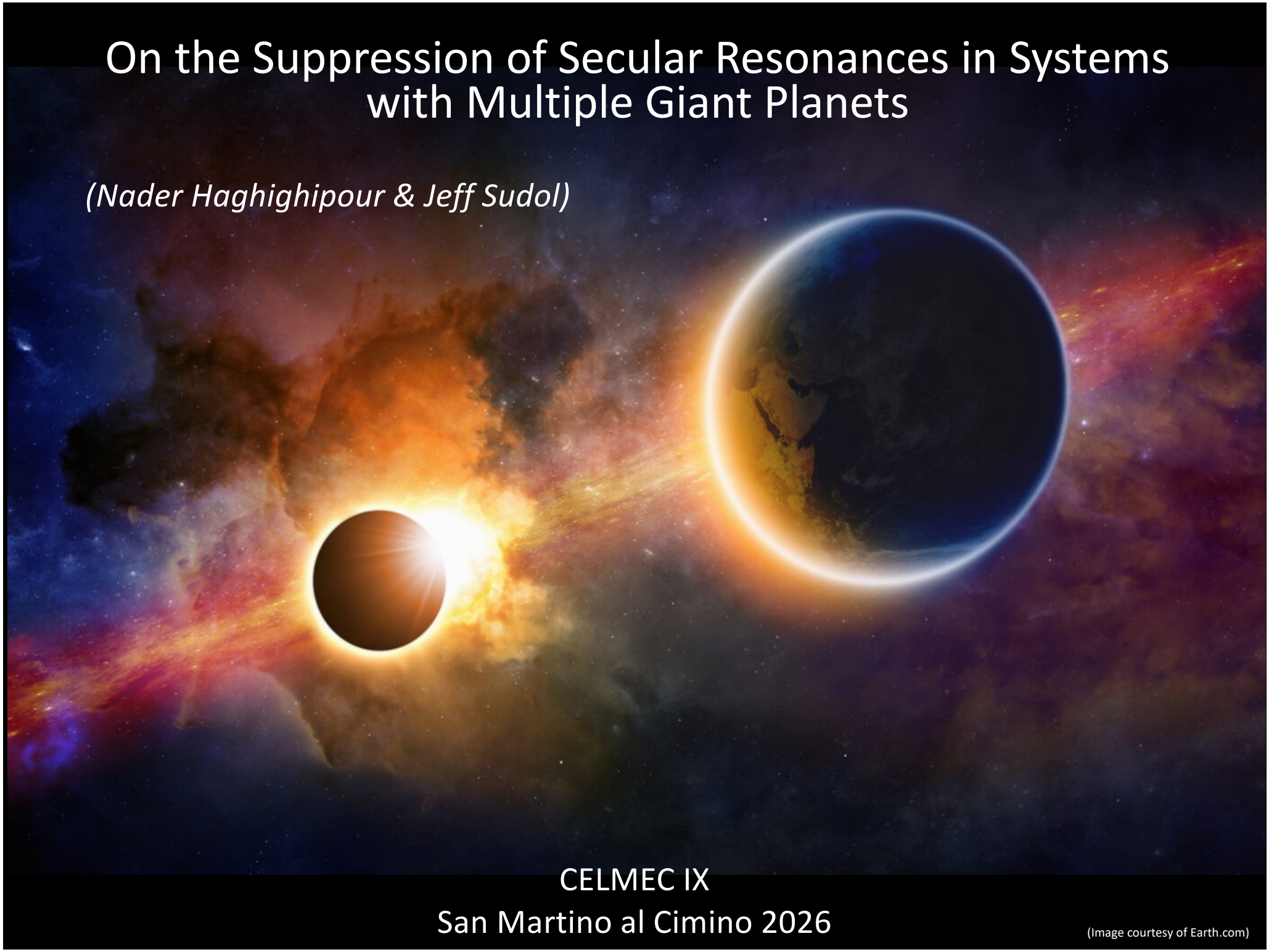
On the Suppression of Secular Resonances in Systems with Multiple Giant Planets

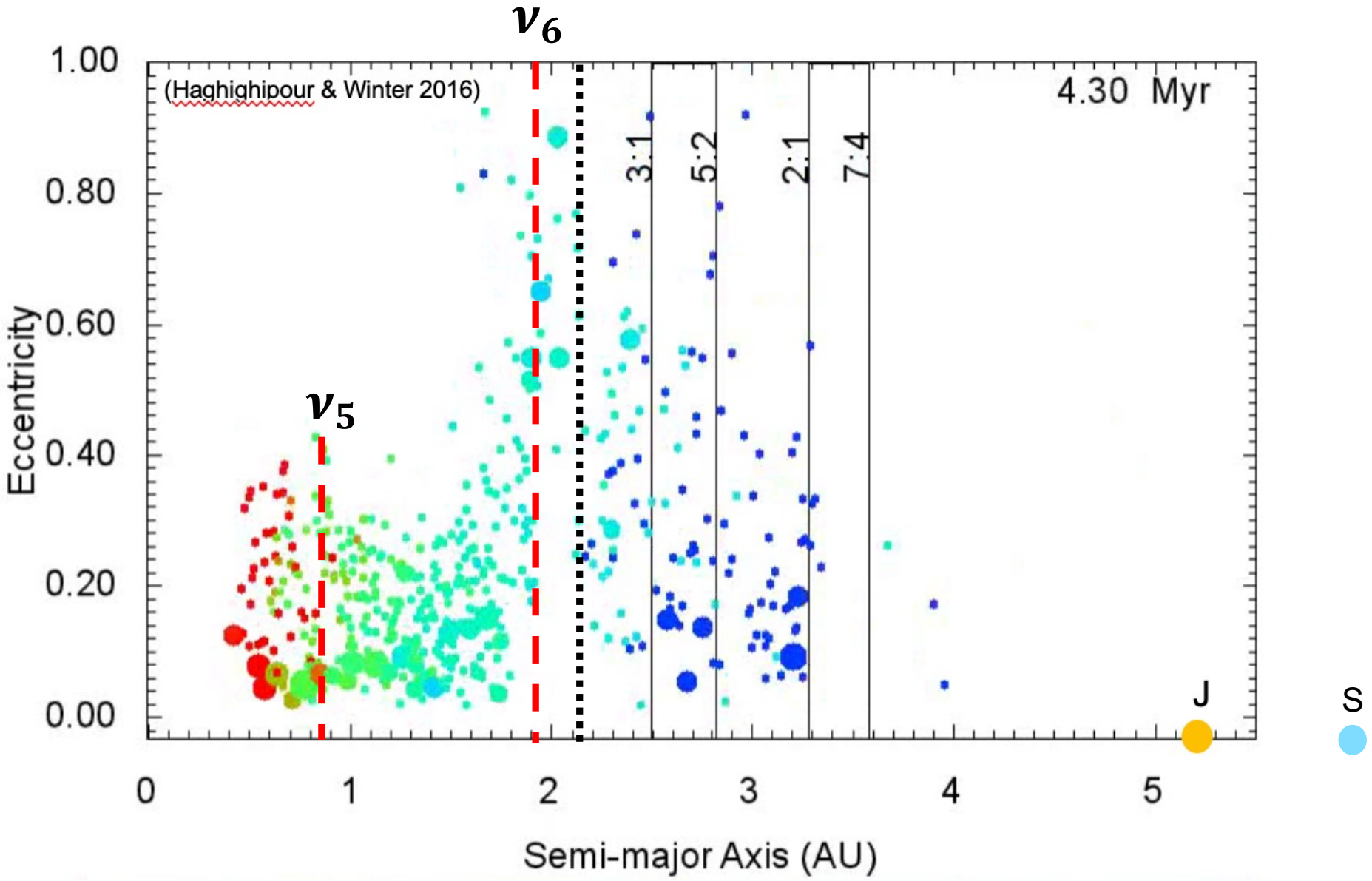
(Nader Haghighipour & Jeff Sudol)



CELMEC IX

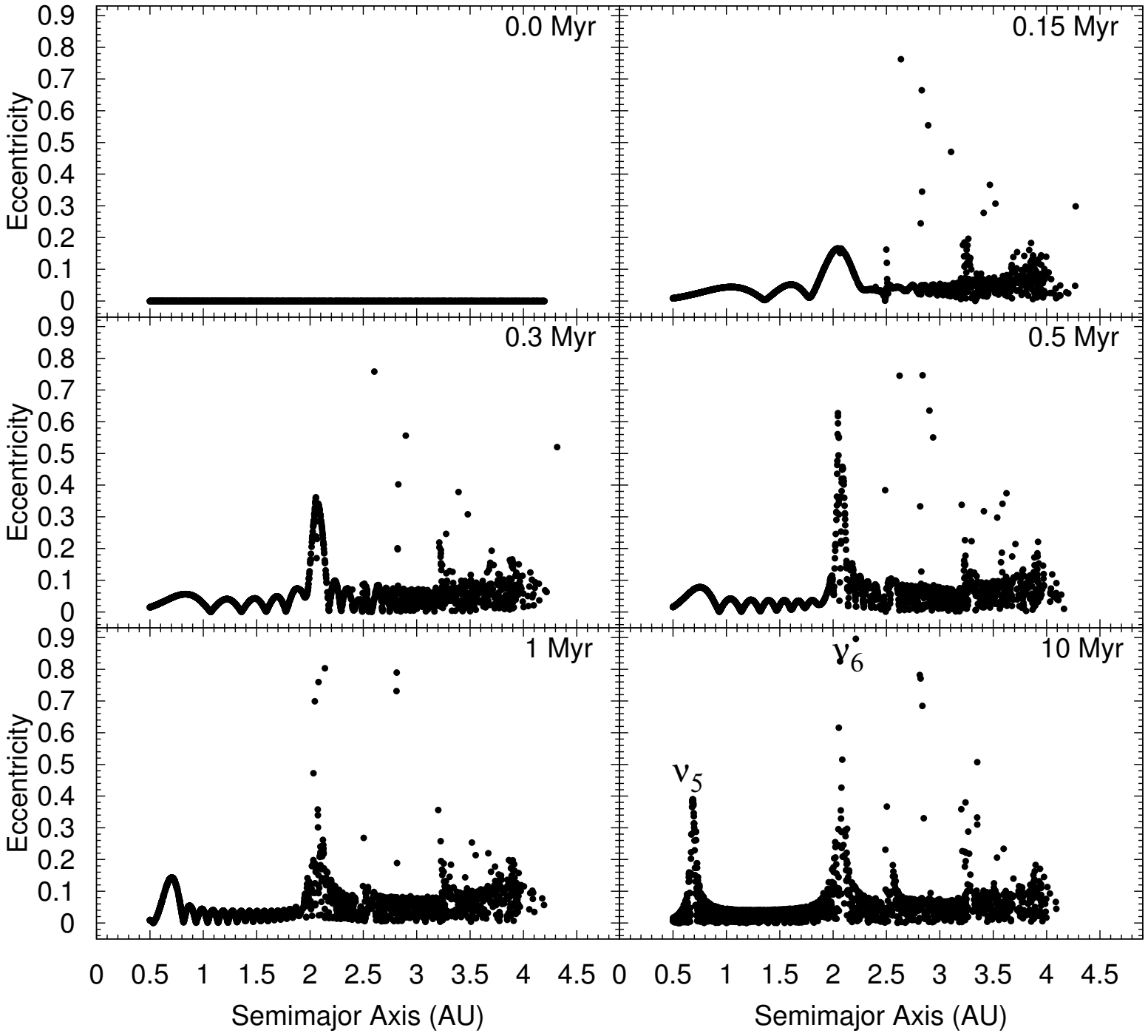
San Martino al Cimino 2026

(Image courtesy of Earth.com)

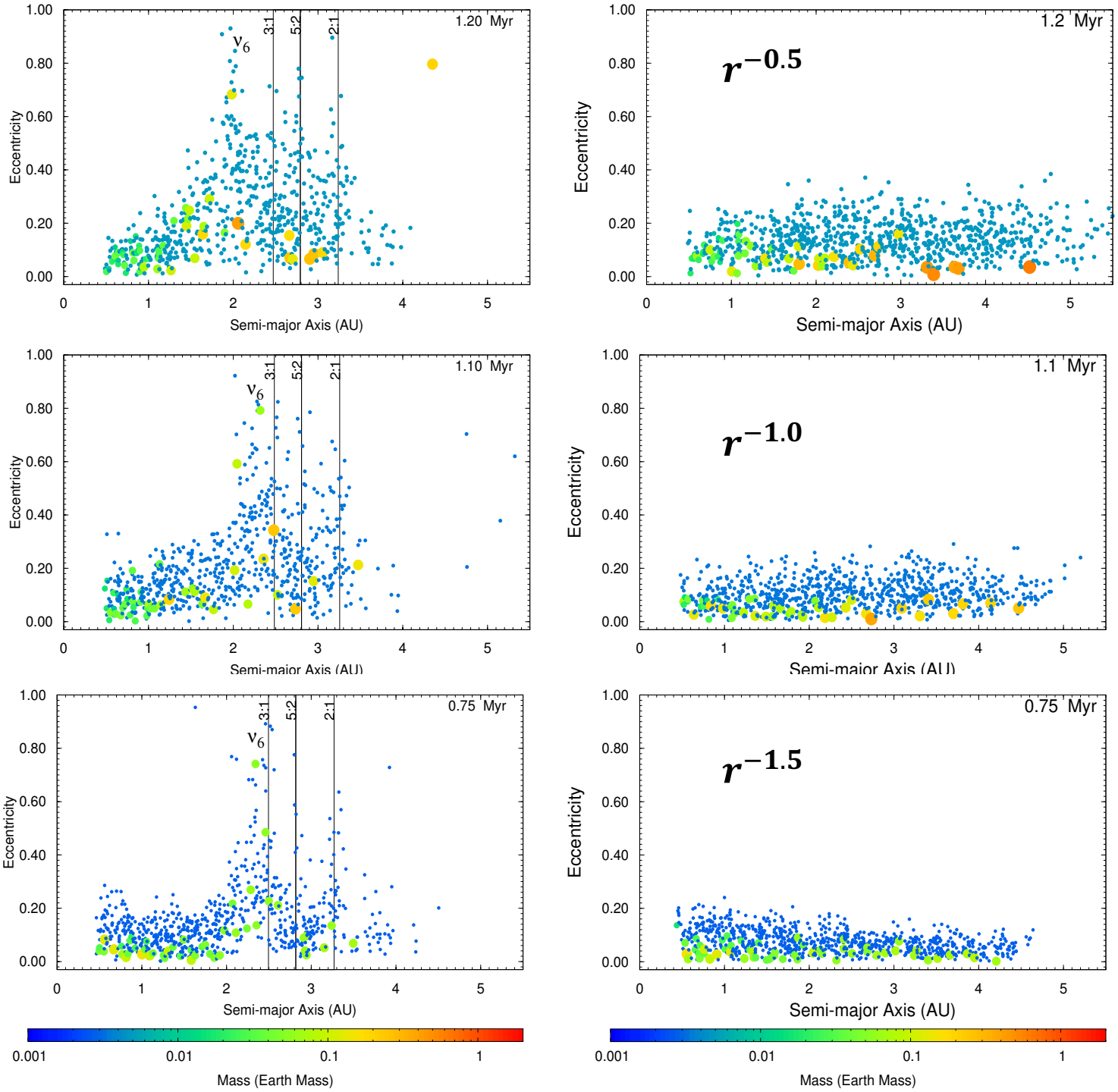


While Jupiter is 3 times more massive than Saturn, the effect of its secular resonance is weaker.

(Haghighipour & Winter, 2016, CMDA, 124:235–268)

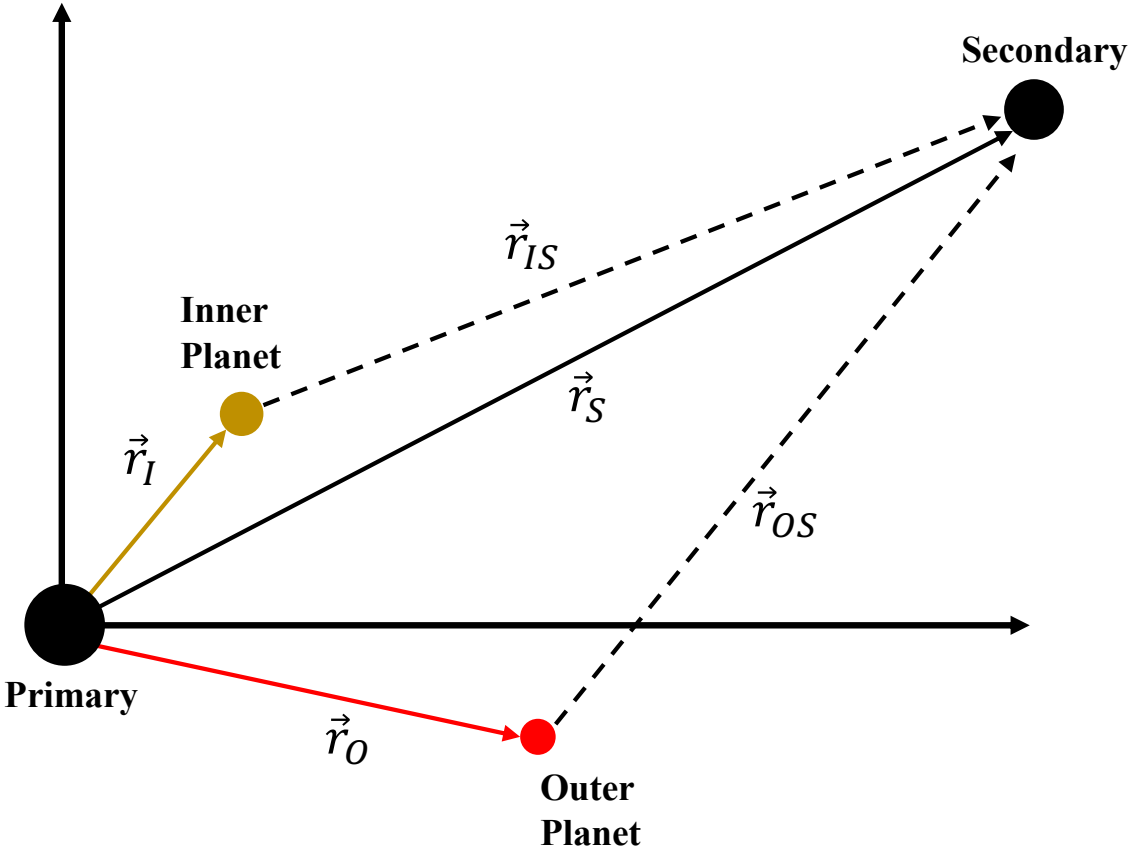


(Haghighipour & Winter, 2016, CMDA, 124:235–268)



$$\frac{de_j}{dt} = -\frac{1}{n_j e_j a_j^2}\frac{\partial R_j}{\partial \varpi_j}$$

$$\frac{d\varpi_j}{dt} = \frac{1}{n_j e_j a_j^2}\frac{\partial R_j}{\partial e_j}$$



$$e_{\mathrm{E}} \sin \varpi_{\mathrm{E}} = e_{\mathrm{E}} \sin(\Omega_{\mathrm{E}} t + \theta_{\mathrm{E}})$$

$$e_{\mathrm{E}} \cos \varpi_{\mathrm{E}} = e_{\mathrm{E}} \cos(\Omega_{\mathrm{E}} t + \theta_{\mathrm{E}})$$

$$R_j = \left[\frac{\mu_k}{|\boldsymbol{r}_k - \boldsymbol{r}_j|} + \frac{\mu_\ell}{|\boldsymbol{r}_\ell - \boldsymbol{r}_j|}\right] - \left[\mu_k \frac{\boldsymbol{r}_k \cdot \boldsymbol{r}_j}{r_k^3} + \mu_\ell \frac{\boldsymbol{r}_\ell \cdot \boldsymbol{r}_j}{r_\ell^3}\right], \qquad j \neq k \neq \ell = \mathrm{I}, \mathrm{O}, \mathrm{S}.$$

(Haghighipour & Andrews, AJ, 170, 117, 2025)

$$R_{\rm I} = n_{\rm I}^2 a_{\rm I}^2 \left(\frac{m_{\rm O}}{m_{\rm P}+m_{\rm I}}\right)\left[\frac{1}{8}\alpha_{\rm IO}^2 e_{\rm I}^2 b_{3/2}^{(1)}(\alpha_{\rm IO}) - \frac{1}{4}\alpha_{\rm IO}^2 e_{\rm I} e_{\rm O} b_{3/2}^{(2)}(\alpha_{\rm IO})\cos(\varpi_{\rm I}-\varpi_{\rm O})\right]$$
$$+ n_{\rm I}^2 a_{\rm I}^2 \left(\frac{m_{\rm S}}{m_{\rm P}+m_{\rm I}}\right)\left[\frac{1}{8}\alpha_{\rm IS}^2 e_{\rm I}^2 b_{3/2}^{(1)}(\alpha_{\rm IS}) - \frac{1}{4}\alpha_{\rm IS}^2 e_{\rm I} e_{\rm S} b_{3/2}^{(2)}(\alpha_{\rm IS})\cos(\varpi_{\rm I}-\varpi_{\rm S})\right],$$

$$R_{\rm O} = n_{\rm O}^2 a_{\rm O}^2 \left(\frac{m_{\rm I}}{m_{\rm P}+m_{\rm O}}\right)\left[\frac{1}{8}\alpha_{\rm IO} e_{\rm O}^2 b_{3/2}^{(1)}(\alpha_{\rm IO}) - \frac{1}{4}\alpha_{\rm IO} e_{\rm I} e_{\rm O} b_{3/2}^{(2)}(\alpha_{\rm IO})\cos(\varpi_{\rm I}-\varpi_{\rm O})\right]$$
$$+ n_{\rm O}^2 a_{\rm O}^2 \left(\frac{m_{\rm S}}{m_{\rm P}+m_{\rm O}}\right)\left[\frac{1}{8}\alpha_{\rm OS}^2 e_{\rm O}^2 b_{3/2}^{(1)}(\alpha_{\rm OS}) - \frac{1}{4}\alpha_{\rm OS}^2 e_{\rm O} e_{\rm S} b_{3/2}^{(2)}(\alpha_{\rm OS})\cos(\varpi_{\rm O}-\varpi_{\rm S})\right],$$

$$R_{\rm S} = n_{\rm S}^2 a_{\rm S}^2 \left(\frac{m_{\rm I}}{m_{\rm P}+m_{\rm S}}\right)\left[\frac{1}{8}\alpha_{\rm IS} e_{\rm S}^2 b_{3/2}^{(1)}(\alpha_{\rm IS}) - \frac{1}{4}\alpha_{\rm IS} e_{\rm I} e_{\rm S} b_{3/2}^{(2)}(\alpha_{\rm IS})\cos(\varpi_{\rm I}-\varpi_{\rm S})\right]$$
$$+ n_{\rm S}^2 a_{\rm S}^2 \left(\frac{m_{\rm O}}{m_{\rm P}+m_{\rm S}}\right)\left[\frac{1}{8}\alpha_{\rm OS} e_{\rm S}^2 b_{3/2}^{(1)}(\alpha_{\rm OS}) - \frac{1}{4}\alpha_{\rm OS} e_{\rm O} e_{\rm S} b_{3/2}^{(2)}(\alpha_{\rm OS})\cos(\varpi_{\rm O}-\varpi_{\rm S})\right].$$

(Haghighipour & Andrews, AJ, 170, 117, 2025)

$$R_j = \frac{1}{2} n_j a_j^2 \left[A_{jk}\right] e_j e_k \cos(\varpi_j - \varpi_k).$$

For a small object (E) such as a planetary embryo, similar calculation results in

$$\Omega_{\rm E} = \sum_j \frac{1}{4} n_{\rm E} \left(\frac{m_j}{m_{\rm P}}\right) \alpha_{{\rm E}j}^2 b_{3/2}^{(1)}(\alpha_{{\rm E}j}).$$

Secular resonances appear where $\Omega_{\rm E} = g_k$ where g_k are the eigenvalues of the matrix $\left[A_{jk}\right]$

$$A_{11} = \frac{n_{\rm I}}{4(m_{\rm p}+m_{\rm I})}\left[m_{\rm O}\alpha_{\rm IO}^2 b_{3/2}^{(1)}(\alpha_{\rm IO}) + m_{\rm S}\alpha_{\rm IS}^2 b_{3/2}^{(1)}(\alpha_{\rm IS})\right],$$
$$A_{12} = -\frac{n_{\rm I}}{4(m_{\rm p}+m_{\rm I})}\left[m_{\rm O}\alpha_{\rm IO}^2 b_{3/2}^{(2)}(\alpha_{\rm IO})\right],$$
$$A_{13} = -\frac{n_{\rm I}}{4(m_{\rm p}+m_{\rm I})}\left[m_{\rm S}\alpha_{\rm IS}^2 b_{3/2}^{(2)}(\alpha_{\rm IS})\right],$$
$$A_{21} = -\frac{n_{\rm O}}{4(m_{\rm p}+m_{\rm O})}\left[m_{\rm O}\alpha_{\rm IO} b_{3/2}^{(2)}(\alpha_{\rm IO})\right],$$
$$A_{22} = \frac{n_{\rm O}}{4(m_{\rm p}+m_{\rm O})}\left[m_{\rm I}\alpha_{\rm IO} b_{3/2}^{(1)}(\alpha_{\rm IO}) + m_{\rm S}\alpha_{\rm OS}^2 b_{3/2}^{(1)}(\alpha_{\rm OS})\right],$$
$$A_{23} = -\frac{n_{\rm O}}{4(m_{\rm p}+m_{\rm O})}\left[m_{\rm S}\alpha_{\rm OS}^2 b_{3/2}^{(2)}(\alpha_{\rm OS})\right],$$
$$A_{31} = -\frac{n_{\rm S}}{4(m_{\rm p}+m_{\rm S})}\left[m_{\rm O}\alpha_{\rm IS} b_{3/2}^{(2)}(\alpha_{\rm IS})\right],$$
$$A_{32} = -\frac{n_{\rm S}}{4(m_{\rm p}+m_{\rm S})}\left[m_{\rm O}\alpha_{\rm 2S}^2 b_{3/2}^{(2)}(\alpha_{\rm 2S})\right],$$
$$A_{33} = \frac{n_{\rm S}}{4(m_{\rm p}+m_{\rm S})}\left[m_{\rm I}\alpha_{\rm IS} b_{3/2}^{(1)}(\alpha_{\rm IS}) + m_{\rm O}\alpha_{\rm OS}^2 b_{3/2}^{(1)}(\alpha_{\rm OS})\right],$$

(Haghighipour & Andrews, AJ, 170, 117, 2025)

Binary

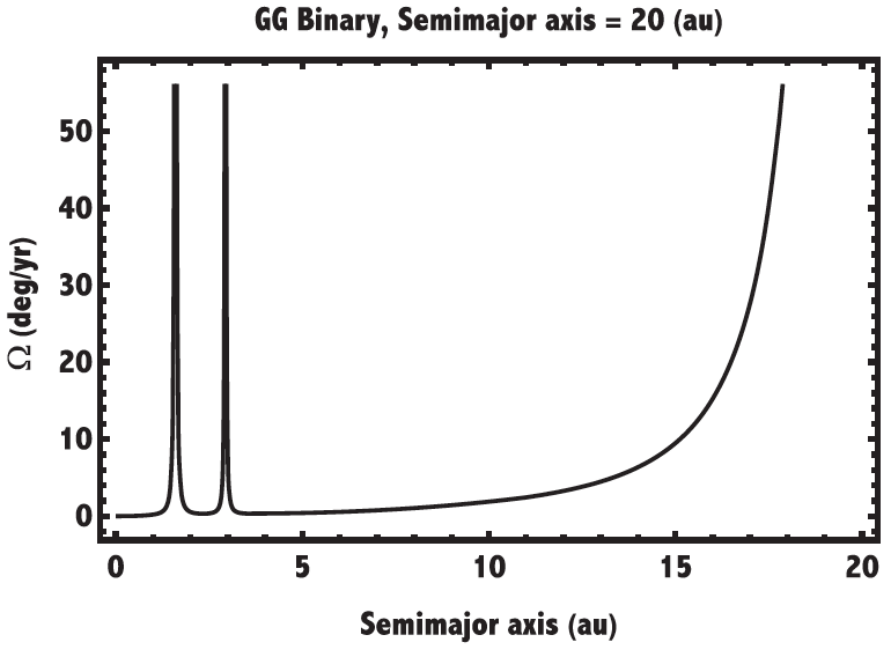
Primary = G star with solar mass
Secondary = M, K, G, F (0.4, 0.7, 1.0, 1.3 solar mass)
Semimajor axis = 20, 30, 40, 50 (AU)
Eccentricity = 0, 0.2, 0.4, 0.5

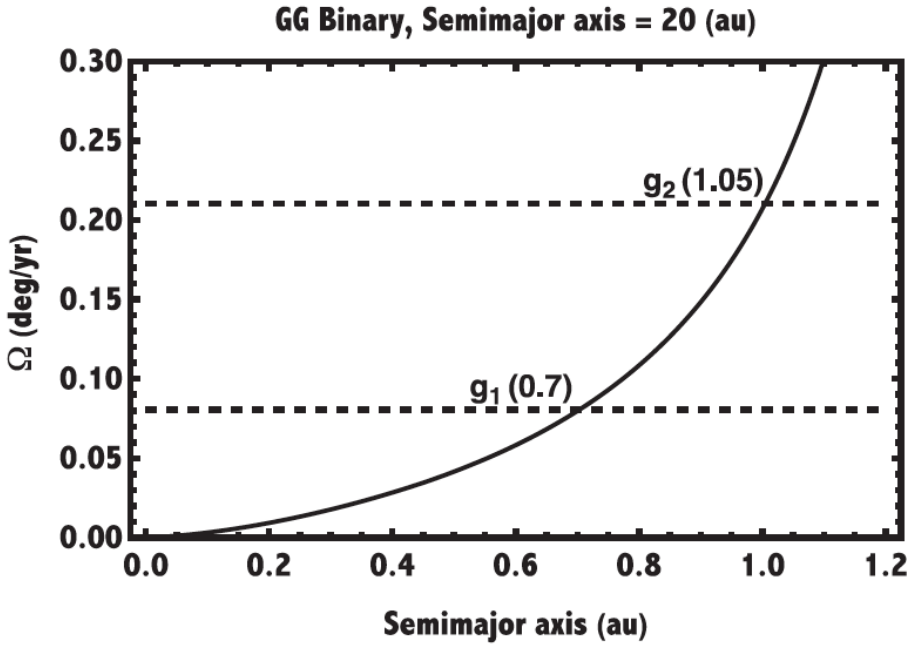
Planets

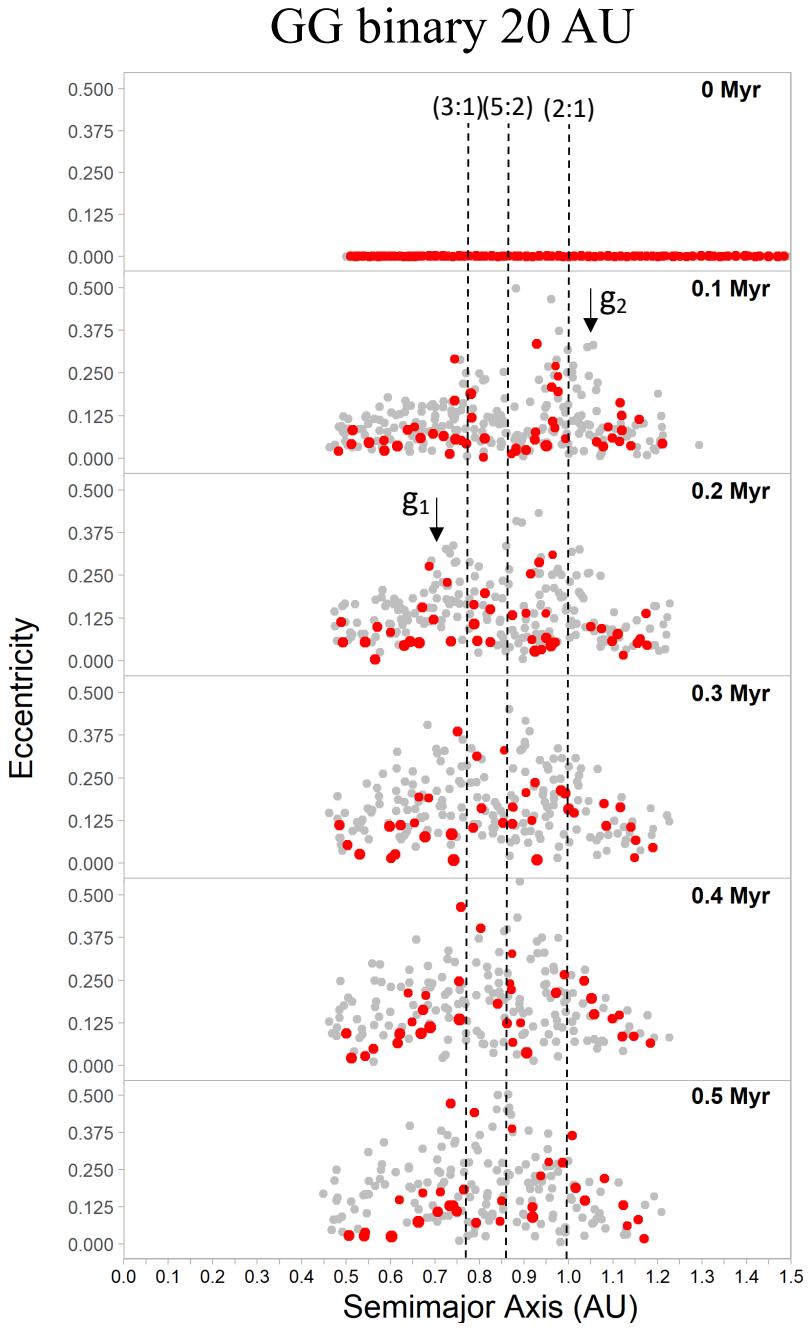
Inner planets = Jupiter-mass at 1.36, 1.6, 1.9, 2.17 AU
Outer planet = Saturn-mass at 2.5, 2.94, 3.5, 4 AU
Mutual inclination = 0, 5, 10 deg.

Disk

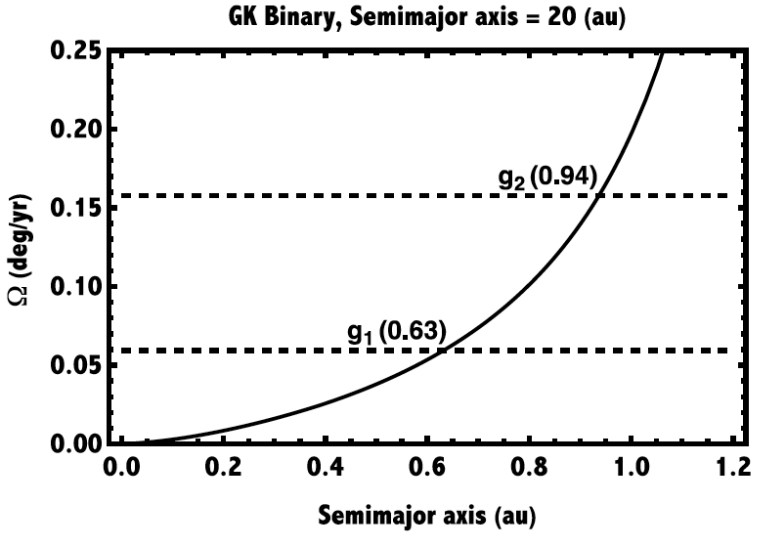
120 planetary embryos from 0.5 (AU) to 2.5 (AU)
2000 planetesimals
Disk radial profile = $r^{-3/2}$

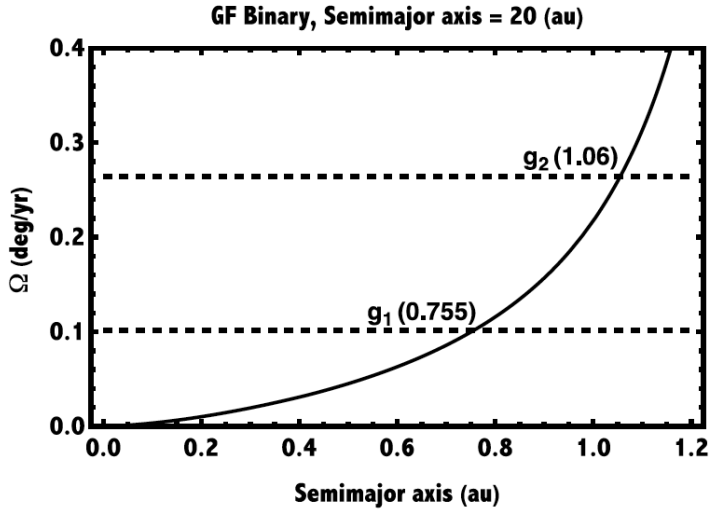


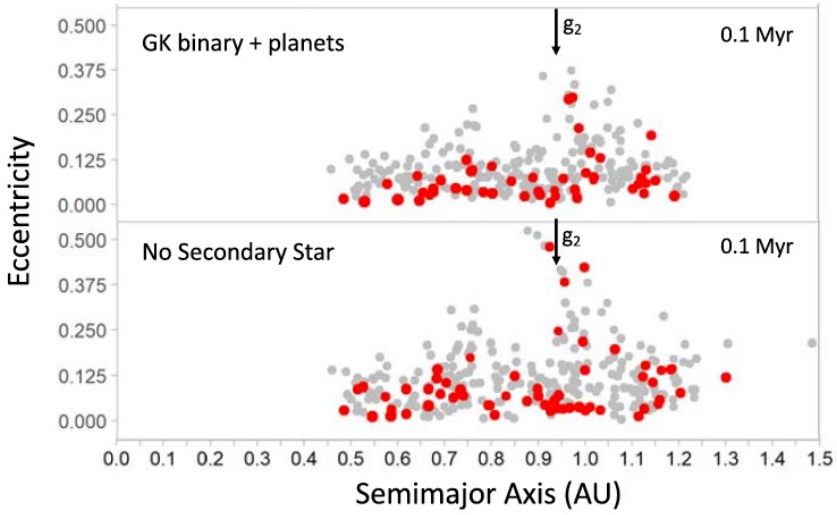


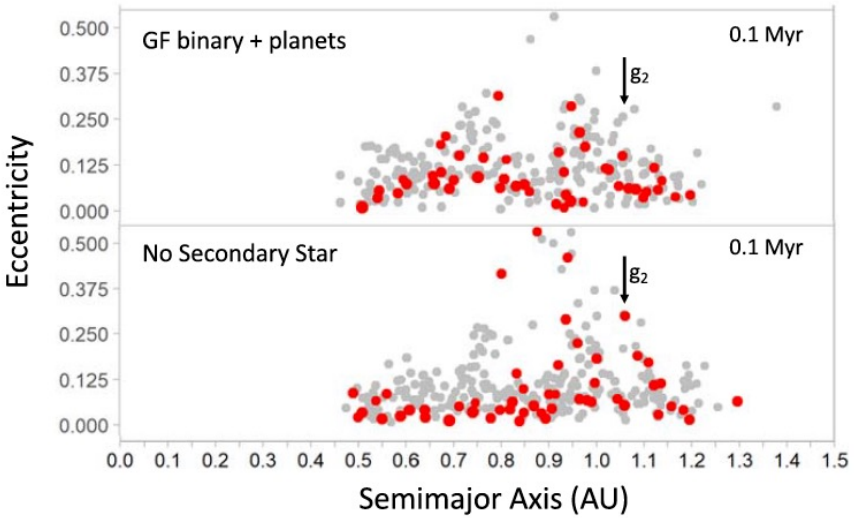


(Haghighipour & Andrews, AJ, 170, 117, 2025)

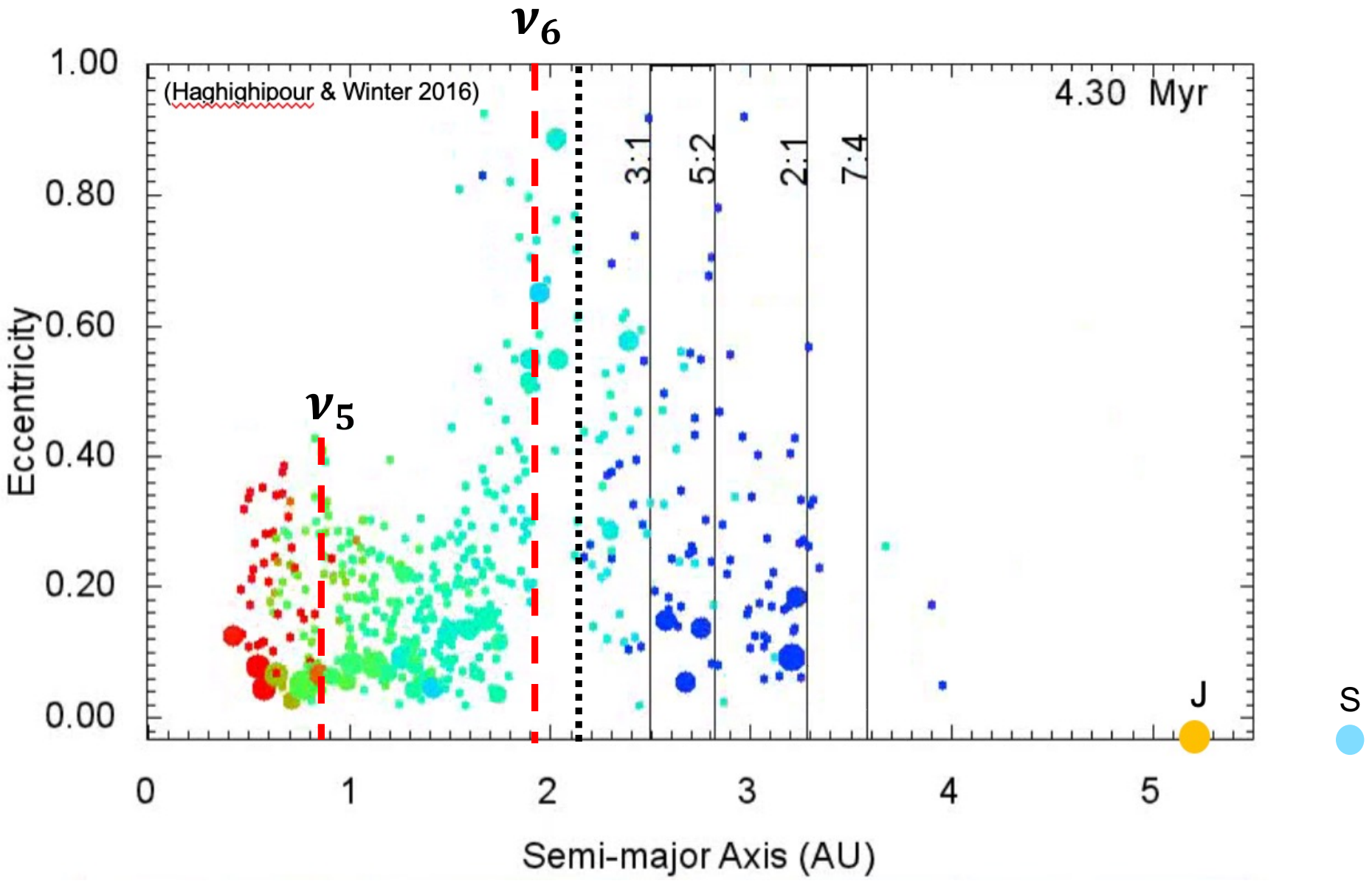


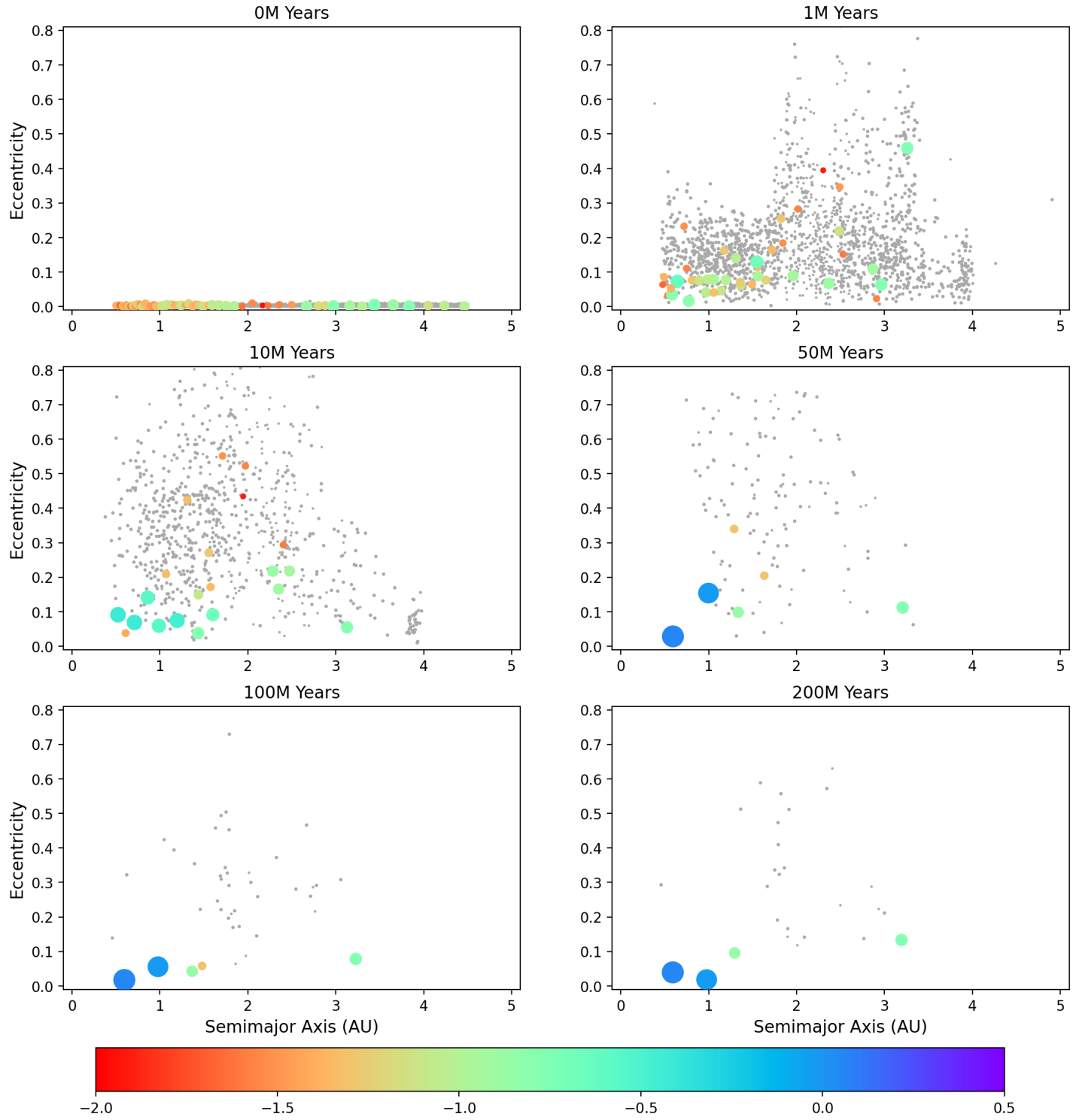




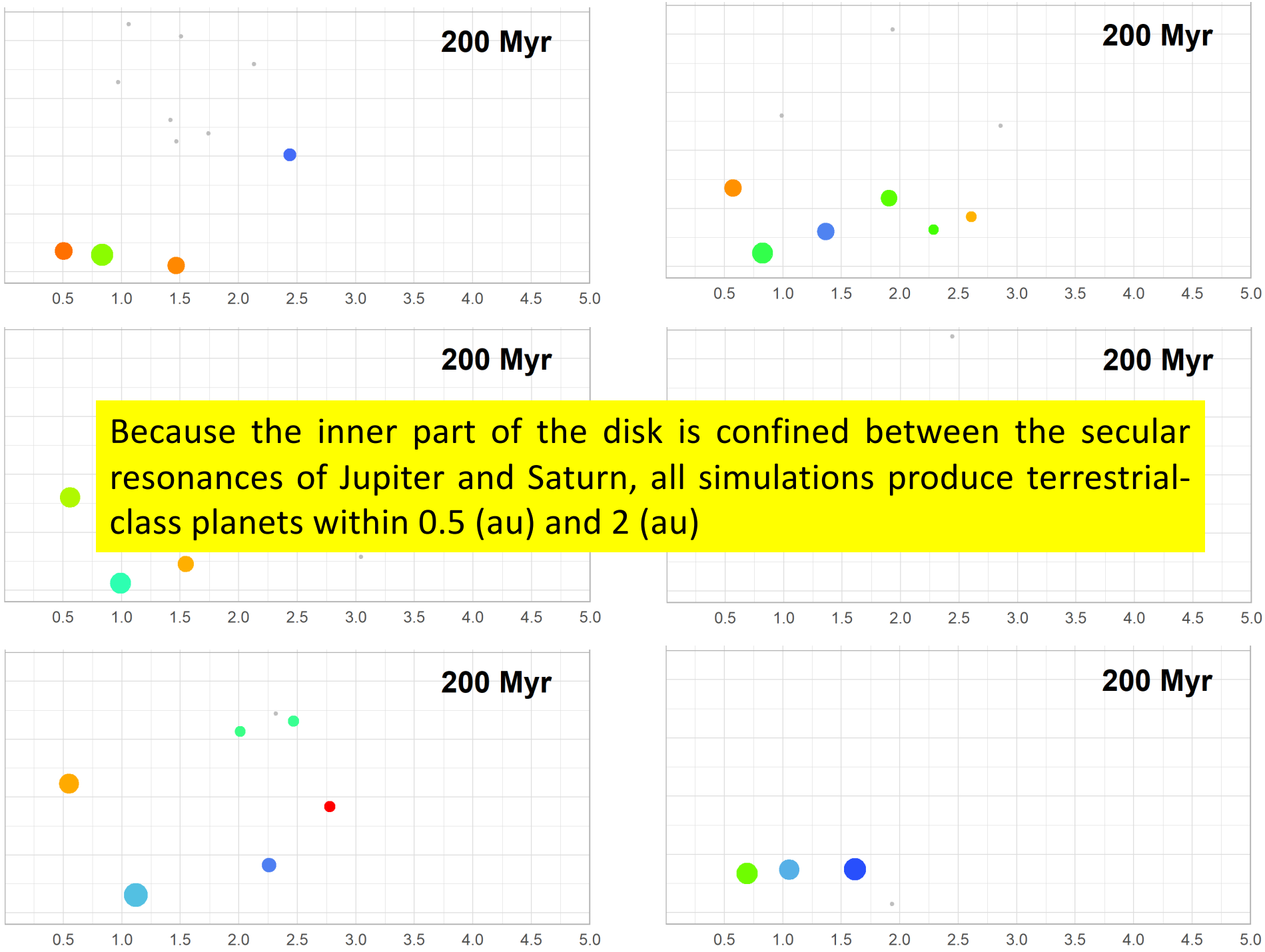


(Haghighipour & Andrews, AJ, 170, 117, 2025)

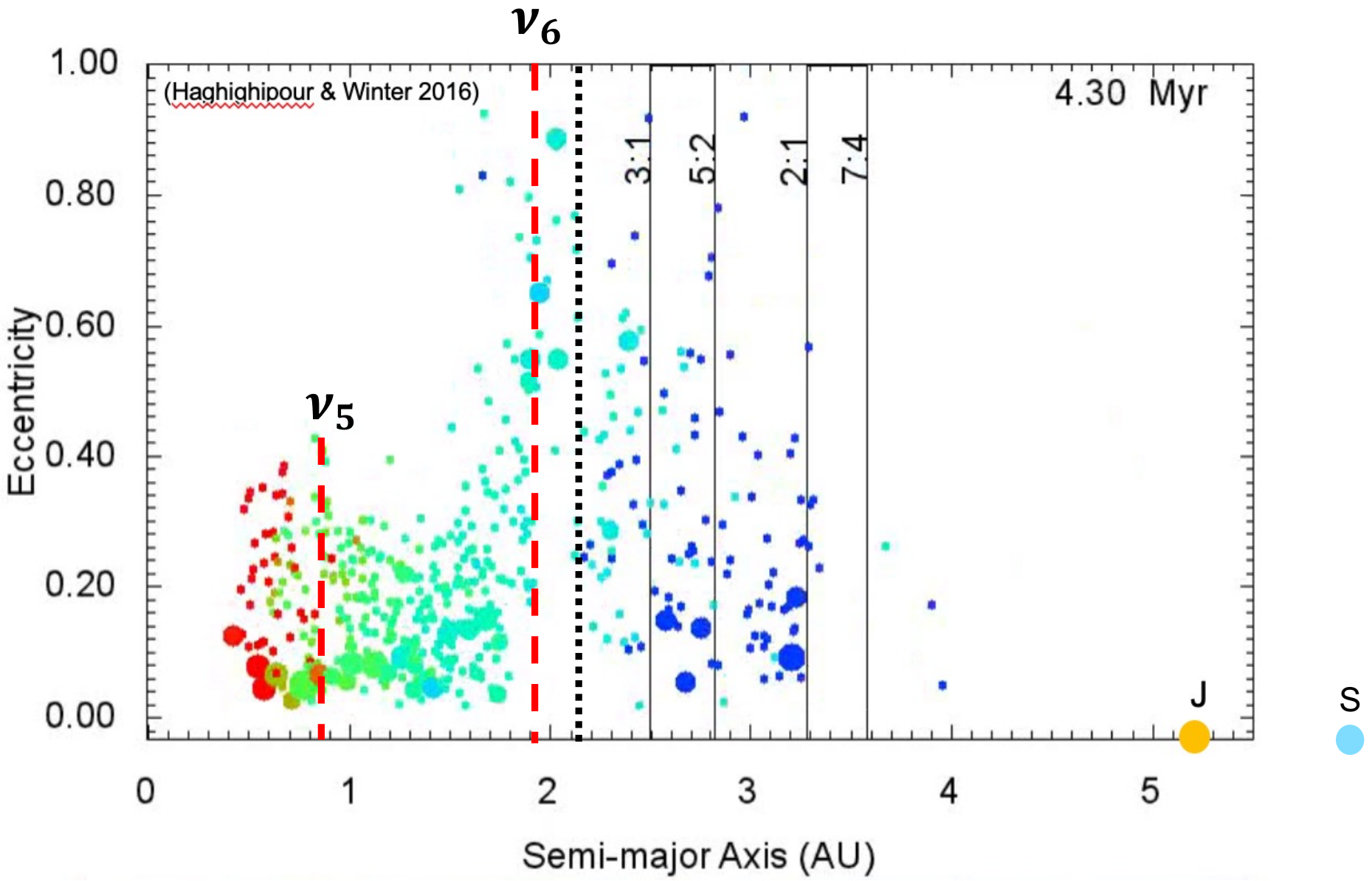

ν_6
ν_5
(Haghighipour & Winter 2016)
4.30 Myr
3:1
5:2
2:1
7:4
J
S
Eccentricity
Semi-major Axis (AU)
1.00
0.80
0.60
0.40
0.20
0.00
0
1
2
3
4
5



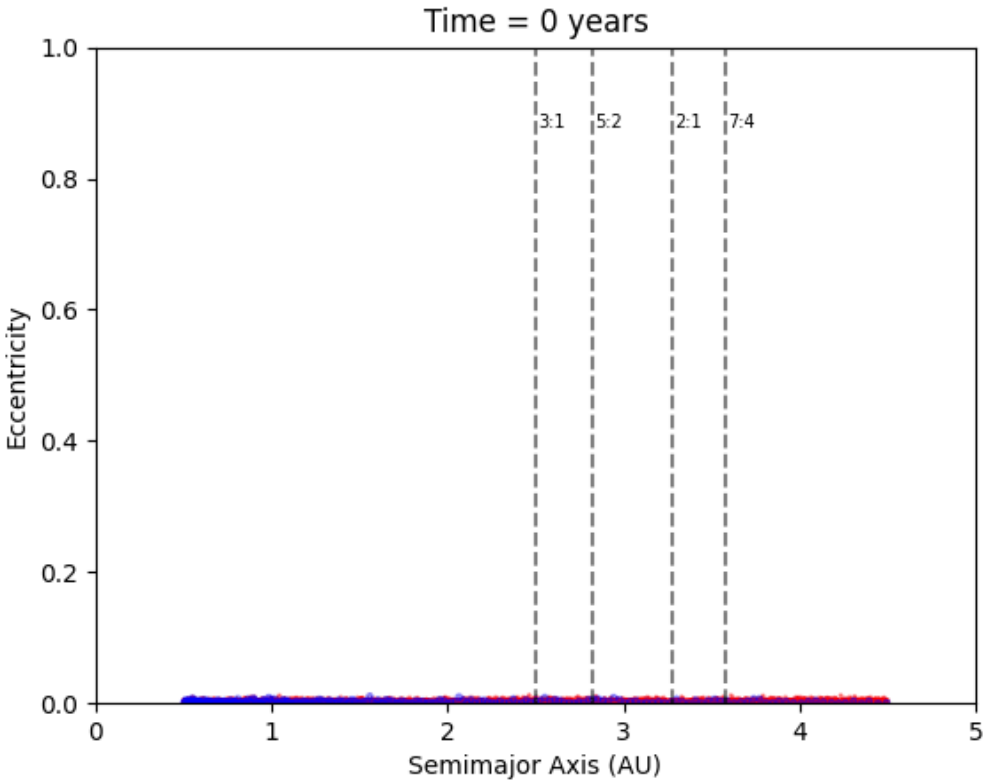
Haghighipour, Sudol, (in prep)

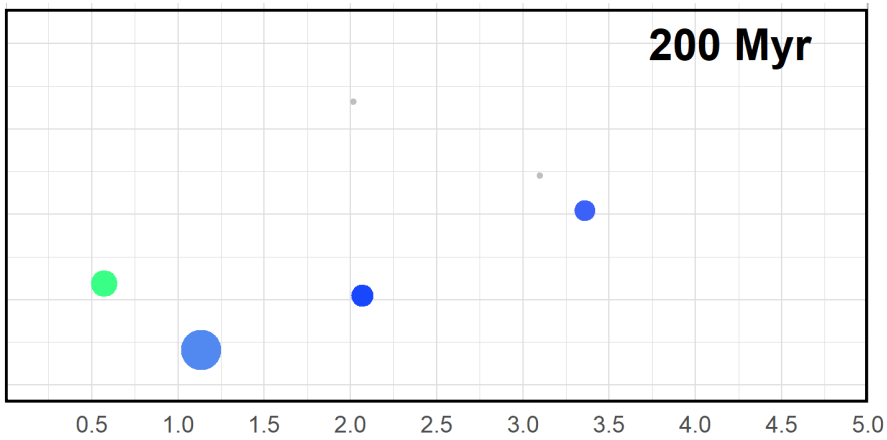


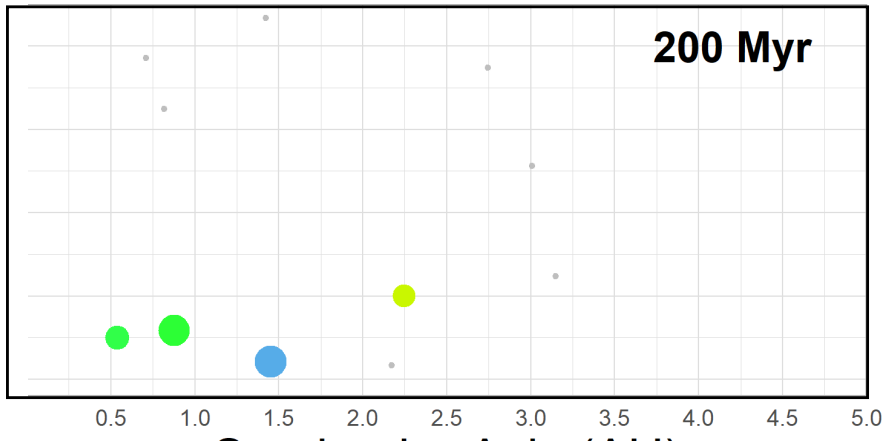
Because the inner part of the disk is confined between the secular resonances of Jupiter and Saturn, all simulations produce terrestrial-class planets within 0.5 (au) and 2 (au)

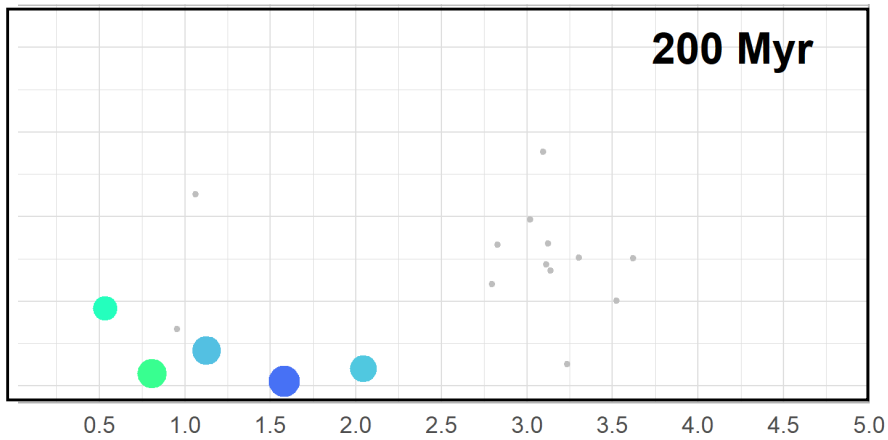

ν_6
ν_5
(Haghighipour & Winter 2016)
4.30 Myr
3:1
5:2
2:1
7:4
J
S
Eccentricity
Semi-major Axis (AU)
1.00
0.80
0.60
0.40
0.20
0.00
0
1
2
3
4
5

Jupiter only





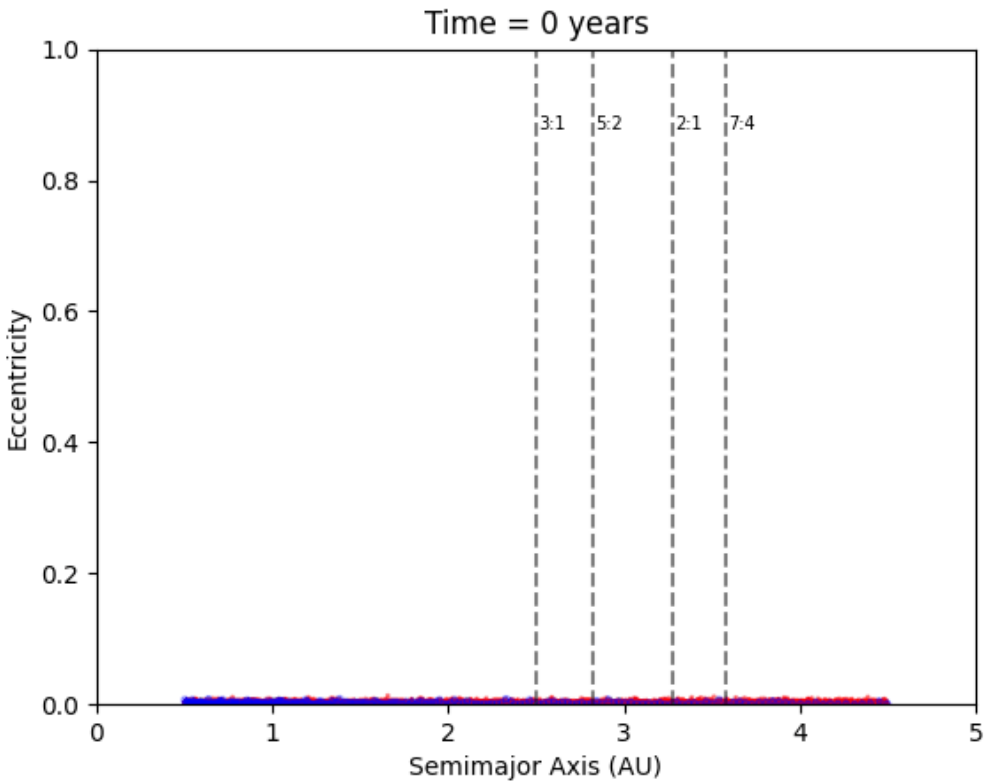


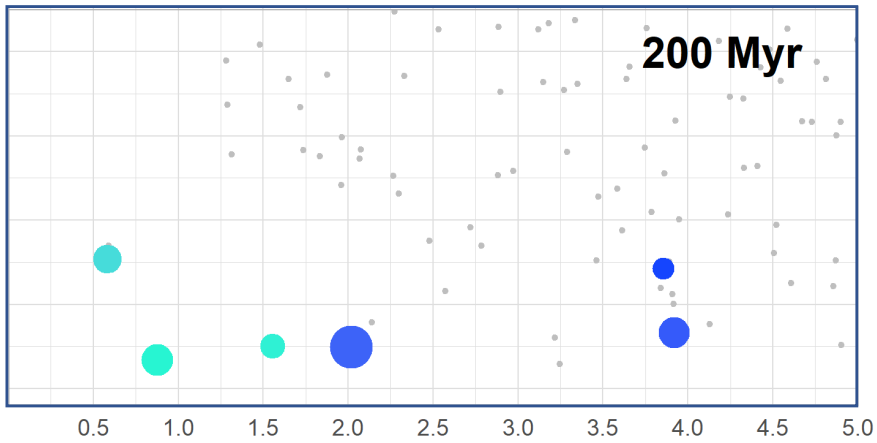


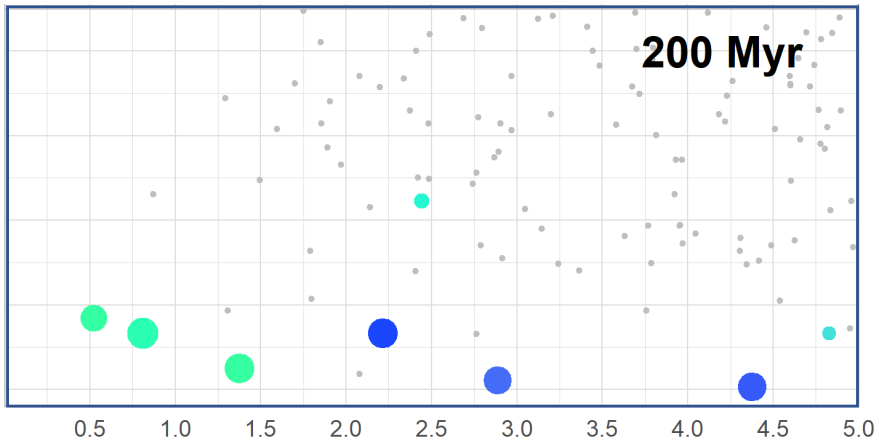
The confinement of the inner disk is merely due to the mean-motion resonances.

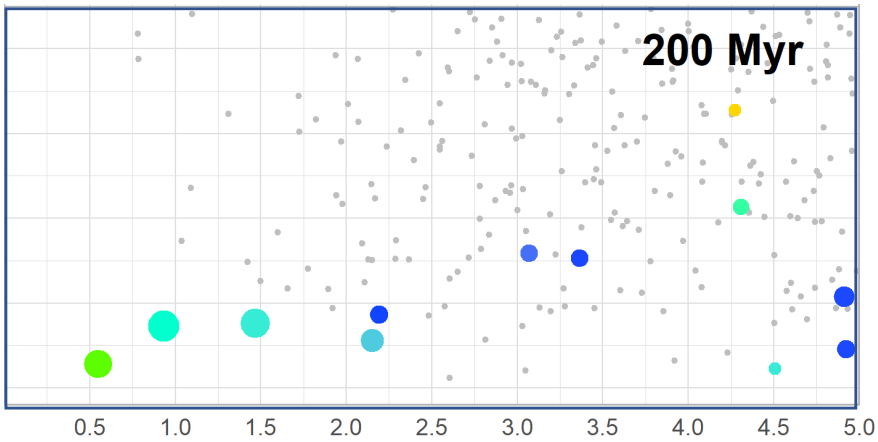
Now, super-Earths can also form.

Saturn only





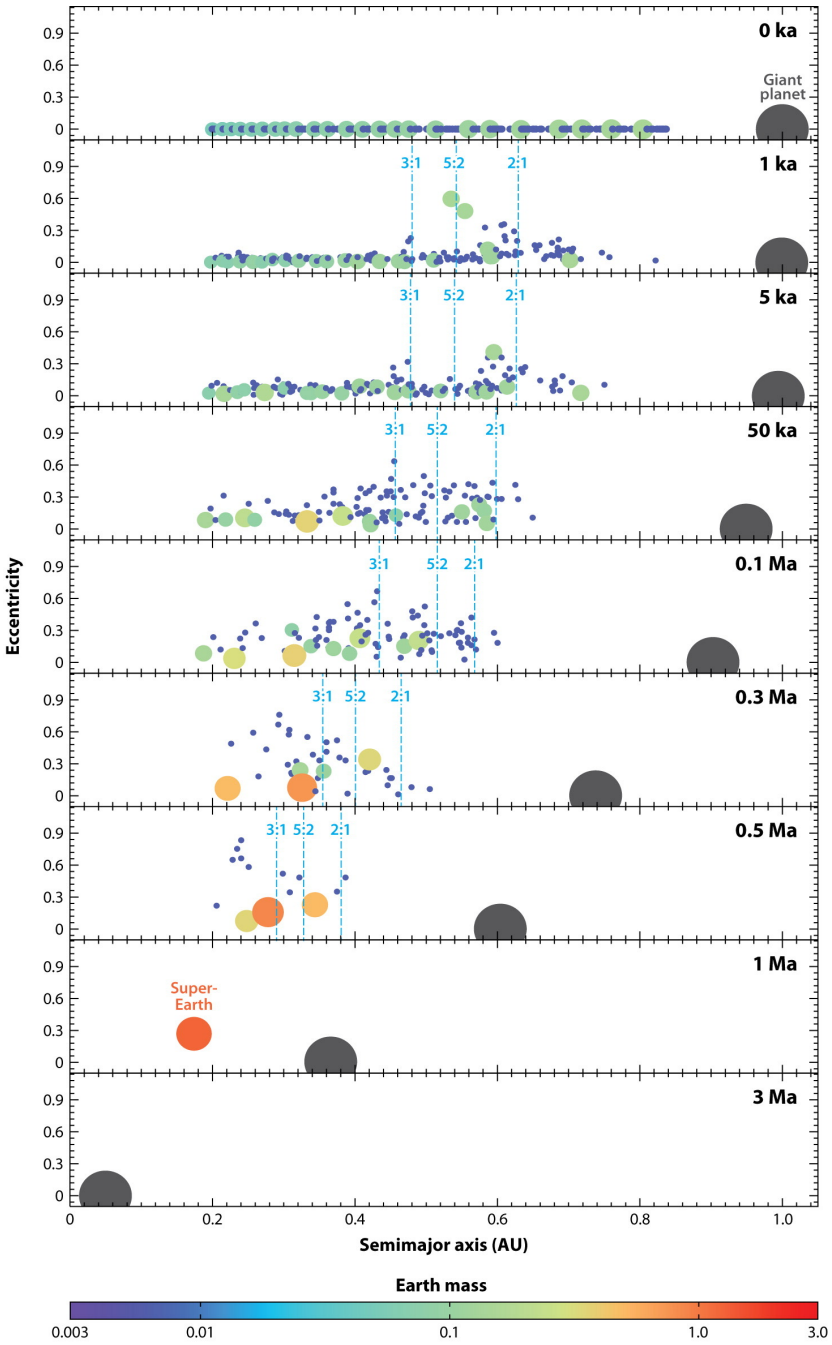




The inner disk is no longer confined.

Super-Earth and terrestrial-class planets routinely form.

Migrating Giant Planets



Haghighipour N. 2013.
Annu. Rev. Earth Planet. Sci. 41:469–95

- Confinement due to moving secular and mean-motion resonances

- Resonances enhance collisions among protoplanetary bodies

- The efficiency of growth and final architecture of the system strongly depend on the rate of migration

→ Terrestrial planets

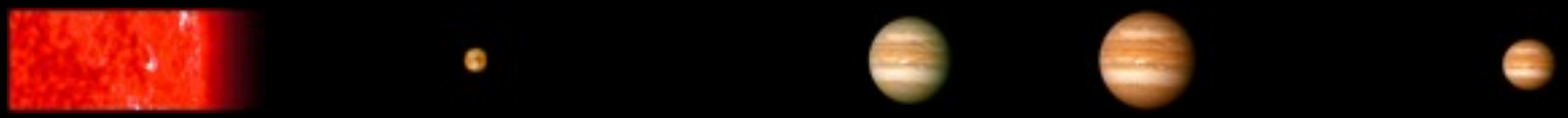
→ Super-Earths

→ Hot Jupiters

(Haghighipour, ARE&PS, 41: 469-495, 2013)

Gliese 876

A Three-Planet Resonant System



Giant planet migration + planet-planet scattering of an in-situ formed super-Earth

- Two Jovian-type planets: 30 & 60 days
- A Uranus-mass planet: ~ 124 days
- Three-planet Laplace resonance
- Dynamically full
- Super-Earth (6.7 M_E) in ~ 2-day orbit

(Gerlach & Haghighipour 2012)

