# Orbit determination procedure on large datasets

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# Introduction

Large amount of observations thanks also to new observational campaigns $\Longrightarrow$ Discovery of many new objects in our solar system

To make this happen it is necessary to have orbit determination procedures that can sift through the data to find observations that are useful to determine an object's orbit.

Our procedure is based on the one developed and tested by Rodriguez, Gronchi, Baù, Jedicke - Icarus (2024)

Isolated Tracklet File (ITF): long list of tracklets that have never been attributed to a known objects, nor have been used to compute the orbit of a new object.

On $31^{st}$ July 2026, the ITF contained 9358852 observations

# Linking pairs of tracklets

Consider a set of $n$ tracklets. To each pair of tracklets we apply the preliminary orbit determination method based on the conservation of the Keplerian Integrals (KI) of Gronchi, Baù, Marò (2015)

- solutions are given by solving a polynomial equation
- can be applied also to tracklets separated by a long time interval
- dependence on observational accuracy

The authors were able to select an overdetermined system made of 7 polynomial equations that is generically consistent and that leads to a univariate polynomial equation of degree 9.

If we have two tracklets at times $t_1, t_2$, we write

$$\boldsymbol{c}_1 - \boldsymbol{c}_2 = \boldsymbol{0}, \qquad \mu(\boldsymbol{L}_1 - \boldsymbol{L}_2) = \boldsymbol{0}, \qquad \mathcal{E}_1 - \mathcal{E}_2 = 0 \tag{1}$$

where the unknowns are $\rho_1, \rho_2, \dot{\rho}_1, \dot{\rho}_2$. We introduce the auxiliary variables

$$z_j = \frac{\mu}{|\boldsymbol{r}_j|}, \qquad j = 1, 2$$

and the polynomials

$$\begin{aligned}
\mu\widetilde{\boldsymbol{L}}_j &= (|\dot{\boldsymbol{r}}_j|^2 - z_j)\boldsymbol{r}_j - (\dot{\boldsymbol{r}}_j \cdot \boldsymbol{r}_j)\dot{\boldsymbol{r}}_j, \\
\widetilde{\mathcal{E}}_j &= \frac{1}{2}|\dot{\boldsymbol{r}}_j|^2 - z_j \\
\zeta_j &= z_j^2|\boldsymbol{r}_j|^2 - \mu^2,
\end{aligned}$$

We obtain the polynomial system

$$\boldsymbol{c}_1 - \boldsymbol{c}_2 = \mathbf{0}, \quad \mu(\widetilde{\boldsymbol{L}}_1 - \widetilde{\boldsymbol{L}}_2) = \mathbf{0}, \quad \widetilde{\mathcal{E}}_1 - \widetilde{\mathcal{E}}_2 = 0, \quad \zeta_1 = 0, \quad \zeta_2 = 0 \tag{2}$$

of 9 equations in 6 unknowns $(\rho_1, \rho_2, \dot{\rho}_1, \dot{\rho}_2, z_1, z_2)$ that is inconsistent.

In Grassi, Gronchi, CMDA (2025): *Preliminary orbits with overdetermined systems of Keplerian conservation laws* we considered all polynomial systems that originate from considering different subsets of (2).

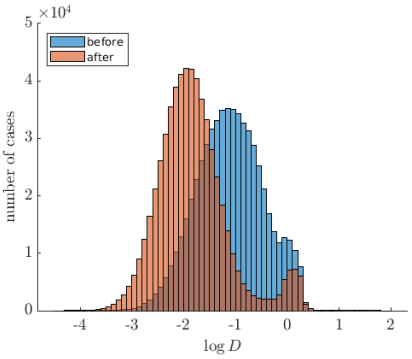
**Results**:

- we found two additional overdetermined systems that are consistent and both lead to univariate polynomial equations of degree 18
- all other combinations lead to systems that are either inconsistent or result in polynomials of degree $> 18$

For each pair of tracklets that gives a solution corresponding to an elliptic orbit, we apply 2-body differential corrections to refine the preliminary orbit.

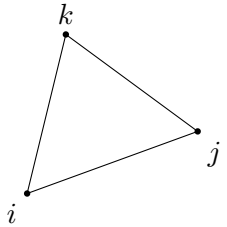
Keplerian dynamics $\Longrightarrow$ all functions and derivatives at each iteration can be easily computed, making 2-body differential corrections faster.

Each preliminary orbit is kept if the rms after 2-body differential corrections is below a chosen bound.



# Forming 3-cycles

3-cycle: a set of tracklets $(i, j, k)$ such that we have preliminary orbits $\mathcal{O}_{ij}, \mathcal{O}_{ik}, \mathcal{O}_{jk}$ and we can compute an orbit $\mathcal{O}_{ijk}$ using all three tracklets.



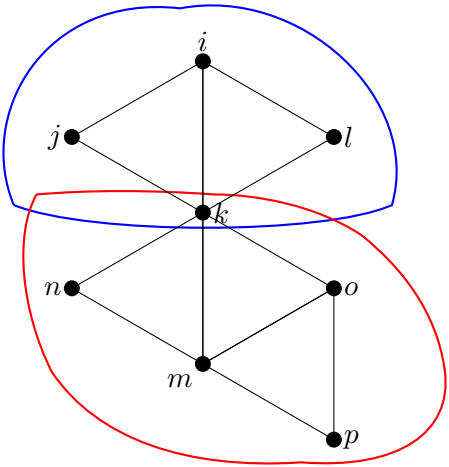

To decide if a set of three tracklets is interesting, we use a norm that measures the accuracy of the preliminary orbits $\mathcal{O}_{ij}, \mathcal{O}_{ik}, \mathcal{O}_{jk}$ and how close they are to each other.

Each 3-cycle is kept if the rms after 2-body differential corrections is low enough.

# Obtaining identifications

Apply full n-body differential corrections within connected components to obtain identifications.

Identification: a set of at least 4 tracklets that provide an elliptic orbit with rms below a chosen bound.

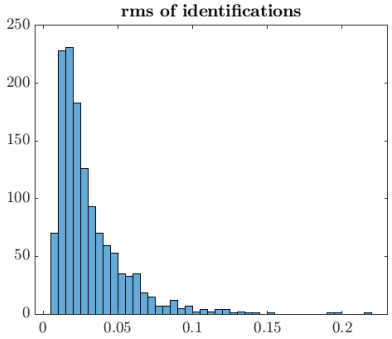


# Test on observations of known objects

Using all observations from the Vera C. Rubin Observatory (LSST) available on the MPC database:

- Total number of observations: 574268
- Total number of tracklets: 87620

| | Total | Correct |
|---|---|---|
| Pairs | 7906925 | 54638 (0.69%) |
| 3-cycles | 15255 | 13772 (90.28%) |
| Ident | 1312 | 1311 (99.92%) |




We kept 0.2% of the total number of pairs to which we apply the KI linking method.

# Test on observations from the ITF

We took all observations contained in the ITF from the PanSTARRS observatories (F51, F52):

- Total number of observations: 3867321
- Total number of tracklets: 1139147
- Number of accepted identifications: 104

**Data storage**: Results of the linkage of pairs are saved using the Compressed Sparse Row method

$$A \in \{0,1\}^{n\times n} \quad \longleftrightarrow \quad \begin{cases} \mathbf{c} = [c_1, c_2, \ldots, c_{n_z}] & \text{(column indices)}, \\ \mathbf{r} = [r_0, r_1, \ldots, r_n] & \text{(row pointers)}. \end{cases}$$

**Computational load**: The linkage of pairs takes by far the longest to run. For this test it took 24 days on 100 cores.

# Future developments

- Looking for pairs of tracklets that can be linked is very expensive. We are working on introducing filters that can quickly select which pairs to try, possibly using machine learning.
- After finding identifications, we can try to attribute more tracklets to the computed orbit. We are working on developing faster methods for attribution.
- This same procedure can be applied to the search for new interstellar objects on hyperbolic trajectories.

# Interstellar objects

Three interstellar objects have been confirmed

- 1I - Oumuamua (2017)
- 2I - Borisov (2019)
- 3I - Atlas (2025)

| Object | $e$ | $n_{tra}$ | $n_{link}$ | $rms$ |
|---|---|---|---|---|
| 1I | 1.20 | 11 | 11 | 0.21 |
| 2I | 3.36 | 11 | 11 | 0.29 |
| 3I | 6.12 | 20 | 19 | 0.35 |

Questions:

- How many interstellar objects should we expect?
- Have others been missed by surveys?
- What does this low number of interstellar objects tells us?

On-going work with G.F. Gronchi, R. Jedicke, D. Marčeta, D. Seligman, A. Gibbs.

# Conclusions

- We have a procedure that is able to compute orbits from large datasets of observations
- Steps have been taken to reduce the computational load, for example with 2-body differential corrections and the definition of connected components
- A proper way to store data is necessary
- Further improvements could make the procedure even more frequently applicable

# Thank you!