The 2:1 resonance between Ariel and Umbriel in a fast-migration scenario of the Uranian system

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Joint work with G. Lari, M. Falletta, C. Grassi, S. R. A. Gomes, & M. Saillenfest

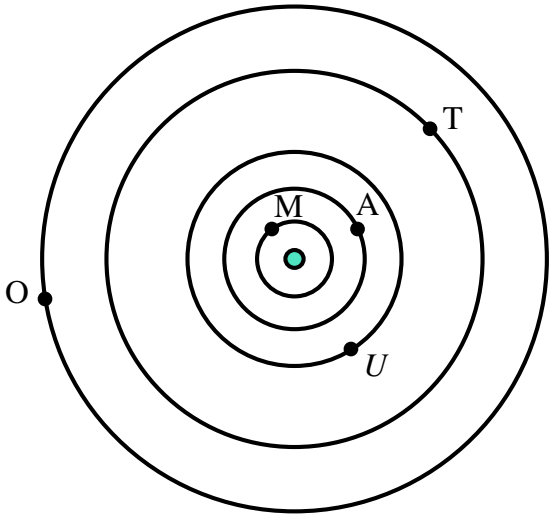
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The Uranian satellite system



Miranda Ariel Umbriel Titania Oberon



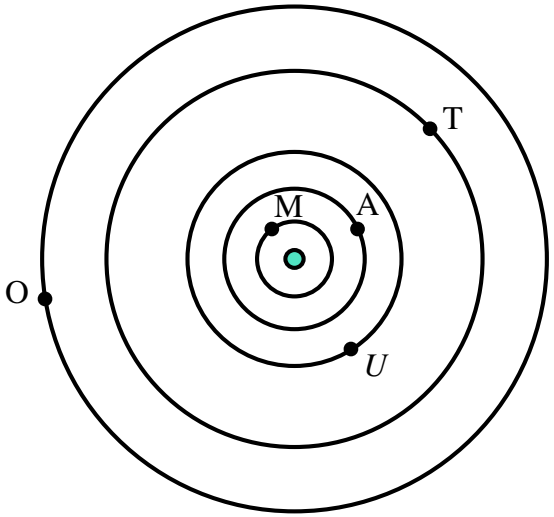
Moon	$a(R_U)$	e	$I(^\circ)$	$m(\times 10^{-5} m_U)$
Miranda (1)	5.081	0.0014	4.407	0.075
Ariel (2)	7.470	0.0012	0.017	1.441
Umbriel (3)	10.407	0.0039	0.080	1.469
Titania (4)	17.070	0.0012	0.113	3.917
Oberon (5)	22.828	0.0014	0.148	3.544

Mean values at J2000 epoch, Uranus's equatorial frame
(*Jacobson 2014*; *Gomes & Correia 2024*)

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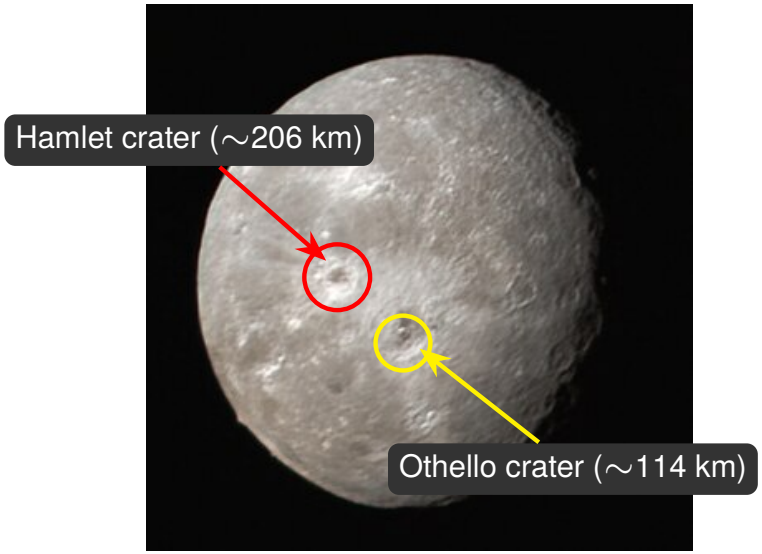
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Present-day state of the moons

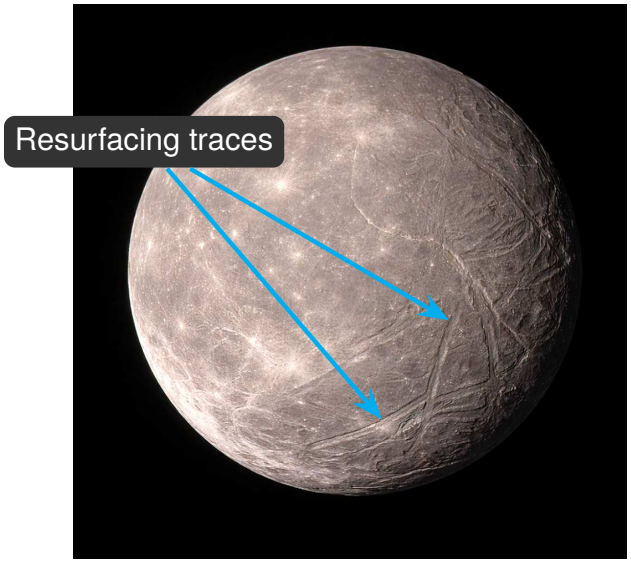
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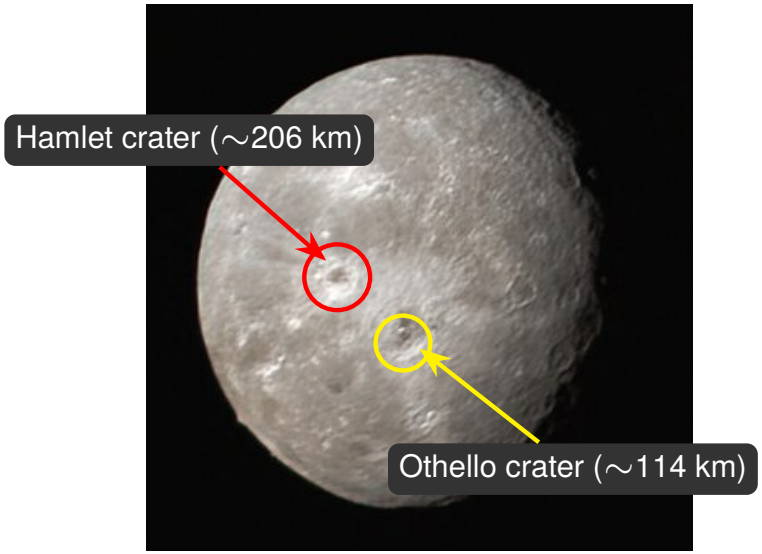
Oberon



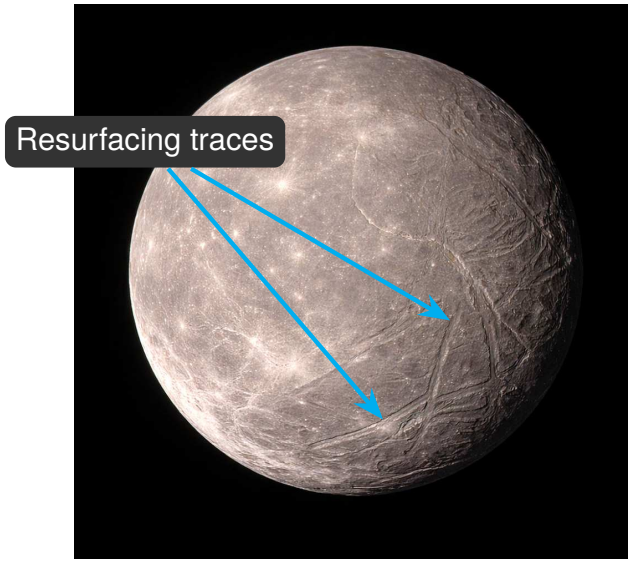
Ariel

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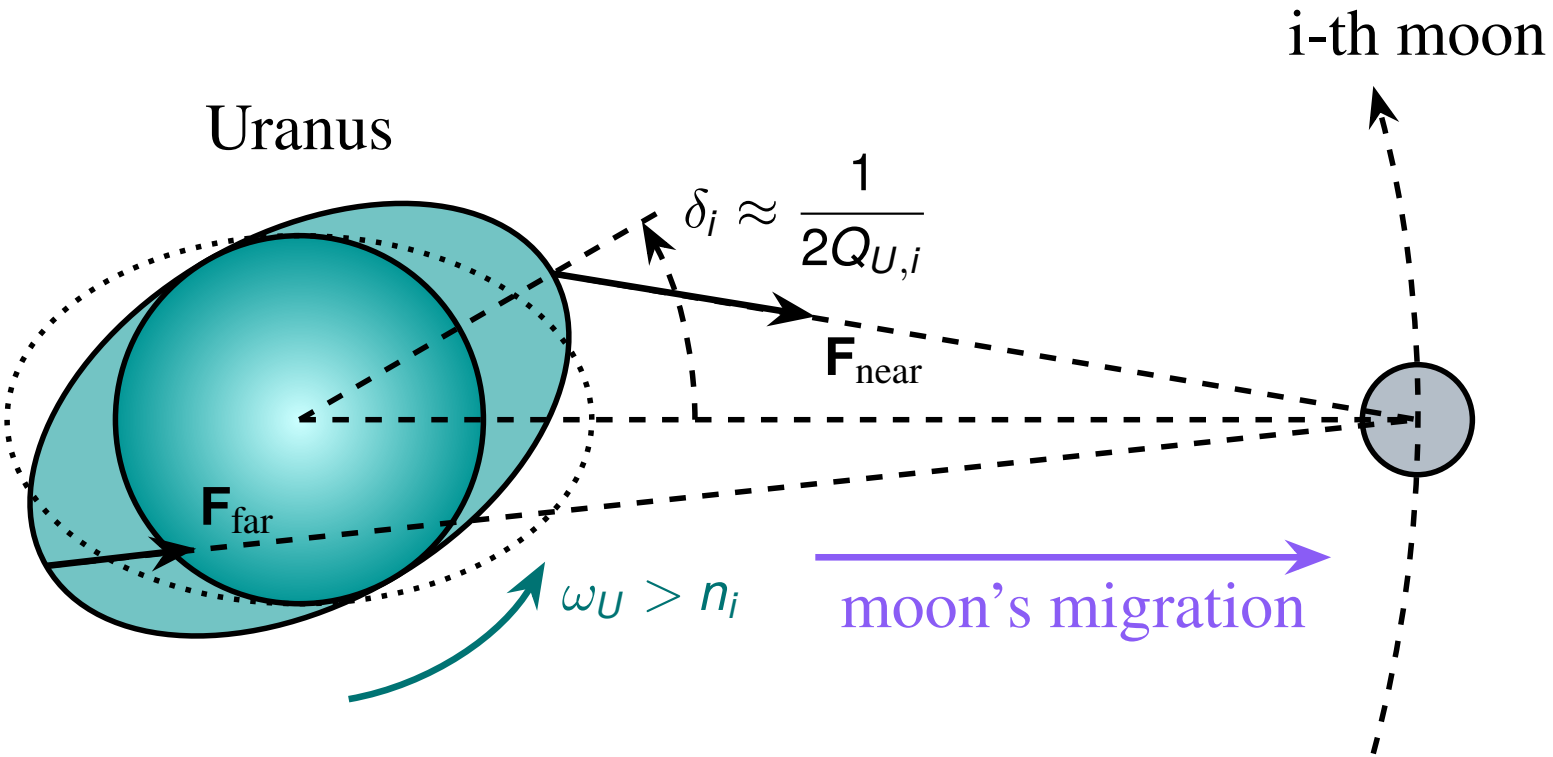
Ariel

Explain:

- current **free eccentricities** and **inclinations** of the moons;
- **resurfacing events** (especially for Ariel).

Tides on the planet

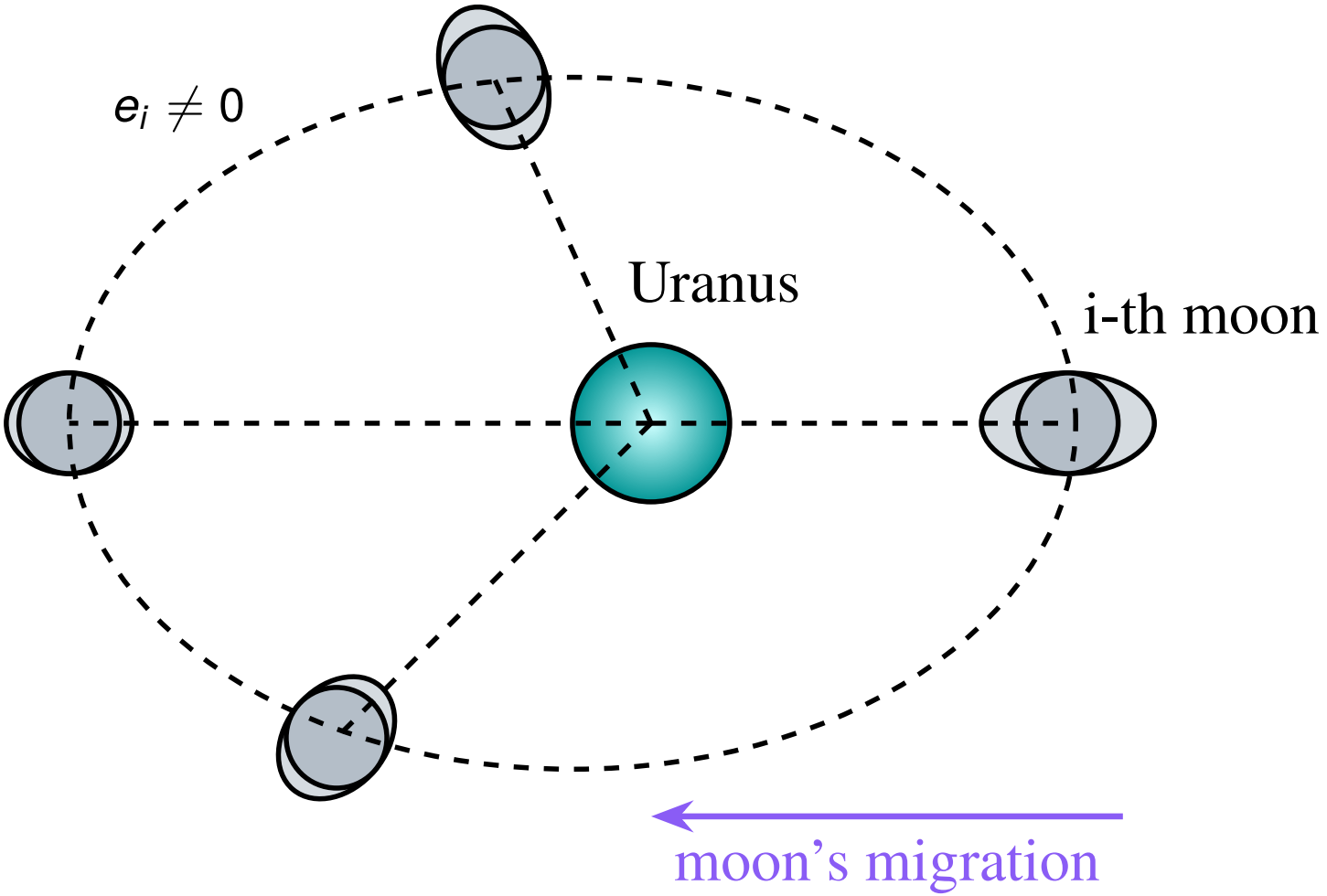
Tides on Uranus ($Q_{U,i}$): i-th moon's momentum gain from the planet's spin → main effect → $a_i \nearrow$



Tides on the satellite

Tides on moon (Q_i): moon's energy loss because of internal friction in an eccentric orbit

main effect → $e_i \searrow$ + heat



Past and recent literature

Previous studies (e.g., *Tittemore & Wisdom 1988-'89-'90*) adopted a **medium-high** $Q_U = 10\,000 - 39\,000$ to avoid the 2:1 A-U MMR.

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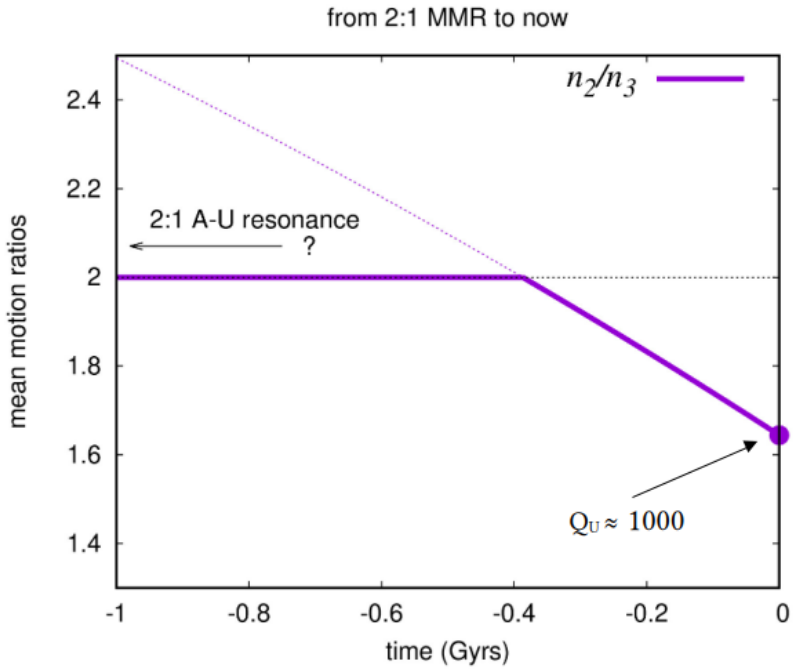
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Crossing 2:1 A-U MMR highly probable, about 200 – 600 Myrs ago

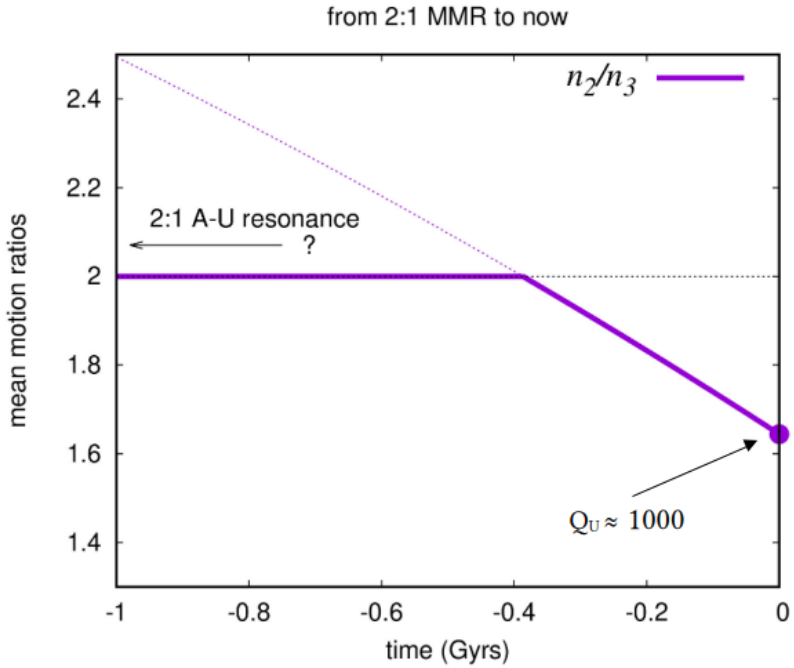


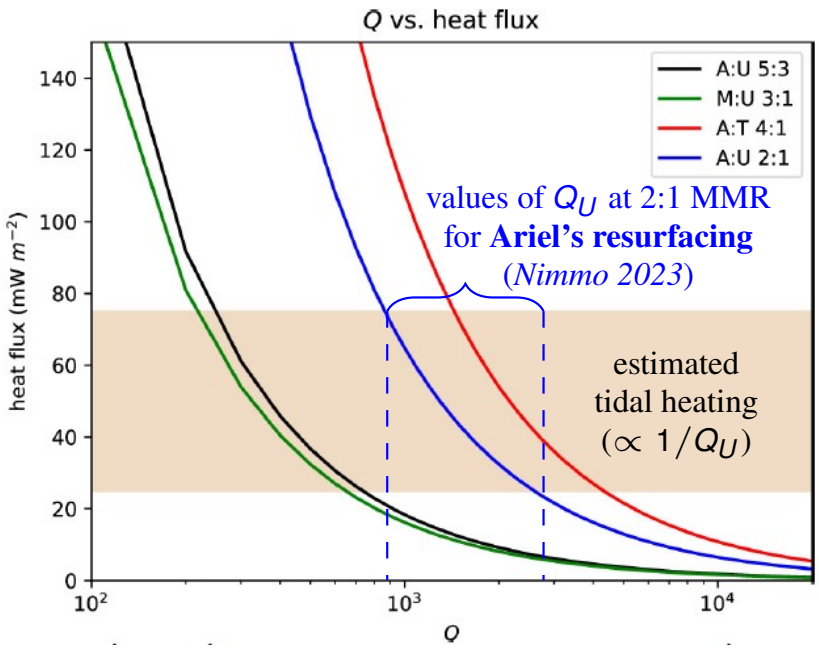
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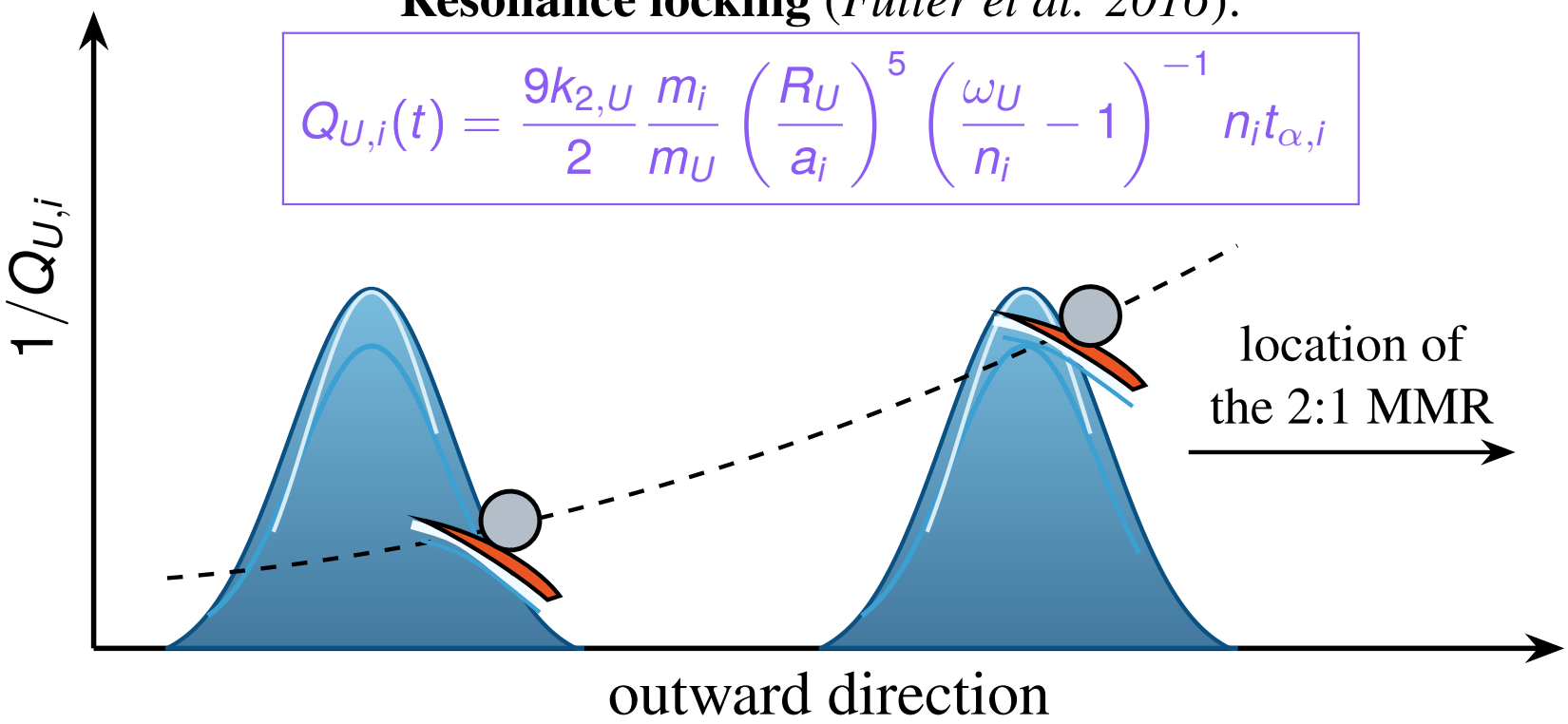
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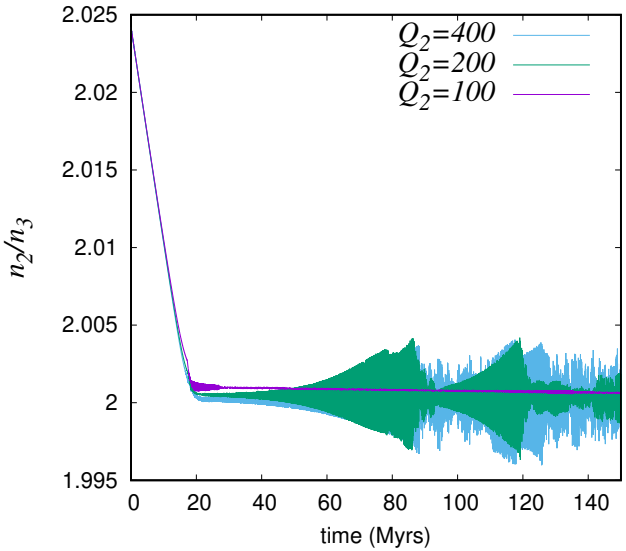
Resonance locking (*Fuller et al. 2016*):

$$Q_{U,i}(t) = \frac{9k_{2,U}}{2}\frac{m_i}{m_U}\left(\frac{R_U}{a_i}\right)^5\left(\frac{\omega_U}{n_i}-1\right)^{-1} n_i t_{\alpha,i}$$

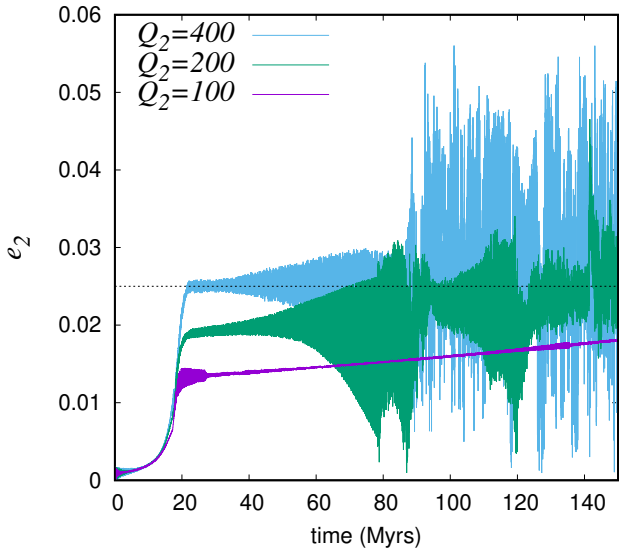


2:1 Ariel-Umbriel MMR revisited

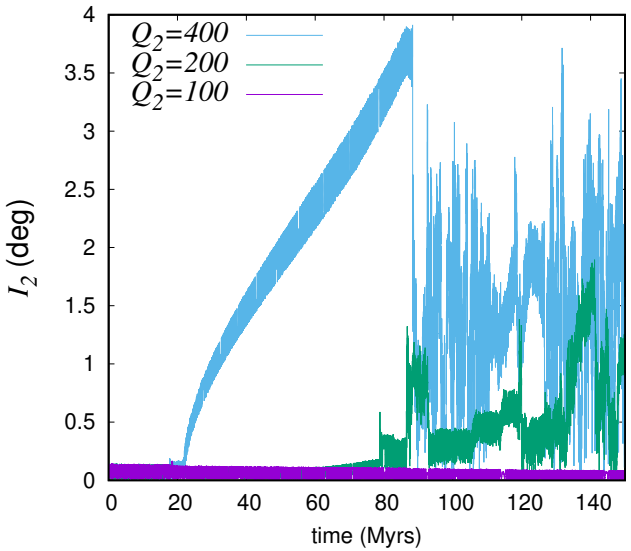
A-U mean motion ratio



eccentricity of Ariel



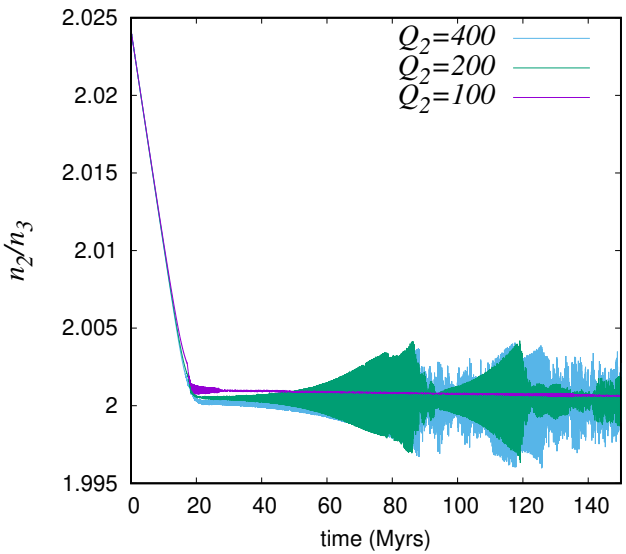
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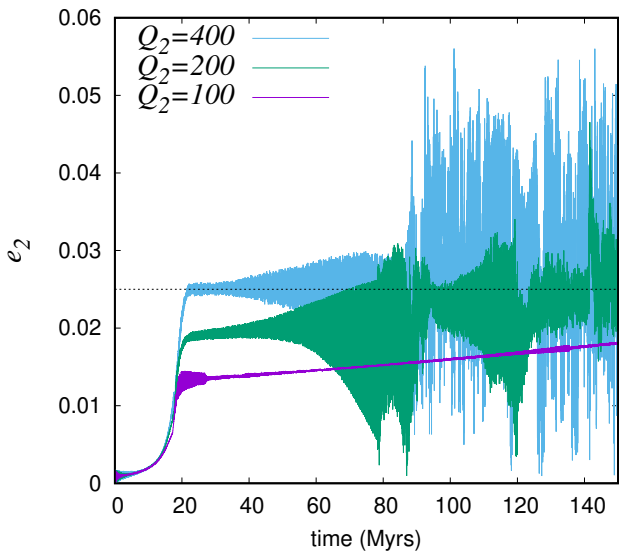
Resonance-locking simulations parametrized by Ariel's quality factor Q_2.

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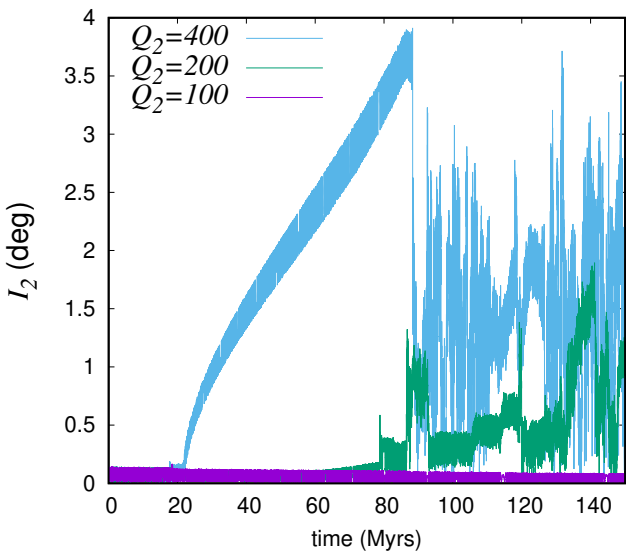
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Resonance-locking simulations parametrized by Ariel's quality factor Q_2.

Regardless of the value of Q_2 and the type of evolution, **the 2:1 A-U MMR persists** after capture (cf. also *Tittemore & Wisdom 1990*).

Breaking the 2:1 resonance

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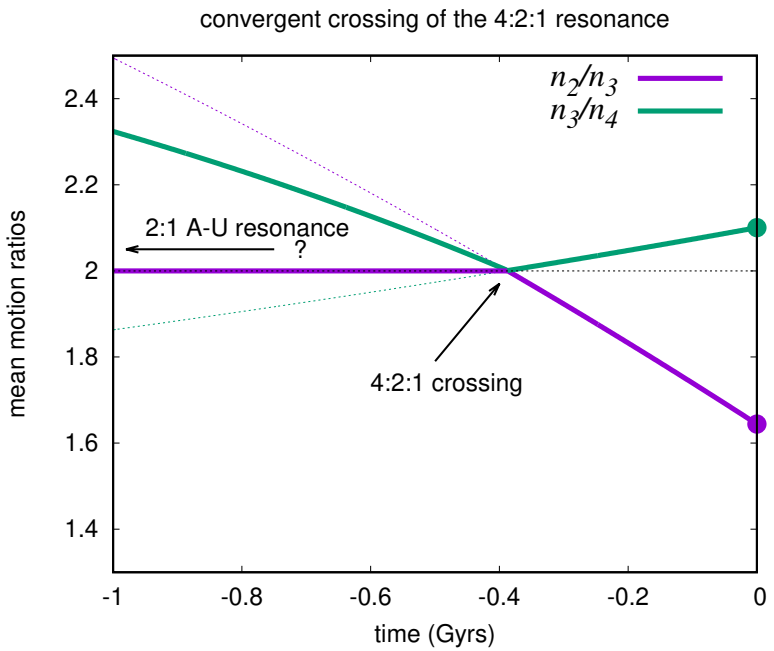
Peale (1987): disruption of the resonance by the **perturbation of Titania**.

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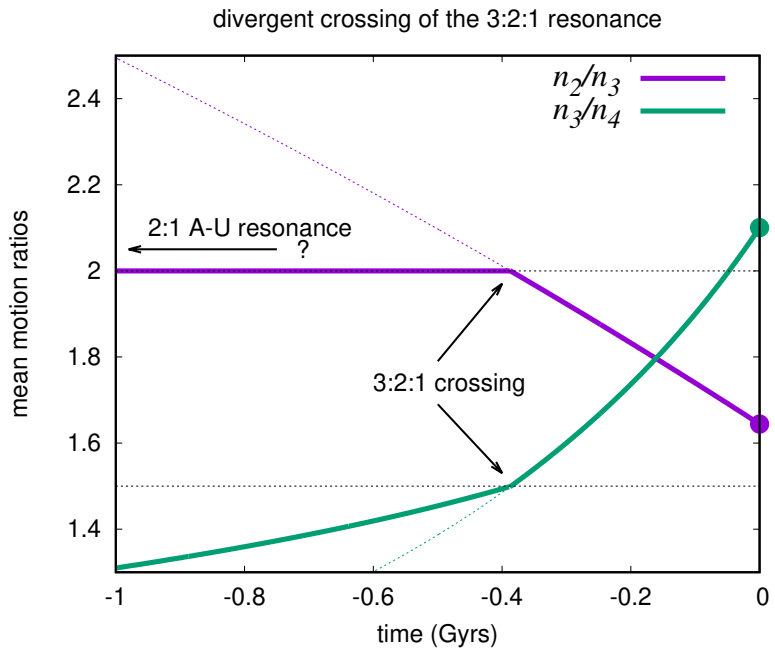
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Peale (1987): disruption of the resonance by the **perturbation of Titania**. ➡ We investigated **two possibilities** (need of **low** $Q_{U,4}$):

4:2:1 A-U-T



3:2:1 A-U-T



Dynamical model

To this purpose, we developed an **averaged model** that includes:

- all the 5 satellites;
- a 3-body MMR chain (namely, Ariel-Umbriel-Titania);
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We start from the Hamiltonian in planetocentric coordinates

$$\mathcal{H} = \mathcal{H}_K + \mathcal{H}_I + \mathcal{H}_D + \mathcal{H}_O \ ,$$

where

- $\mathcal{H}_K$ is the Keplerian part,
- $\mathcal{H}_I$ and $\mathcal{H}_D$ are the indirect and direct parts,
- $\mathcal{H}_O$ is the effect of the non-sphericity of Uranus (J_2).

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- write $\mathcal{H}$ in elliptical elements $(a_i, e_i, I_i, \lambda_i, \varpi_i, \Omega_i)$, $i = 1, \ldots, 5$;

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$$\sigma_j = \lambda_j \, (j = 1, 4, 5) \, , \; \sigma_2 = 2\lambda_3 - \lambda_2 \, , \; \sigma_3 = \begin{cases} 2\lambda_4 - \lambda_3 & (4{:}2{:}1) \\ 3\lambda_4 - 2\lambda_3 & (3{:}2{:}1) \end{cases} ;$$

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- take the average over fast angles λ_i except σ_2, σ_3:

$$\langle \mathcal{H} \rangle_{\sigma_2, \sigma_3} = \frac{1}{(2\pi)^5} \int_{\mathbb{T}^5} \mathcal{H} \mathrm{d}\lambda_1 \ldots \mathrm{d}\lambda_5 \; ;$$

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- pass to Poincaré variables (y_i, v_i, x_i, u_i) and complete the **canonical transformation** with conjugate pairs (σ_i, Σ_i).

Dynamical model

Setting $\beta_i = m_U m_i/(m_U + m_i)$, the four contributions read

$$\langle \mathcal{H}_K \rangle_{\sigma_2,\sigma_3} = -\sum_{i=1}^{5} \frac{\mathcal{G} m_0 m_i}{2a_i} ,$$

$$\langle \mathcal{H}_O \rangle_{\sigma_2,\sigma_3} = \sum_{i=1}^{5} \frac{\mathcal{G} m_0 m_i}{a_i} \left[J_2 \left(\frac{R_0}{a_i} \right)^2 \frac{1}{4} (12 s_i^2 - 3e_i^2 - 2) \right] ,$$

$$\langle \mathcal{H}_I \rangle_{\sigma_2,\sigma_3} = \begin{cases} \dfrac{\beta_2 a_2 n_2 \beta_3 a_3 n_3}{m_0} e_3 \cos(2\lambda_3 - \lambda_2 - \varpi_3) & \\ \quad + \dfrac{\beta_3 a_3 n_3 \beta_4 a_4 n_4}{m_0} e_4 \cos(2\lambda_4 - \lambda_3 - \varpi_4) & \text{(4:2:1)} \\ \dfrac{\beta_2 a_2 n_2 \beta_3 a_3 n_3}{m_0} e_3 \cos(2\lambda_3 - \lambda_2 - \varpi_3) & \\ \quad + \dfrac{9\beta_2 a_2 n_2 \beta_4 a_4 n_4}{8 m_0} e_4^2 \cos(3\lambda_4 - \lambda_2 - 2\varpi_4) & \text{(3:2:1)} \end{cases} ,$$

Dynamical model

$$\langle \mathcal{H}_D \rangle_{\sigma_2,\sigma_3} = \mathcal{H}_{D,sec} + \mathcal{H}_{D,2:1\mathrm{AU}} + \begin{cases} \mathcal{H}_{D,2:1\mathrm{UT}} & (4{:}2{:}1) \\ \mathcal{H}_{D,3:2\mathrm{UT}} + \mathcal{H}_{D,3:1\mathrm{AT}} & (3{:}2{:}1) \end{cases} .$$

E.g.:

$$\begin{aligned} \mathcal{H}_{D,2:1\mathrm{AU}} = -\frac{\mathcal{G}m_2m_3}{a_3} \Big(& F_0 e_2 \cos(2\lambda_3 - \lambda_2 - \varpi_2) \\ & + F_1 e_3 \cos(2\lambda_3 - \lambda_2 - \varpi_3) + F_2 e_2^2 \cos(4\lambda_3 - 2\lambda_2 - 2\varpi_2) \\ & + F_3 e_3^2 \cos(4\lambda_3 - 2\lambda_2 - 2\varpi_3) \\ & + F_4 e_2 e_3 \cos(4\lambda_3 - 2\lambda_2 - \varpi_2 - \varpi_3) \\ & + F_5 s_2^2 \cos(4\lambda_3 - 2\lambda_2 - 2\Omega_2) + F_6 s_3^2 \cos(4\lambda_3 - 2\lambda_2 - \Omega_3) \\ & + F_7 s_2 s_3 \cos(4\lambda_3 - 2\lambda_2 - \Omega_2 - \Omega_3) \Big) , \end{aligned}$$

where $F_j = F_j(\alpha_{23})$, $\alpha_{23} = a_2/a_3 < 1$, depend on Laplace coefficients.

Tidal model

Assuming synchronously rotating satellites and zero obliquity, we use **secular variations** of a_i, e_i, s_i **for the tidal dissipation** (*Malhotra 1991*; *Ferraz-Mello et al. 2008*):

$$\frac{\dot{a}_i}{a_i} = \frac{2}{3} c_i \left(1 + \left(\frac{51}{4} - 7D_i\right) e_i^2\right) ,$$
$$\frac{\dot{e}_i}{e_i} = -\frac{1}{3} c_i \left(7D_i - \frac{19}{4}\right) ,$$
$$\frac{\dot{s}_i}{s_i} = -\frac{1}{6} c_i ,$$

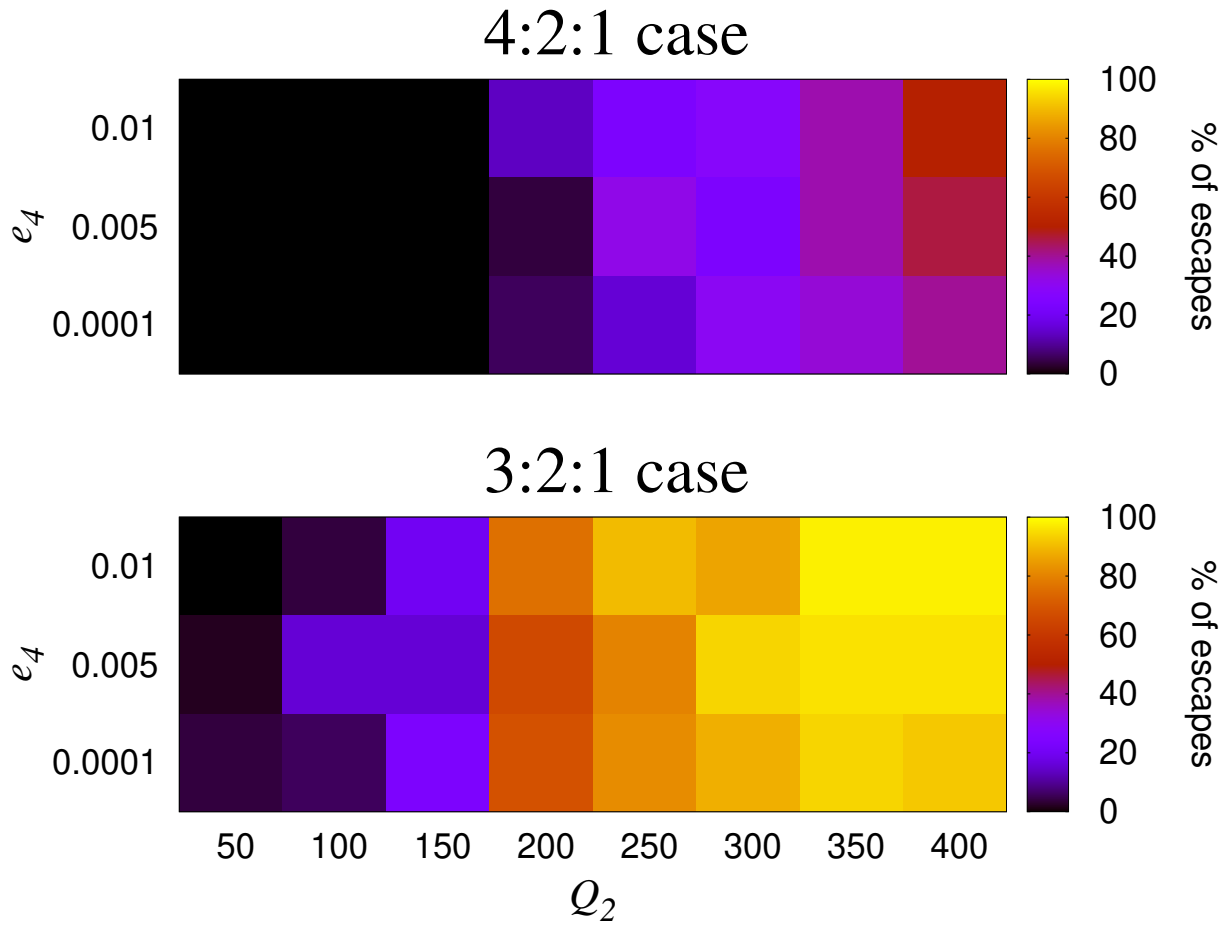
with

$$c_i = \frac{9}{2} \frac{k_{2,U}}{Q_{U,i}(t)} \frac{m_i}{m_U} \left(\frac{R_U}{a_i}\right)^5 n_i , \quad D_i = \frac{k_{2,i}}{Q_i} \frac{Q_{U,i}(t)}{k_{2,U}} \left(\frac{R_i}{R_U}\right)^5 \left(\frac{m_U}{m_i}\right)^2 .$$

These are added to Hamilton equations using canonical variables.

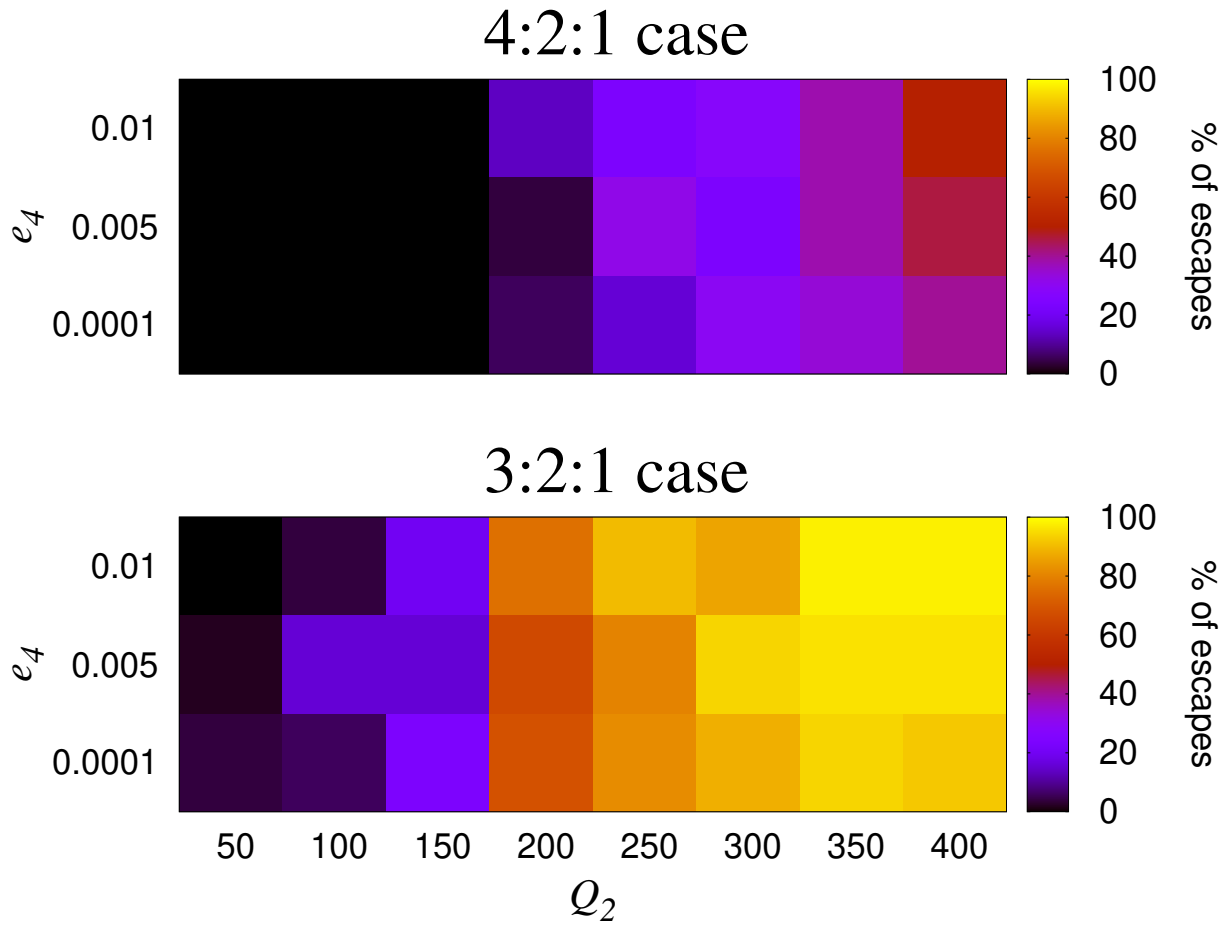
Escaping the 2:1 resonance

$e_2(0)$ and $e_3(0)$ **forced by the 2:1 MMR** ➡ Analysis on $e_4(0)$ and Q_2



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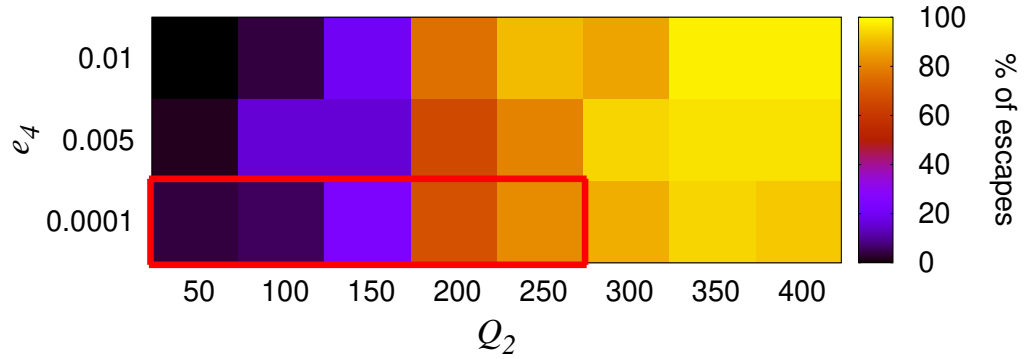
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3:2:1 better than 4:2:1: exit probability $\gtrsim 60\%$ for $Q_2 \geq 200$

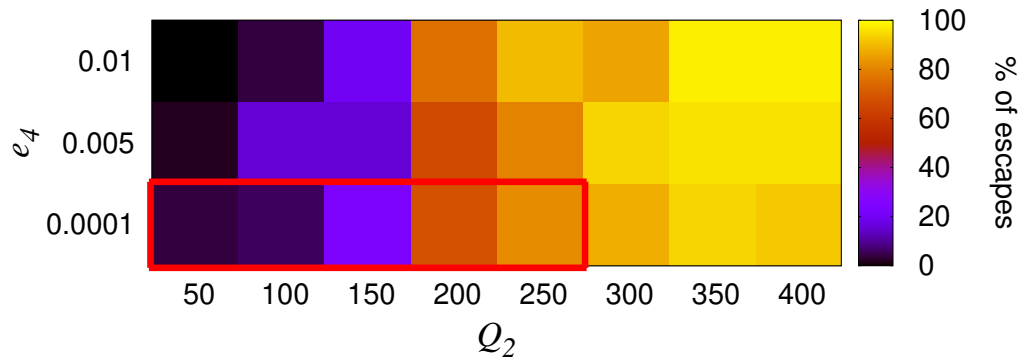
Mean orbital elements (I_i, e_i) at resonance exit

3:2:1 case ($e_4(0) = 0.0001, Q_2 \leq 250$)

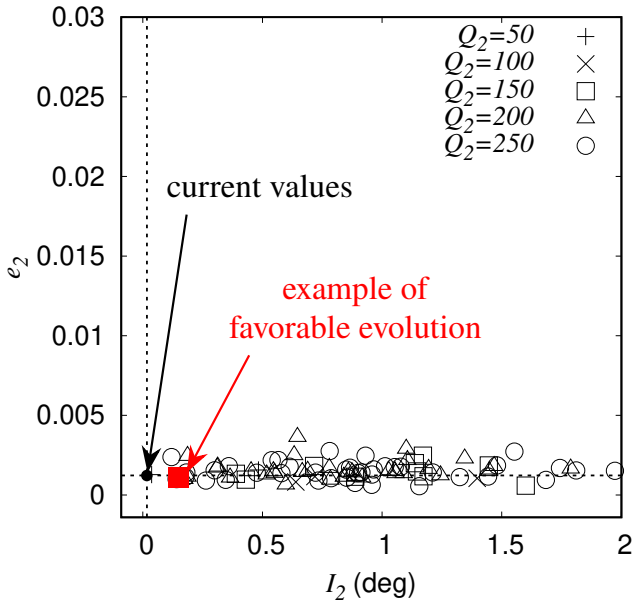


Mean orbital elements (I_i, e_i) at resonance exit

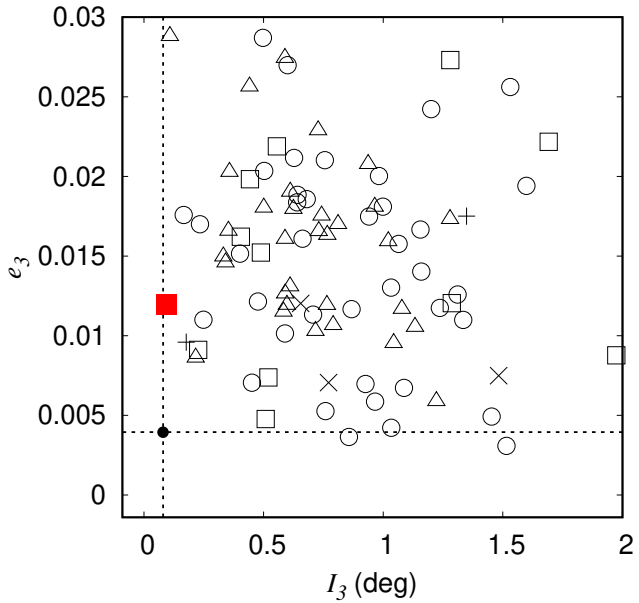
3:2:1 case ($e_4(0) = 0.0001$, $Q_2 \leq 250$)



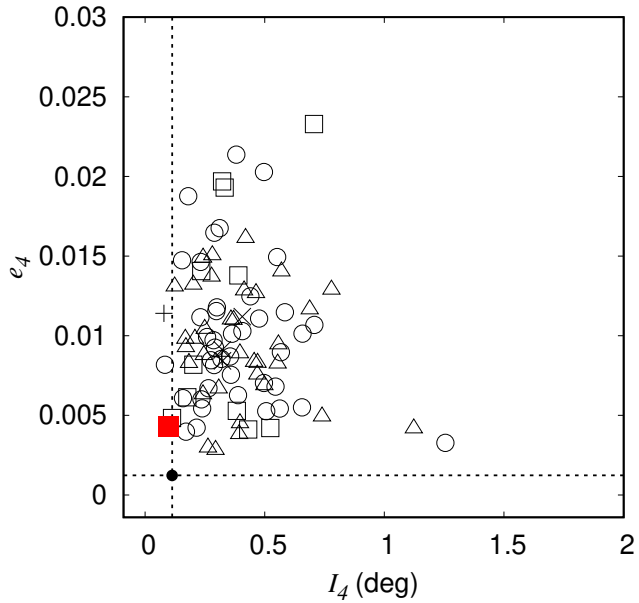
Ariel



Umbriel

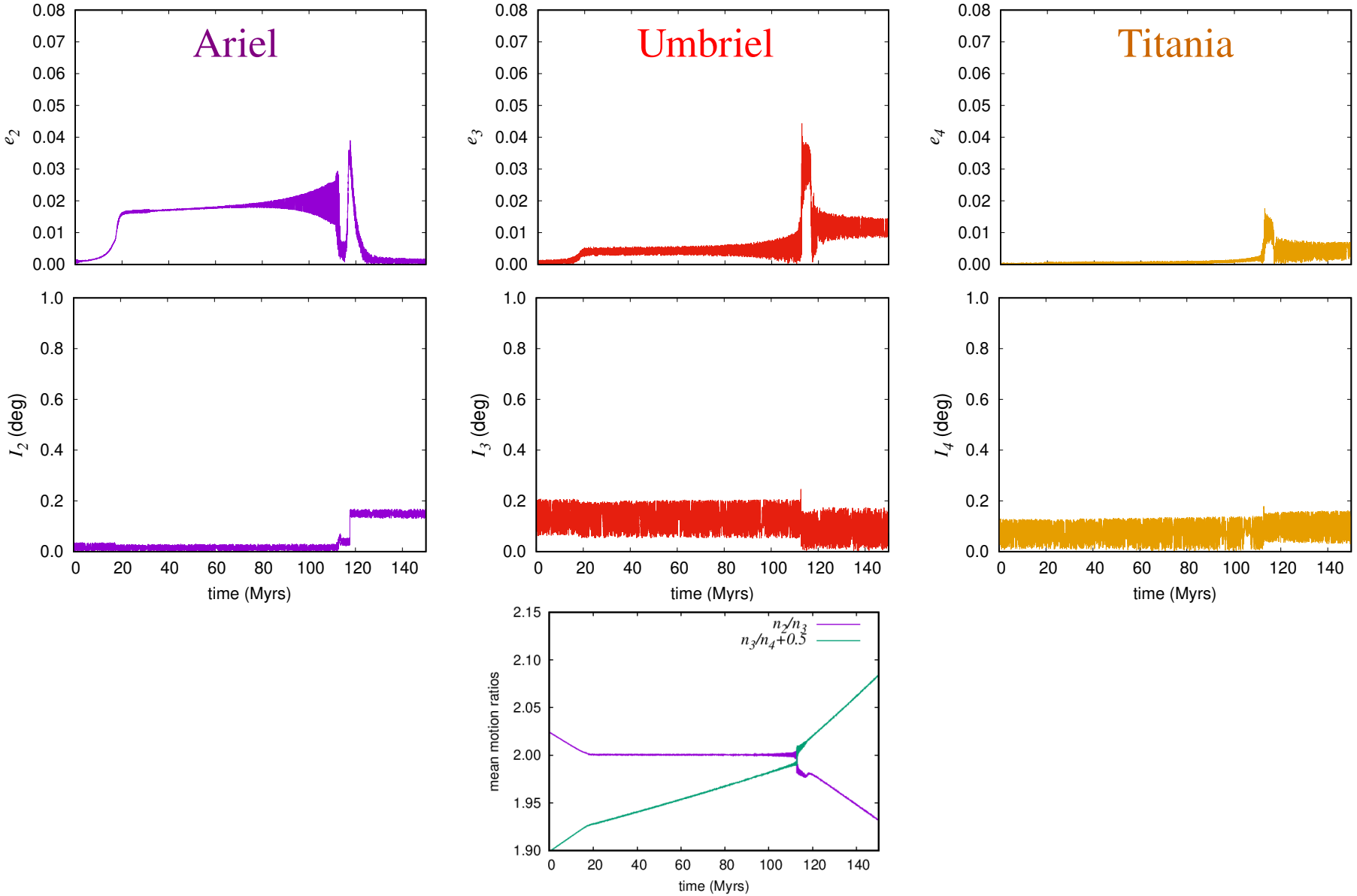


Titania



Example of favorable evolution in the 3:2:1 case

eccentricities and inclinations ($Q_2 = 150$)



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For more details: *Rossi, M., et al. "Capture and escape from the 2:1 resonance between Ariel and Umbriel in a fast-migration scenario of the Uranian system". A&A 705 (2026): A48.*

