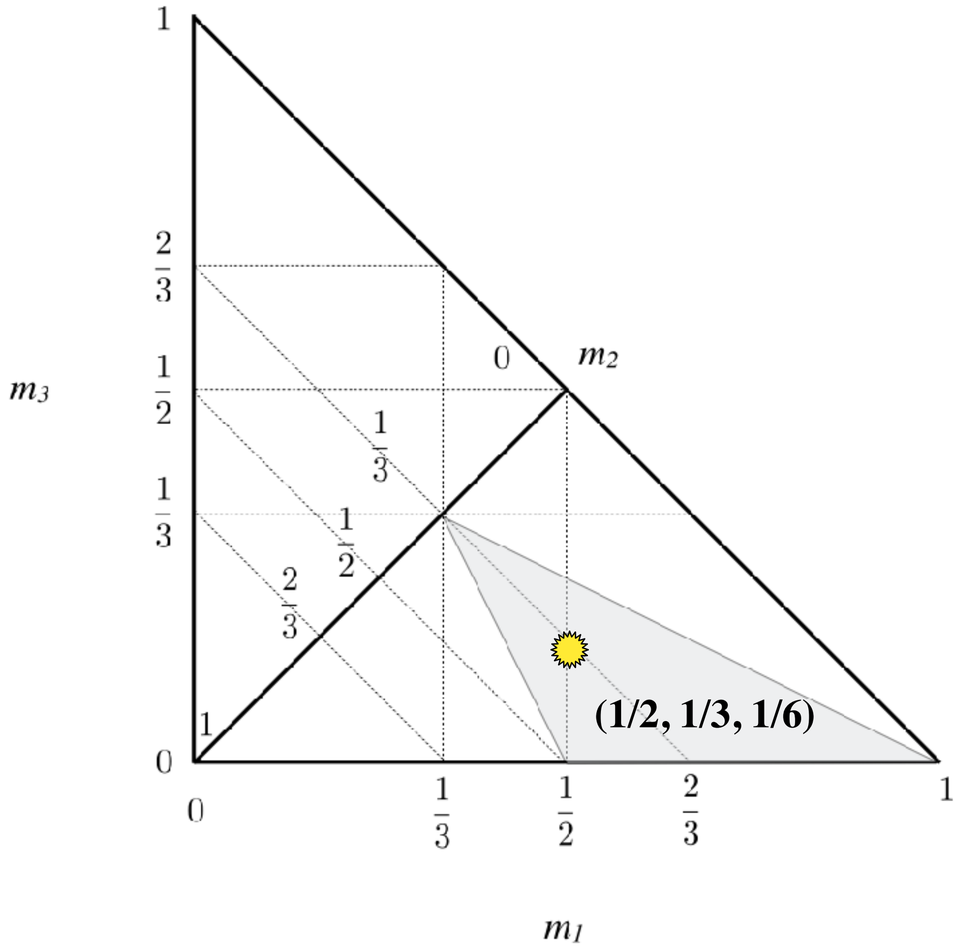
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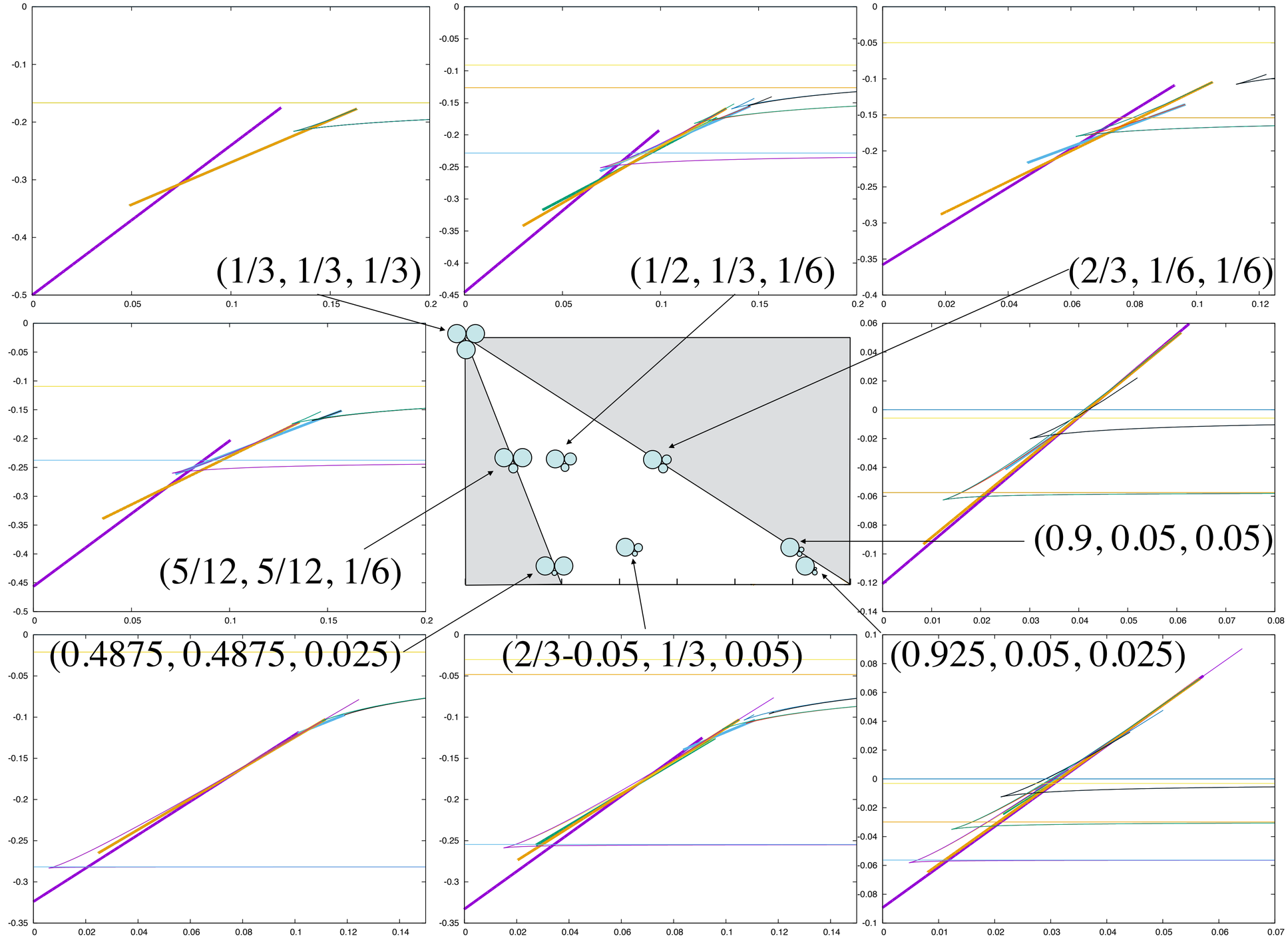

Fraction of initial conditions that end by ejecting a body
Ejection Efficiency (%)
Normalized Energy Level
H2 = 2.50
= 3.33
= 4.16
= 5.00
= 5.83
= 6.66
= 7.50
= 8.33
= 9.16
= 10.0
Increasing H2
60
50
40
30
20
10
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

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Mass Ratio Variations Change Energy Thresholds for Escape



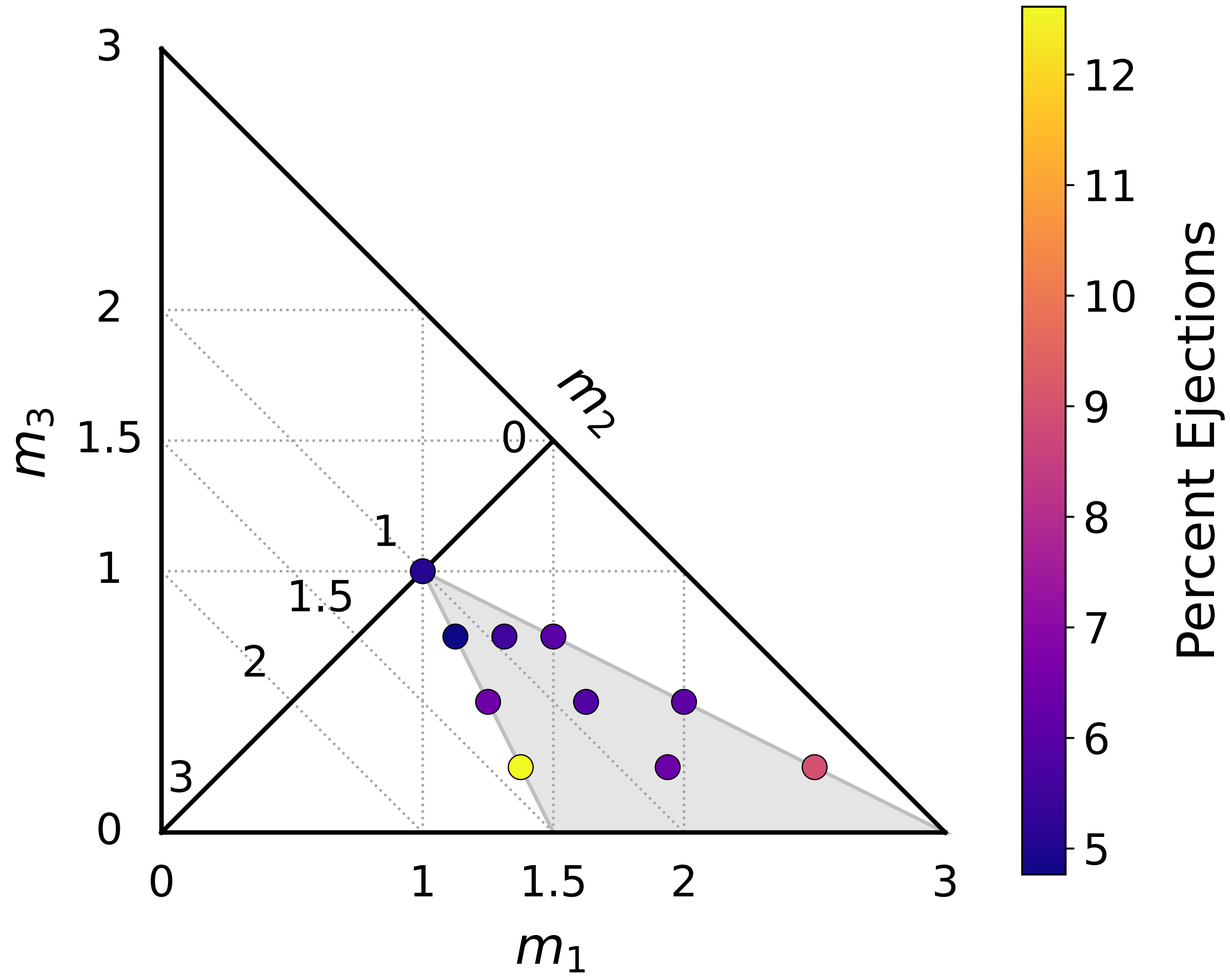



(1/3, 1/3, 1/3)
(1/2, 1/3, 1/6)
(2/3, 1/6, 1/6)
(5/12, 5/12, 1/6)
(0.9, 0.05, 0.05)
(0.4875, 0.4875, 0.025)
(2/3-0.05, 1/3, 0.05)
(0.925, 0.05, 0.025)

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Ejection Efficiency Sensitivity to Mass Distribution

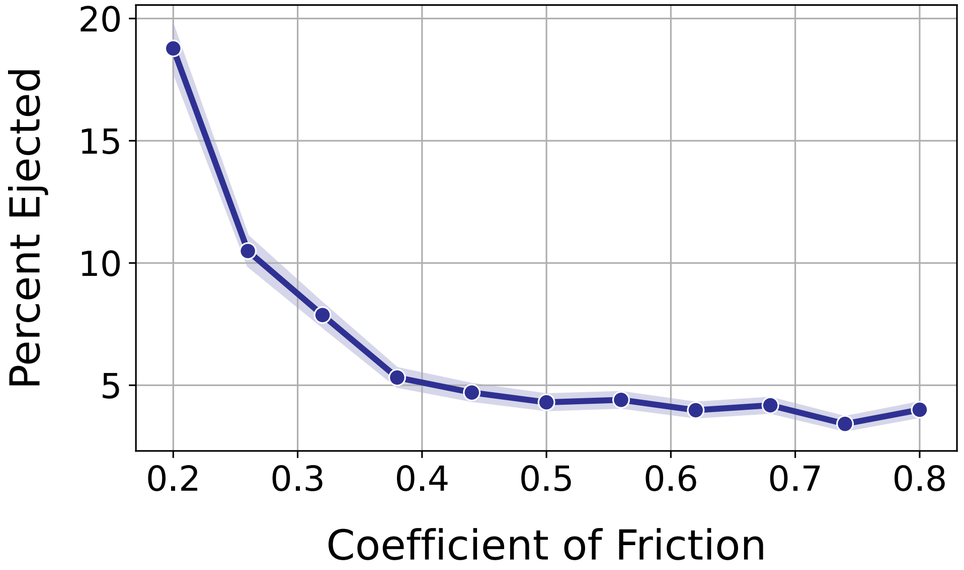
Chosen for $H^2 = 20/3$ and $E = -1/6$

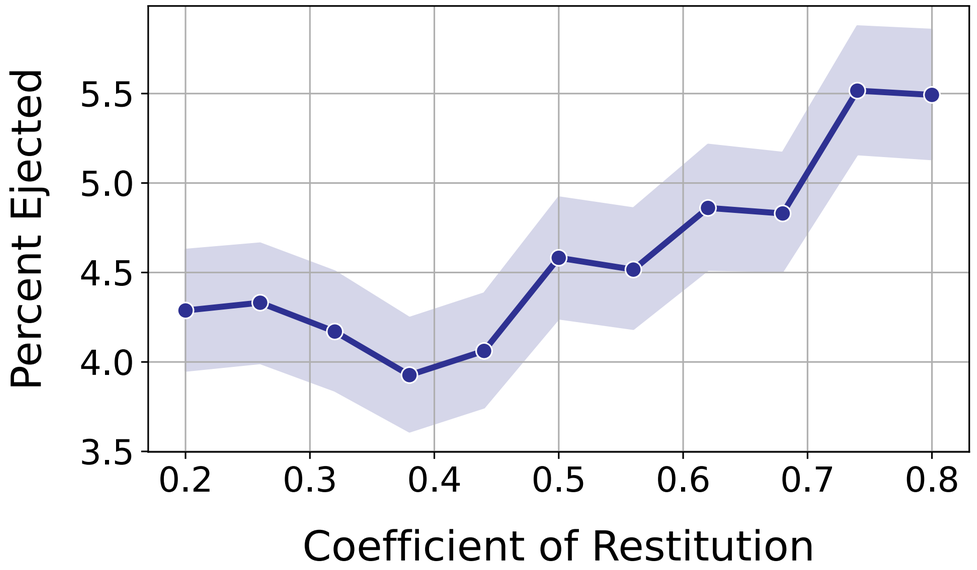


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Ejection Efficiency Sensitivity to Model Parameters

Chosen for $H^2 = 20/3$ and $E = -1/6$





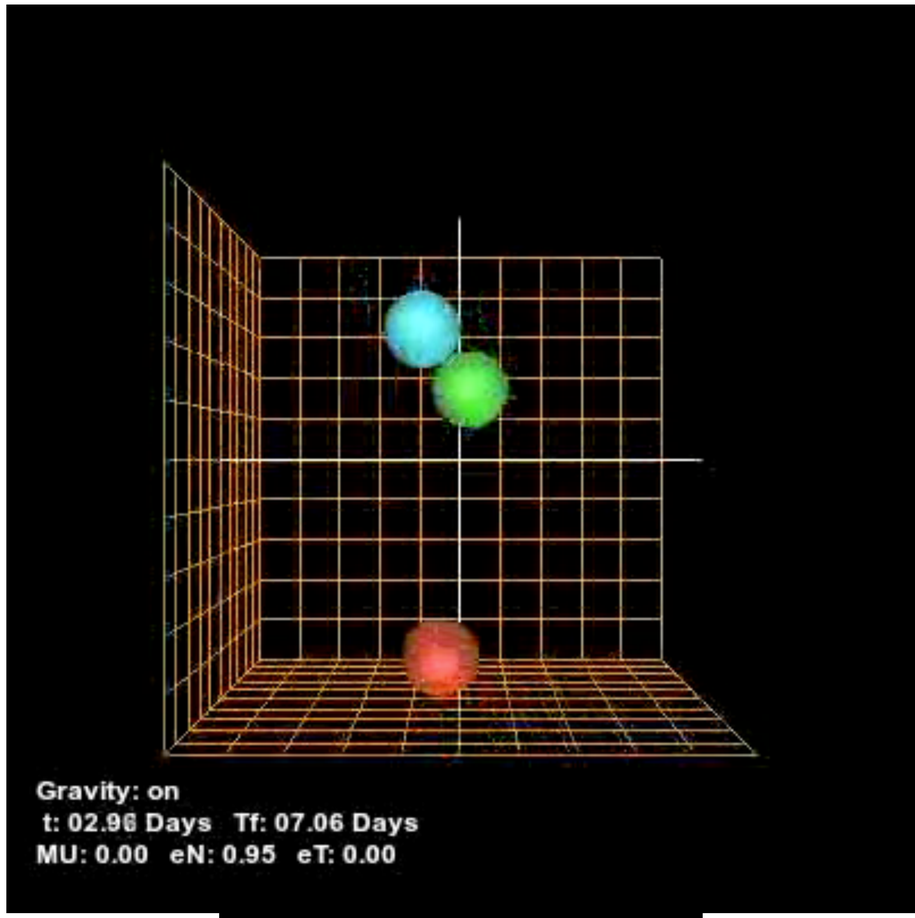
Sensitivity with respect to model parameters is also seen, but fit with the model that increased energy dissipation leads to decreased ejection efficiency

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Conclusions and Future Work

- The mechanics of multi-component asteroid systems are studied as interacting rigid bodies
  - Current studies assume the system is at an early stage of its life
  - Future studies will also look at the evolution of a collapsed system
- Simple 3-body interactions efficiently create asteroid pairs, contact binaries and binary asteroids
  - Across a large range of initial angular momentum and energy we find ejection efficiencies from 5-50%
- Future simulations will explore:
  - Additional bodies
  - Different relative sizes and shapes
  - Initial conditions driven by formation and evolutionary states

Dissipating 3-body interaction
Will end up as  or as 



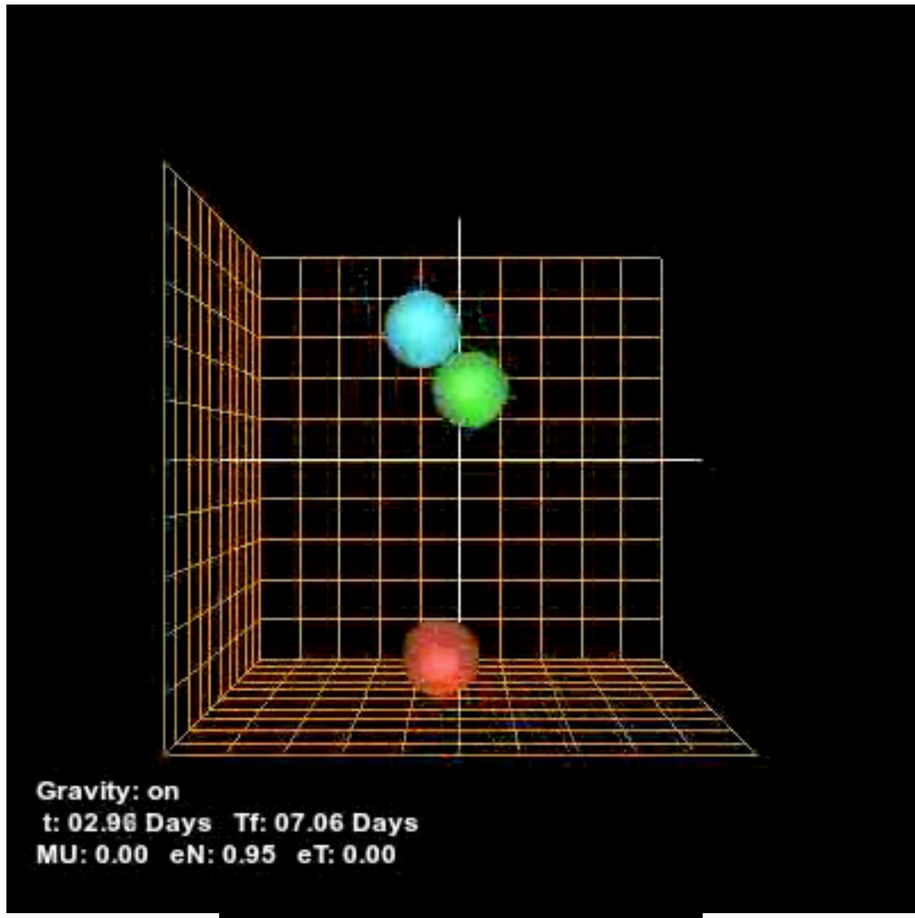
Movie by T. Gabriel

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Supporting Papers



– D.J. Scheeres. 2025. “An Energy-Angular Momentum Phase Function for Rubble Pile Asteroids,” Icarus 436 (2025) 116563.

– A.J. Meyer and D.J. Scheeres. 2024. “The Strength and Shapes of Contact Binary Objects,” Astrophysics Journal Letters 963:L14.

– D.J. Scheeres & G. Brown. 2023. “Bounds on Energy and Angular Momentum Loss in the Full n-Body Problem,” Celestial Mechanics and Dynamical Astronomy 132:4.

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– D.J. Scheeres. 2018. “Stability of the Euler Resting N-Body Relative Equlilbria,” Celestial Mechanics and Dynamical Astronomy 130:26.

– D.J. Scheeres. 2017. “Constraints on Bounded Motion and Mutual Escape for the Full 3-Body Problem,” Celestial Mechanics and Dynamical Astronomy 128(2-3): 131-148.

– T. Gabriel and D.J. Scheeres. 2016. “Energy Dissipation End States of the Sphere Restricted Planar Three Body Problem with Collisional Interaction,” Monthly Notices of the Royal Astronomical Society 463, 794-801.

– D.J. Scheeres. 2016. “Relative Equilibria in the Spherical, Finite Density 3-Body Problem,” Journal of Nonlinear Science 26: 1445-1482.

– D.J. Scheeres. 2012. “Minimum Energy Configurations in the N-Body Problem and the Celestial Mechanics of Granular Systems,” Celestial Mechanics and Dynamical Astronomy 113: 291-320.