Detecting Hamiltonian chaos from scalar time series via recurrence plot divergence

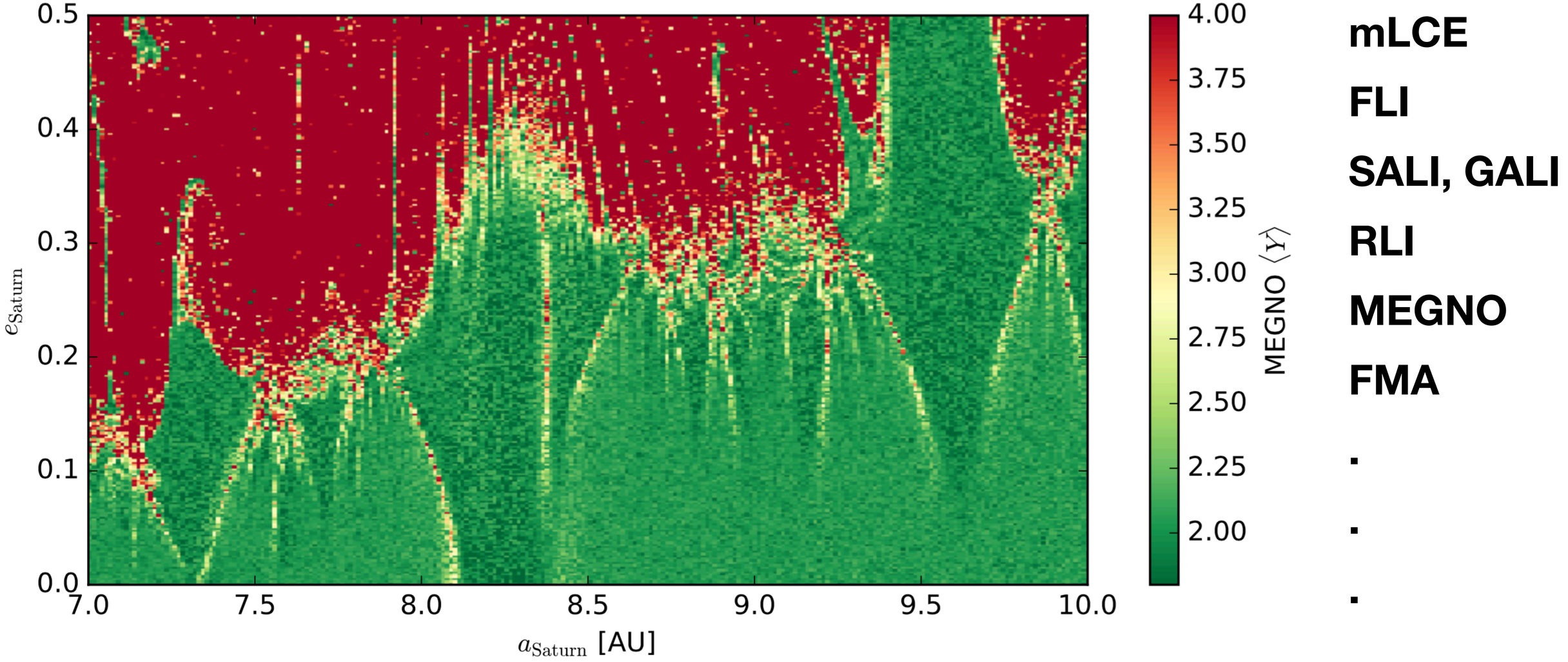
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[2]Eötvös University, Budapest

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Daquin, J., Kovács, T.; Eur. Phys. J. Spec. Top. (2026) [https://doi.org/10.1140/epjs/s11734-026-02183-4]

Sun - Jupiter - Saturn (2D)



mLCE

FLI

SALI, GALI

RLI

MEGNO

FMA

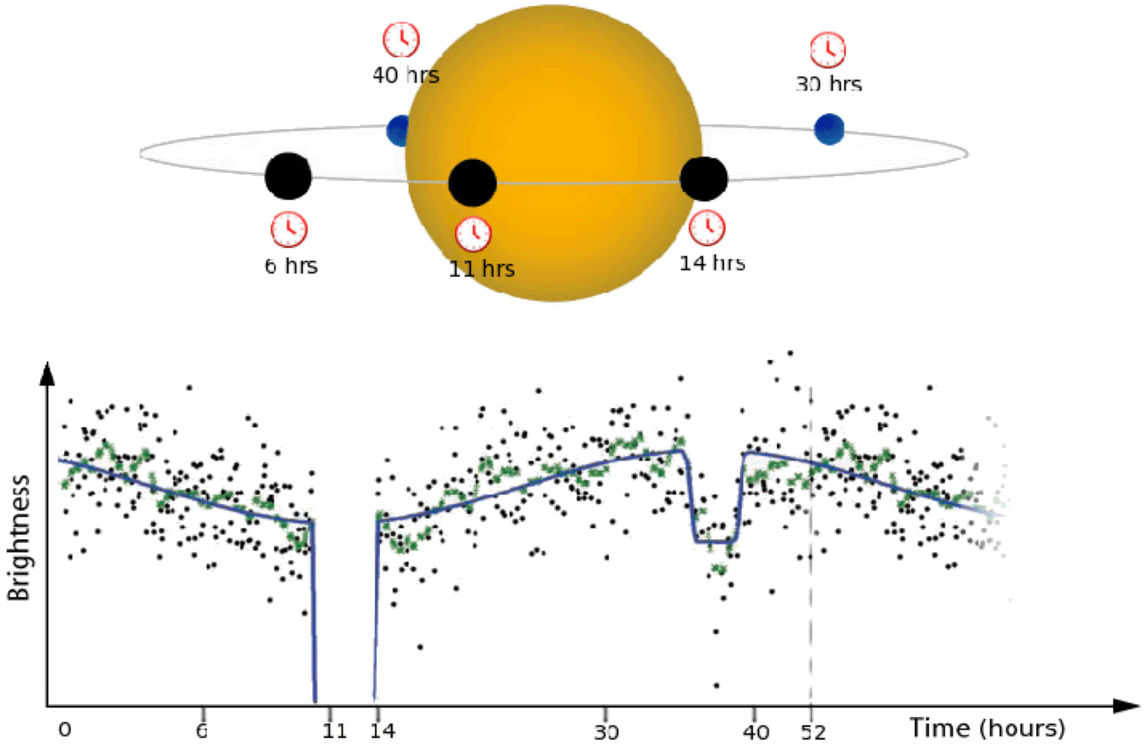
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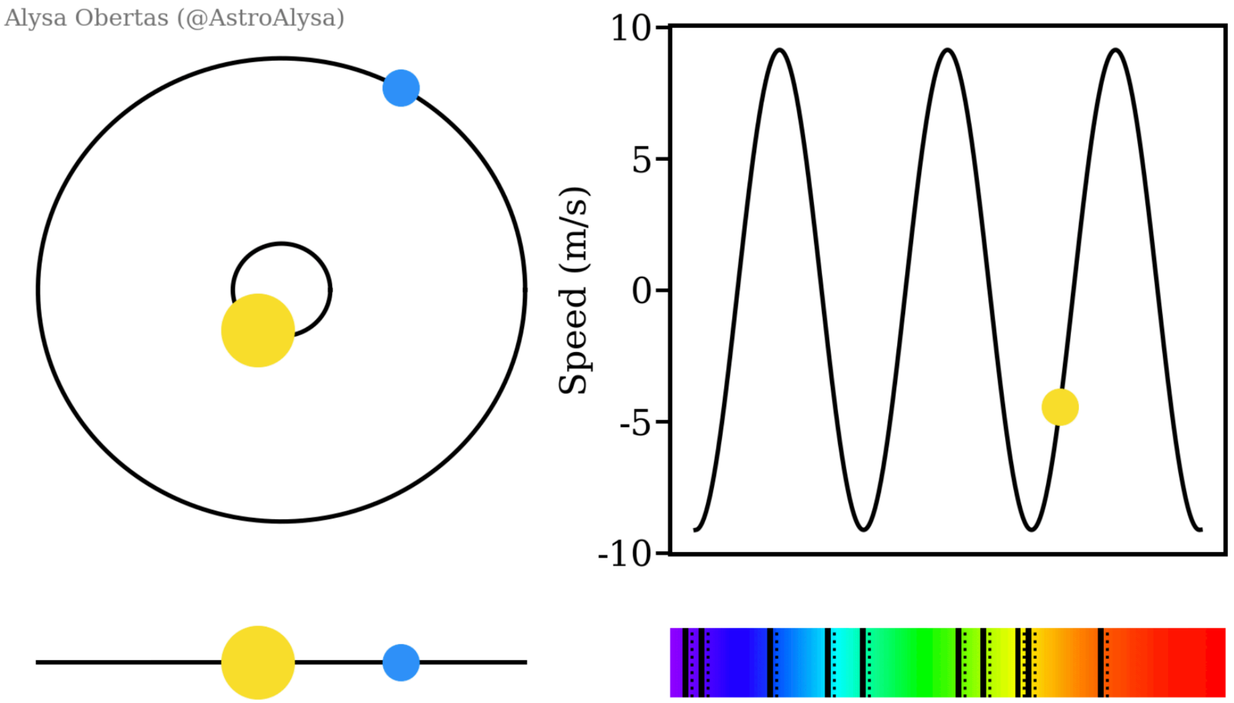
Observations

transit time signal



https://arxiv.org/pdf/2506.22839

radial velocity measurement

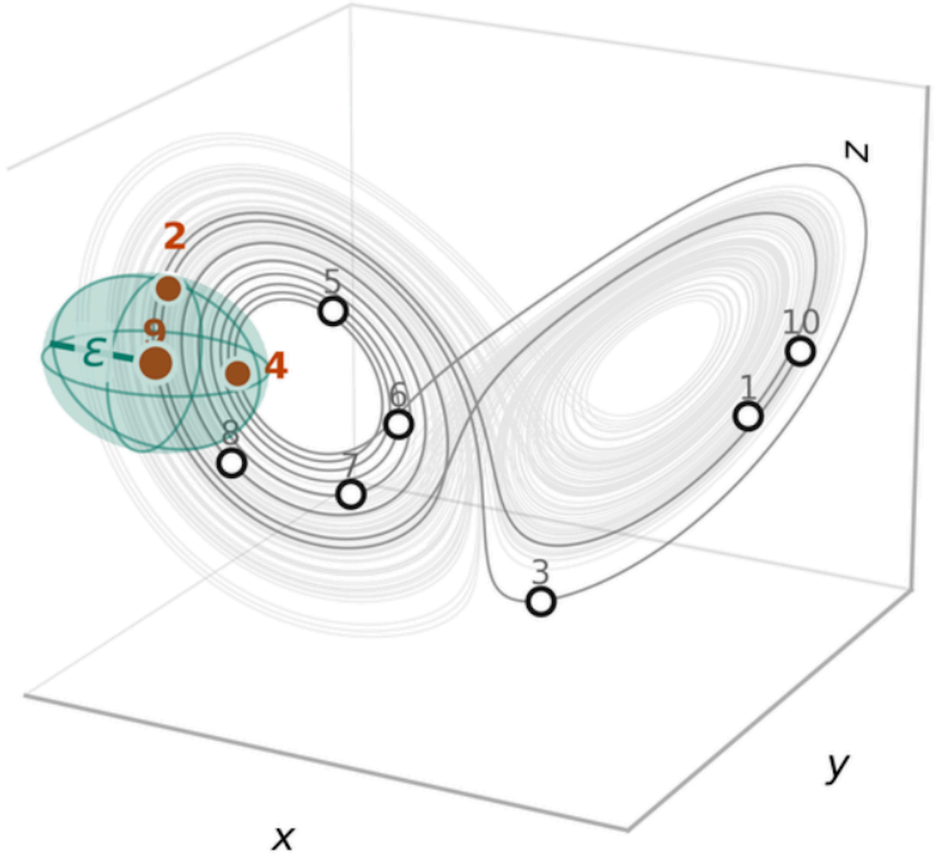


https://www.communitystories.ca

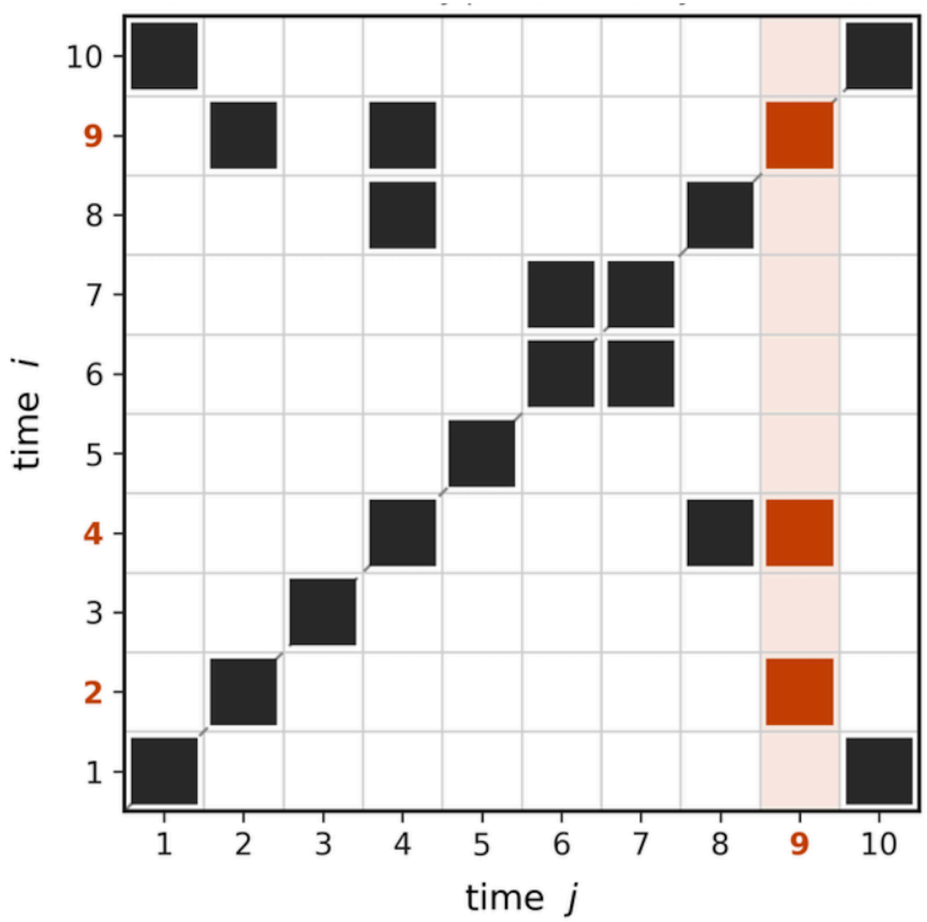
Recurrence plots

$$R_{i,j}(\epsilon) = \Theta(\epsilon - ||\mathbf{x}_i - \mathbf{x}_j||)$$

phase space trajectory

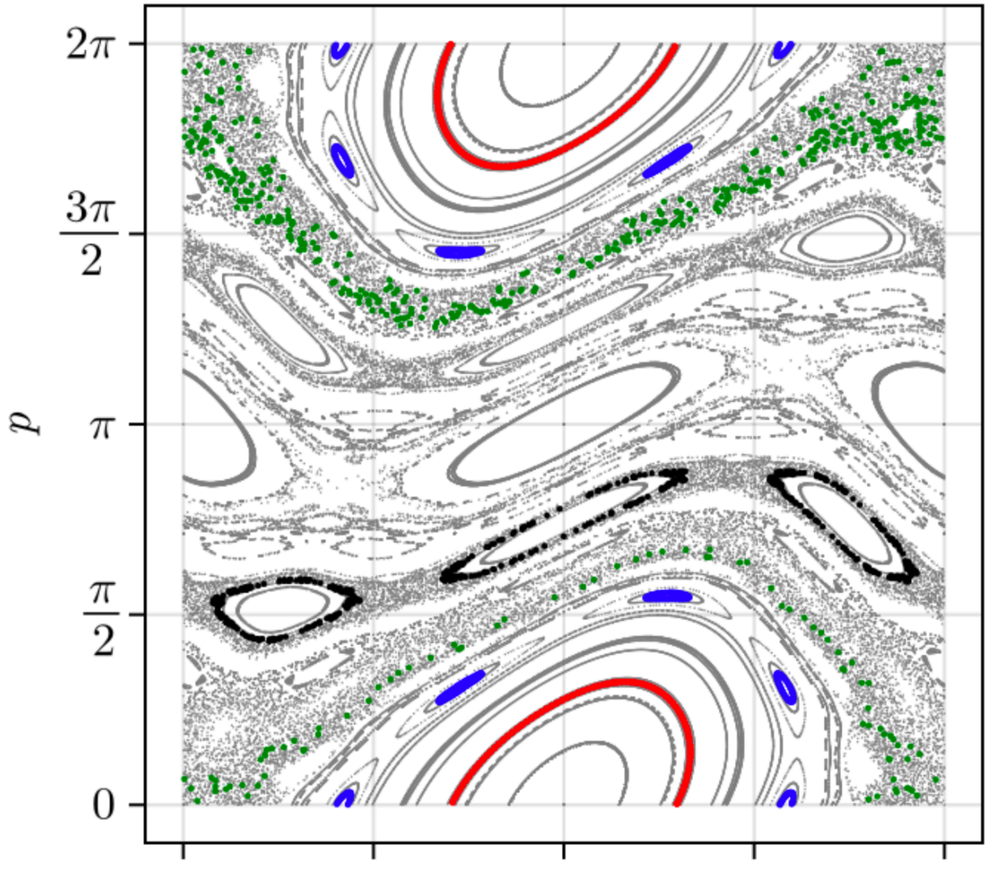


recurrence matrix

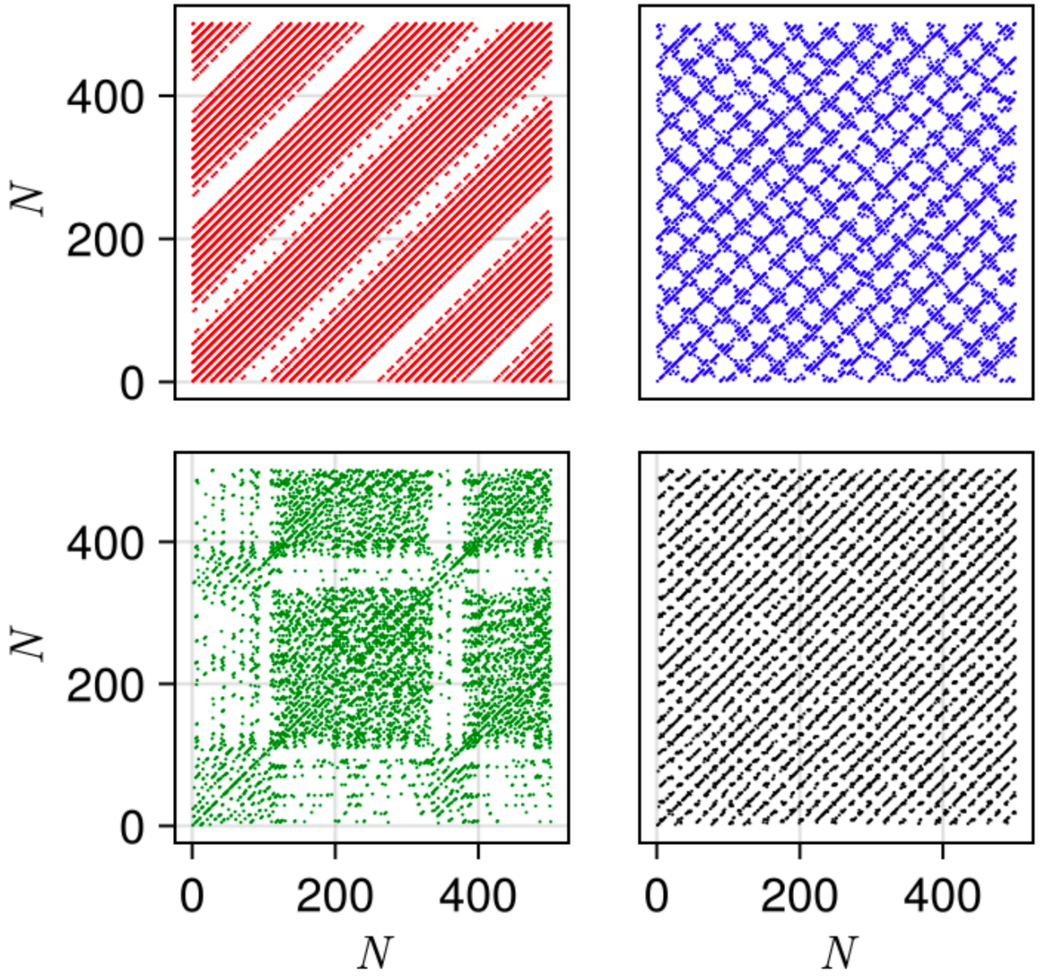


Eckmann, Oliffson Kamphorst, Ruelle; *Europhys. Lett.*, **4** (9) pp. 973-977. (1987)

Recurrence plots

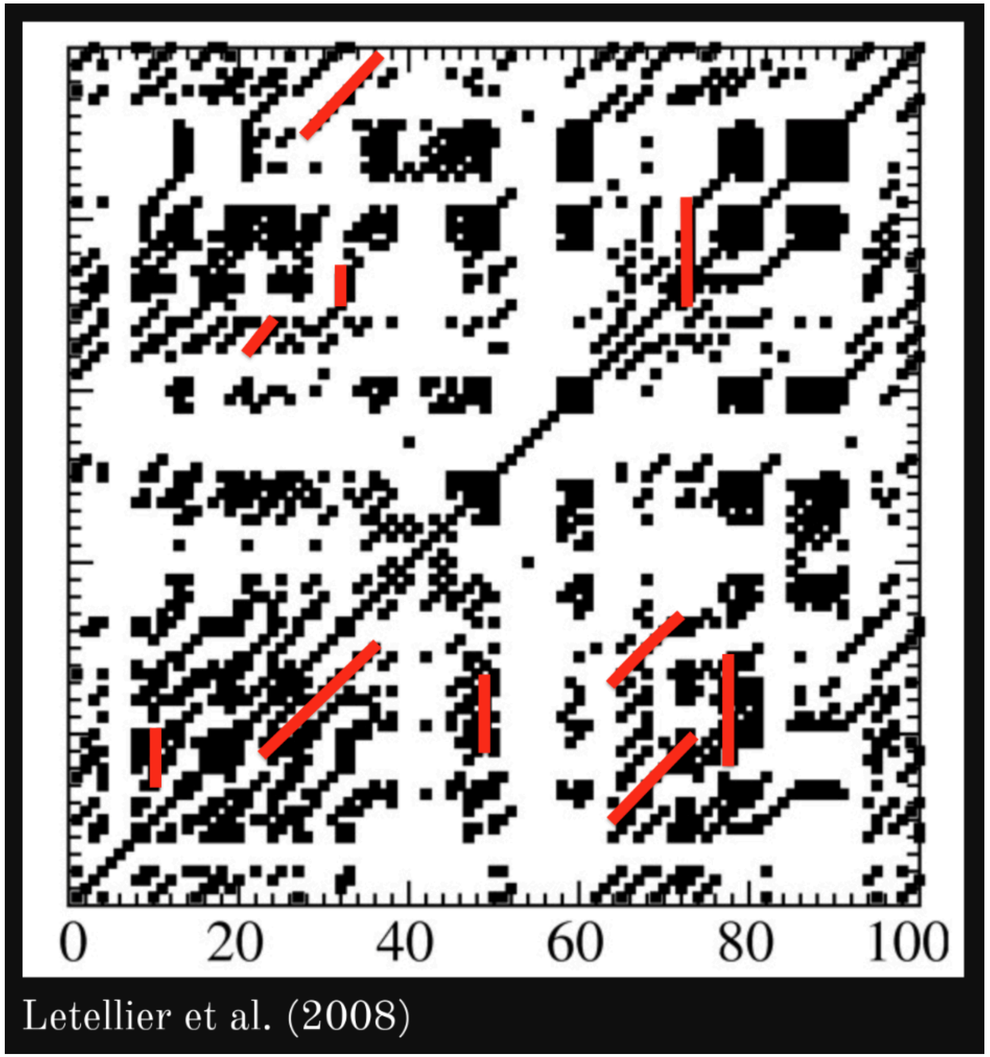


$$\begin{cases} p_{n+1} = p_n + K \sin\theta_n \pmod{2\pi}, \\ \theta_{n+1} = \theta_n + p_{n+1} \pmod{2\pi} \end{cases}$$



Daquin, Kovács; *Eur. Phys. J. Spec. Top.* (2026)

Texture of RPs

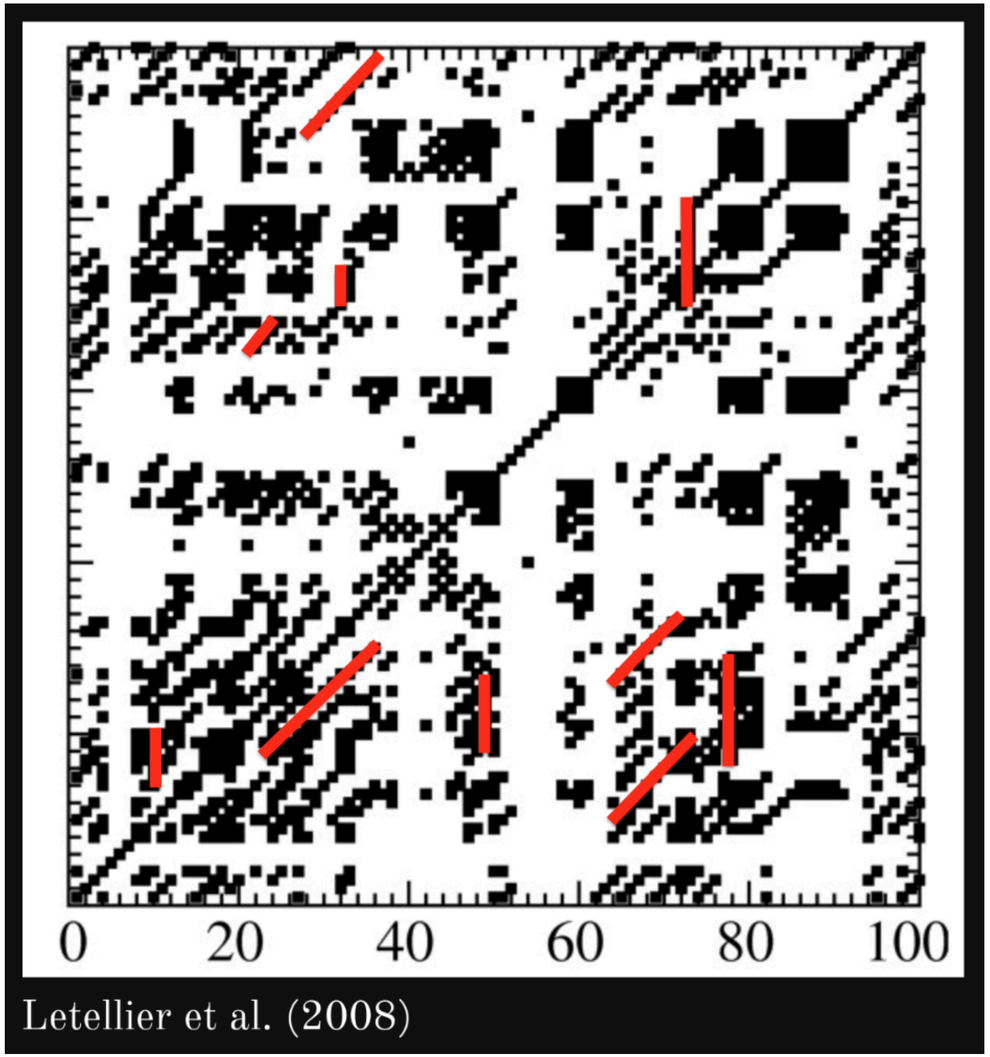


Letellier et al. (2008)

Quantitative measures of RP:

- density of points
- distribution of diagonal line length
- distribution of vertical line length
- related entropies, etc.

Texture of RPs



Letellier et al. (2008)

Quantitative measures of RP:

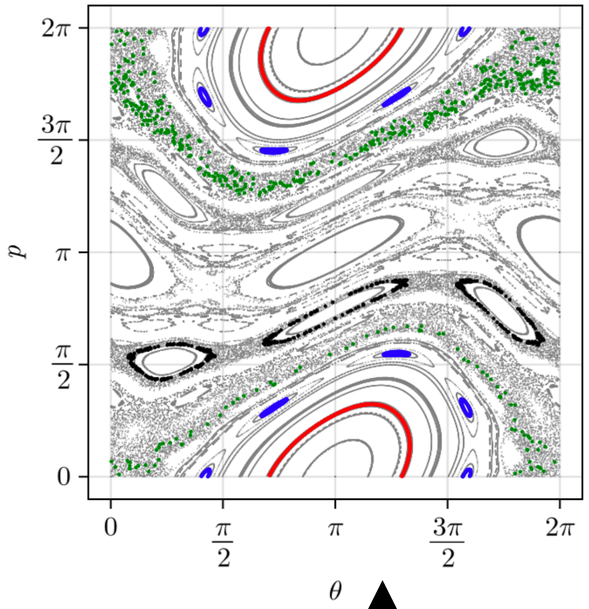
Diagonal of length ℓ

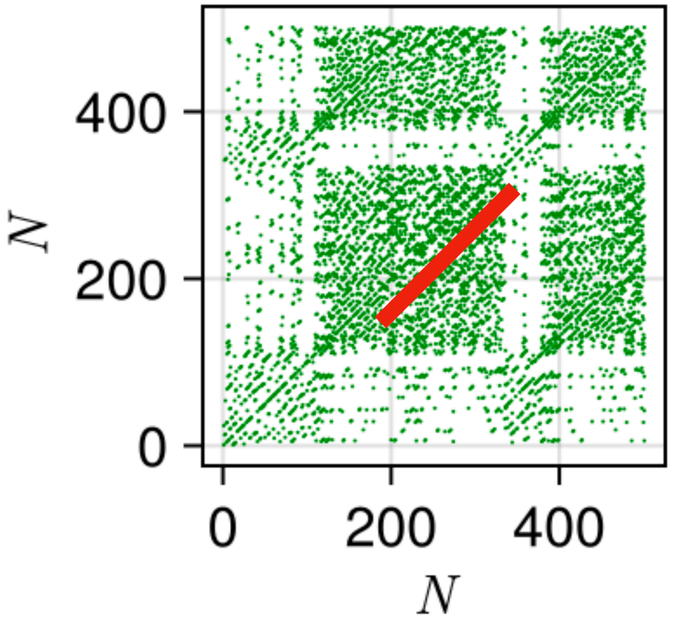
$$\begin{cases} r_{i-1,j-1} = 0, \\ r_{i,j} = 1, \\ r_{i+1,j+1} = 1, \\ \dots \\ r_{i+\ell-1,j+\ell-1} = 1, \\ r_{i+\ell,j+\ell} = 0. \end{cases}$$

$$\boxed{DIV = 1/\ell_{\max}}$$

Workflow

(p, θ)





$$DIV = 1/\ell_{\max}$$

repeat for different length *N*

plot *DIV* vs. *N*

DIV(N) - asymptotic behavior

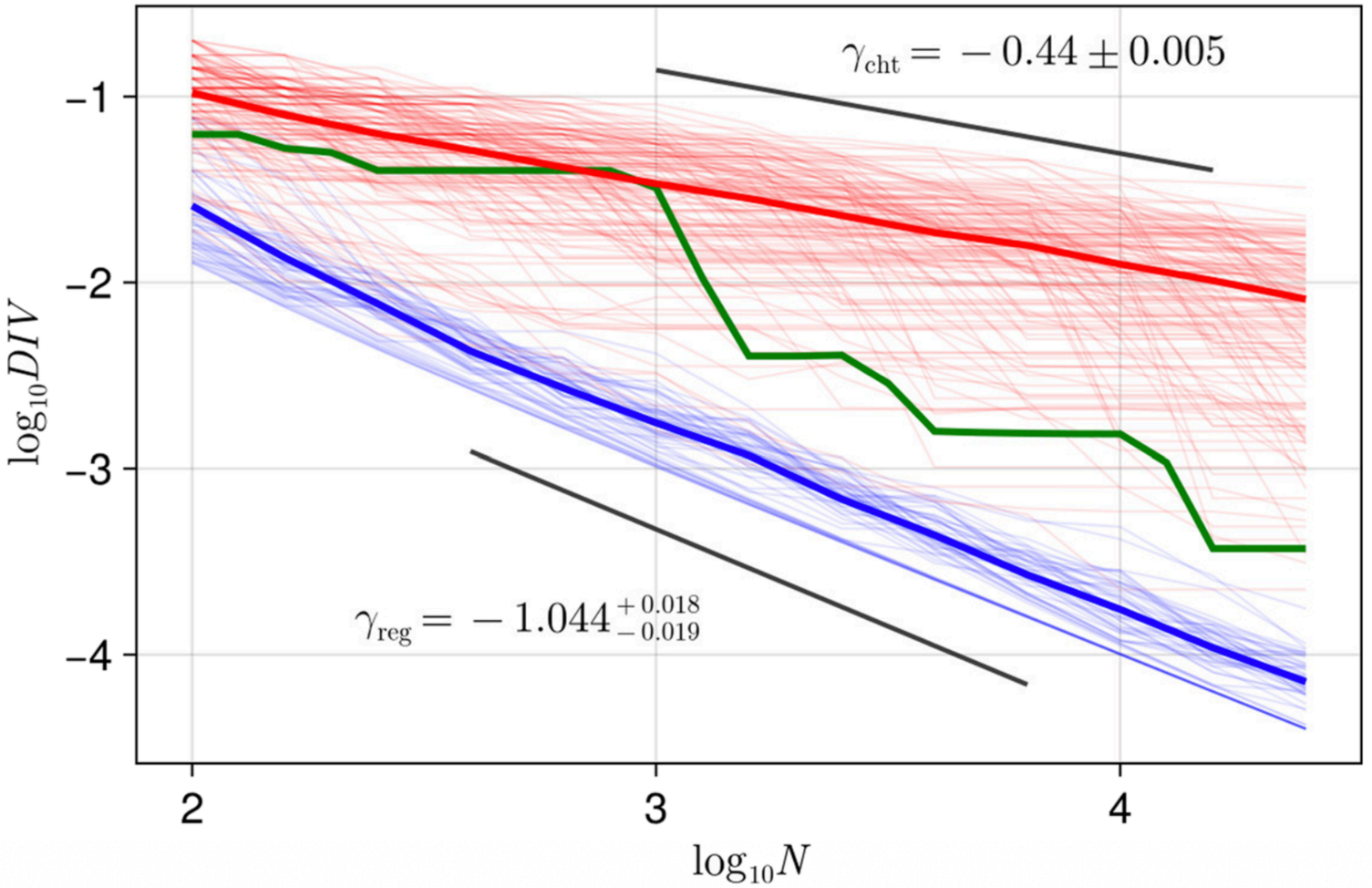
Random *K<6*,

Random ICs,

FLI chaotic,

FLI regular,

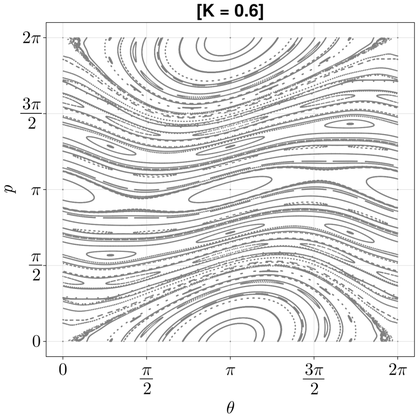
Sticky.

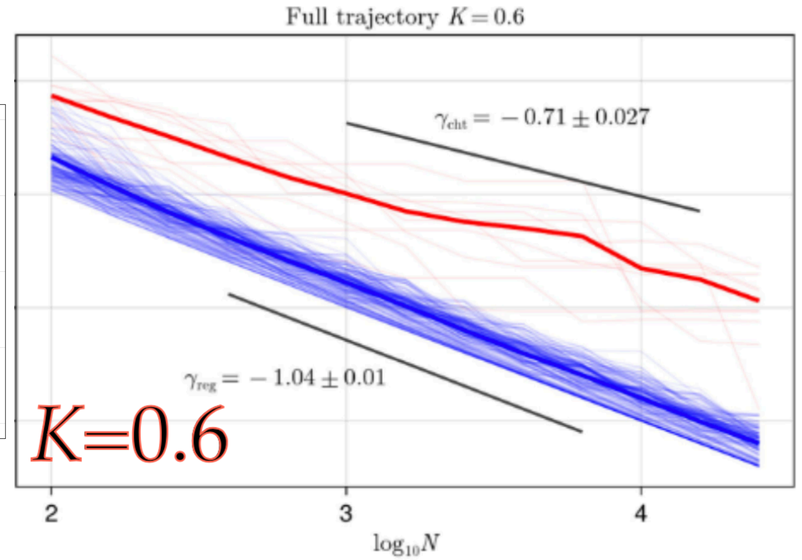


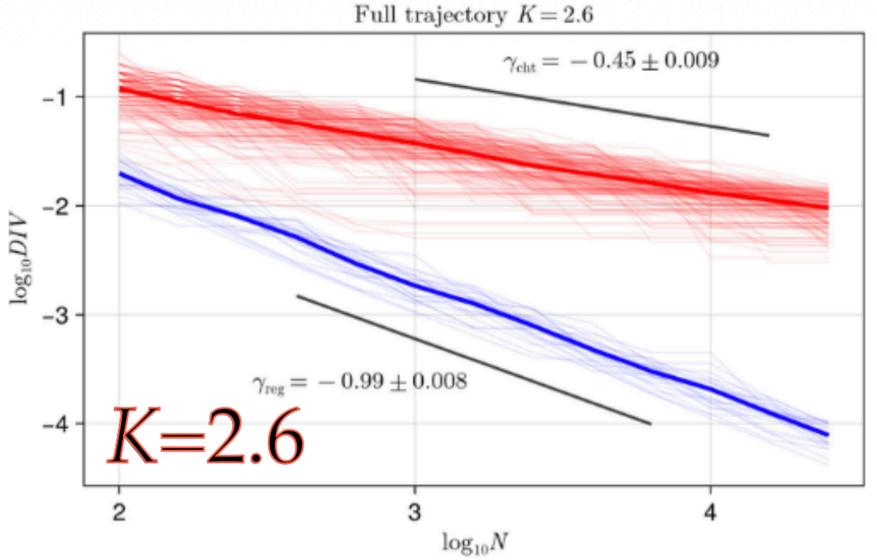
$\langle DIV(N) \rangle_{\rm reg.} \propto N^{\gamma_{\rm reg.}}, \quad \gamma_{\rm reg.} \sim -1,$

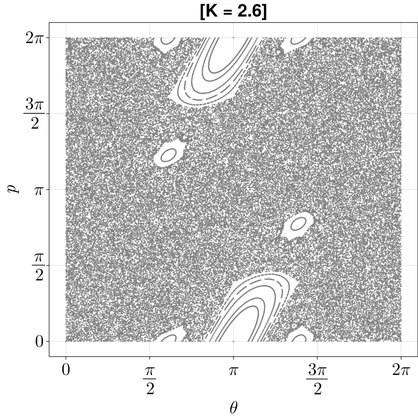
$\langle DIV(N) \rangle_{\rm cht.} \propto N^{\gamma_{\rm cht.}}, \quad \gamma_{\rm cht.} \sim -1/2,$

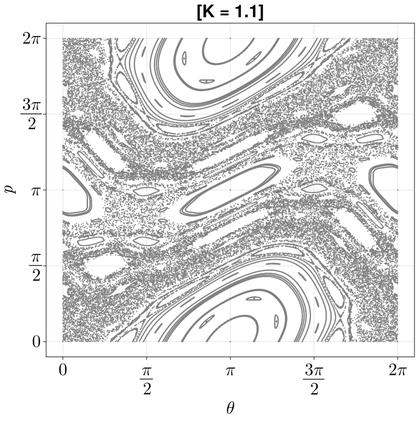
DIV(N) - asymptotic behavior

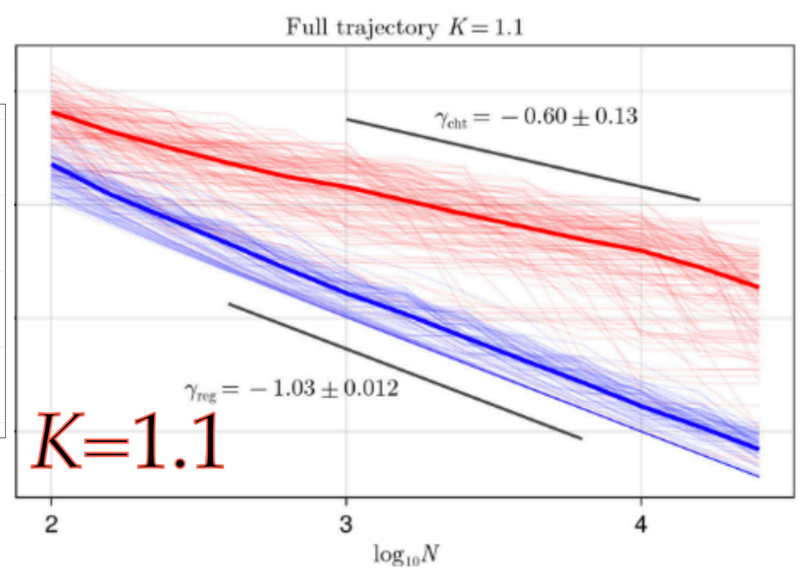


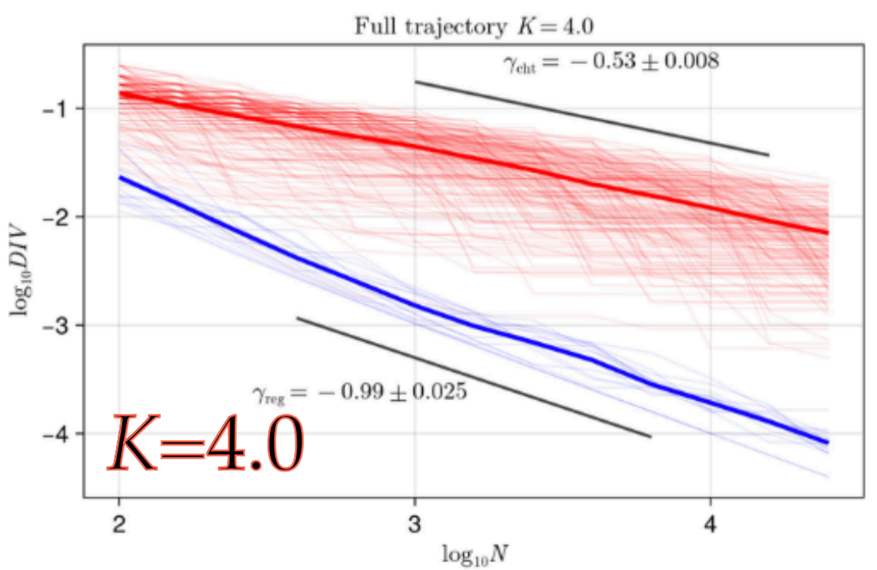


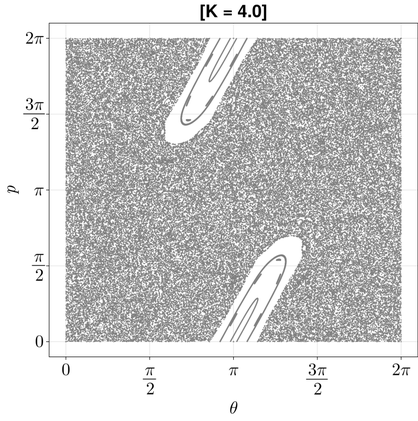








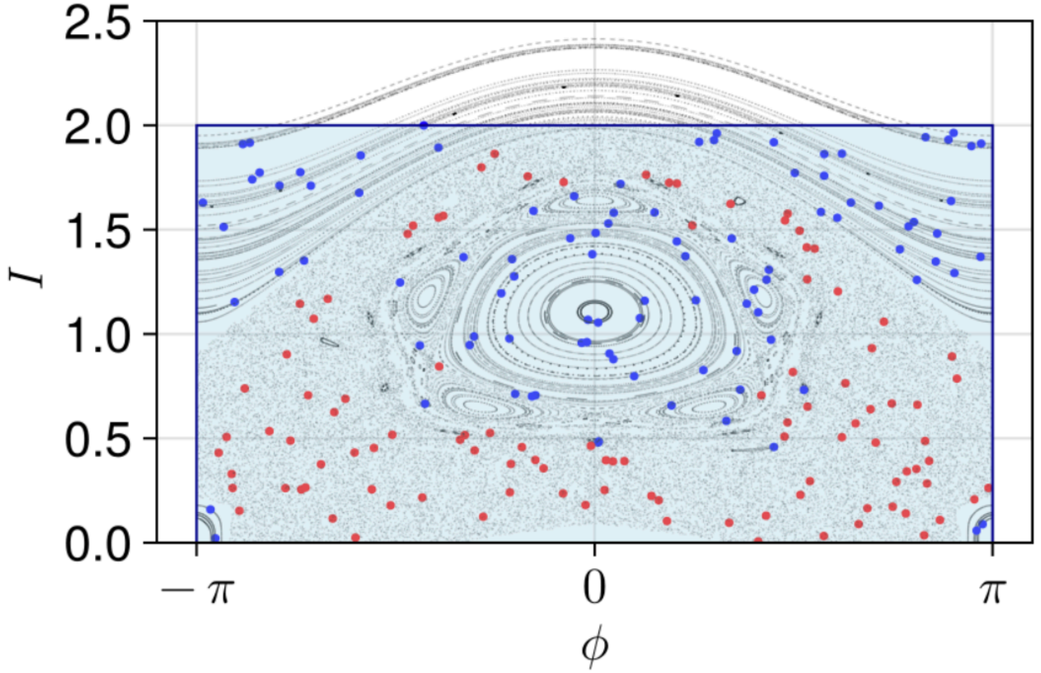


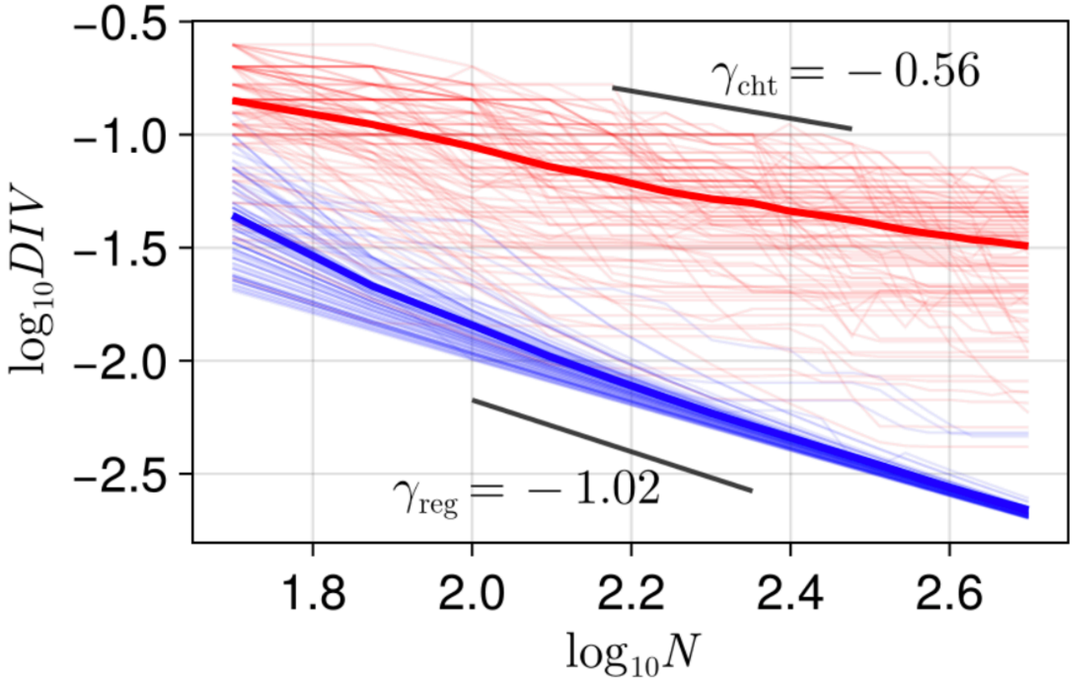


DIV(N) - asymptotic behavior

$$\mathcal{K}(I,\phi,t) = \frac{I^2}{2} - (\alpha_1 \cos(\phi - t) + \alpha_2 \cos(\phi + t))$$

$$\alpha_1 = \alpha_2 = 1/5$$





Time-delay embedding

1 | true state space $M \subset \mathbb{R}^d$

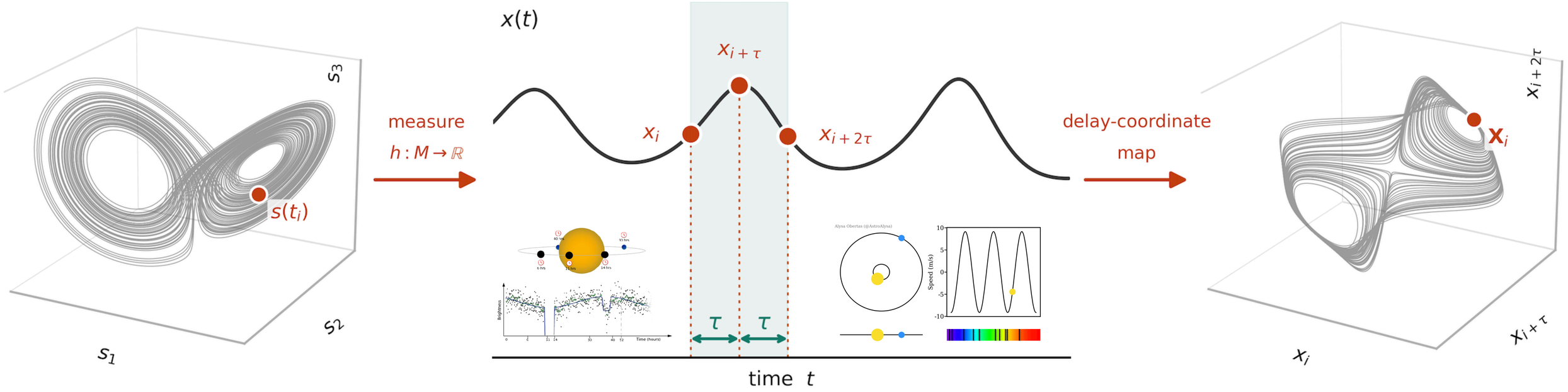
$\dot{s} = F(s)$ — unknown and inaccessible

2 | the measurement: one scalar observable

$x(t) = h(s(t))$ — a single generic function of the state

3 | reconstructed state space $\mathbb{R}^m$

$\mathbf{X}_i = (x_i, x_{i+\tau}, \ldots, x_{i+(m-1)\tau})$ — here $m = 3$



the three sampled values are the three coordinates of the single point $\mathbf{X}_i$

τ - delay time

m - embedding dimension

Takens, *Lecture Notes in Mathematics,* **898**, pp. 366-381. (1981)

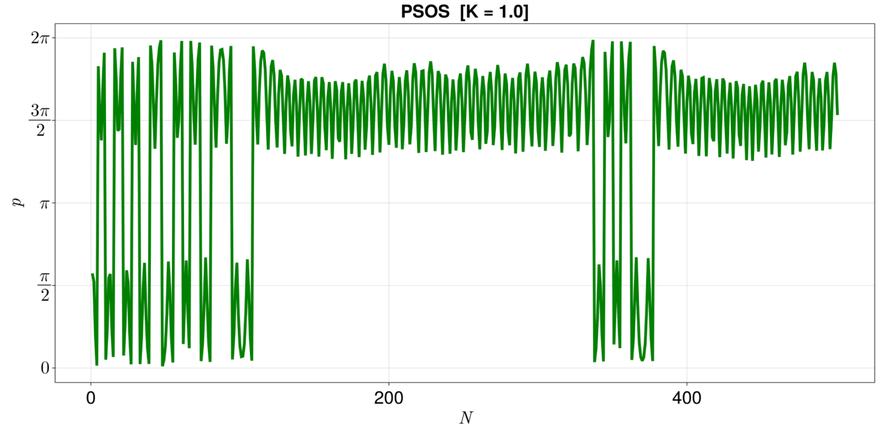
Time series perspective

original phase space

$$\begin{cases} p_{n+1} = p_n + K \sin\theta_n \pmod{2\pi}, \\ \theta_{n+1} = \theta_n + p_{n+1} \pmod{2\pi} \end{cases}$$

“observation”

$$\{p_1, p_2, \cdots, p_N\}$$
$$\{\theta_1, \theta_2, \cdots, \theta_N\}$$



$$f(p,\theta) = p$$
$$g(p,\theta) = \theta$$

no reconstruction

reconstruction

$$\mathbb{R} \quad |\bullet|$$

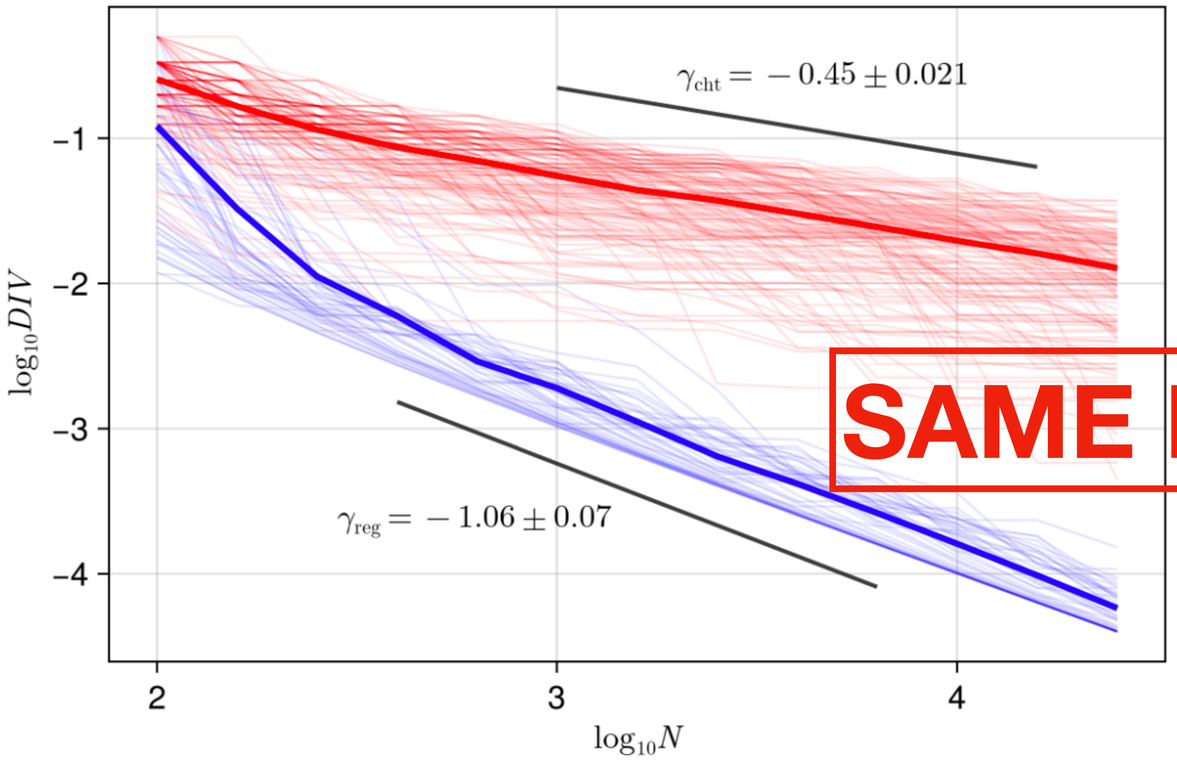
$$\mathbb{R}^m \quad \ell^2 - \text{norm}$$

Iwanski, Bradley, *CHAOS*, **8**, pp 861-871. (1998)

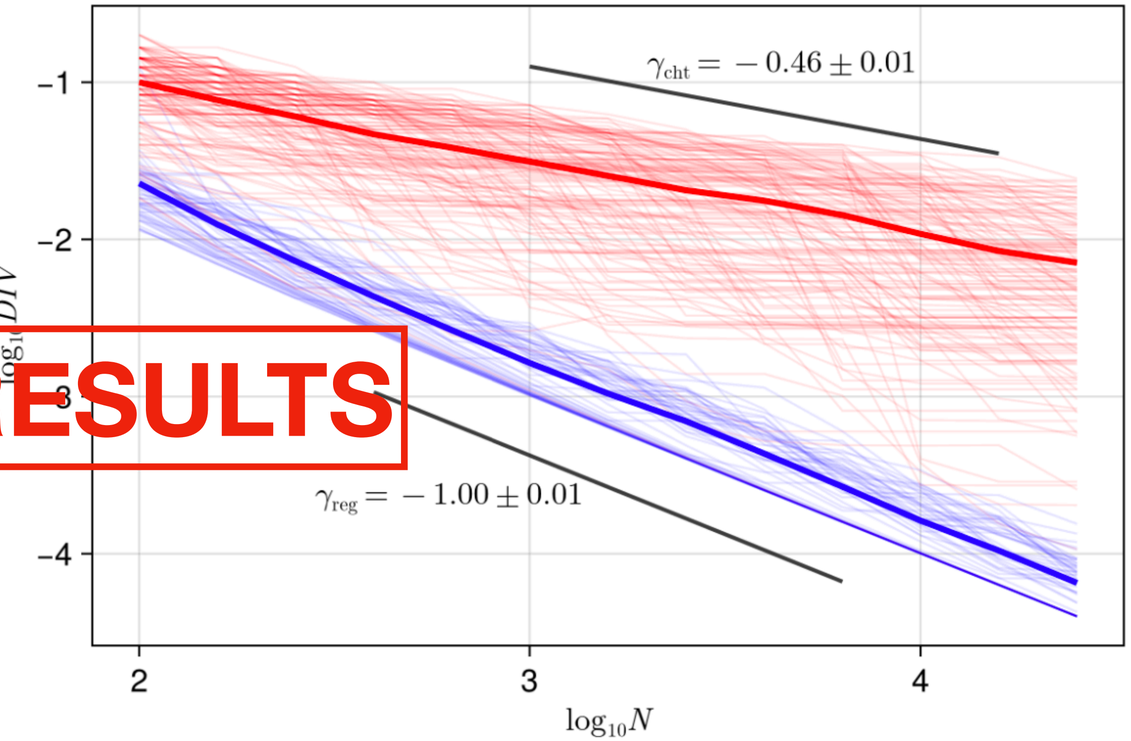
Takens, *Lecture Notes in Mathematics,* **898**, pp. 366-381. (1981)

Time series perspective, DIV(N)

based on action *p*



based on reconstructed traj.



SAME RESULTS

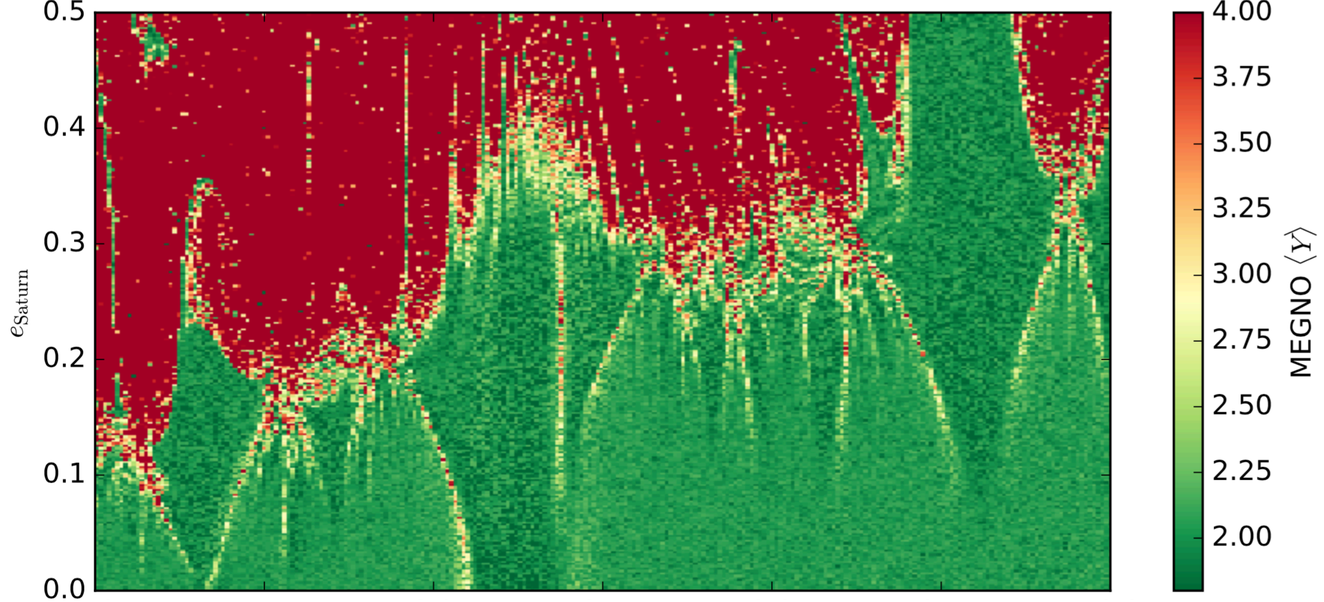
Daquin, J., Kovács, T.; Eur. Phys. J. Spec. Top. (2026) [https://doi.org/10.1140/epjs/s11734-026-02183-4]

Detecting Hamiltonian chaos from scalar time series via recurrence plot divergence

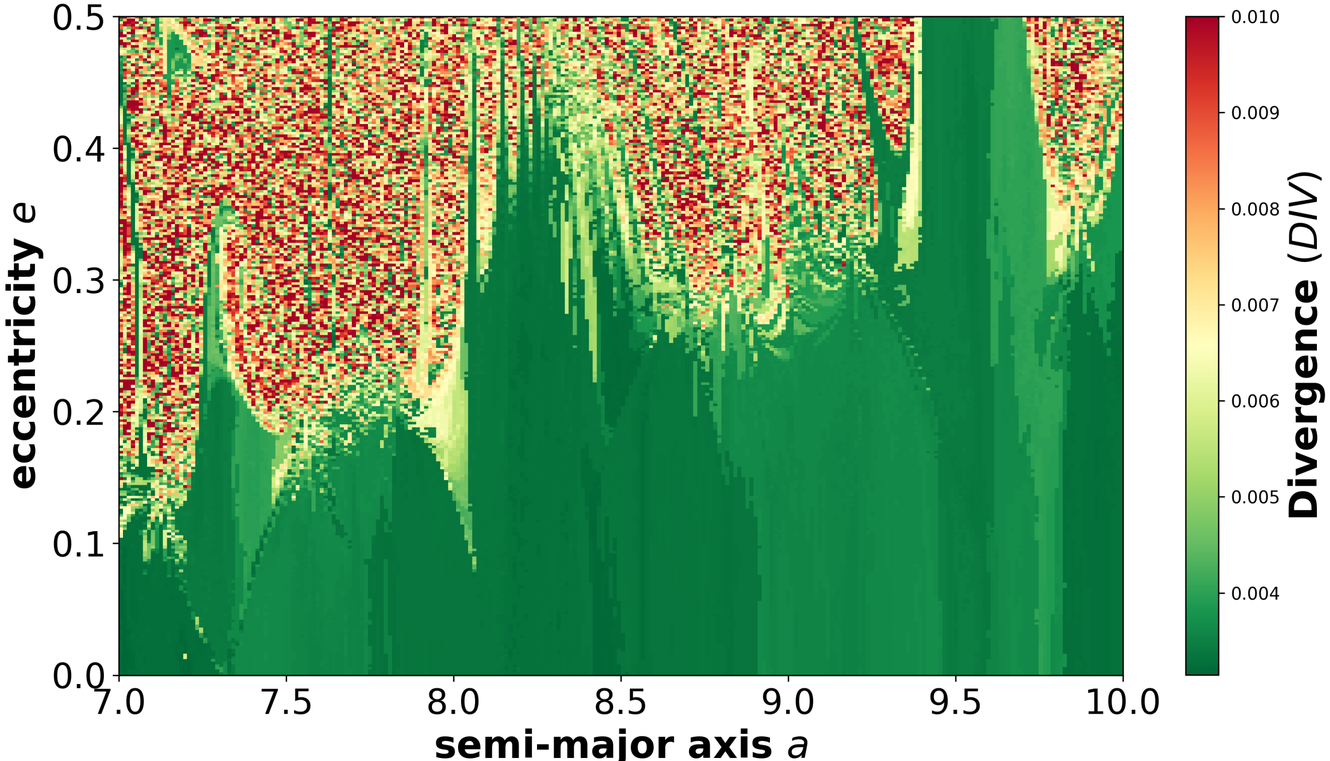
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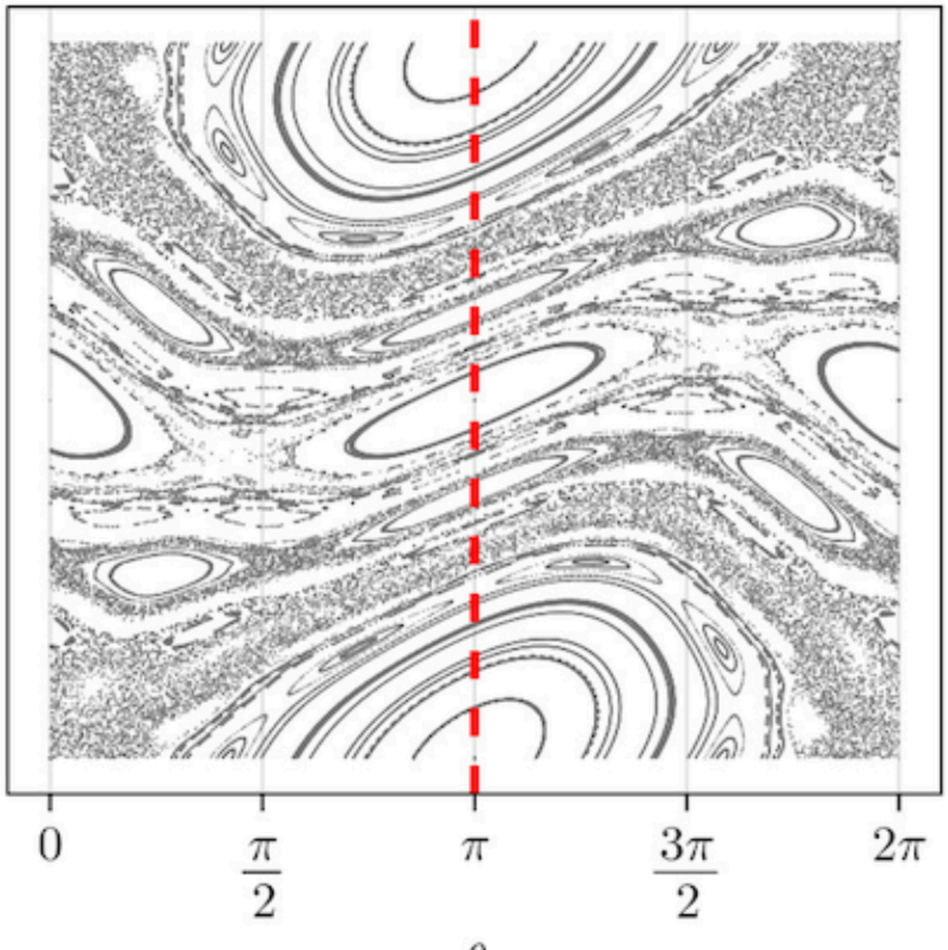
MEGNO:
based on numerical integration

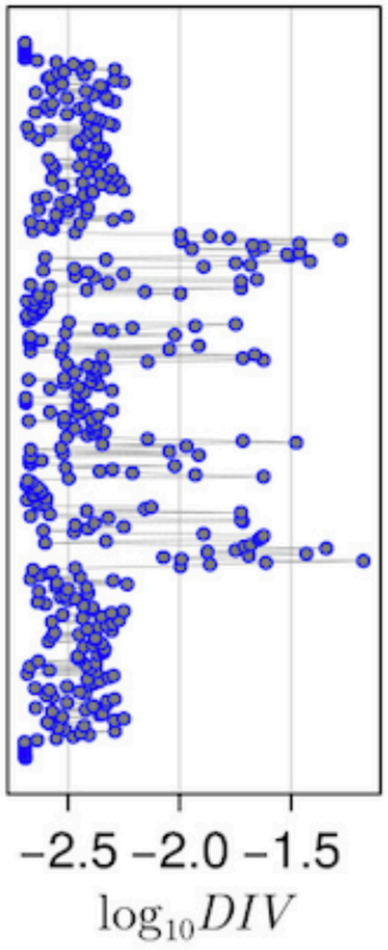


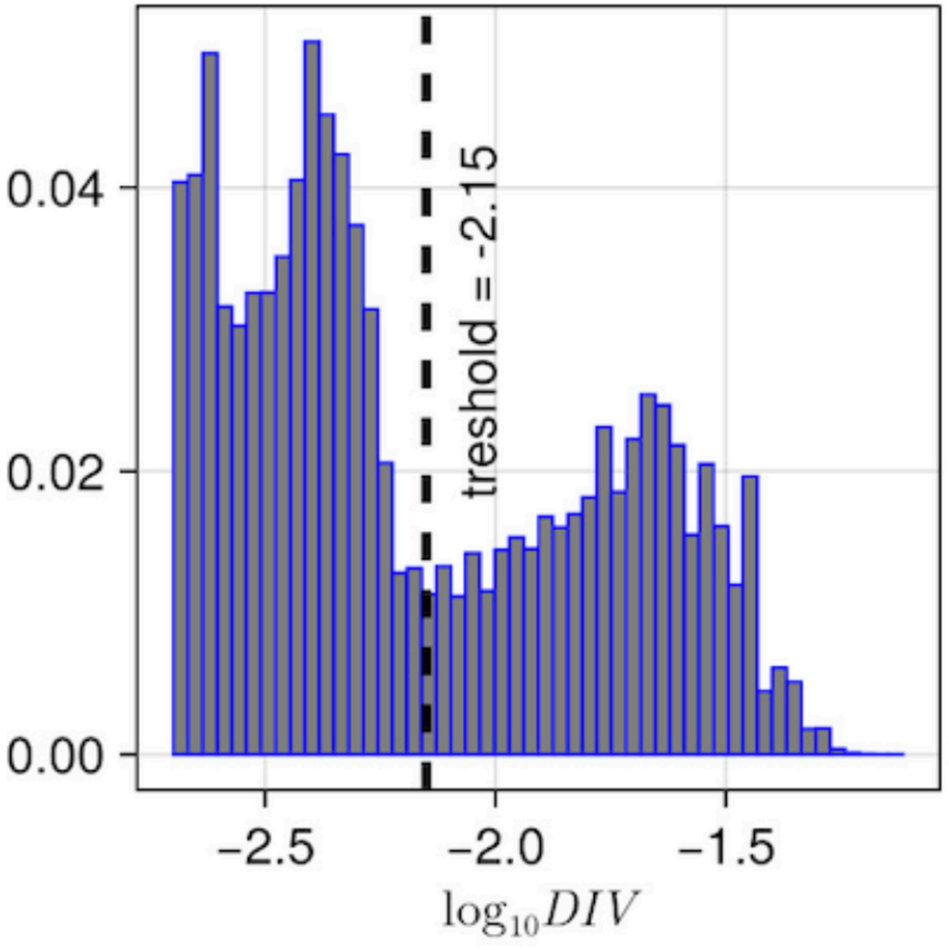
DIV:
based on sun's radial velocity data
v_X component
(phase space reconstruciton)

Label depending on the final DIV:

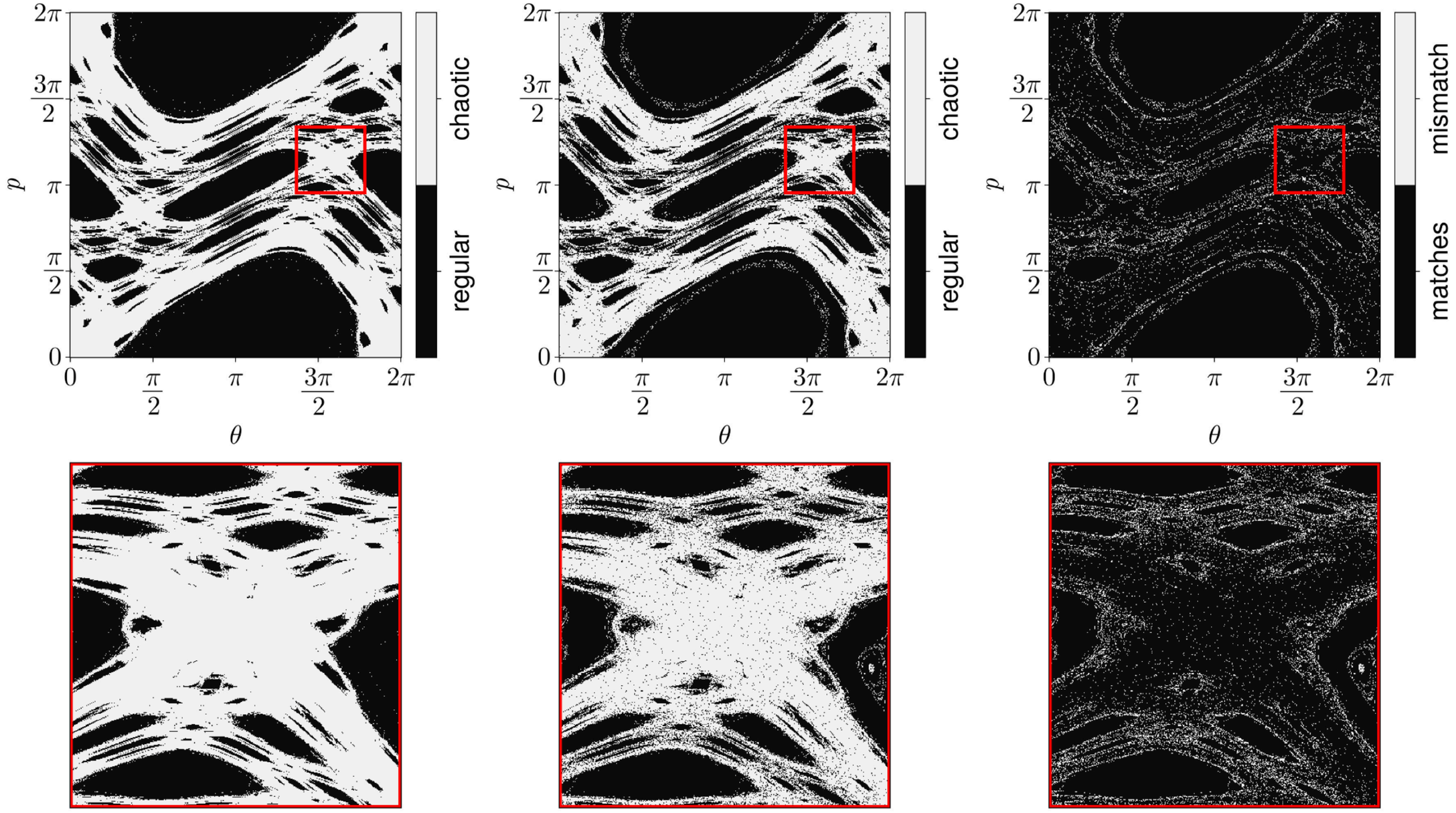
$$\begin{cases} DIV(x_0; N) \leq \alpha \Rightarrow \text{“The orbit stemming from } x_0 \text{ is regular,”} \\ DIV(x_0; N) > \alpha \Rightarrow \text{“The orbit stemming from } x_0 \text{ is chaotic.”} \end{cases}$$





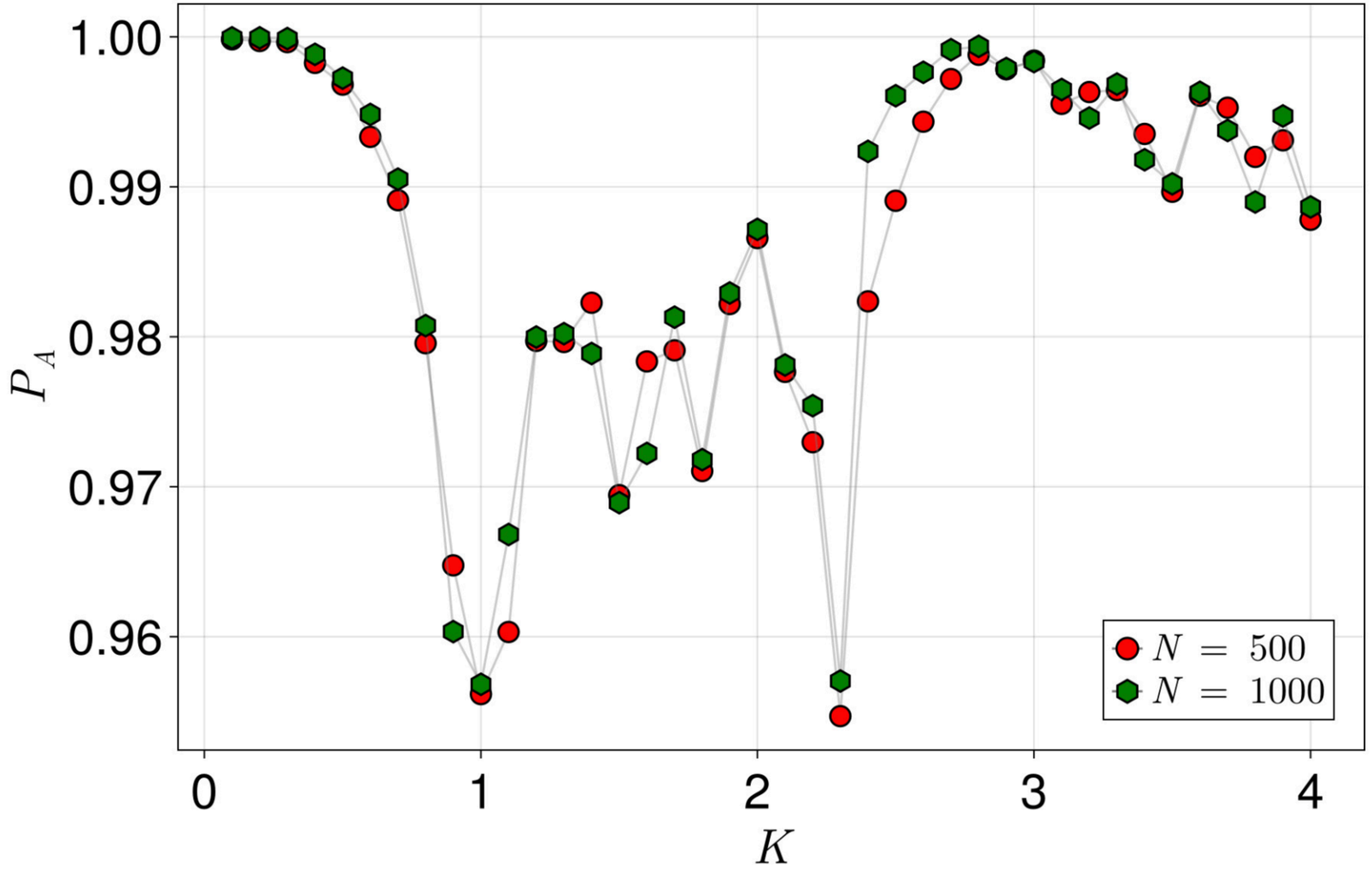


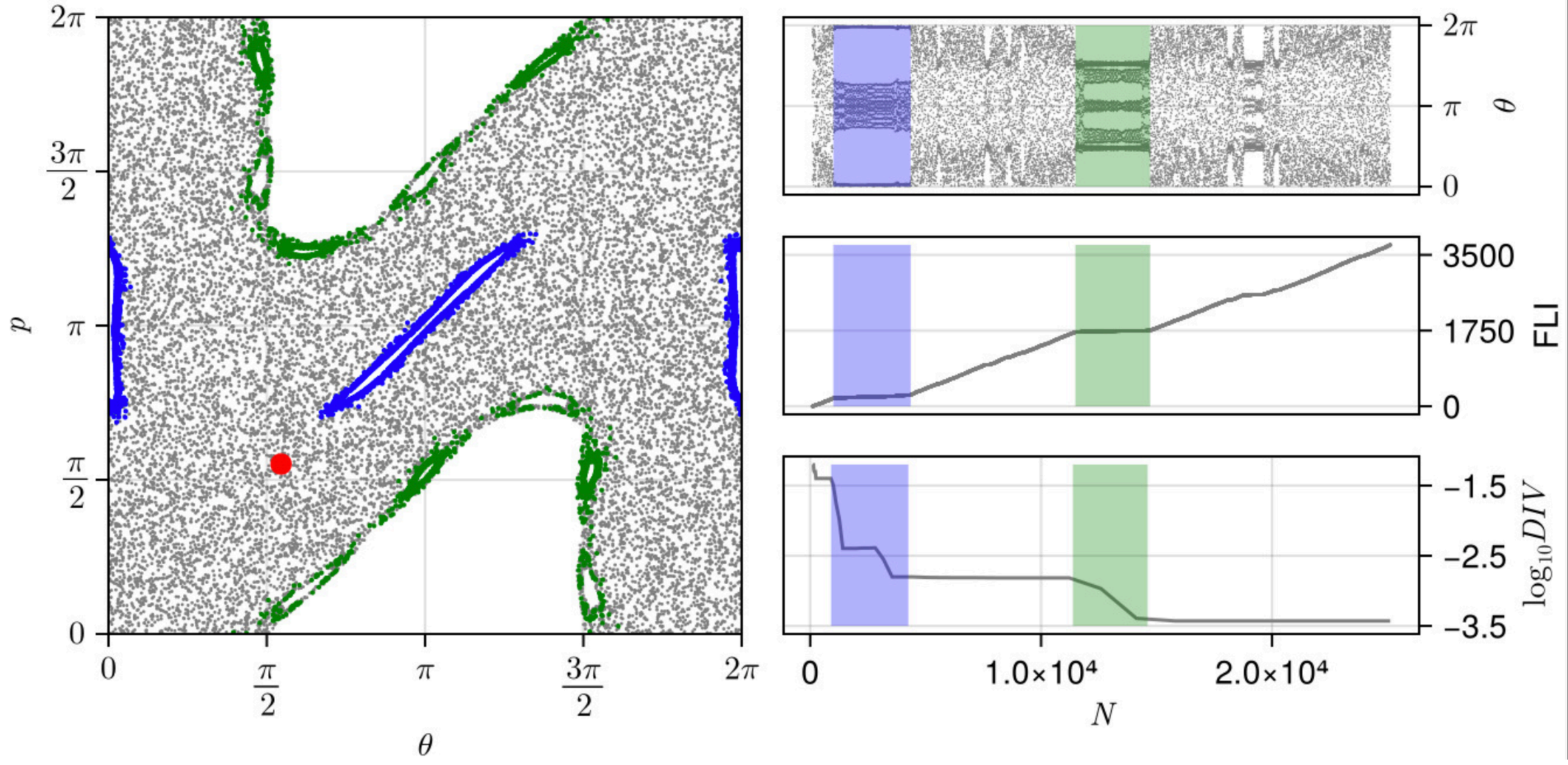
Qualitative comparison of dynamical maps



Quantitative assessments of dynamical maps

$$P_A = \frac{\#\text{Labels in agreement}}{\#\text{Initial conditions in } A}.$$




2π
3π/2
π
π/2
0
p
0
π/2
π
3π/2
2π
θ
θ
2π
π
0
3500
1750
0
FLI
−1.5
−2.5
−3.5
log10DIV
0
1.0×10⁴
2.0×10⁴
N