# On the catalogs of asteroid synthetic proper elements: comparison of the contents and data accuracy

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# Synthetic method to compute asteroid proper elements:

- Numerical integration of orbits;

- digital filtering of short-periodic perturbations: mean elements, but proper semimajor axis

- Fourier analysis:

- removal of forced perturbations;

- computation of frequencies from time series of slow angles;

- extraction of proper modes from time series of equinoctal elements:

proper eccentricity and (sine of) proper inclination

Synthetic proper elements of asteroids (> 1M) available from:

AstDyS2:

https://newton.spacedys.com/astdys2/index.php?pc=5

AFP:

http://asteroids.matf.bg.ac.rs/fam/properelements.php

PDS A&D subnode:
https://sbnarchive.psi.edu/pds4/non_mission/ast.nesvorny.families_V2_0/data/

Using basically the same software: Orbfit (orb507) – different integration time spans and dynamical models

Knežević and Milani, Synthetic proper elements for outer main belt asteroids, CMDA, 78, 17–46, 2000

# Comparison of proper elements catalogs:

- Contents of catalogs
- Proper elements computation
- Data analysis – values, accuracy, long term reliability
- Failures, optimization
- Conclusions

# **Contents** of proper elements catalogs

AstDyS: numbered (699991) and multiopposition, 1.17 Million

Hungaria, Main belt, Trojans, TNOs; (1.8<a<5.4, q(H)>1.6; q(MB)>1.75; Q(MB)<4.2). resonant, low eccentricity;

AFP: numbered (756999), multi and single opposition, 1.31 Million

Hungaria, Main belt

PDS: numbered (640000) and multiopposition, 1.25 Million

Hungaria, Main belt, (1.6<a<3.8, q>1.3; Q<5)

Number of oppositions, no chaos indicators & errors of frequencies.

# **Computation** of proper elements

## AstDyS (orbfit):

outer belt -2/10My, Trojans 5/50My, TNOs 100My;

dynamical model with 4 outer planets (barycentric correction)

Hungaria -2/10My, inner belt ±1My,

with 7 planets (except Mercury, barycentric correction)

## AFP (orbfit):

all for 10 My

all with 7 planets (as in AstDyS)

## PDS (orbfit like):

all for 10 My

all with 8 planets, largest asteroids, relativistic correction

# Resonant proper elements

For asteroids in or near the secular resonances.

Resonant terms librate with periods of several My.

Synthetic method does not provide stable proper elements.



The need to compute resonant proper elements

specifically adapted to a given secular resonance.

# Resonant proper elements

Definition of a new proper element:

Use the amplitude of libration Δe in eccentricity

associated with the libration of the critical argument.

- Numerical integration over 10My provides mean elements sampled every 500y with perturbations up to 750y of period removed.
- Removal of forced terms

  Period of libration long (>1My), filter again with decimation 100

  Removal of terms with periods up to 75,000y, leaves libration period
- Semiproper elements

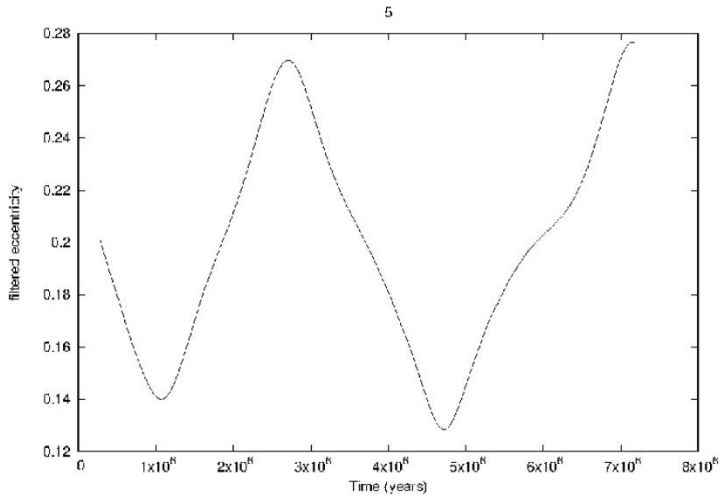
  Compute spectrum in the interval $1-6$~My, find max spectral density.

  Use the corresponding amplitude as proper element!

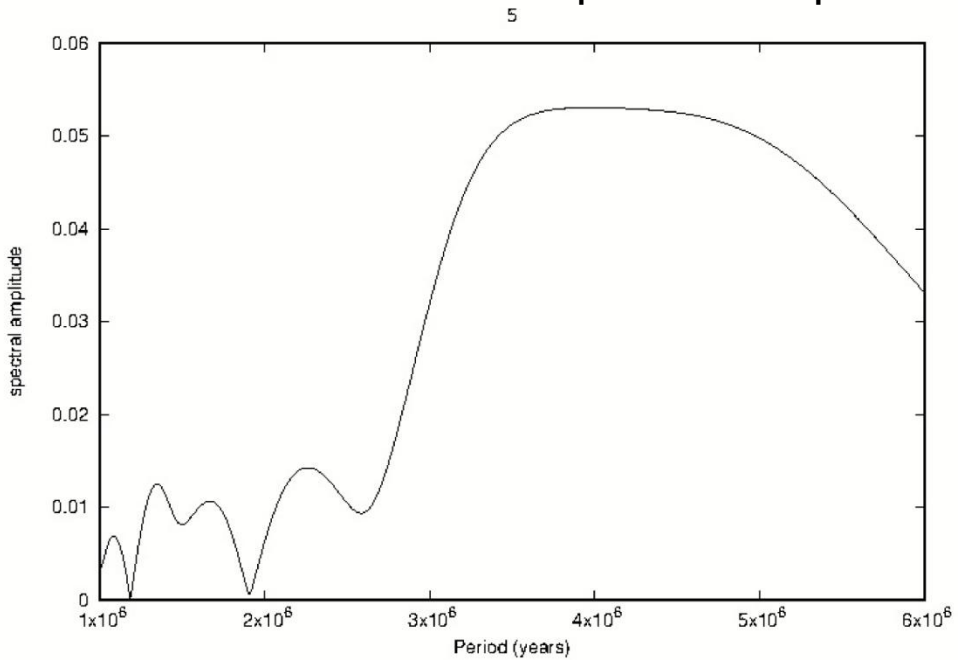
# Resonant proper elements: (5) Astraea

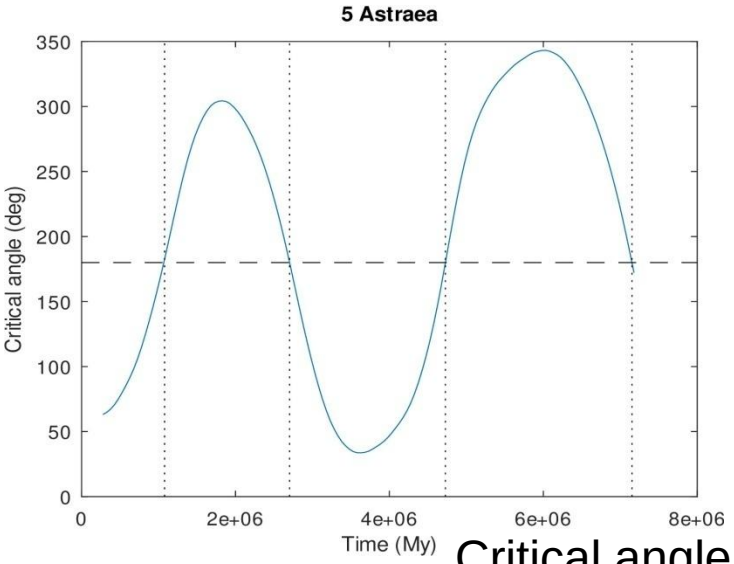
$g+g_5-2g_6$ secular resonance: affects only eccentricity.

Semiproper eccentricity



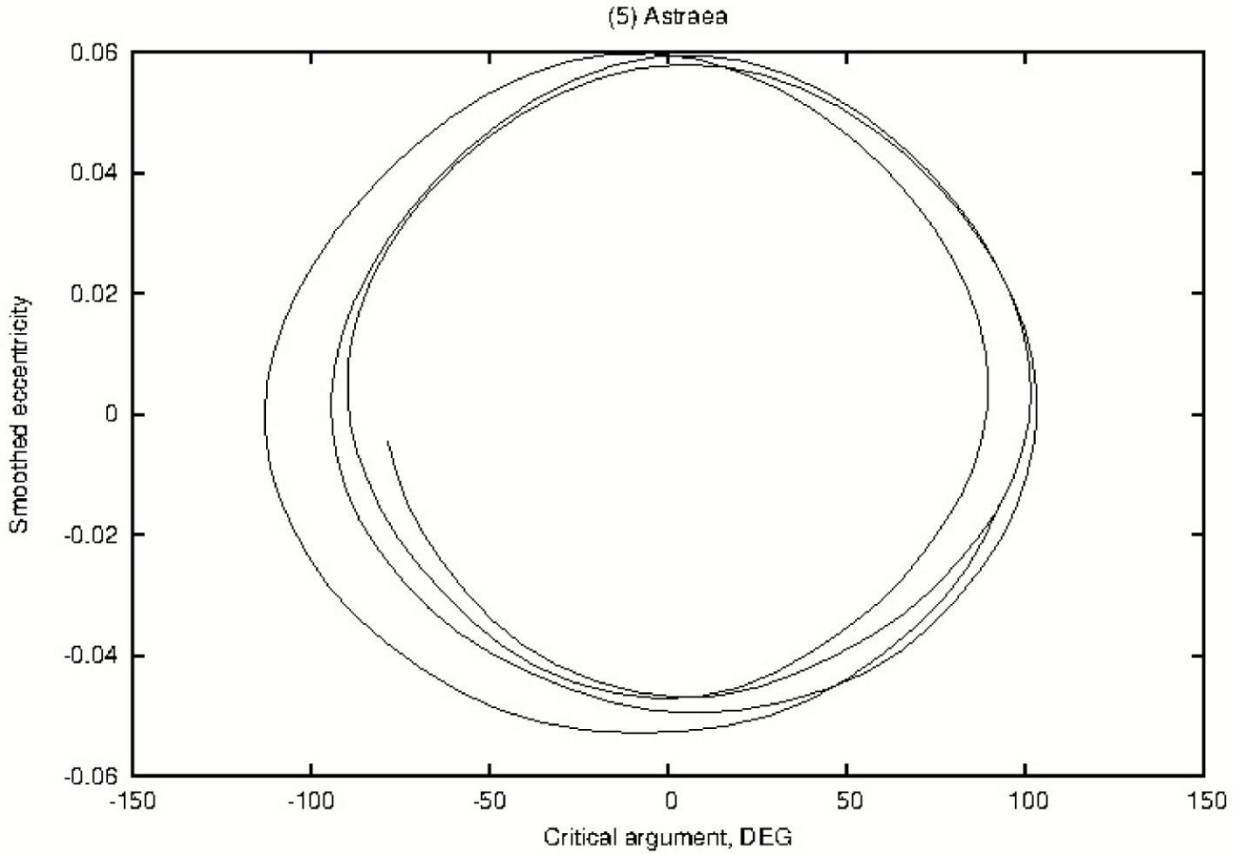

Spectral amplitude




Critical angle in libration

# Resonant proper elements: (5) Astraea




Libration of eccentricity and critical argument: semiproper elements

# **Accuracy** of proper elements

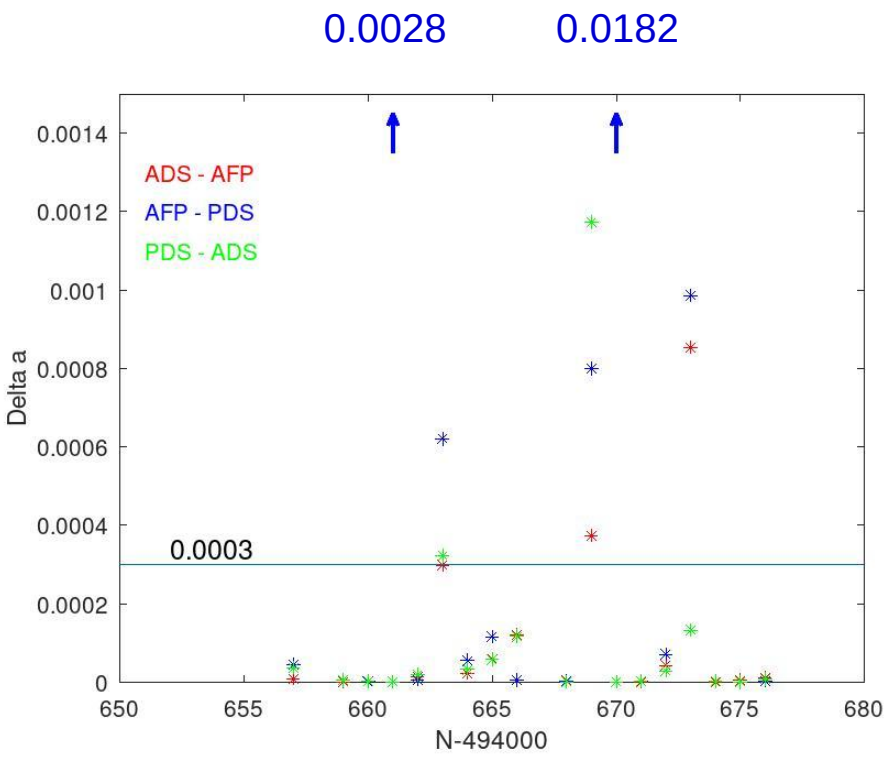
For family identification purposes:

$$\sigma a < 0.0003,\ \sigma e < 0.001,\ \sigma \sin I < 0.001$$

corresponds to the relative velocity of $\sim 15$ m/s

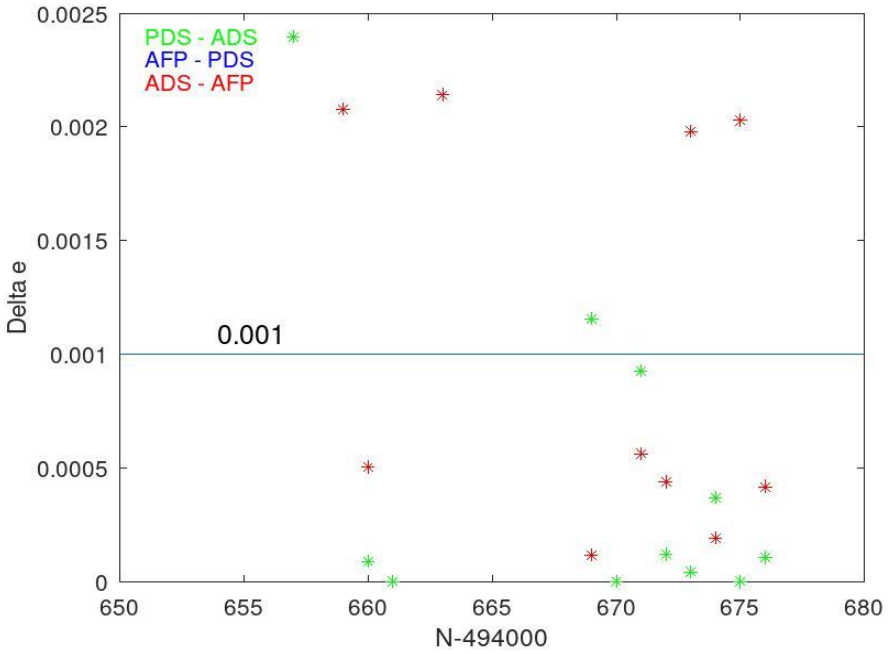
# Comparison: semimajor axis

| No | AstDyS | AFP | PDS |
|---|---|---|---|
| 494657 | 2.7201729 | 2.7201804 | 2.720137x |
| 494659 | 1.8666329 | 1.8666326 | 1.866638 |
| 494660 | 1.8882836 | 1.8882859 | 1.888285 |
| 494661 | - | 2.6309944 | 2.633822 |
| 494662 | 2.5907269 | 2.5907402 | 2.590745 |
| 494663 | 2.2124486 | 2.2121505 | 2.212771 |
| 494664 | 2.7764118 | 2.7764344 | 2.776380 |
| 494665 | 2.5781628 | 2.5781041 | 2.578220 |
| 494666 | 2.5897191 | 2.5895984 | 2.589603 |
| 494668 | 1.8550555 | 1.8550583 | 1.855056 |
| 494669 | 2.7443181 | 2.7439448 | 2.743146 |
| 494670 | - | 2.1891747 | 2.207329 |
| 494671 | 2.4795401 | 2.4795409 | 2.479538 |
| 494672 | 3.1371998 | 3.1371571 | 3.137228 |
| 494673 | 3.0714014 | 3.0722549 | 3.071269 |
| 494674 | 3.1428585 | 3.1428586 | 3.142860 |
| 494675 | 3.9682094 | 3.9682138 | - |
| 494676 | 2.7922118 | 2.7922219 | 2.792219 |



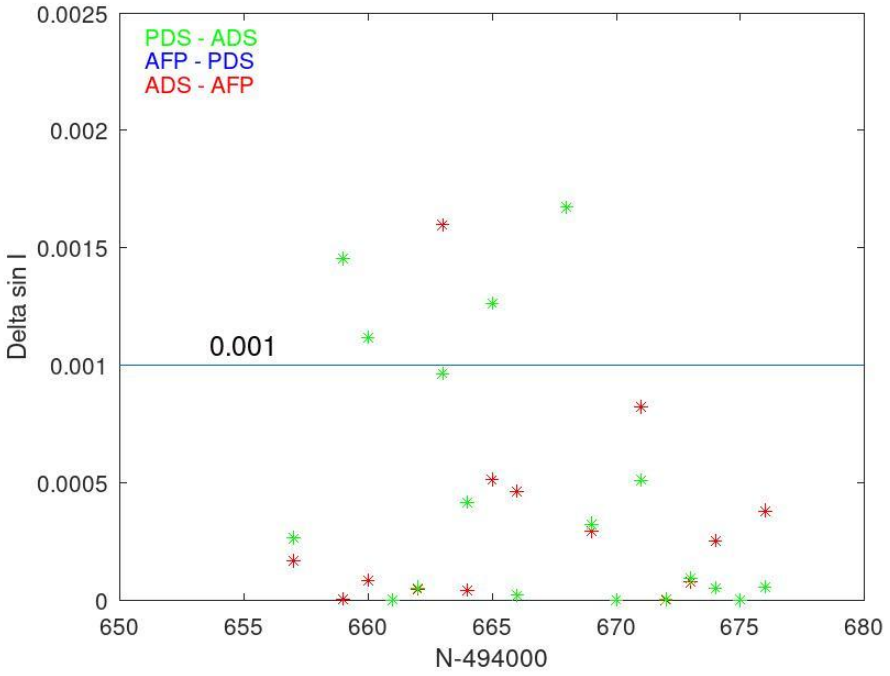
# Comparison: eccentricity

| No | AstDyS | AFP | PDS |
|---|---|---|---|
| 494657 | 0.2475548 | 0.2500854 | 0.249950x |
| 494659 | 0.0848542 | 0.0869315 | 0.090722 |
| 494660 | 0.0940207 | 0.0945241 | 0.094110 |
| 494661 | - | 0.3076288 | 0.318102 |
| 494662 | 0.2407213 | 0.2193800 | 0.216894 |
| 494663 | 0.1097201 | 0.1075787 | 0.106091 |
| 494664 | 0.2594529 | 0.2535153 | 0.252941 |
| 494665 | 0.2401658 | 0.2182282 | 0.216388 |
| 494666 | 0.2233055 | 0.2203520 | 0.219901 |
| 494668 | 0.0778102 | 0.0746742 | 0.073648 |
| 494669 | 0.2892673 | 0.2893836 | 0.288111 |
| 494670 | - | 0.3141487 | 0.312330 |
| 494671 | 0.1494170 | 0.1499794 | 0.148491 |
| 494672 | 0.1133840 | 0.1138230 | 0.113504 |
| 494673 | 0.1647375 | 0.1627565 | 0.164695 |
| 494674 | 0.0576030 | 0.0577944 | 0.057972 |
| 494675 | 0.2605993 | 0.2626286 | - |
| 494676 | 0.1795975 | 0.1800146 | 0.179703 |



# Comparison: inclination

| No | AstDyS | AFP | PDS |
|---|---|---|---|
| 494657 | 0.1585576 | 0.1583916 | 0.158294x |
| 494659 | 0.3649891 | 0.3649850 | 0.363535 |
| 494660 | 0.3868455 | 0.3869296 | 0.387963 |
| 494661 | - | 0.1275028 | 0.127014 |
| 494662 | 0.0618165 | 0.0617684 | 0.061764 |
| 494663 | 0.1308729 | 0.1324706 | 0.131837 |
| 494664 | 0.2091307 | 0.2090882 | 0.208714 |
| 494665 | 0.0628566 | 0.0633691 | 0.064120 |
| 494666 | 0.0727268 | 0.0731899 | 0.072747 |
| 494668 | 0.3792716 | 0.3760588 | 0.377599 |
| 494669 | 0.3166336 | 0.3169256 | 0.316310 |
| 494670 | - | 0.3695953 | 0.376044 |
| 494671 | 0.0550881 | 0.0559100 | 0.055599 |
| 494672 | 0.0190155 | 0.0190153 | 0.019019 |
| 494673 | 0.0657957 | 0.0657186 | 0.065703 |
| 494674 | 0.1484857 | 0.1487371 | 0.148434 |
| 494675 | 0.1194909 | 0.1165424 | - |
| 494676 | 0.2337449 | 0.2341245 | 0.233690 |



# Missing objects

Out of 20 random objects from the catalog with fairly stable orbits:

494658 (q=0.79) all

494667 (q=1.63, Q= 5.95) all

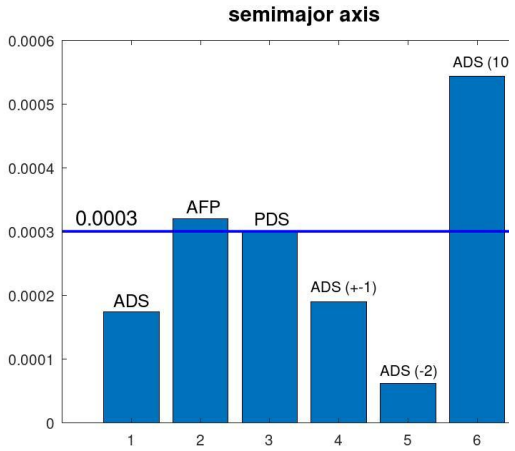
494661 (q=1.60) AstDyS

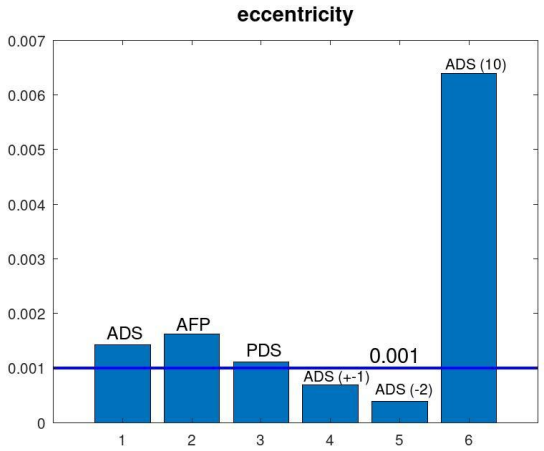
494670 (q=1.44) AstDyS

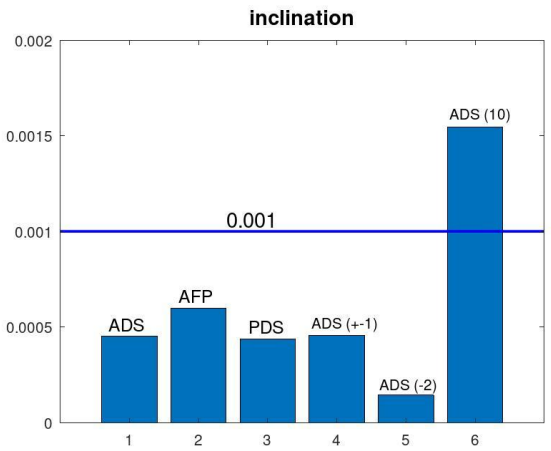
494675 (Q=5.30) PDS

Accuracy of proper elements: the average standard deviations of proper elements in the AstDyS, AFP and PDS data bases

| Data | $\sigma a$ (au) | $\sigma e$ | $\sigma \sin I$ |
|---|---|---|---|
| AstDys all | 0.000174 | 0.001429 | 0.000454 |
| AFP all | 0.000320 | 0.001620 | 0.000600 |
| PDS all | 0.000300 | 0.001106 | 0.000437 |
| AstDyS ±1My | 0.000190 | 0.000693 | 0.000456 |
| AstDys -2My | 0.000062 | 0.000391 | 0.000144 |
| AstDys 10My | 0.000544 | 0.006399 | 0.001546 |

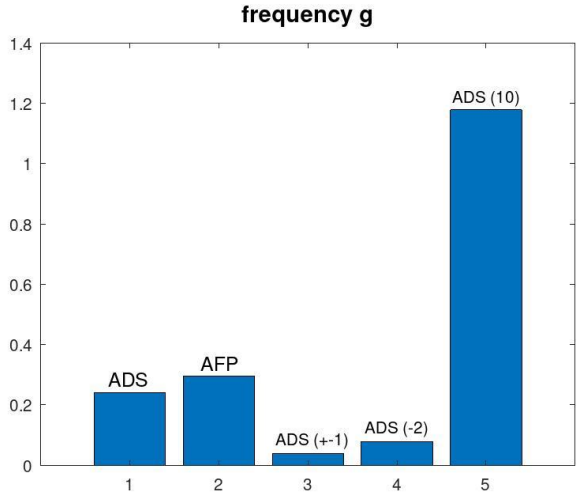


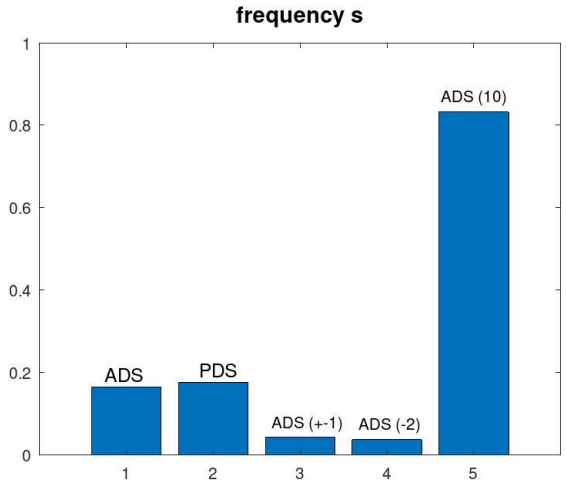



# Accuracy of proper frequencies: the average standard deviations of proper frequencies in the AstDyS and AFP data bases

| | $\sigma g$ ("/y) | $\sigma s$ ("/y) |
|---|---|---|
| AstDys all | 0.2415 | 0.1645 |
| AFP all | 0.2963 | 0.1747 |
| AstDyS ±1My | 0.0387 | 0.0428 |
| AstDys -2My | 0.0782 | 0.0365 |
| AstDys 10My | 1.1779 | 0.8325 |



# **Optimization** of the procedure to compute asteroid proper elements

Procedure demanding in terms of the computing power:

- Integration time span
- Completeness of the dynamical model

Are the more demanding procedures of Afp and PDS justified ?

Whether the optimized approach of AstDyS provides good enough results ?

Proper elements computed using short (−2 My) and long (10 My) integration time span, and the dynamical models including either 4 or 7 planets

| Asteroid | $a$ (au) | $e$ | sin $I$ | $n$ (deg/y) | $g$ ("/y) | $s$ ("/y) | My | pl |
|---|---|---|---|---|---|---|---|---|
| 494657 | 2.720173 | 0.247555 | 0.158558 | 80.22406 | 53.82266 | −71.25520 | −2 | 4 |
| 494657 | 2.720174 | 0.247899 | 0.158592 | 80.22399 | 53.83364 | −71.15010 | 10 | 4 |
| 494657 | 2.720179 | 0.249468 | 0.158464 | 80.22447 | 54.08581 | −71.62373 | −2 | 7 |
| 494662 | 2.590734 | 0.221024 | 0.061548 | 86.31341 | 53.18891 | −61.00469 | −2 | 4 |
| 494662 | 2.590727 | 0.240721 | 0.061816 | 86.31335 | 53.45665 | −63.47093 | 10 | 4 |
| 494662 | 2.590741 | 0.225743 | 0.061585 | 86.31384 | 53.50577 | −61.55593 | −2 | 7 |

10000 asteroids Nos. 494657 - 509585

$2.5 < a < 3.3$; $T_L > 10^5$ y ⇒ 7450/7448

Integration for -2 My with 4 planets

Integrated for 10 My with 4 planets
and for -2 My with 7 planets

# 2 My vs. 10 My

| Mean | −2 My | 10 My | $\sigma_{-2} > \sigma_{10}$ | $\sigma_{-2} < \sigma_{10}$ |
|---|---|---|---|---|
| $\bar{\sigma} a$ | 0.0000020 | 0.0000081 | 2763(37.1%) | 4685(62.9%) |
| $\bar{\sigma} e$ | 0.0019252 | 0.0019915 | 2884(38.7%) | 4564(61.3%) |
| $\bar{\sigma} sI$ | 0.0001811 | 0.0003102 | 5367(72.1%) | 2081(27.9%) |
| $\bar{\sigma} g$ | 0.1432 | 0.1349 | 2001(26.9%) | 5447(73.1%) |
| $\bar{\sigma} s$ | 0.1415 | 0.1681 | 1914(25.7%) | 5534(74.3%) |

# 4 planets vs. 7 planets

| Mean | 4 planets | 7 planets | $\sigma_4 > \sigma_7$ | $\sigma_4 < \sigma_7$ |
|---|---|---|---|---|
| $\bar{\sigma} a$ | 0.0000020 | 0.0000125 | 3085(41.4%) | 4365(58.6%) |
| $\bar{\sigma} e$ | 0.0019252 | 0.0017843 | 3948(53.0%) | 3502(47.0%) |
| $\bar{\sigma} sI$ | 0.0001811 | 0.0001900 | 3376(45.3%) | 4074(54.7%) |
| $\bar{\sigma} g$ | 0.1432 | 0.1588 | 4295(57.6%) | 3155(42.4%) |
| $\bar{\sigma} s$ | 0.1415 | 0.1276 | 4541(61.0%) | 2909(39.0%) |

# Conclusions (1)

- proper elements obtained in different runs are nearly the same;
- no significant improvement of the overall accuracy of results when using the longer integration or more complex dynamical model;
- from the point of view of the application of these elements in classification of asteroids into families, they can be considered as practically equal.

# Failures in computation of proper elements

Errors in numerical integration – model and time span dependent:

- error in elements - e.g. hyperbolic orbits (a>100 au): discarded, no proper elements produced;

- too short integration step – asteroid removed from integration;

Errors manifested in proper elements computation:

- format problems (****) in the output due to too large errors

  if in vprs.sig & vprs.del: asteroid removed

  if in vprs.del only: asteroid kept

- failure of the computation of variational equations – NaN in LCE

# Statistics: numbered asteroids

Total of 390992 asteroids (185655 till April 2008 + 295347 after June 2017)

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130 outer belt discarded in -2 My integration (4 for format problems)

499 outer belt discarded in 10 My integration (15 for format problems)

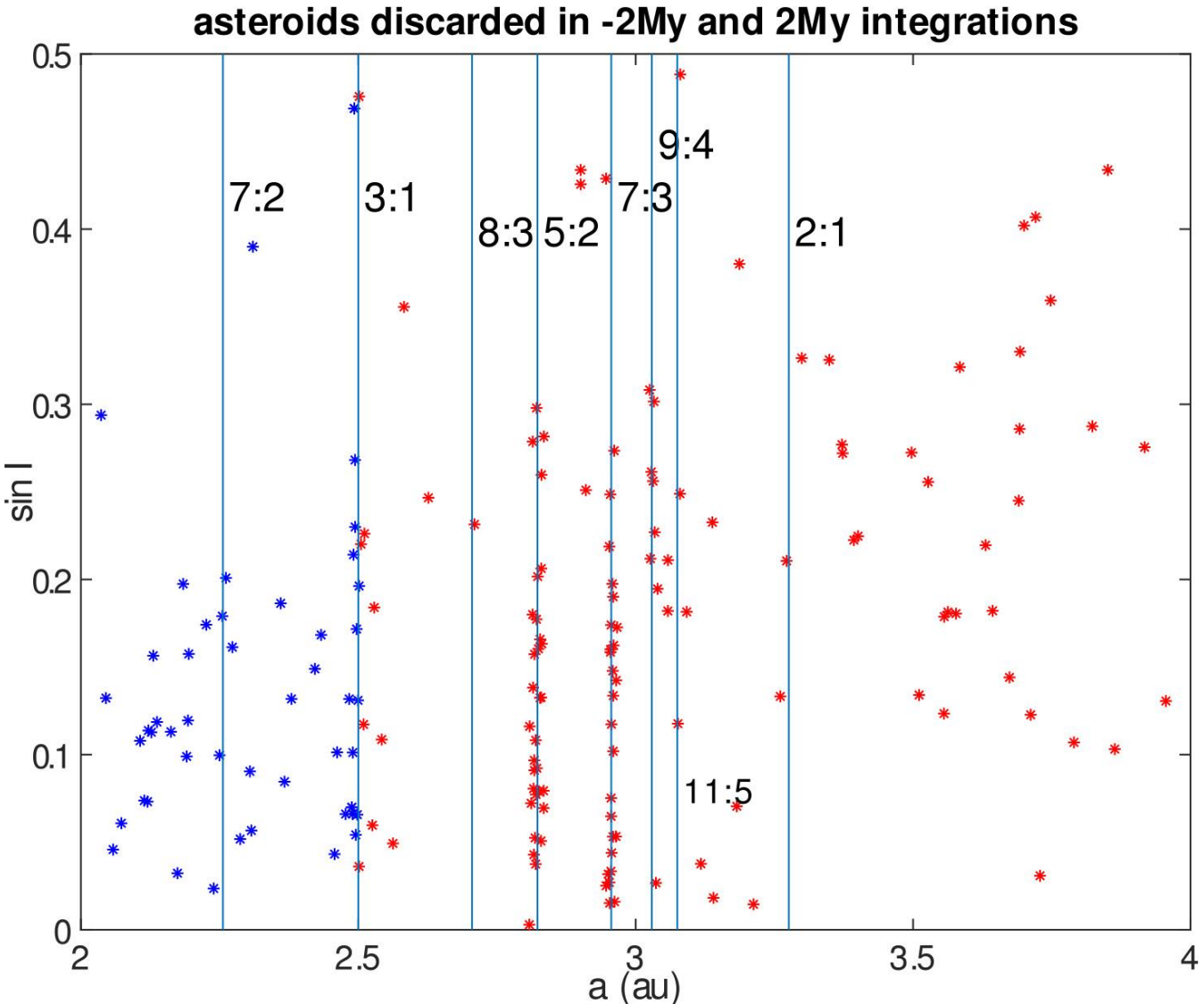
54 inner belt discarded in +-1My (2) integration (5 for format problems)

91 outer belt with LCE set to 999.99 in -2 My integration

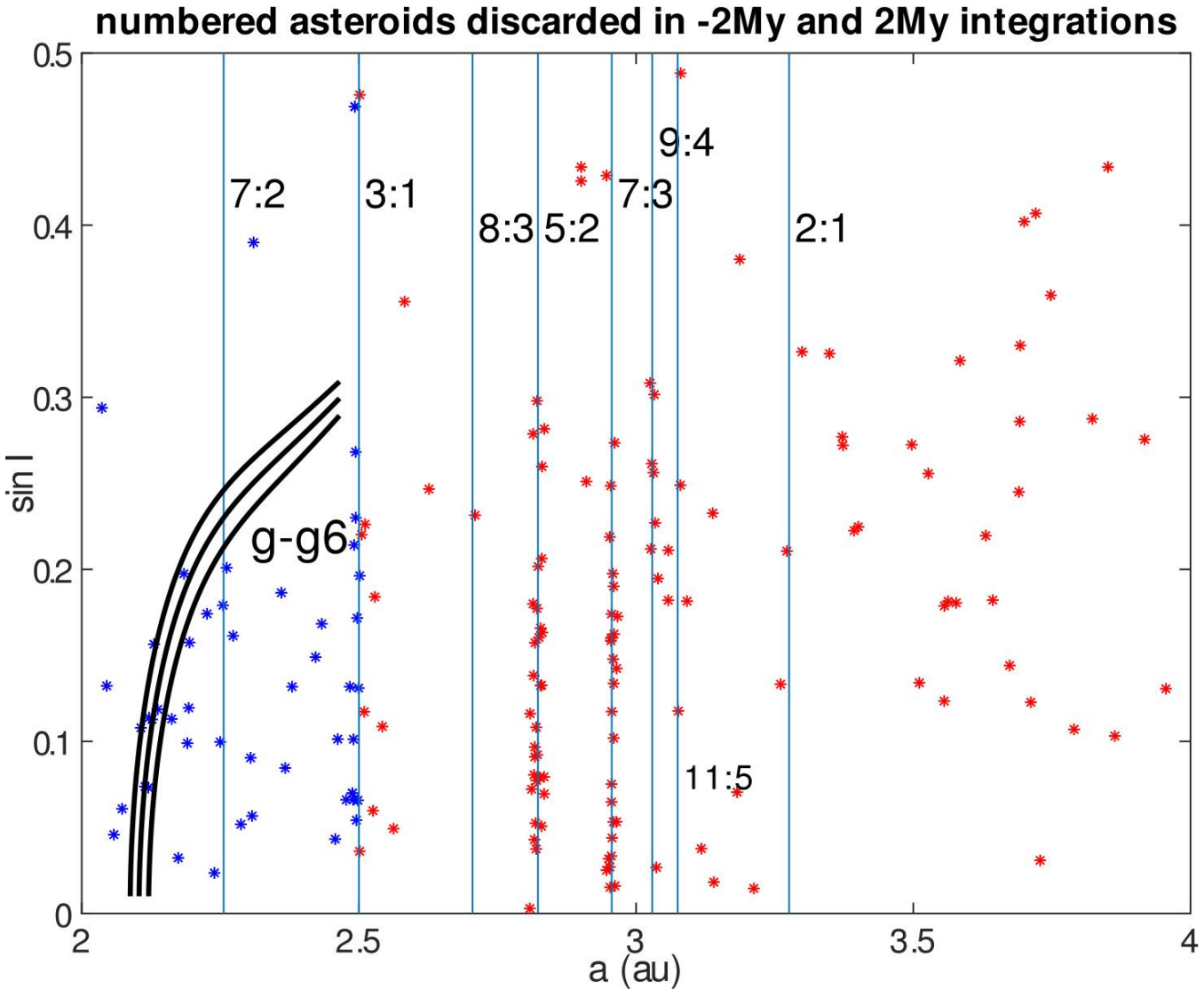
384 outer belt with LCE set to 999.99 in 10 My integration

49 inner belt with LCE set to 999.99 in +-1 My integration

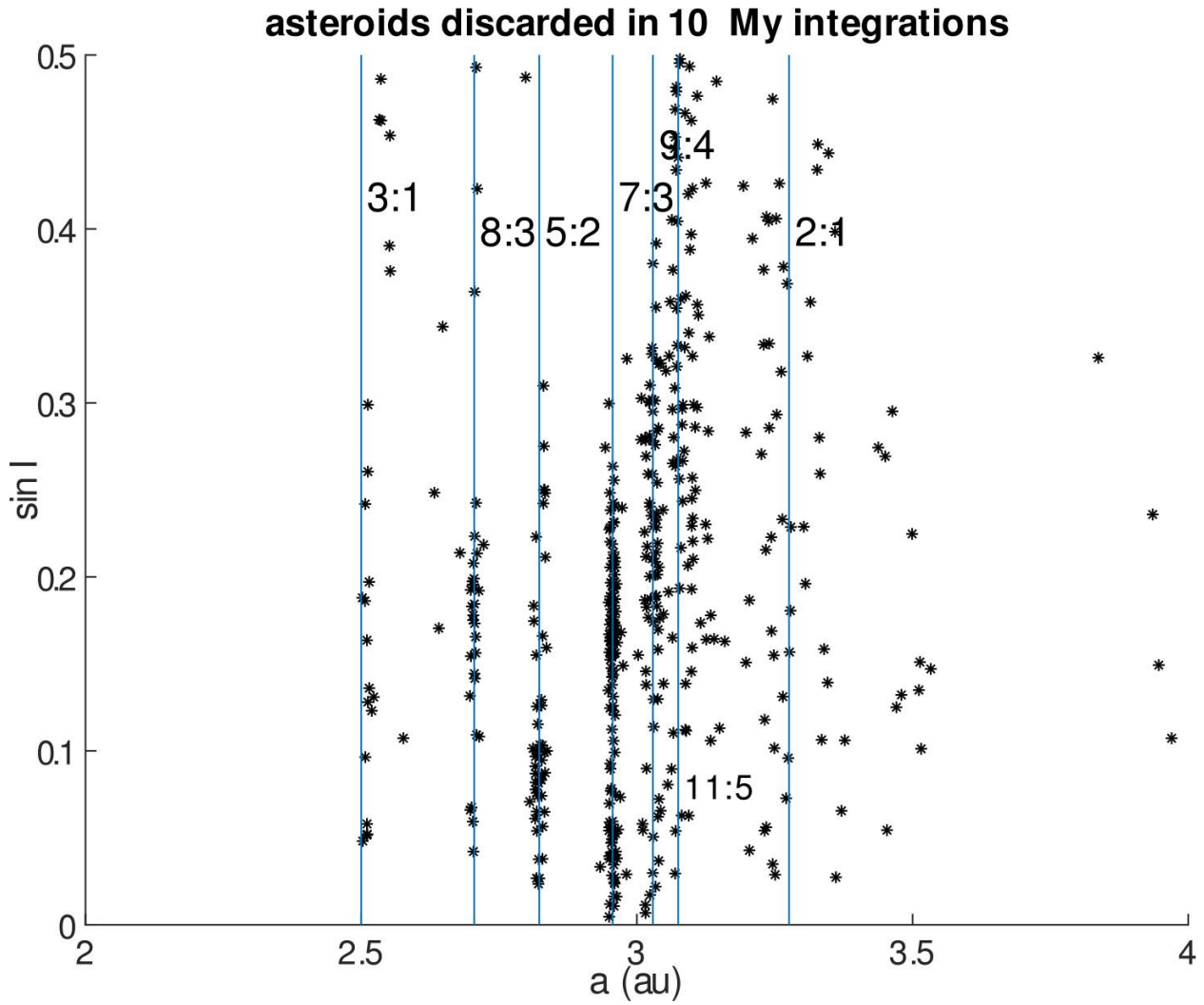
# Overview of the Main Belt (numbered asteroids)



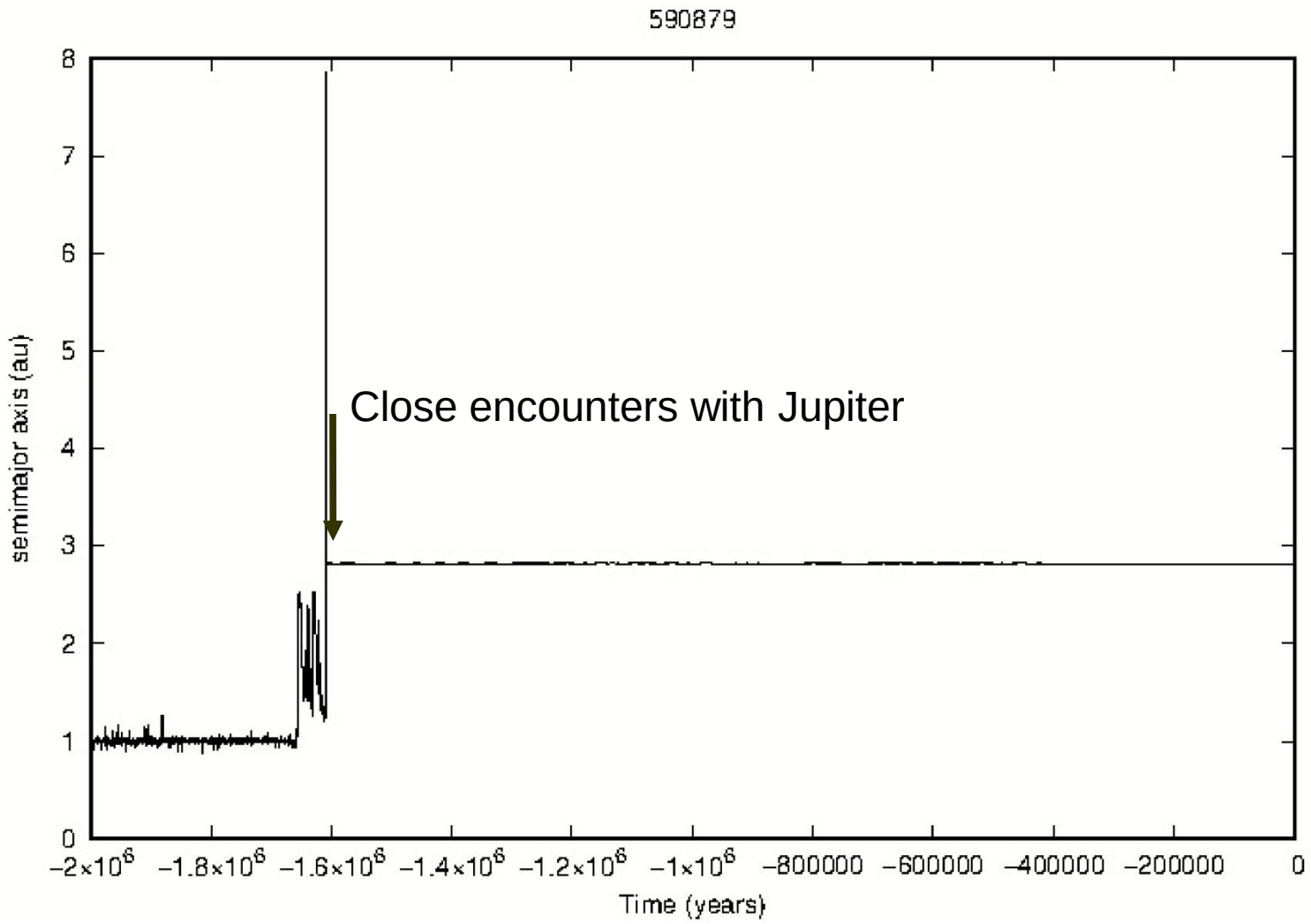
# Overview of the Main Belt (numbered asteroids)



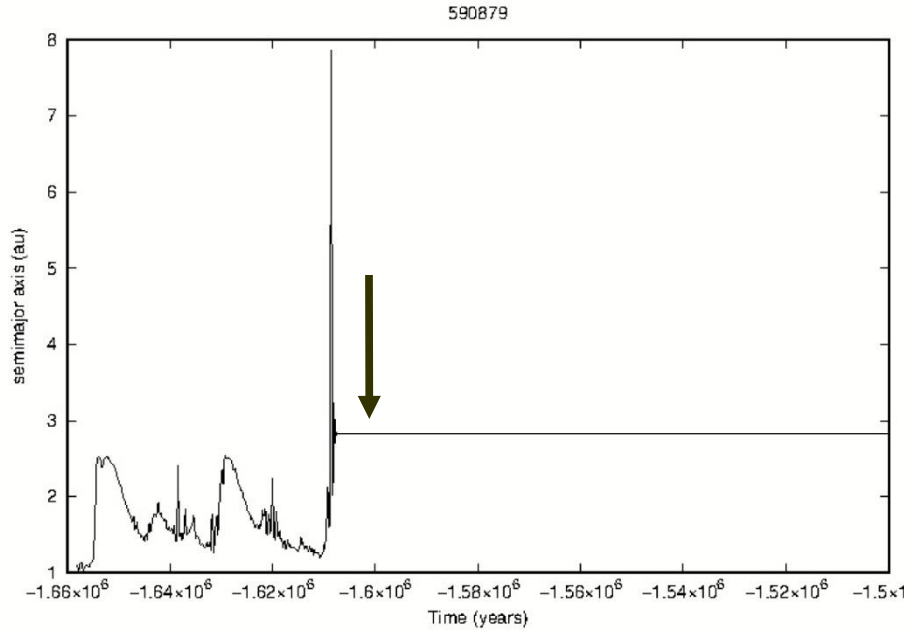
# Overview of the Main Belt (numbered asteroids)



# Examples: (590879) 2012VK14

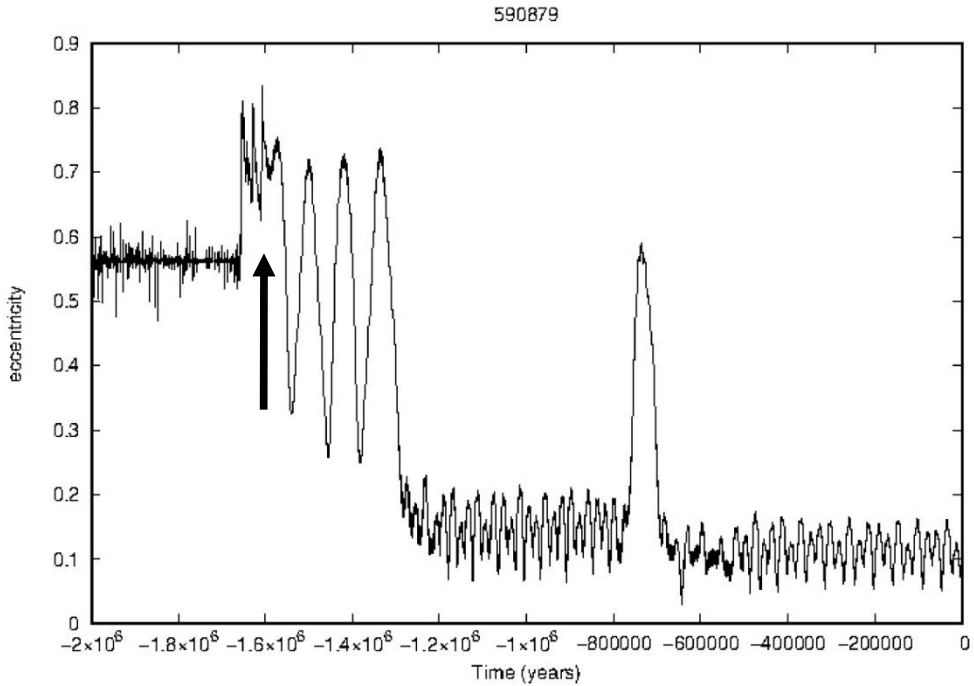


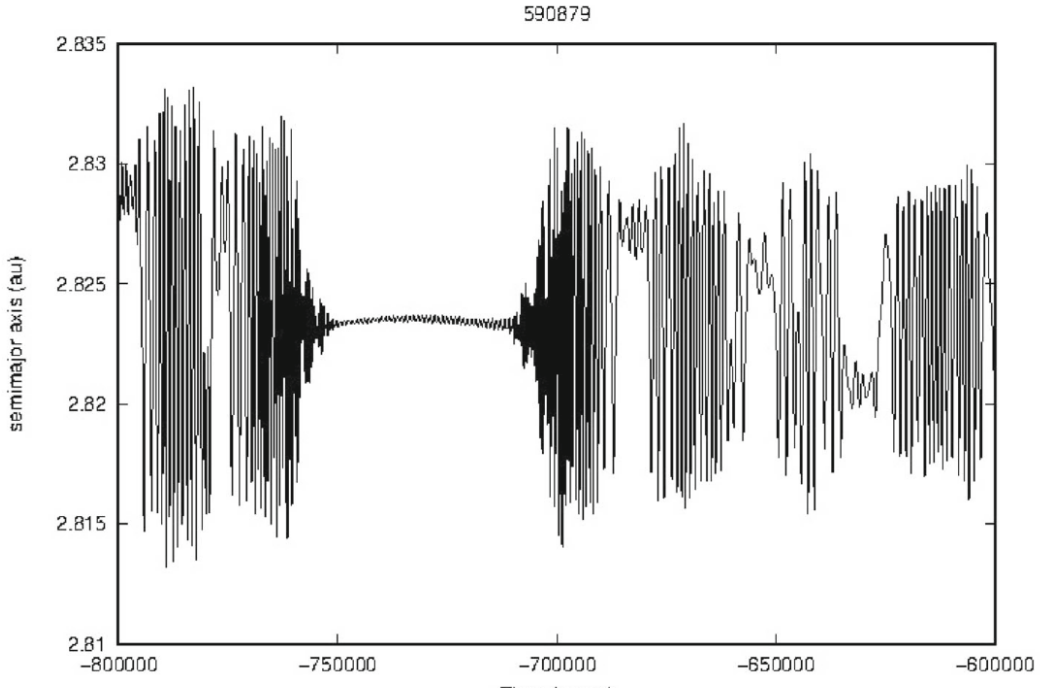
# Examples: (590879) 2012VK14

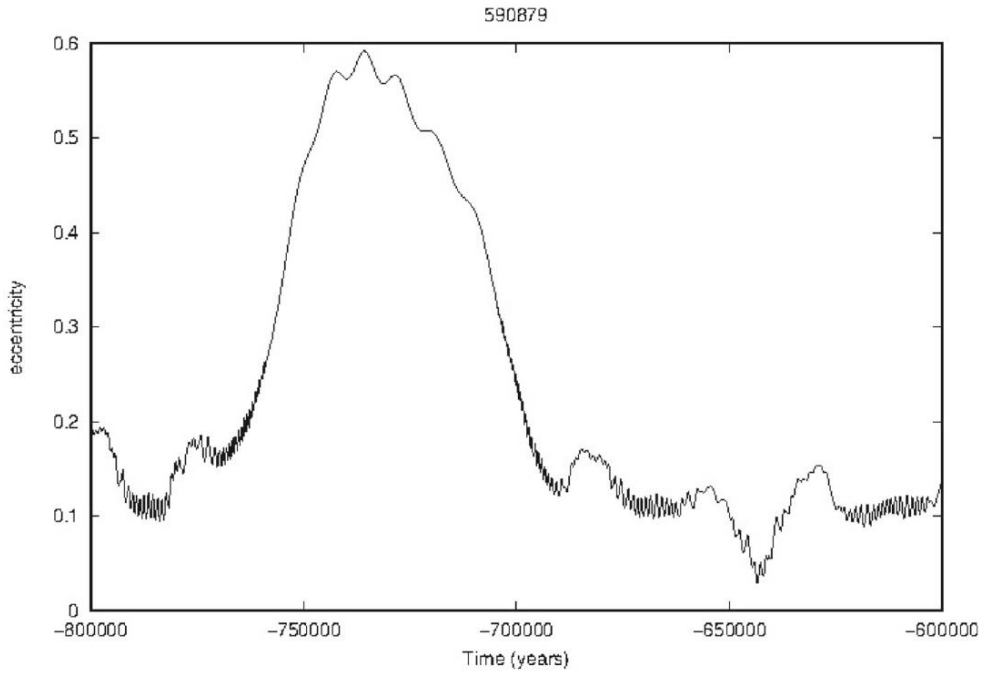



Onset of close encouters with Jupiter at -1.608 My

(shown by arrow)





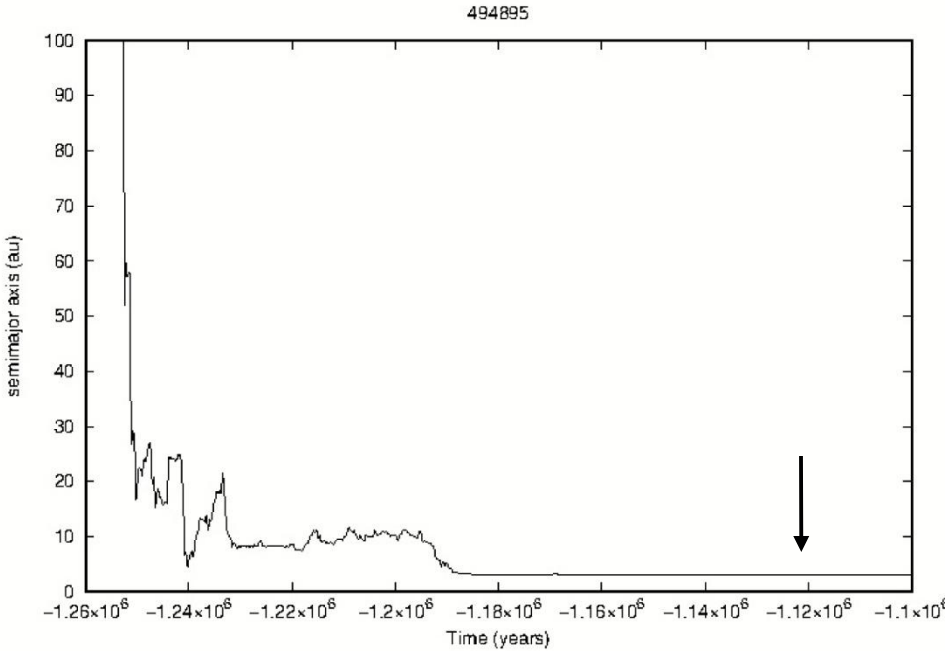




Asteroid

(590879)

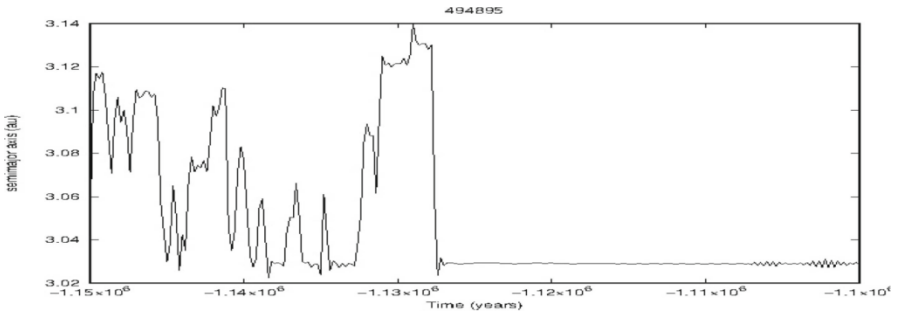
locked in
5:2 MMR

# Examples: (494895) 2008UA53 semimajor axis

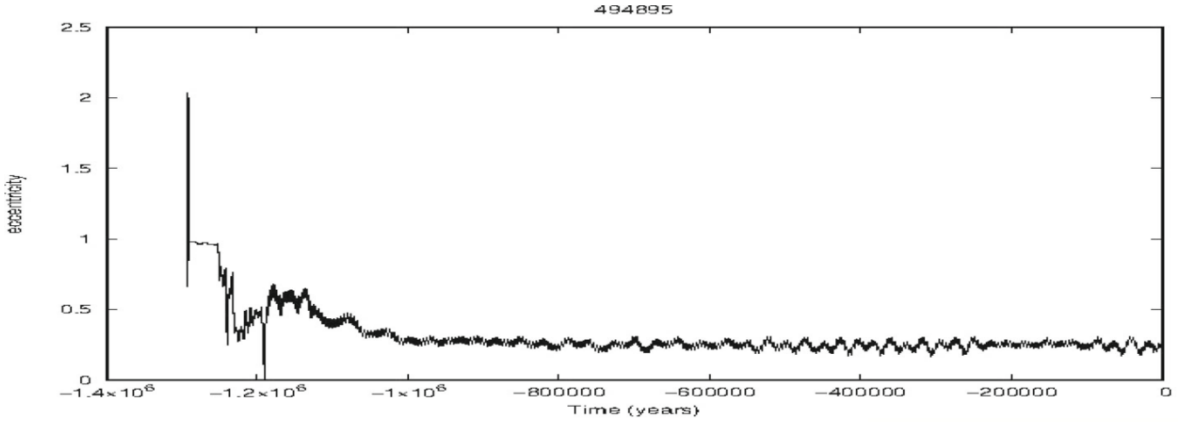


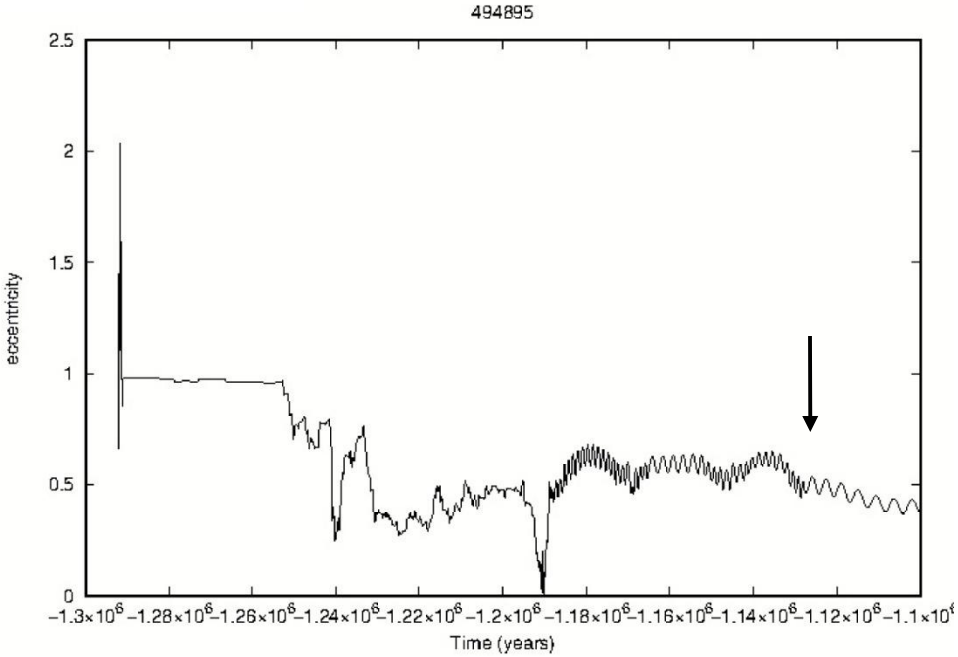

Onset of close encounters with Jupiter at -1.127 My

(shown by arrow)

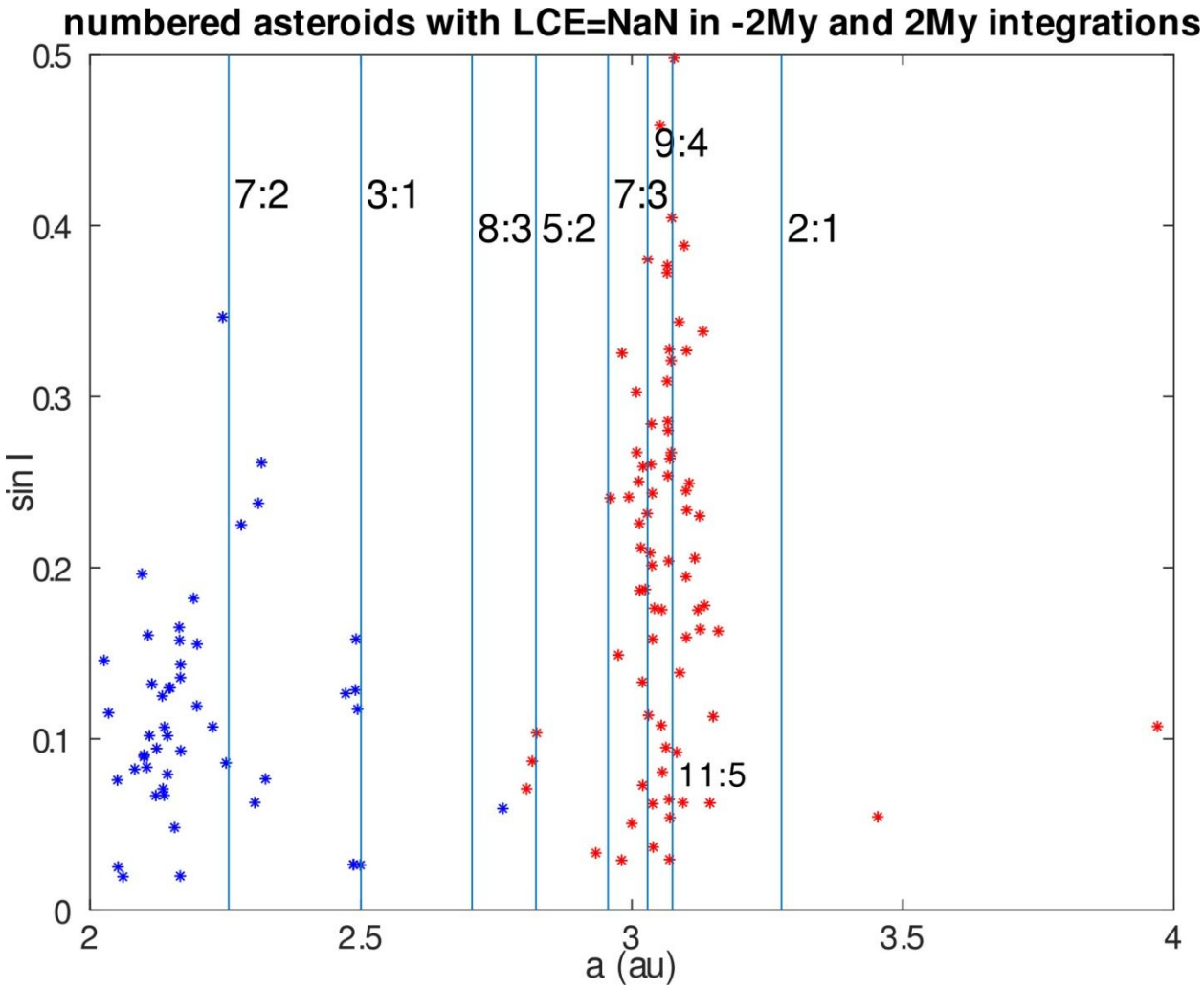


# Examples: (494895) 2008UA53 eccentricity

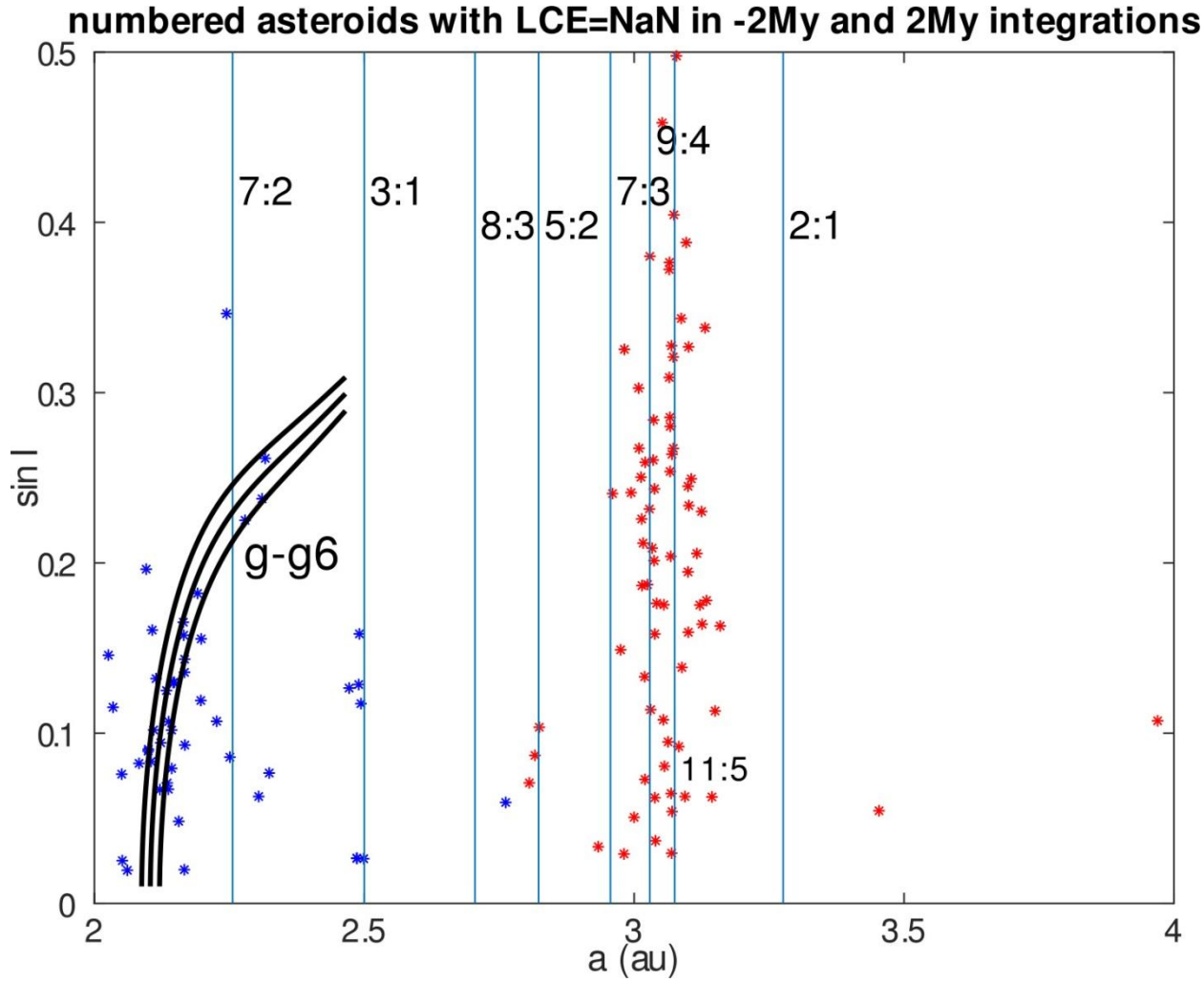


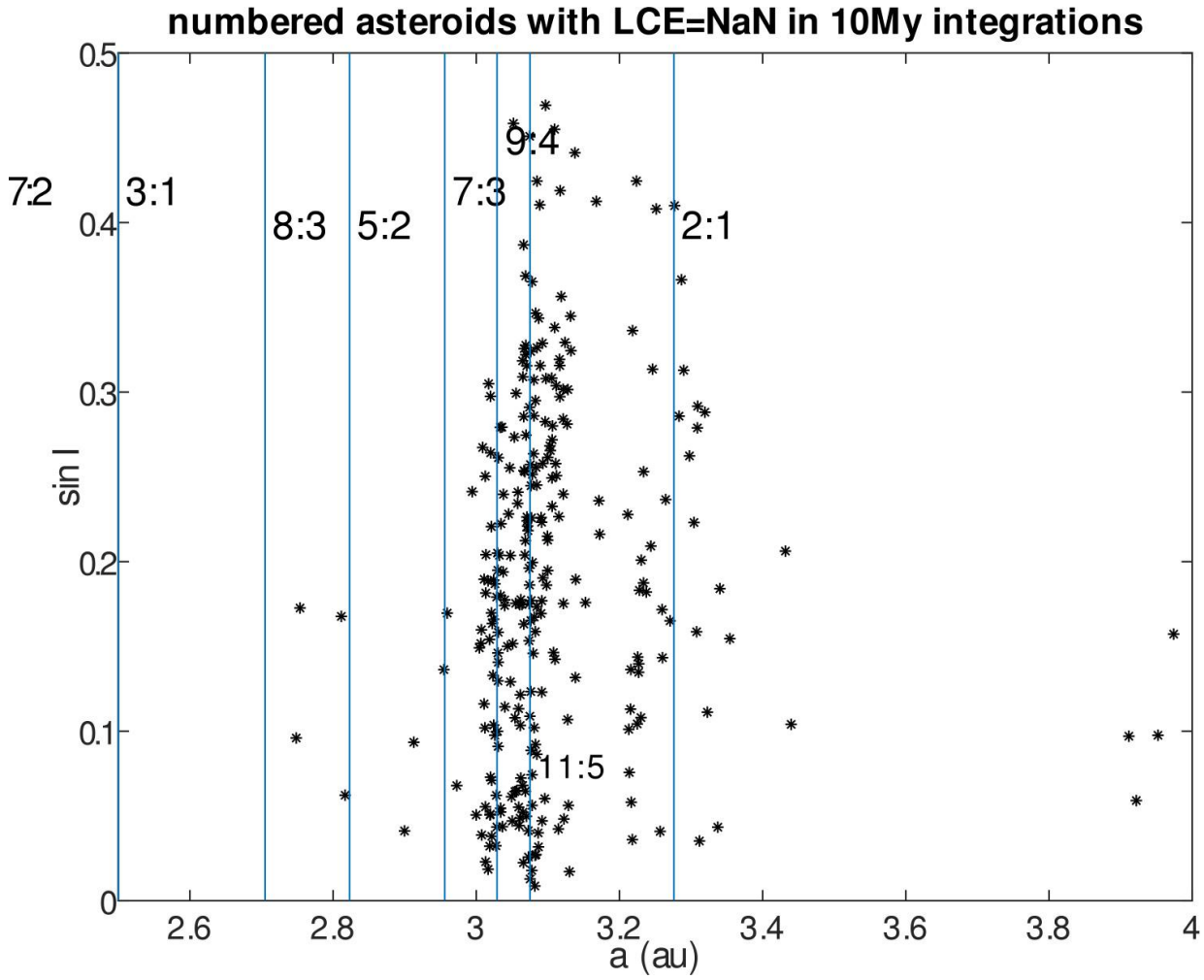
# Overview of the Main Belt (numbered asteroids)



# Overview of the Main Belt (numbered asteroids)



# Overview of the Main Belt (numbered asteroids)



# Conclusions (2)

- different types of failures identified;
- hyperbolic orbits occur for 0.3% asteroids over 10My;
- failures distributed along strong MMRs in the asteroid belt;
- typically, close approaches to Jupiter give rise to instabilities.

Thank you !