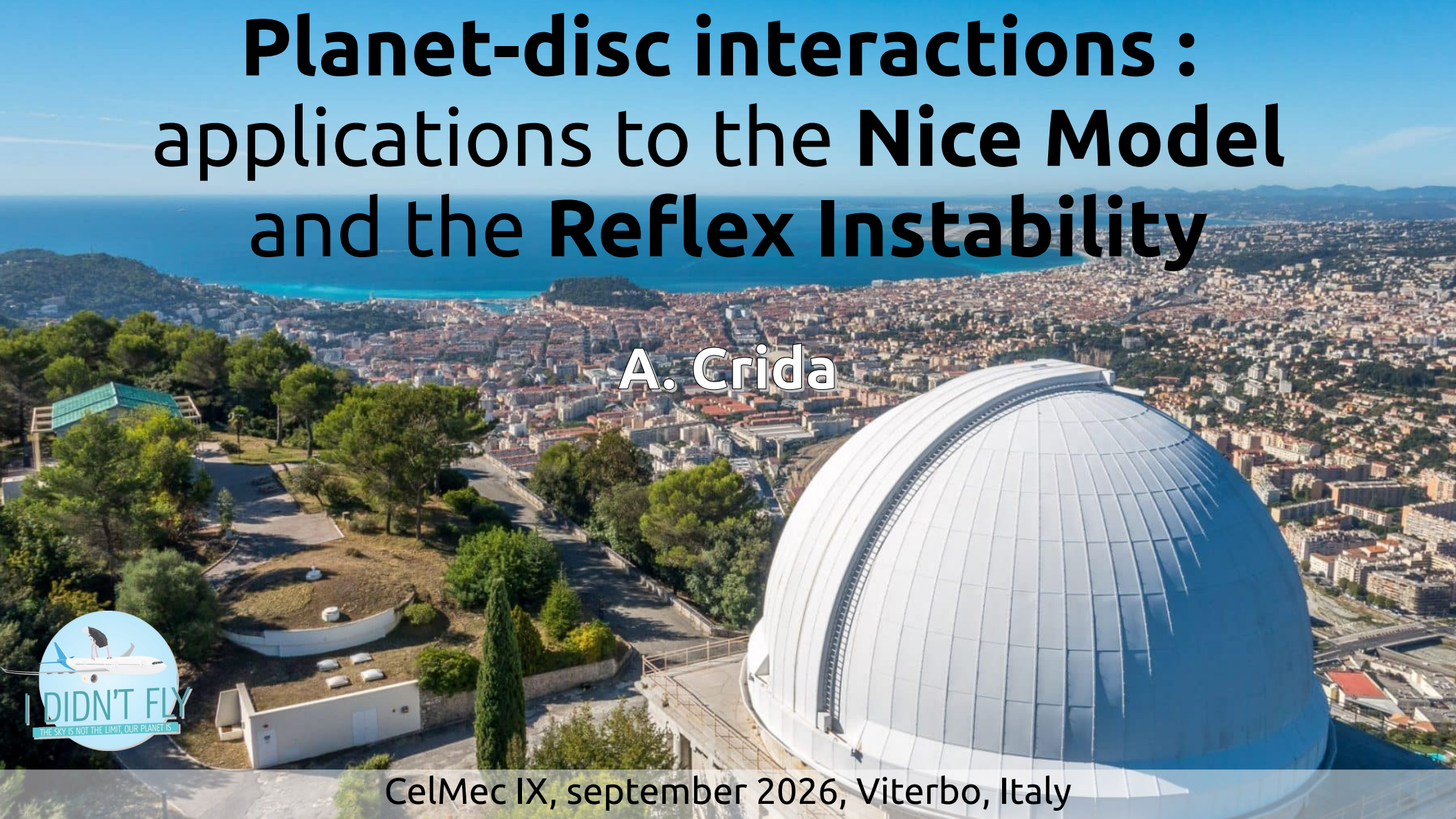
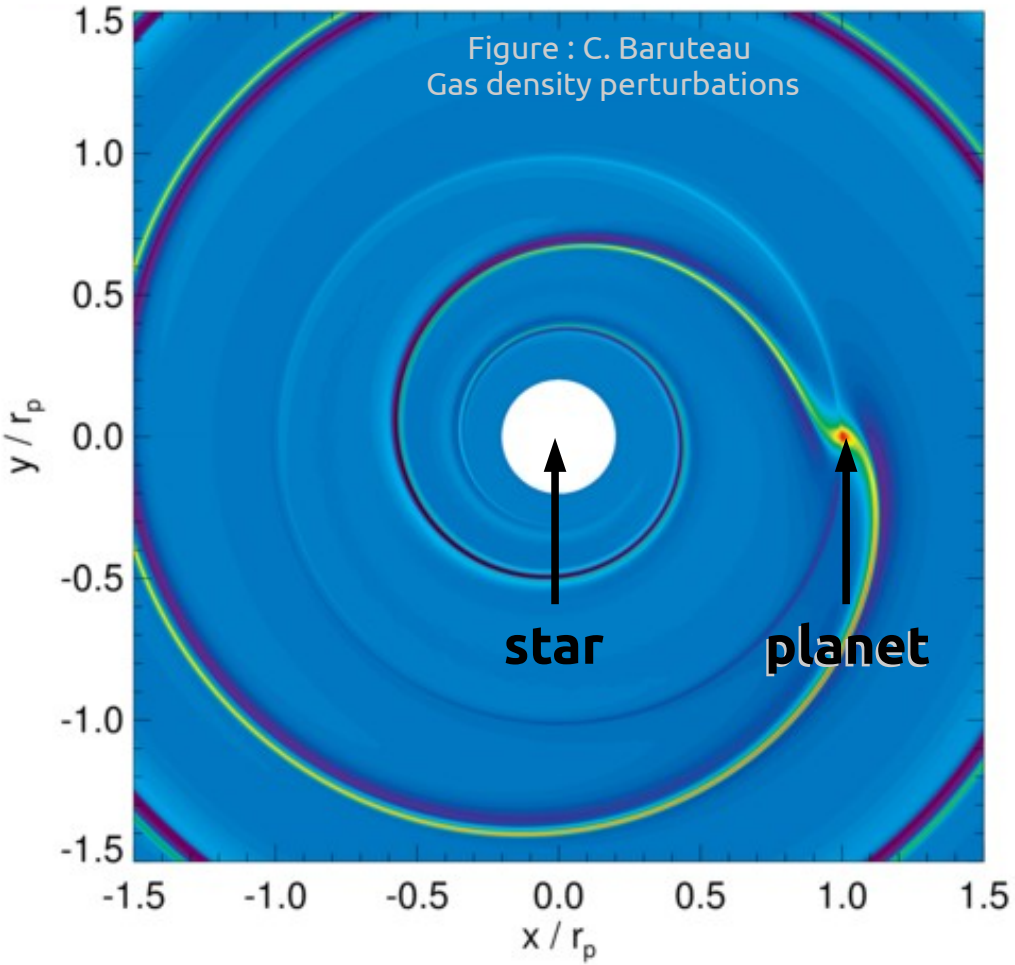
Planet-disc interactions : applications to the **Nice Model** and the **Reflex Instability**



A. Crida

CelMec IX, september 2026, Viterbo, Italy

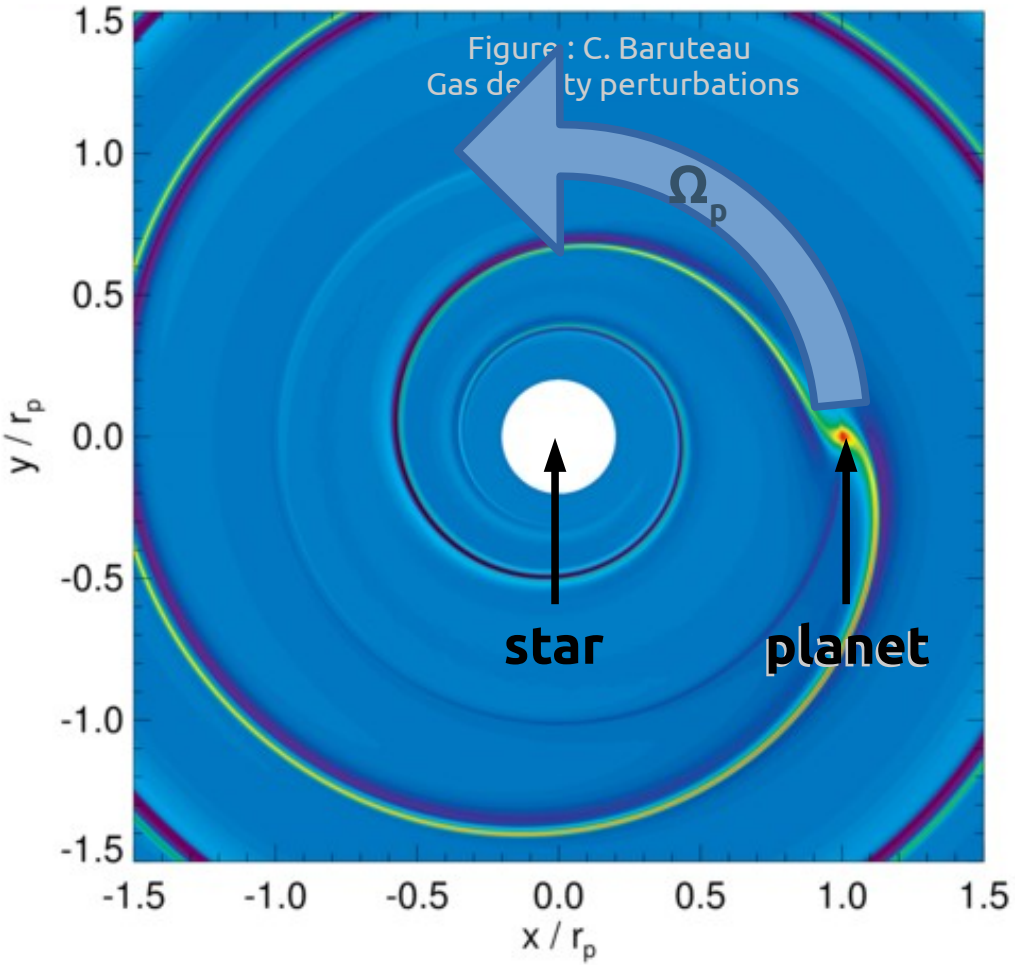
Wake



A planet orbiting in a protoplanetary disk launches a one-armed, spiral wake.

This is a pressure-supported wave, caused by the planet's gravity perturbing the trajectories of the gas particles.

Wake

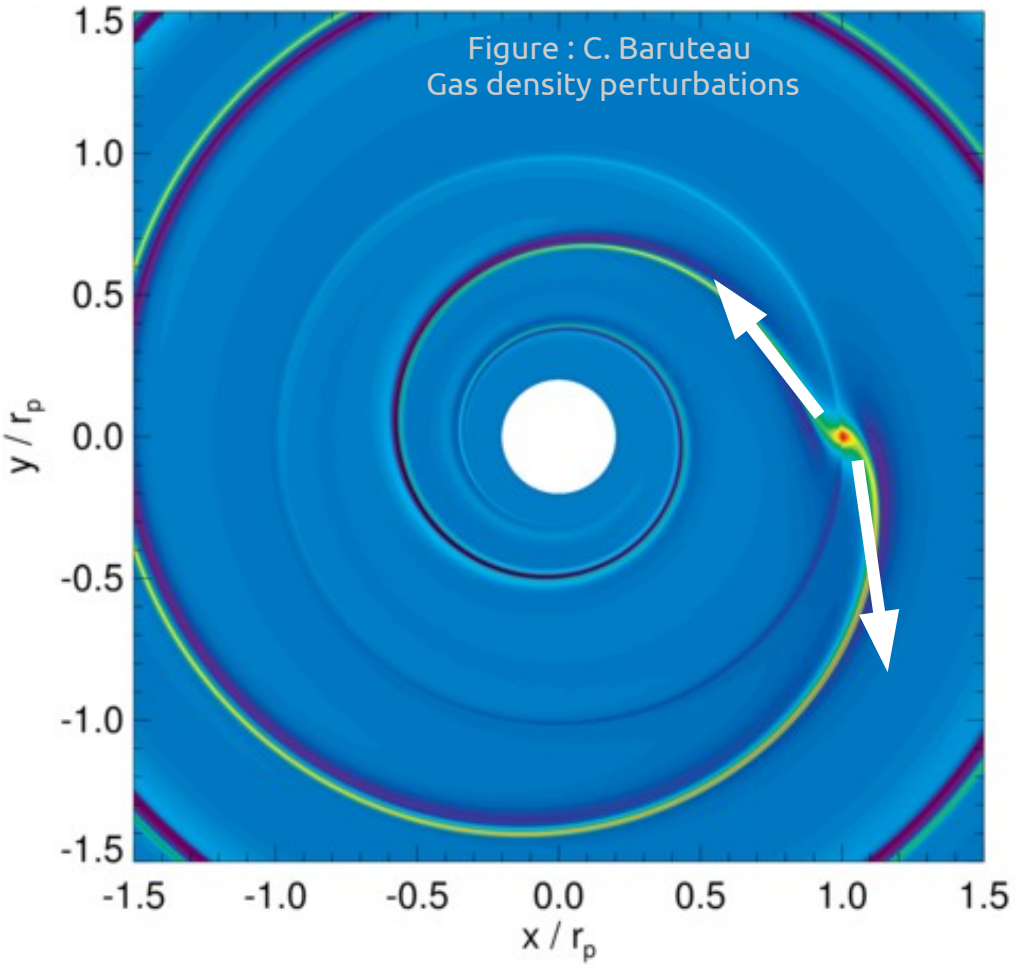


A planet orbiting in a protoplanetary disk launches a one-armed, spiral wake.

This is a pressure-supported wave, caused by the planet's gravity perturbing the trajectories of the gas particles.

The wave corotates with the planet at the same angular velocity $\Omega_p = (GM_*/a_p^3)^{1/2}$.

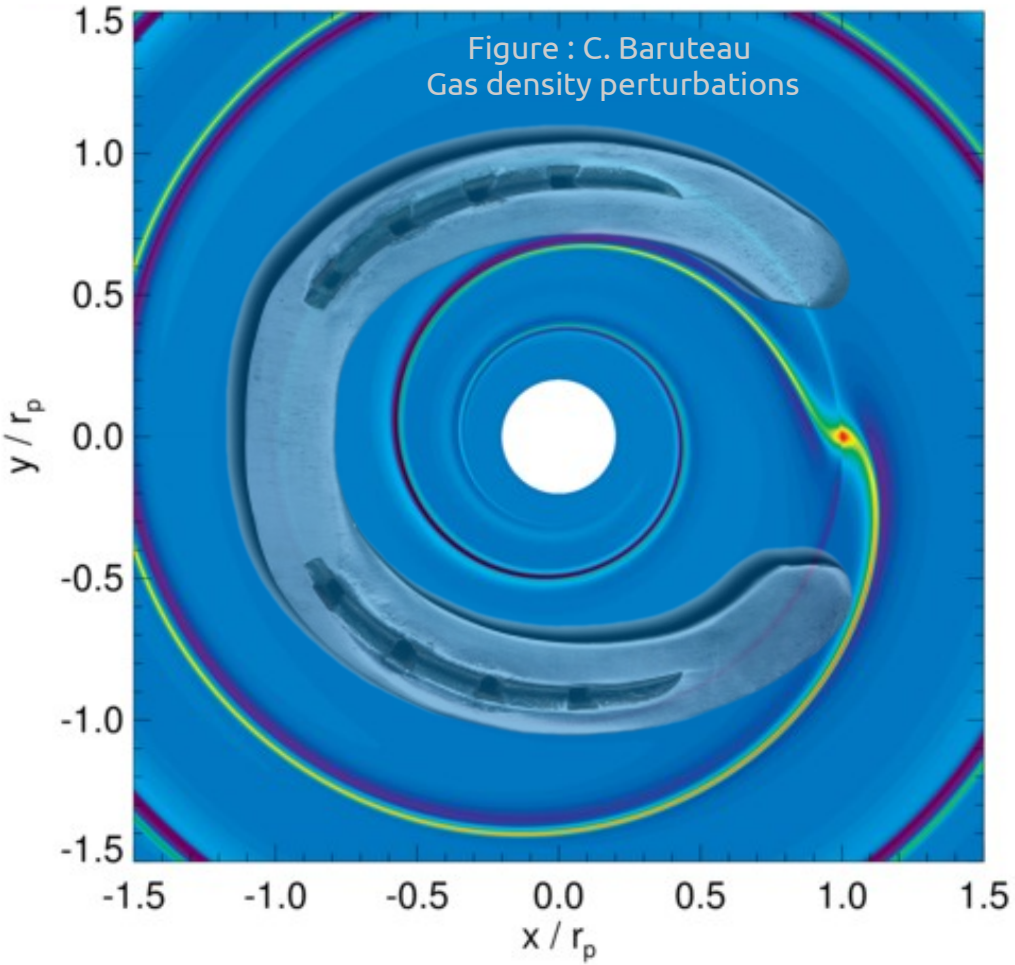
Type I migration



Density perturbation
→ gravity force on the planet.
Momentum w.r.t. star → torque.

Inner wake : positive torque.
Outer wake : negative torque.

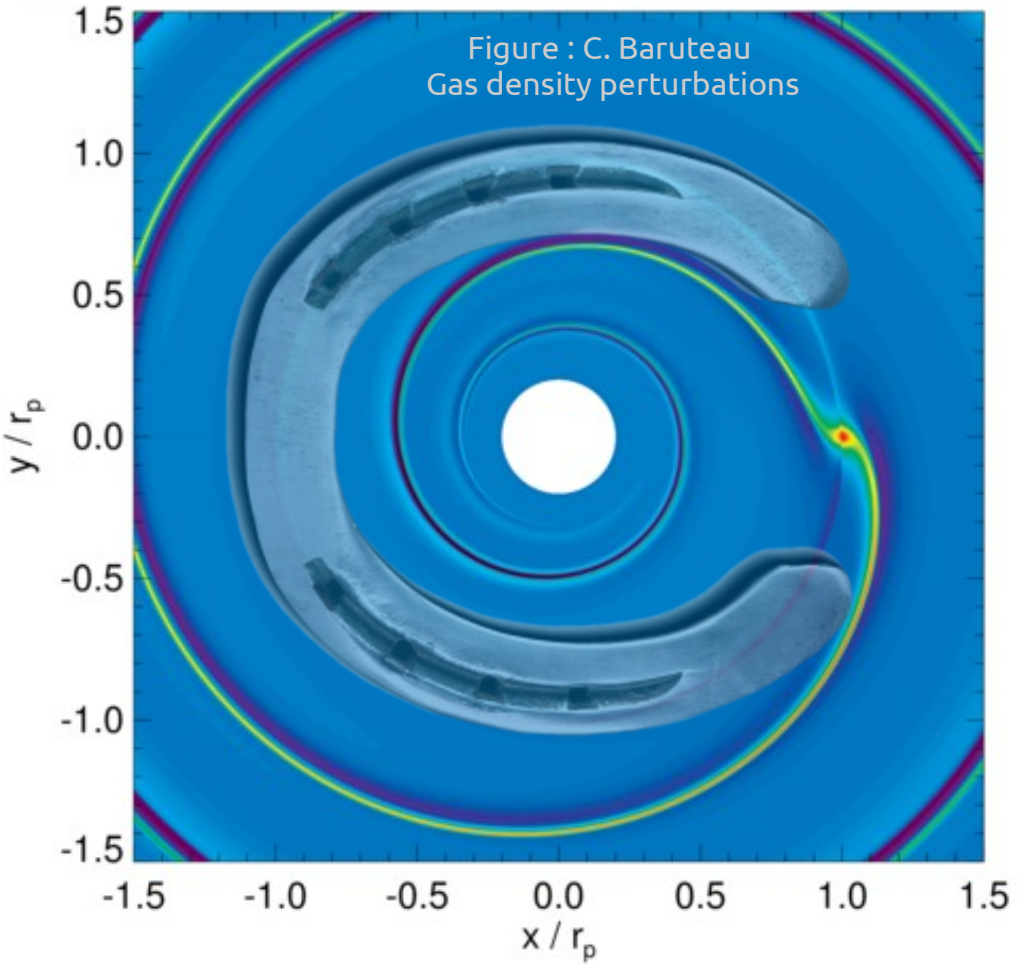
Type I migration



Density perturbation → gravity force on the planet.
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Inner wake : positive torque.
Outer wake : negative torque.
Horseshoe region : often positive.

Type I migration



Density perturbation → gravity force on the planet.
Momentum w.r.t. star → torque.

Inner wake : positive torque.
Outer wake : negative torque.
Horseshoe region : often positive.

Constant force vector *in the frame corotating with the planet* → no net motion, but variation of orbital angular momentum : migration.

(Ward 1997)

Type II migration

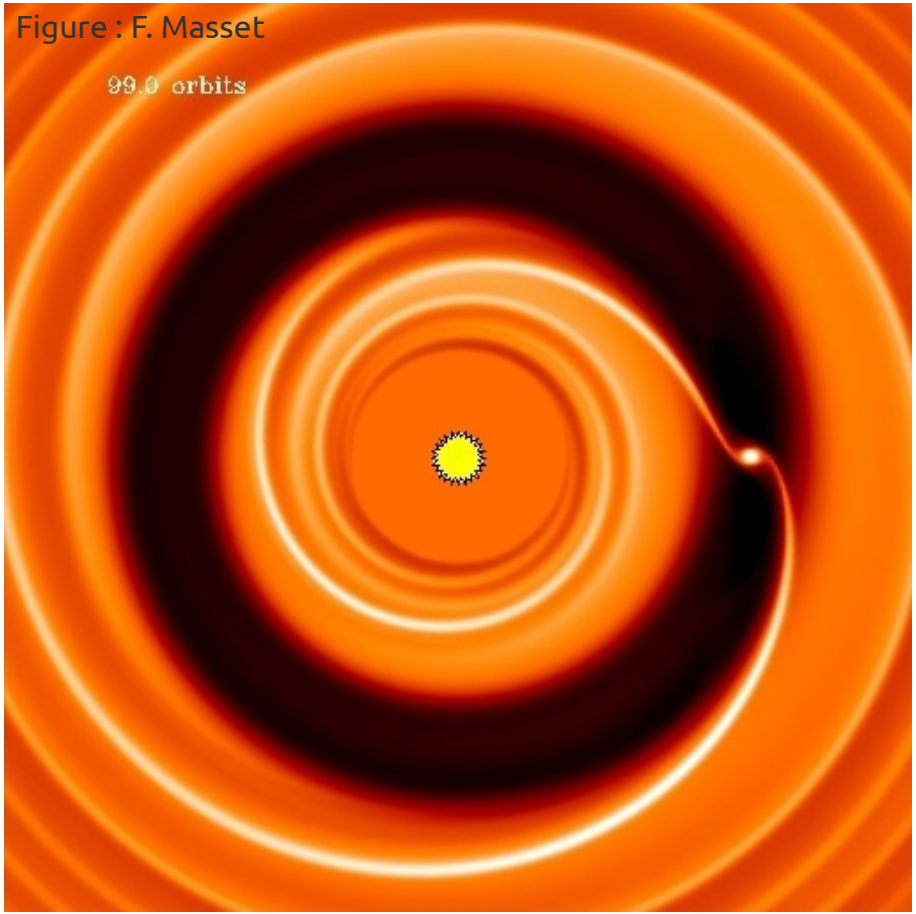
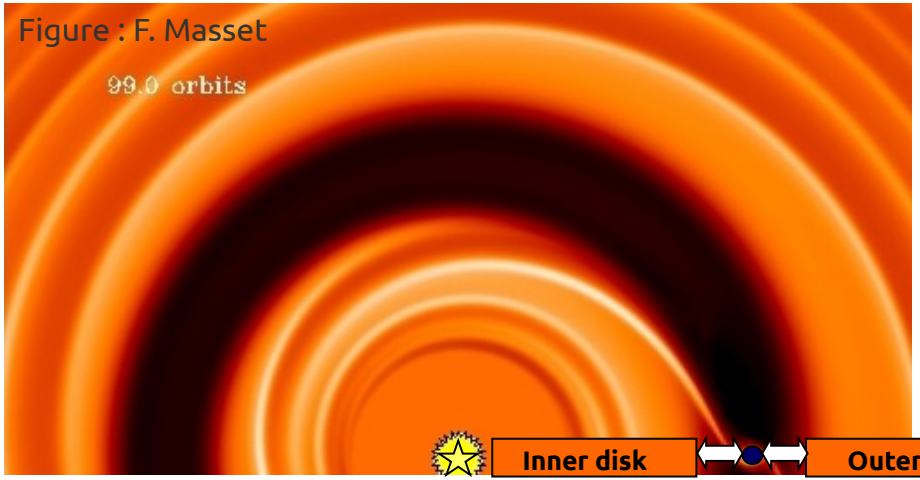


Figure : F. Masset

Reciprocally, the planet repels the inner disk inwards and the outer disk outwards.

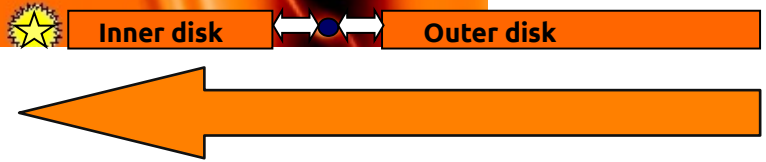
A giant planet opens a gap.(Crida+06)

Type II migration



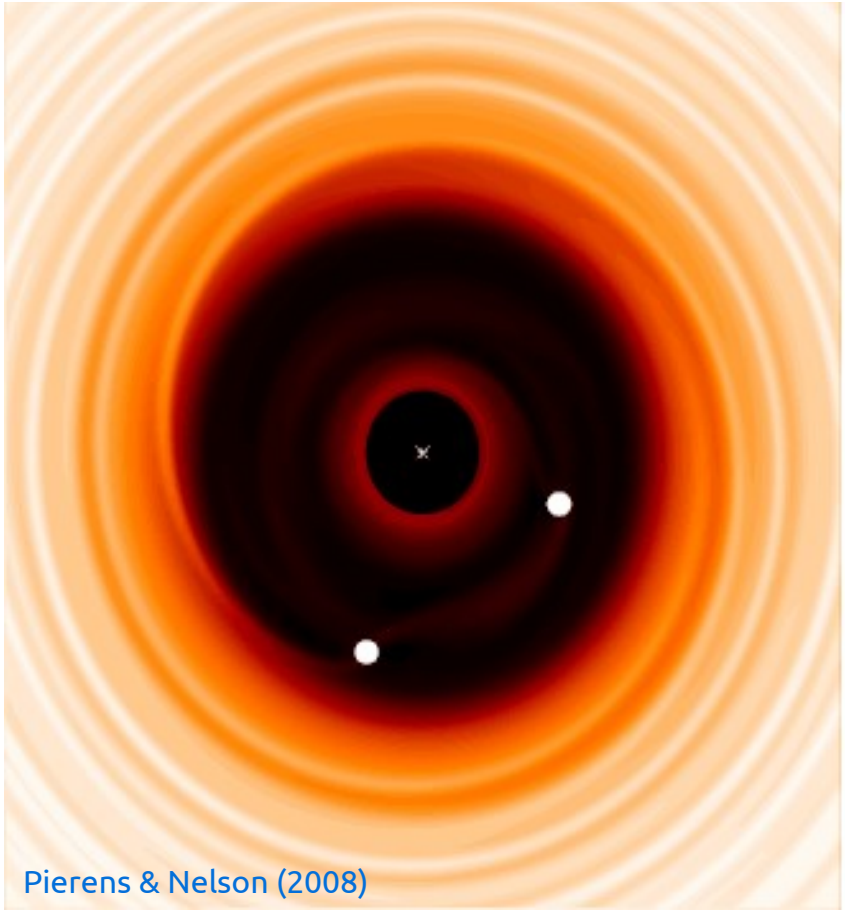
Reciprocally, the planet repels the inner disk inwards and the outer disk outwards.

A giant planet opens a gap.(Crida+06)



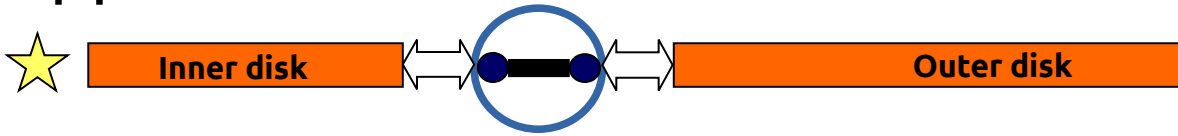
Then, it stays at equilibrium between the inner and the outer disk, and follows the accretion of the gas onto the star.
(Lin & Papaloizou 1986)

Pair of giant planets



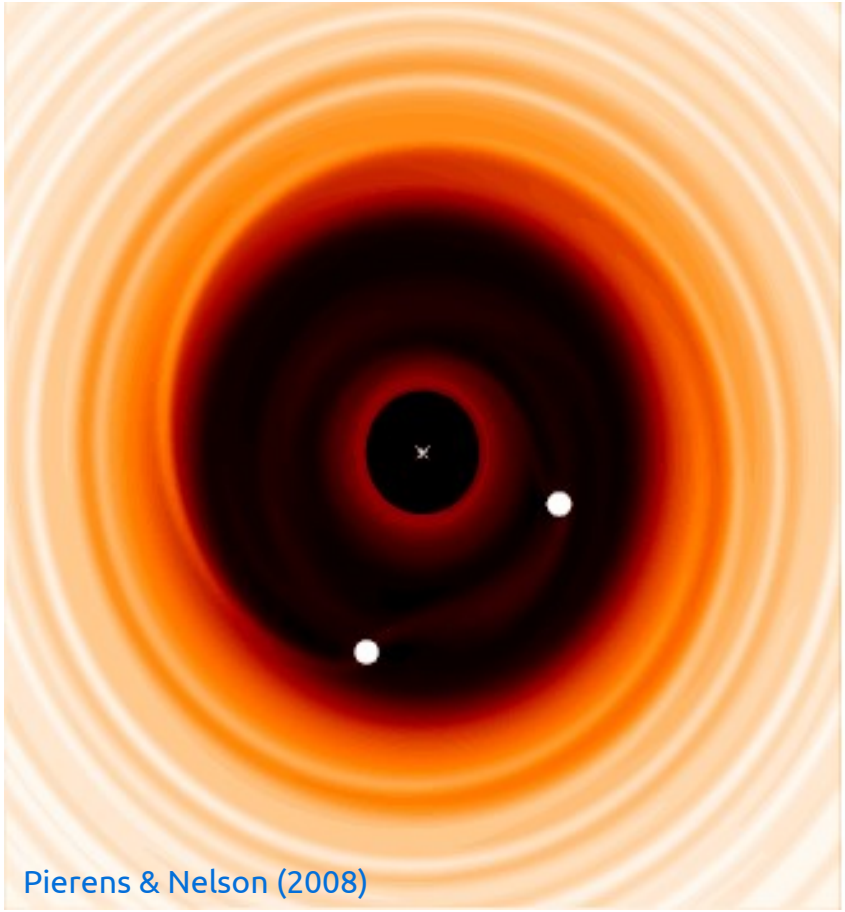
Pierens & Nelson (2008)

Two planets in the same gap approach each other → resonance.



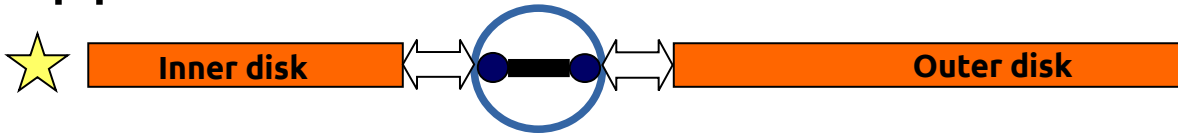
The pair behaves like 1 object.

Pair of giant planets



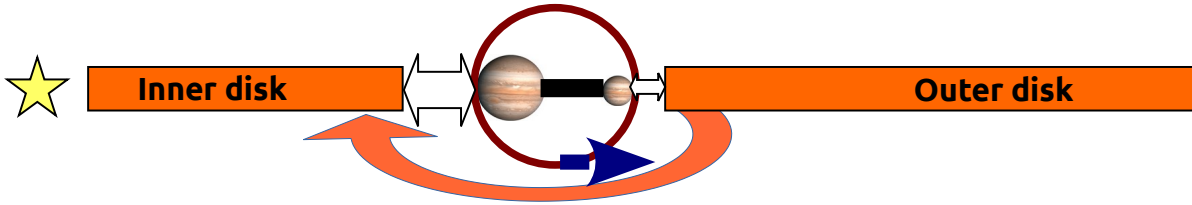
Pierens & Nelson (2008)

Two planets in the same gap approach each other → resonance.



The pair behaves like 1 object.

But if the masses differ, it's not at equilibrium, and can go outwards (Masset & Snellgrove 2001).

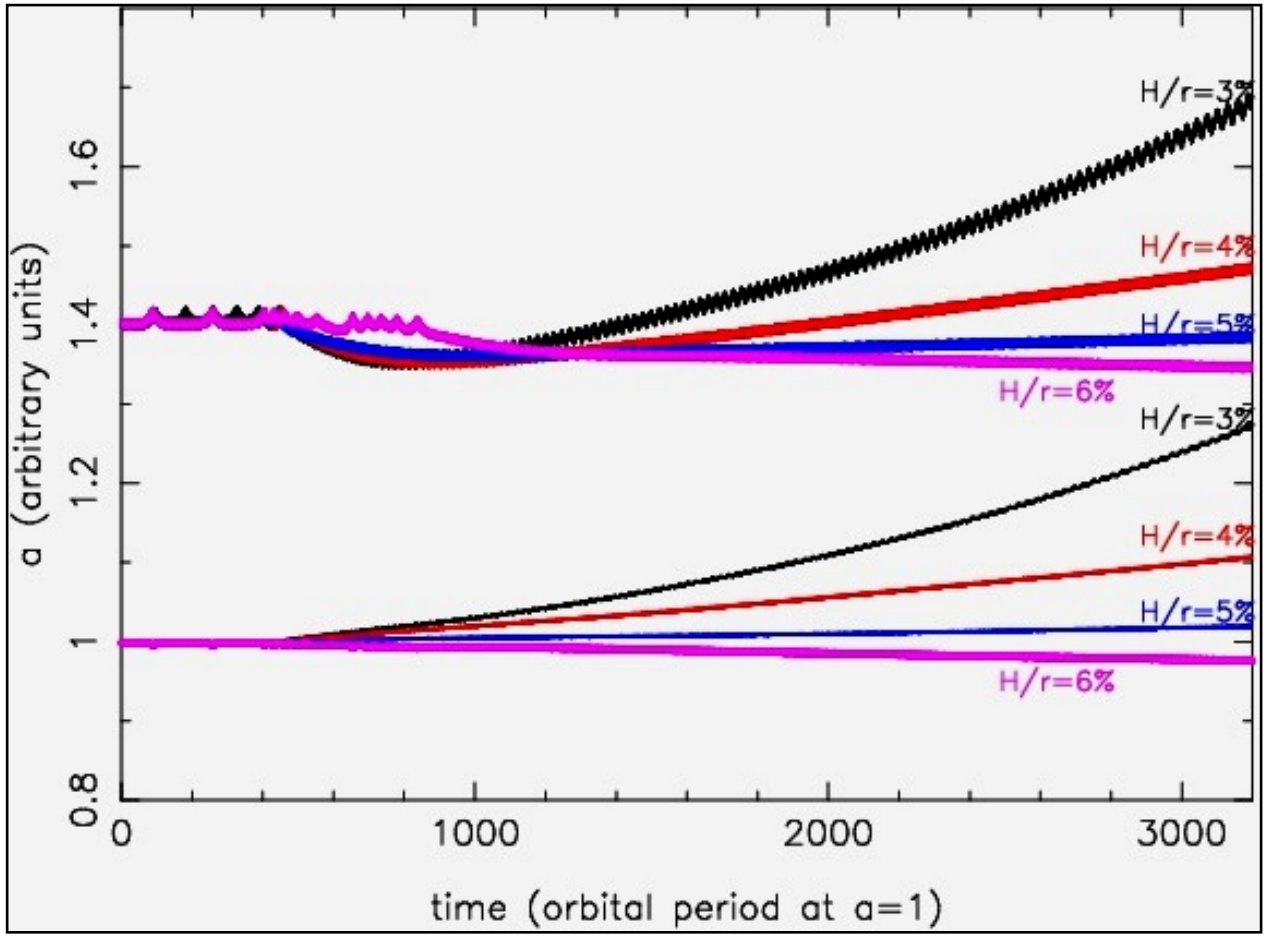


Jupiter-Saturn

Jupiter & Saturn end in 3:2 MMR
(Pierens & Nelson 2008)

Depending on the disk aspect ratio H/r, they may migrate at various speed and direction !

Saturn explains why Jupiter is not « hot »
(Morbidelli & Crida 2007).



Grand Tack

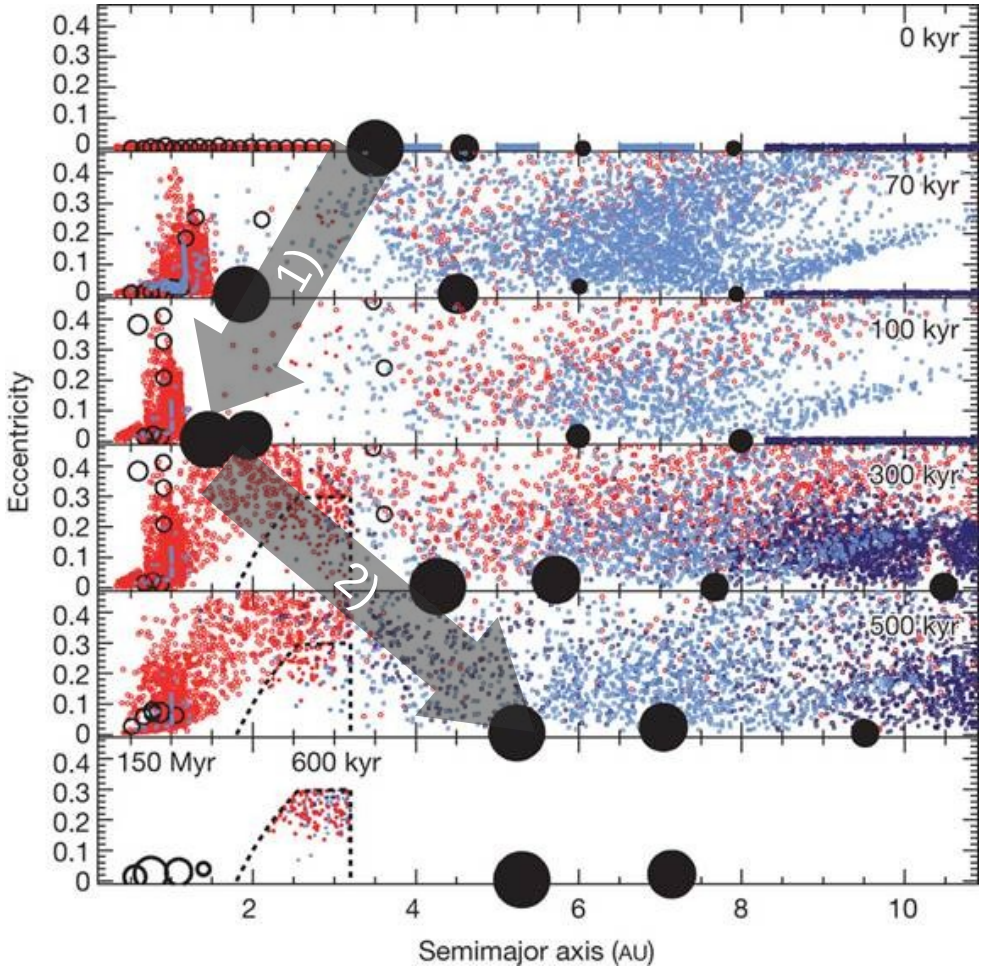
1) Jupiter migrates alone through the asteroid belt down to 1.5 au

2) Saturn joins and makes it tack.

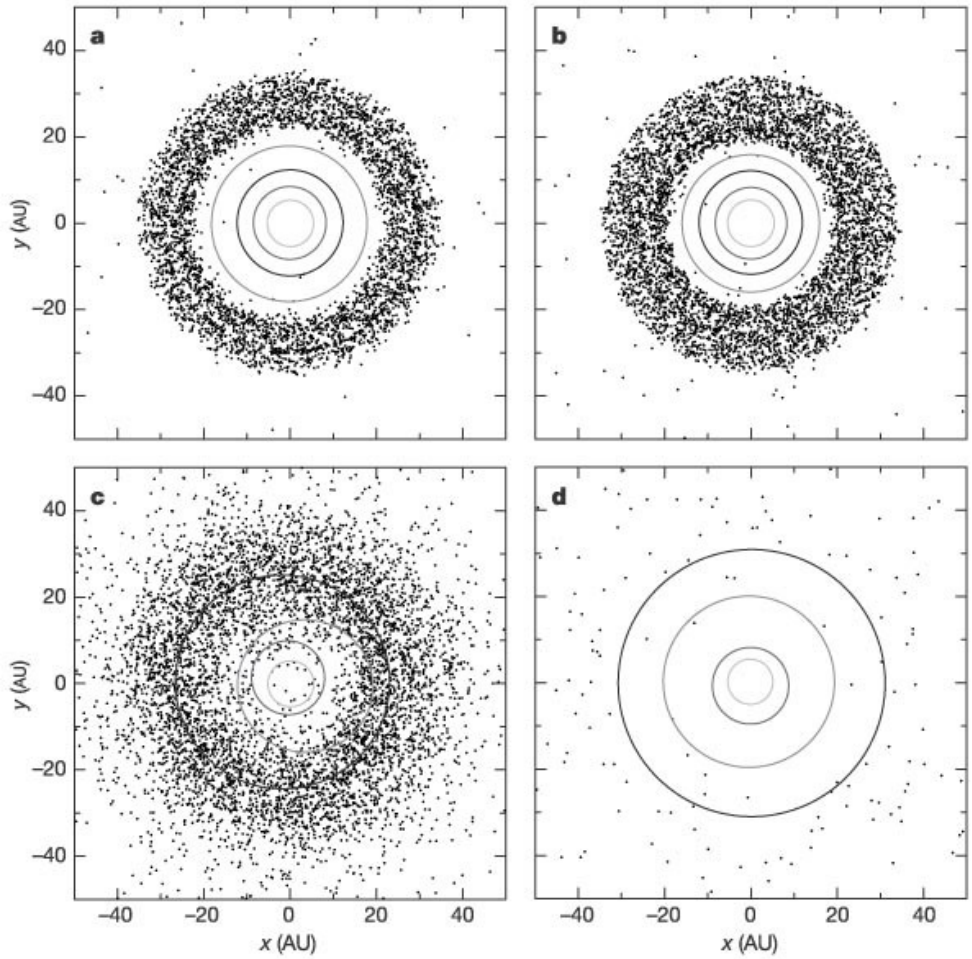
Explains:

- ✓ the small mass of Mars
- ✓ the structure of the M.A.B. !

(Walsh, Morbidelli et al. 2011)



Nice model



The giant planets did not form where they are now !

There has been a (late) global instabilty.

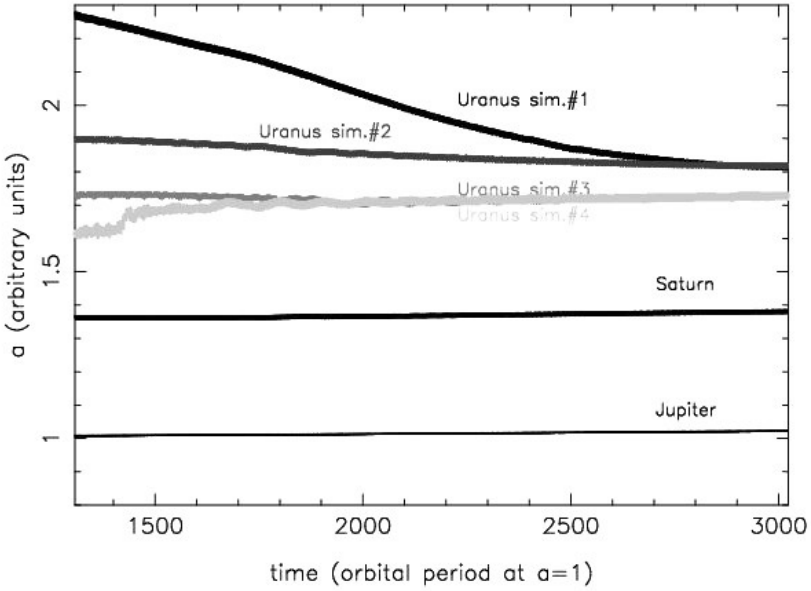
Jupiter and Saturn shoud have been at the end of the disk phase <u>just inside the 2:1 MMR</u>.

The Nice model :
Tsiganis, Gomes, Morbidelli, Levison (2005)
Gomes, Levison, Tsiganis, Morbidelli (2005)
Morbidelli, Levison, Tsiganis, Gomes (2005)

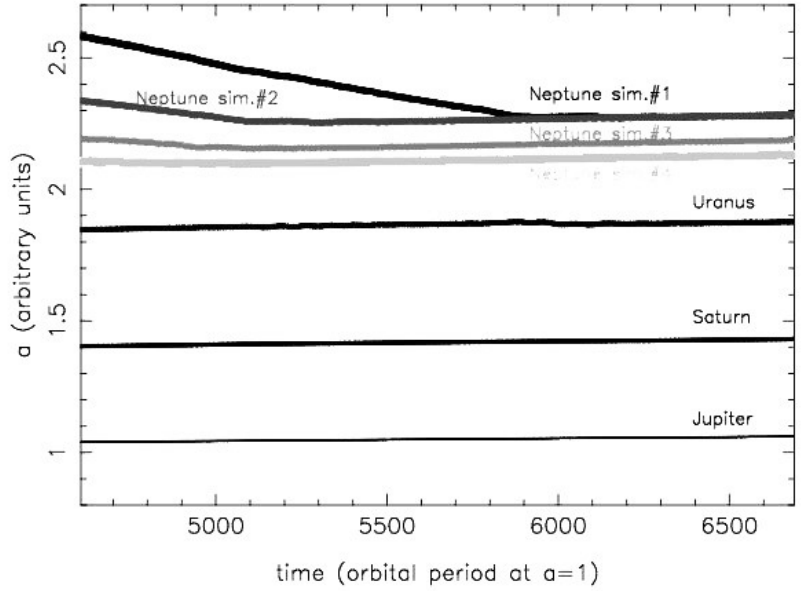
Planetary Migration & Nice model

1) Jupiter & Saturn in 3:2 MMR,
2) add Uranus → capture in 3:2 or 4:3.
3) add Neptune → captured in 3:2, 4:3 or 5:4.

(Morbidelli, Tsiganis, Crida, Levison, Gomes 2007)



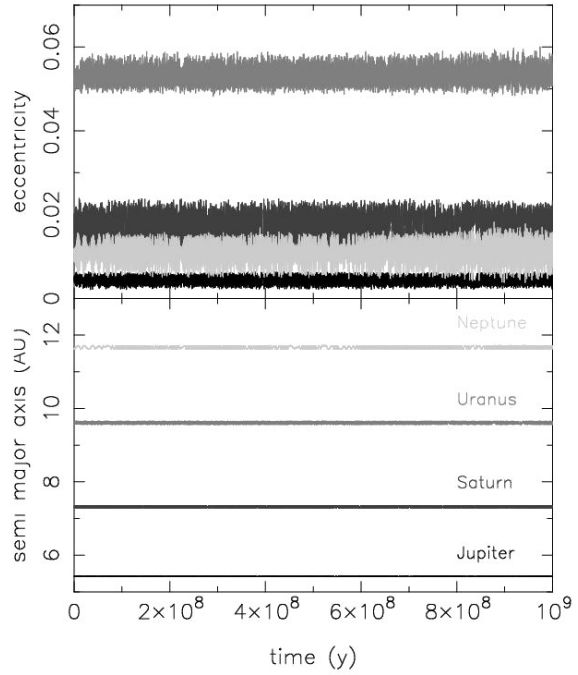
Migration in the gas disk.



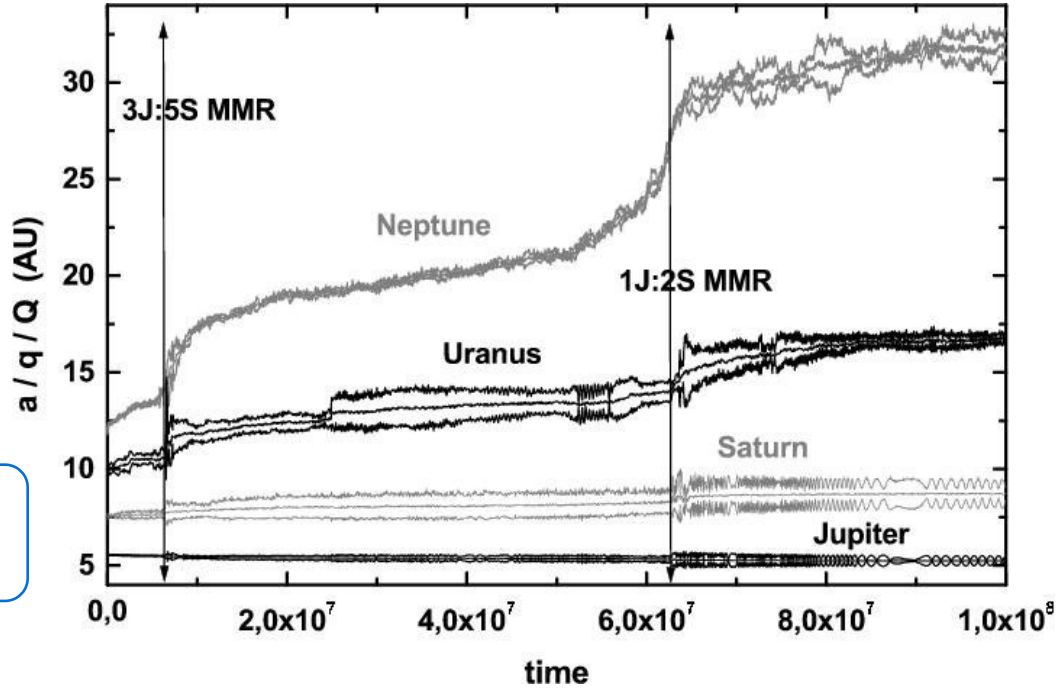
Planetary Migration & Nice model

Fully resonant configurations.
May break late (Levison et al. 2011)

Nice model II



No more gas, pure N-body



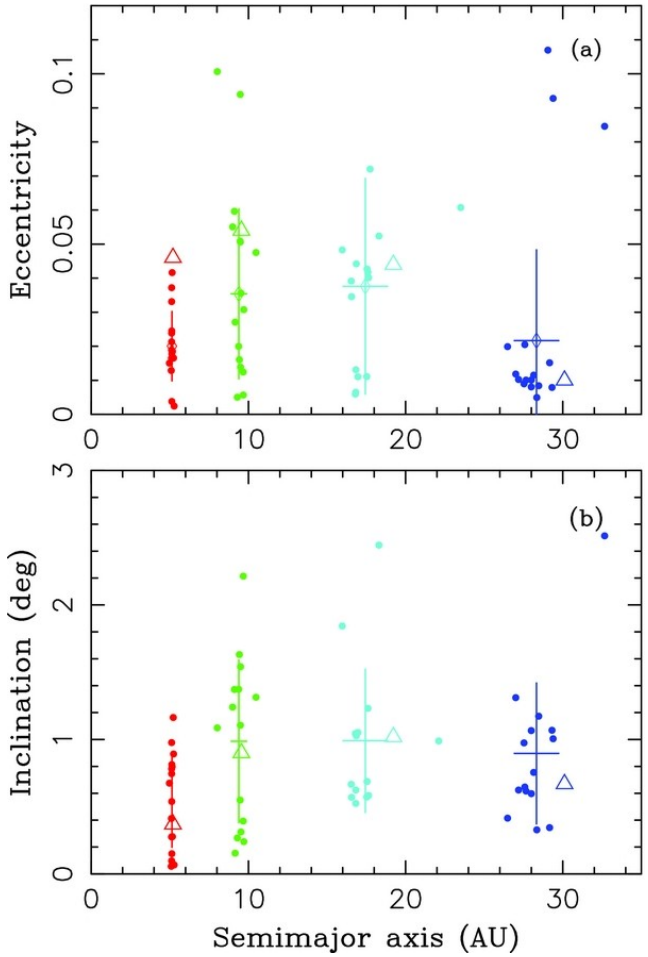
Nice model updates (1)

Better results if Jupiter ejects an ice giant !

Nesvorny & Morbidelli (2012) : *Statistical Study of the Early Solar System's Instability with Four, Five, and Six Giant Planets*

Define criteria for a good Solar System analogue + run N-body simulations from made-up resonant chains.

Nesvorny, Vorkouhlicky, Morbidelli (2013): *Capture of Trojans by Jumping Jupiter*



Nice model updates (2)

The Late Heavy Bombardment doesn't need to be late.



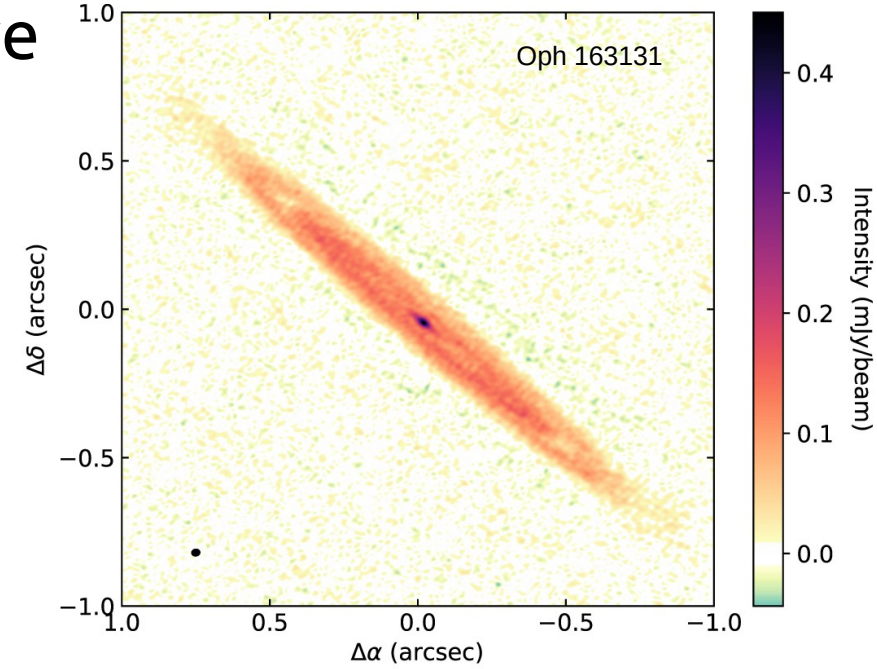
If the lunar mantle remained partially molten until -4.35 Gyr, the crater record and amount of siderophile elements are compatible with a monotonic decline of the bombardment rate (Morbidelli et al. 2018, among others).

Proto-Planetary Disks updates

Theory: The Magneto-Rotational Instabilty actually most often is quenched by non-ideal MHD terms (e.g. Lesur + 2014).

Observations: Pebbles (or **$s_t$ones**) are **very** settled (e.g. Villenave + 2022)

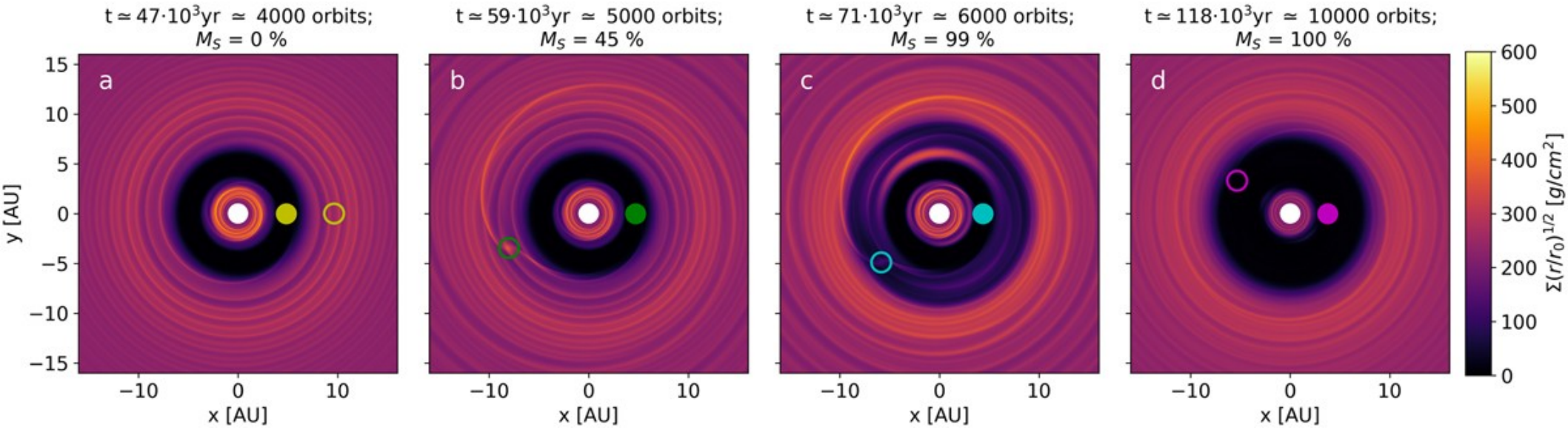
→ Viscosity low : Log(α) ≈ –4 ; – 5 .
(100 × lower than pre-2010)



→ **How do Jupiter and Saturn migrate ?**

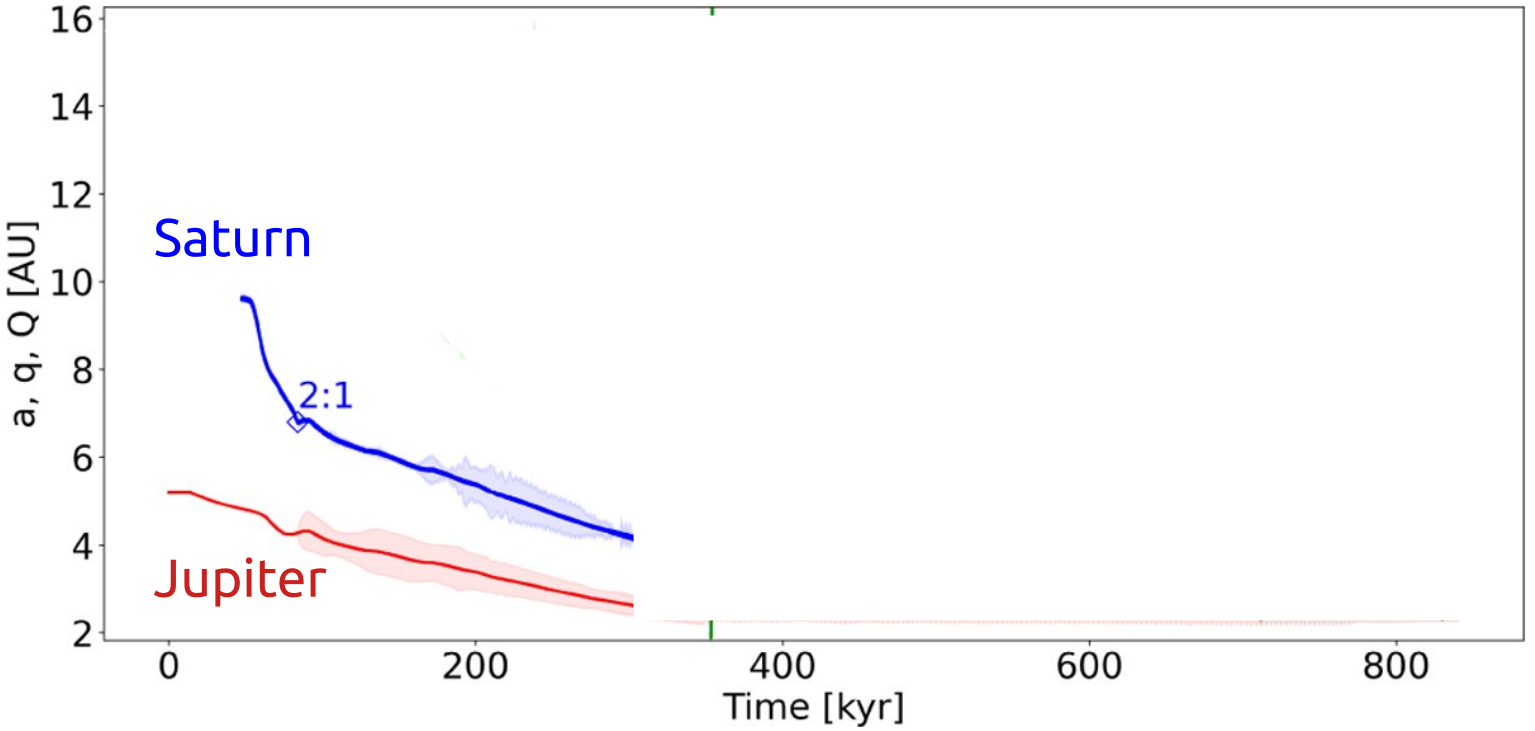
No Grand Tack !

Saturn opens a gap before it migrates fast enough to cross the 2:1. → Jupiter and Saturn always end in the **2:1** MMR.



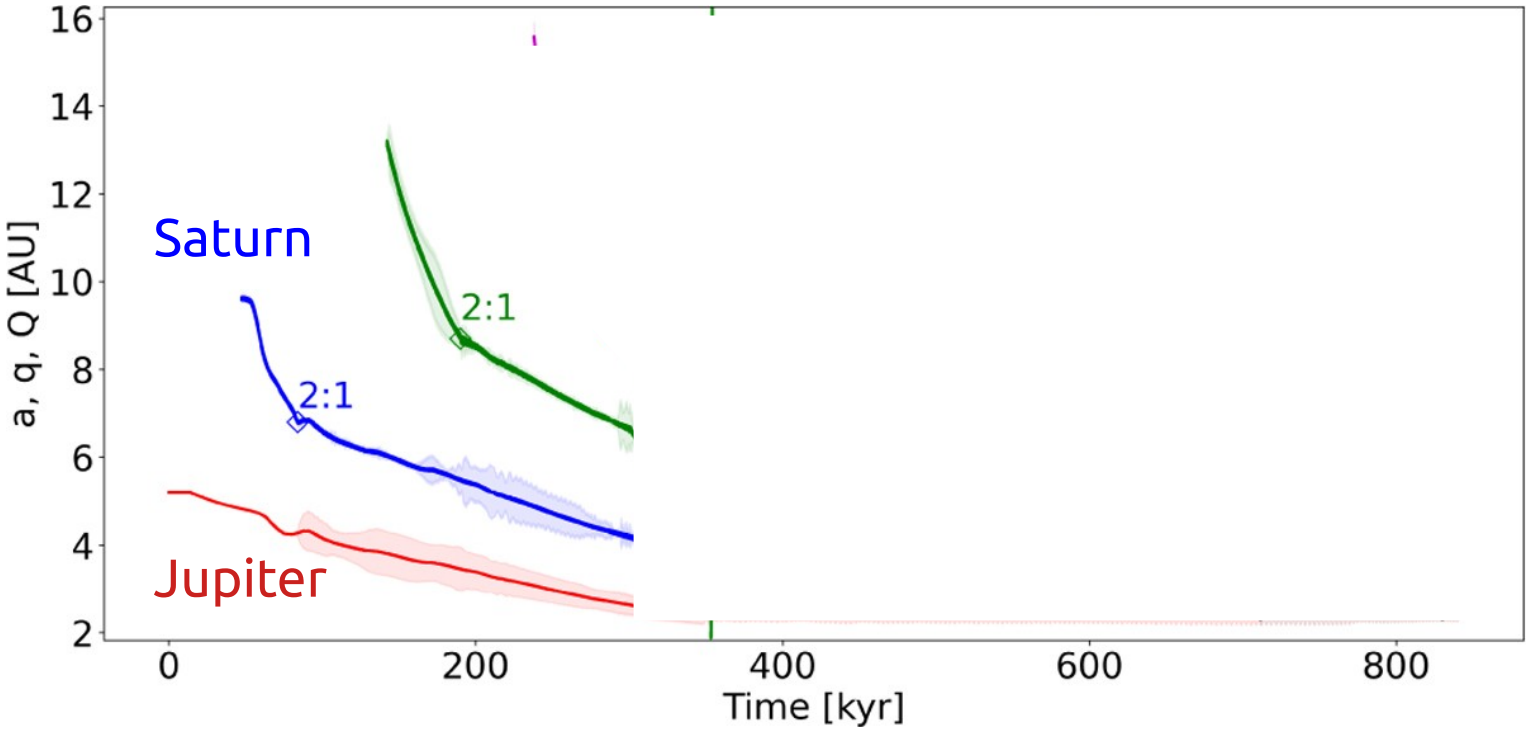
And <u>the pair never tacks</u>… (Griveaud, Crida, Lega 2023)

Planetary Migration (again) and...



Planetary Migration (again) and...

Add Uranus → 2:1.



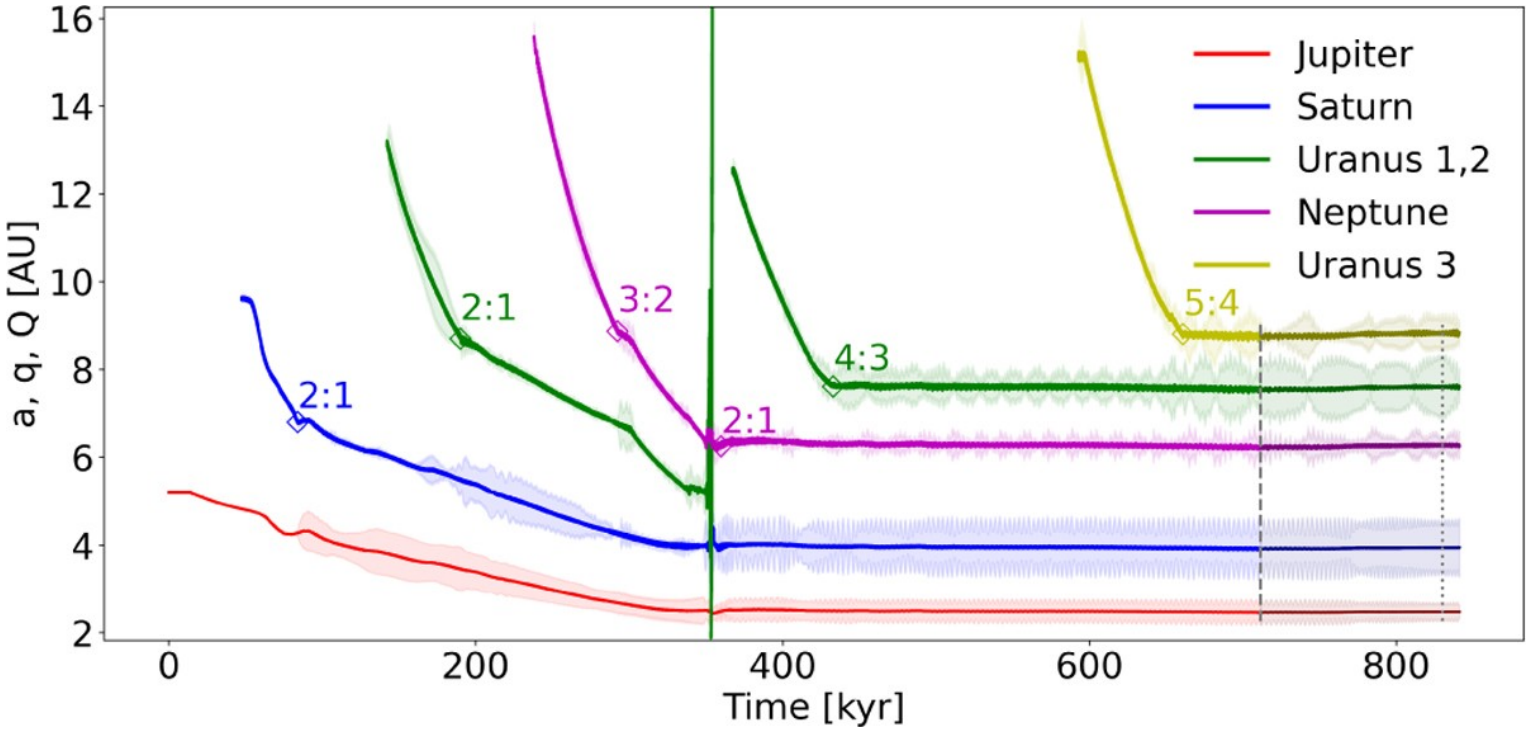
Planetary Migration (again) and…

Add Uranus → 2:1.

Add Neptune → 2:1, lose U.

Re-add Uranus → 4:3.

Add a 3rd giant → 5:4.



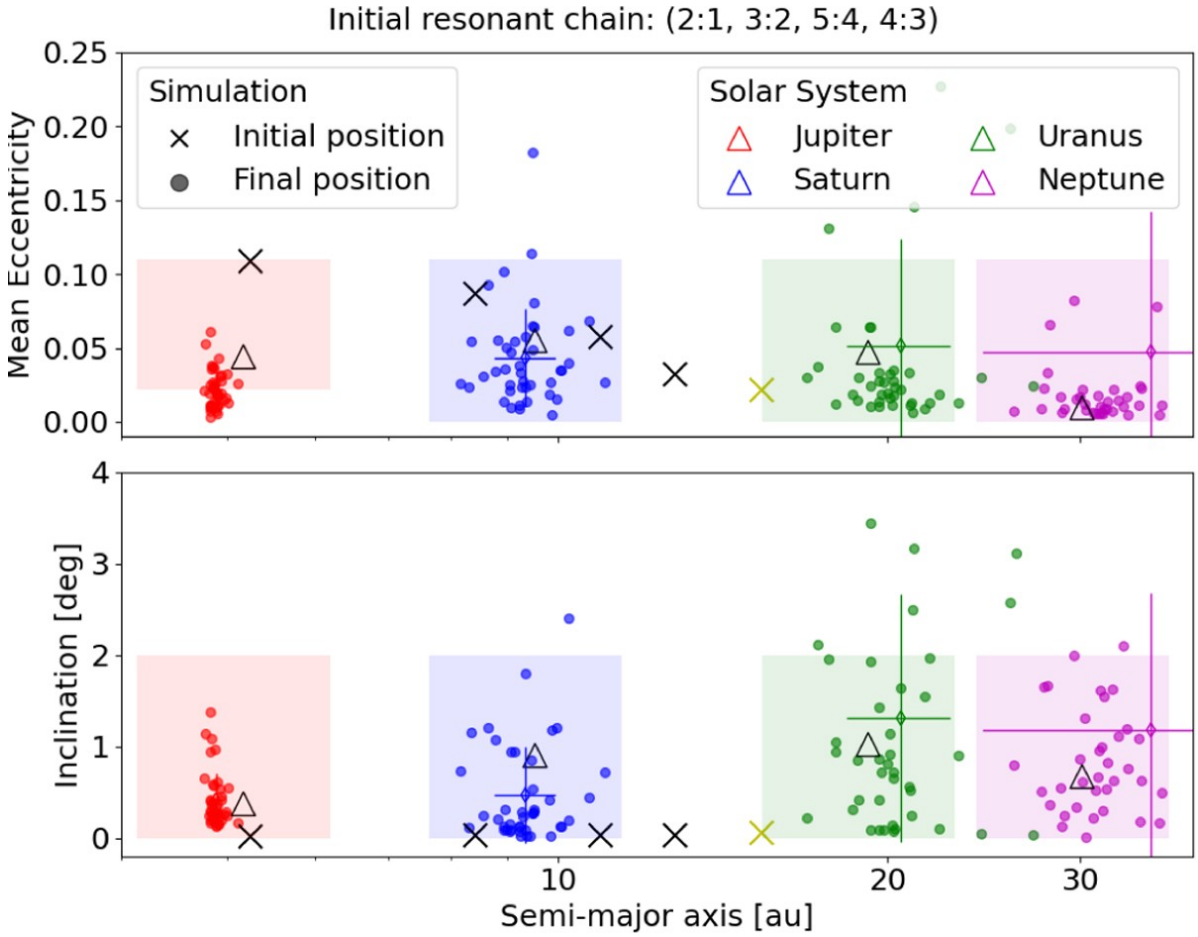
Hard – but possible – to form a stable resonant chain of 5 giant planets ; we found 2. (Griveaud, Crida, Petit, Lega, Morbidelli 2024)

… Nice model (again)

For each resonant chain

- evaporate the gas,
- add a planetesimal disk,
- run N-body simulations,
- analyse the final orbits of the giant planets (using Nesvorny & Morbidelli 2012 criteria)

(Griveaud, Crida, Petit, Lega, Morbidelli 2024)



… Nice model (again)

For each resonant chain

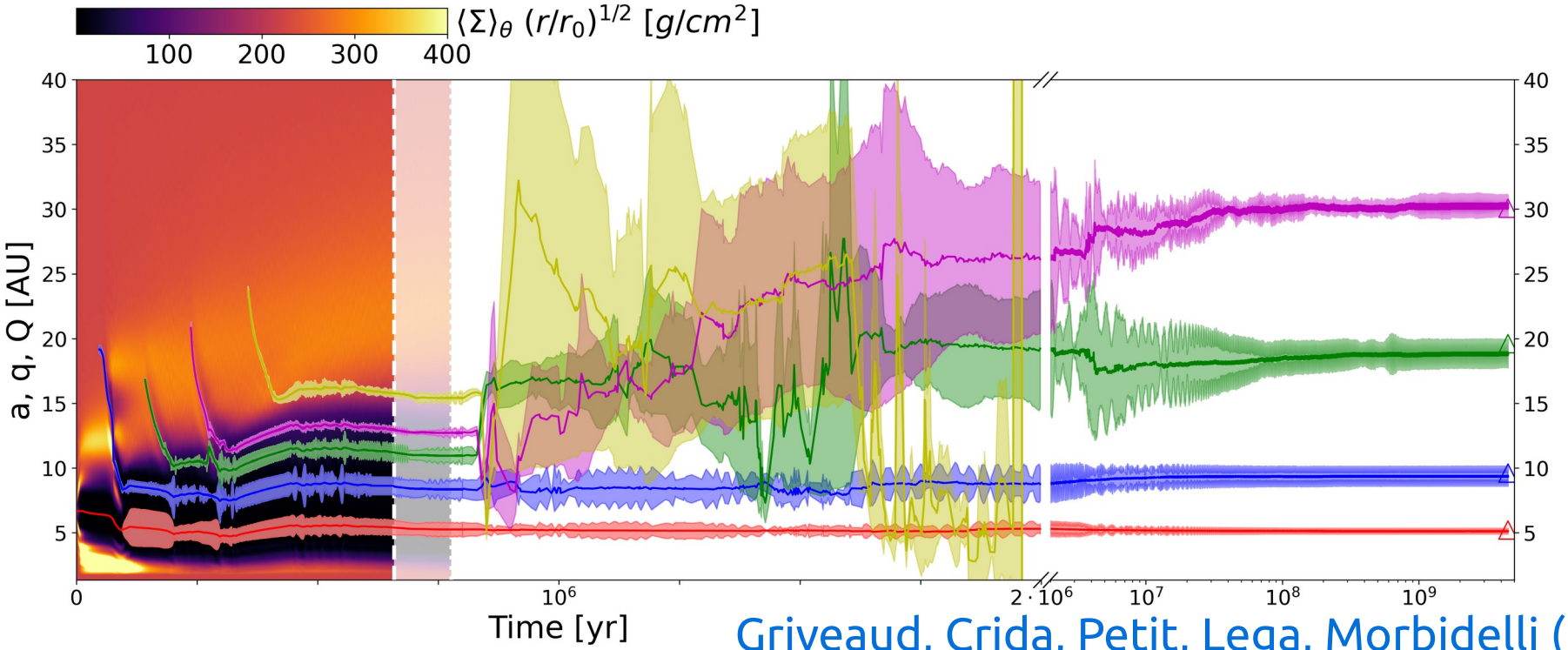
- evaporate the gas,
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- analyse the final orbits of the giant planets (using Nesvorny & Morbidelli 2012 criteria)

(Griveaud, Crida, Petit, Lega, Morbidelli 2024)

Resonant chain	Name	N_{sim}	M_D ($M_\oplus$)	A (%)	B (%)	C (C*) (%)	D; D* (%)
Cold protoplanetary disc (h = (0.035)							
		200	30	7	0.5	0.5 (0.5)	0; 0
(2:1, 3:2, 5:4)	$C4$	200	50	25.5	18	1 (4)	3; 10
(2:1, 3:2, 5:4, 4:3)	$C5$	200	30	9	4	1 (1)	0; 0
		200	50	21	13.5	1.5 (3)	3; 4
Nominal protoplanetary disc (h = 0.05)							
(2:1, 3:2, 4:3)	$N4$	50	30	98	1	0 (0)	0; 0
		100	50	69	29	1 (2)	2; 10
(2:1, 2:1, 4:3)	$N_{bis}4$	50	30	100	100	0 (0)	0; 0
		50	50	100	96	0 (0)	0; 70
(2:1, 2:1, 4:3, 5:4)	$N_{bis}5$	100	30	19	10	0 (2)	3; 0
		100	50	4	3	0 (0)	1; 1

All in all : ~1 % success.
The solar system is rare...

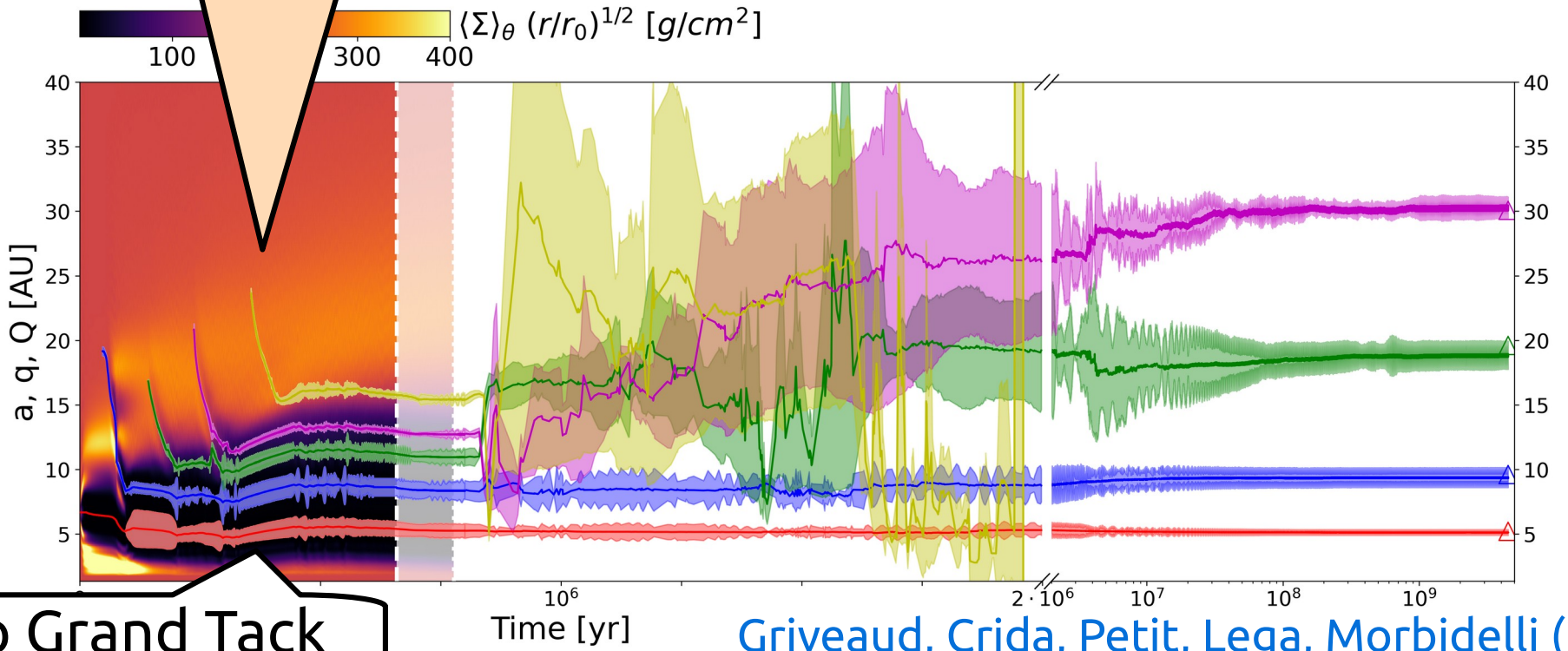
Nice model III



Griveaud, Crida, Petit, Lega, Morbidelli (2024)

Nice model III

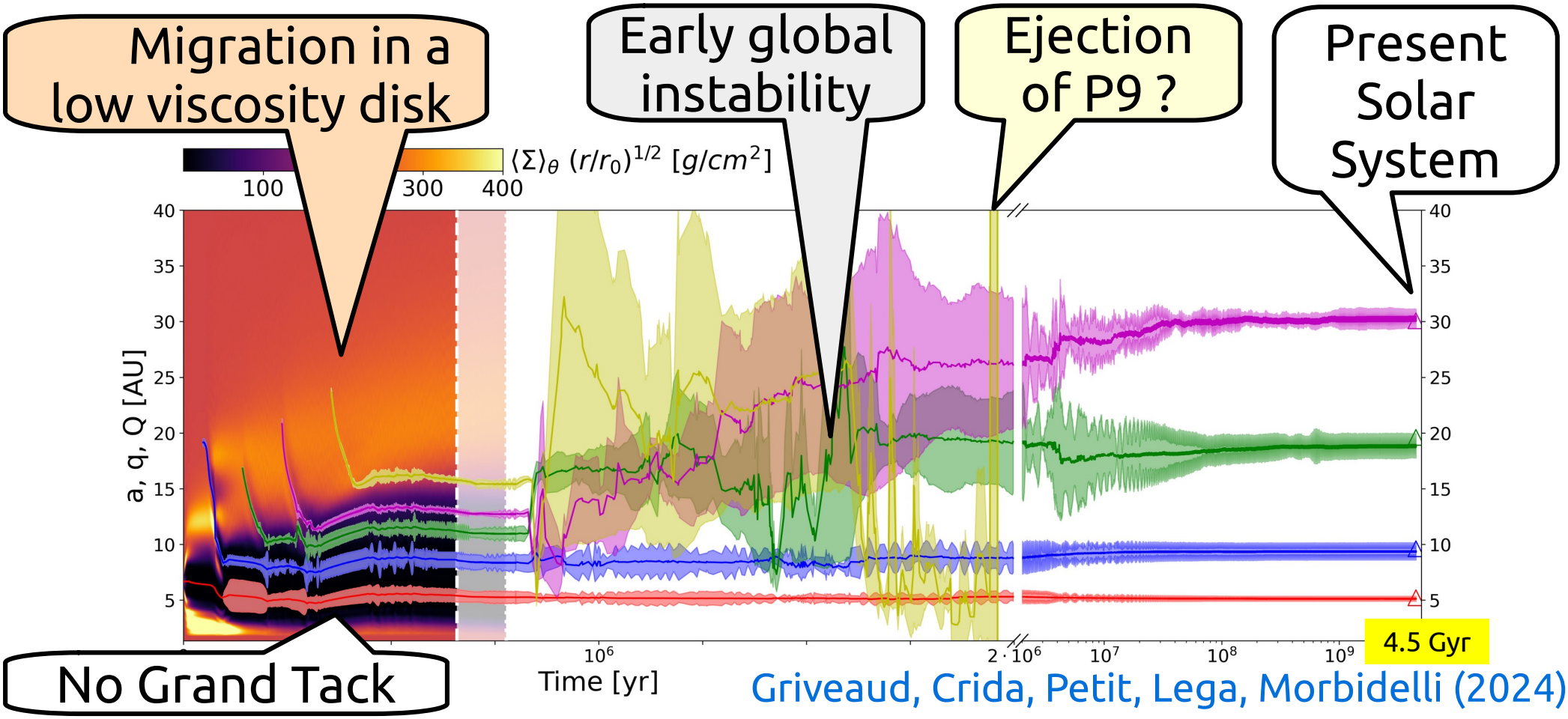
Migration in a low viscosity disk



No Grand Tack

Griveaud, Crida, Petit, Lega, Morbidelli (2024)

Nice model III

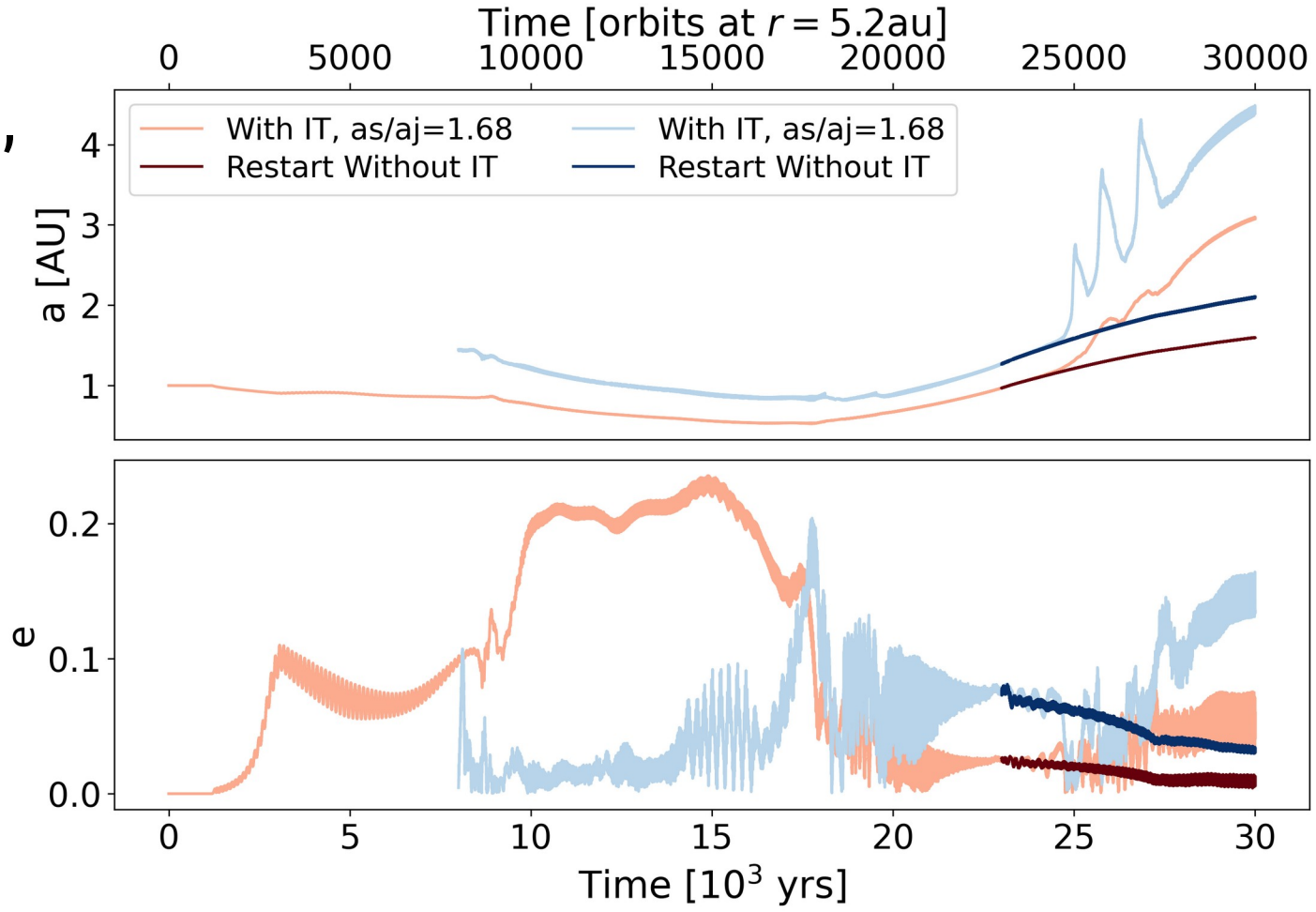


Griveaud, Crida, Petit, Lega, Morbidelli (2024)

An intriguing difference...

Jupiter, Saturn, viscous disk, 3:2 MMR, outward migration.

+ Episodes of type III runaway outwards migration of Saturn.

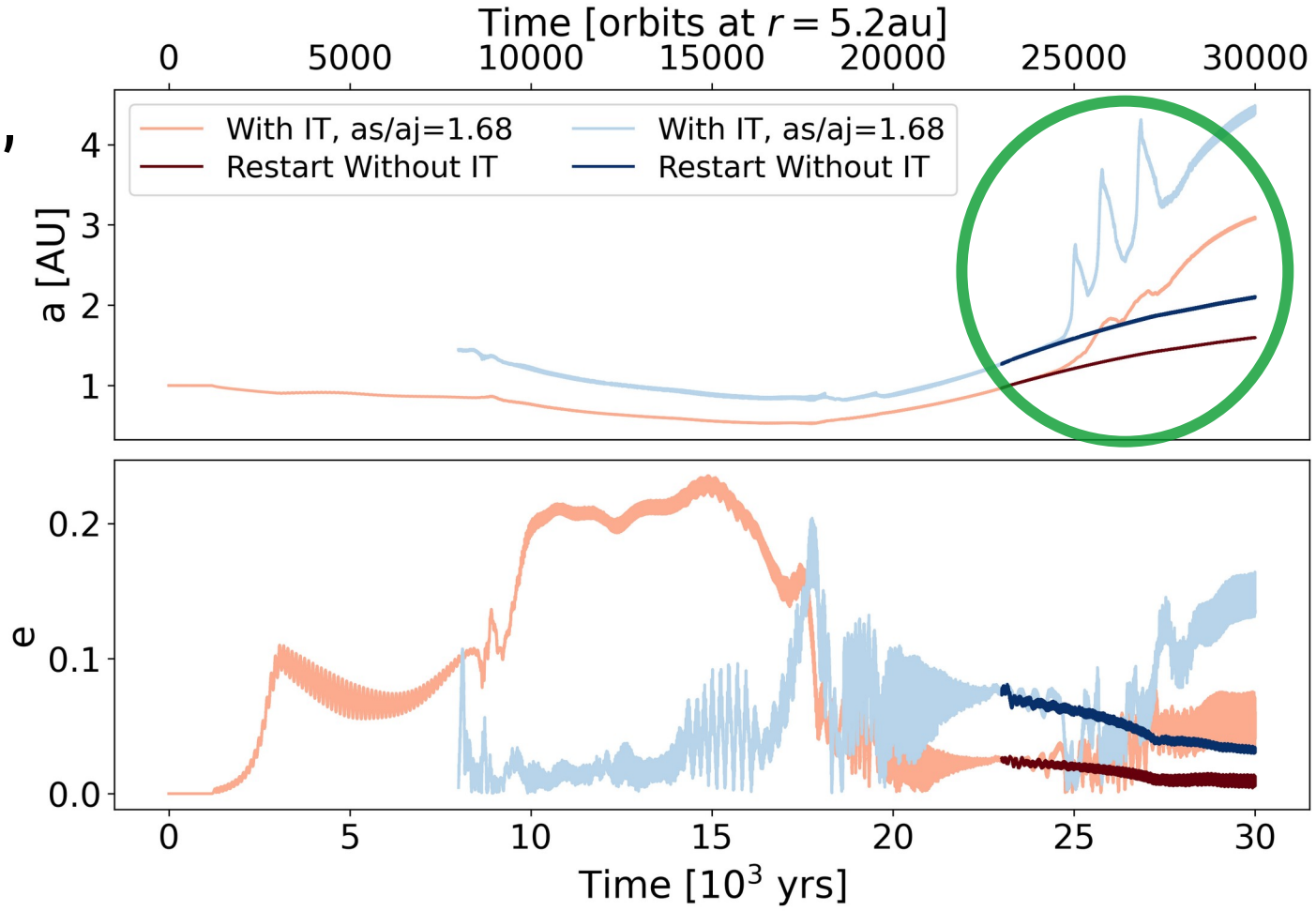


An intriguing difference...

Jupiter, Saturn, viscous disk, 3:2 MMR, outward migration.

+ Episodes of type III runaway outwards migration of Saturn.

These episodes disappear if we remove the « indirect term »...

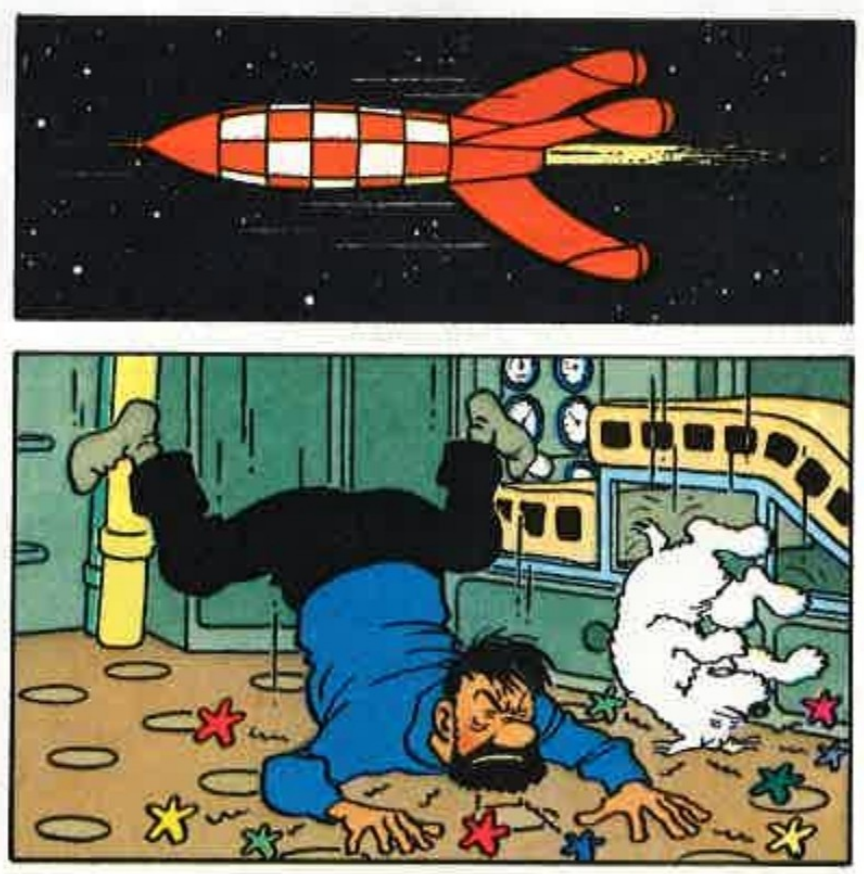


Indirect Term

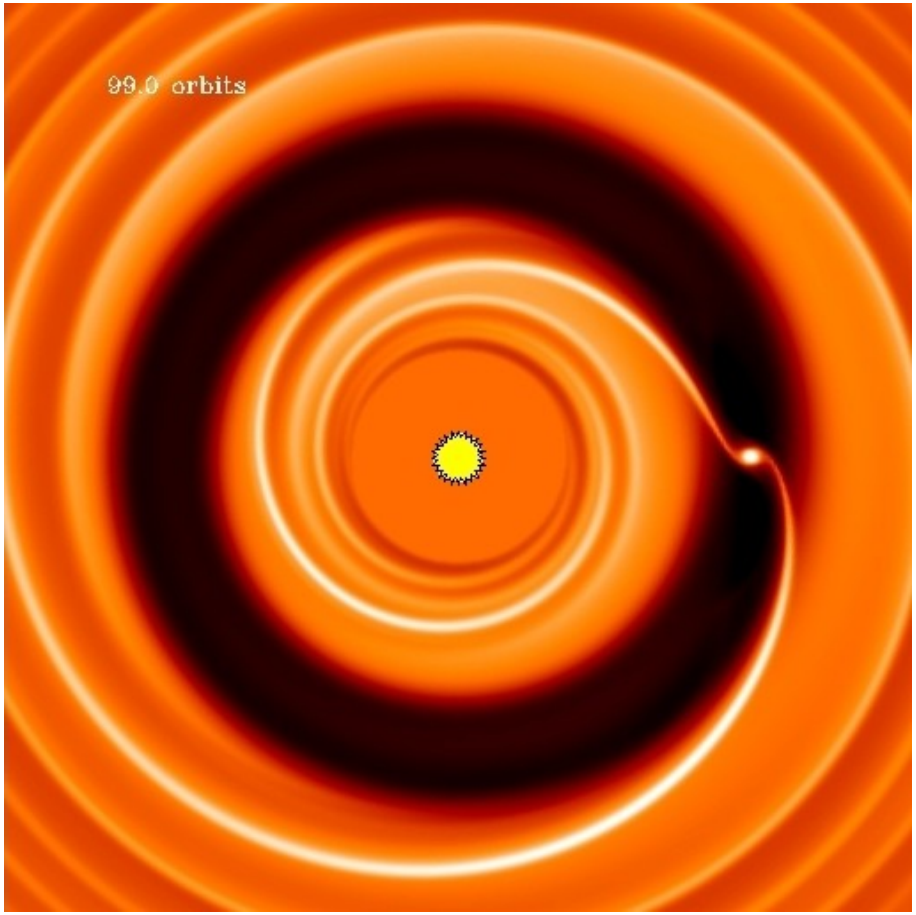
→ What is it ?

A ficticious force to account for the acceleration of the frame (ex : in Tintin's rocket, people feel a kind of gravity, objects « fall ».)

$\mathbf{a}_{\text{indirect}} = - \mathbf{a}_{\text{frame}}$.



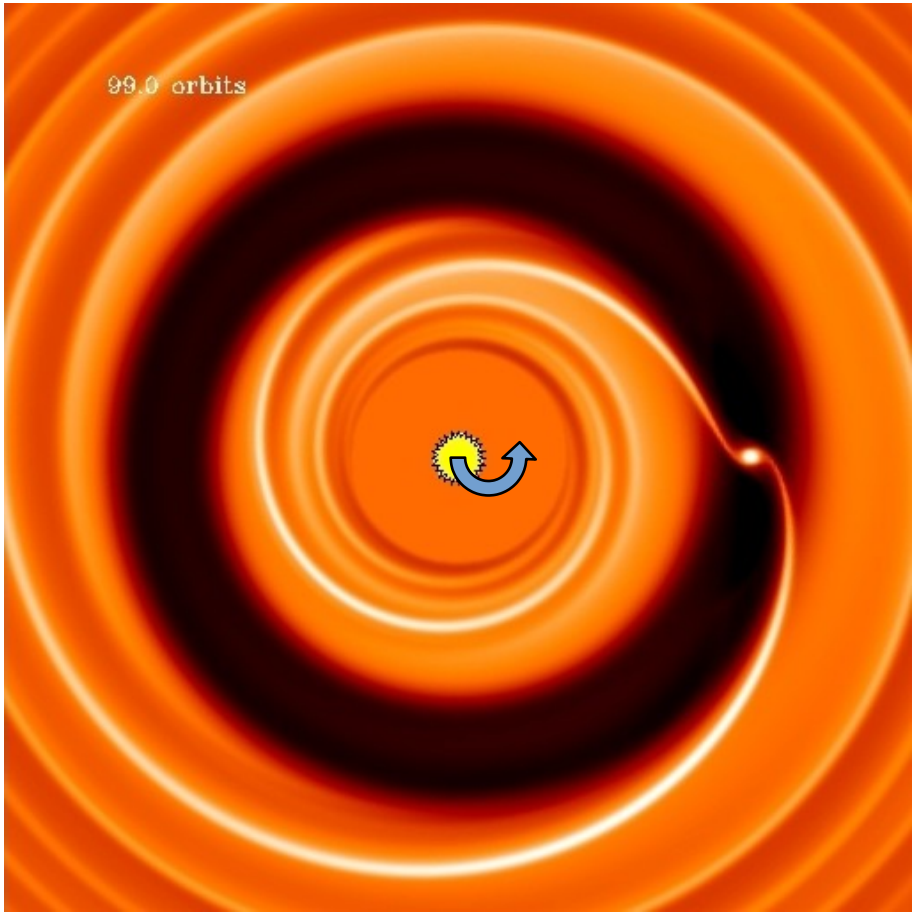
Indirect Term



→ Why do we care ?

def: Star
punctual object of mass 1,
at the centre of the frame.

Indirect Term



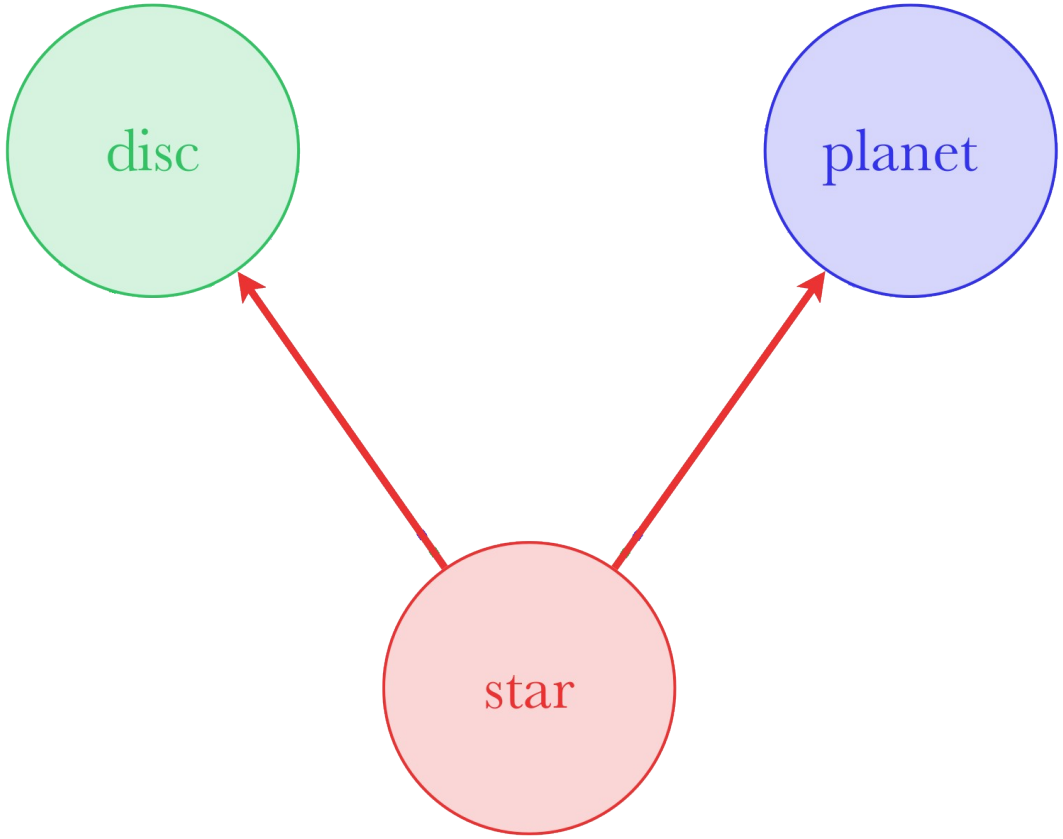
→ Why do we care ?

The star-centred frame is not inertial : the star is accelerated by the planet(s) and disk.

Hence, an inertial force, called

the indirect term

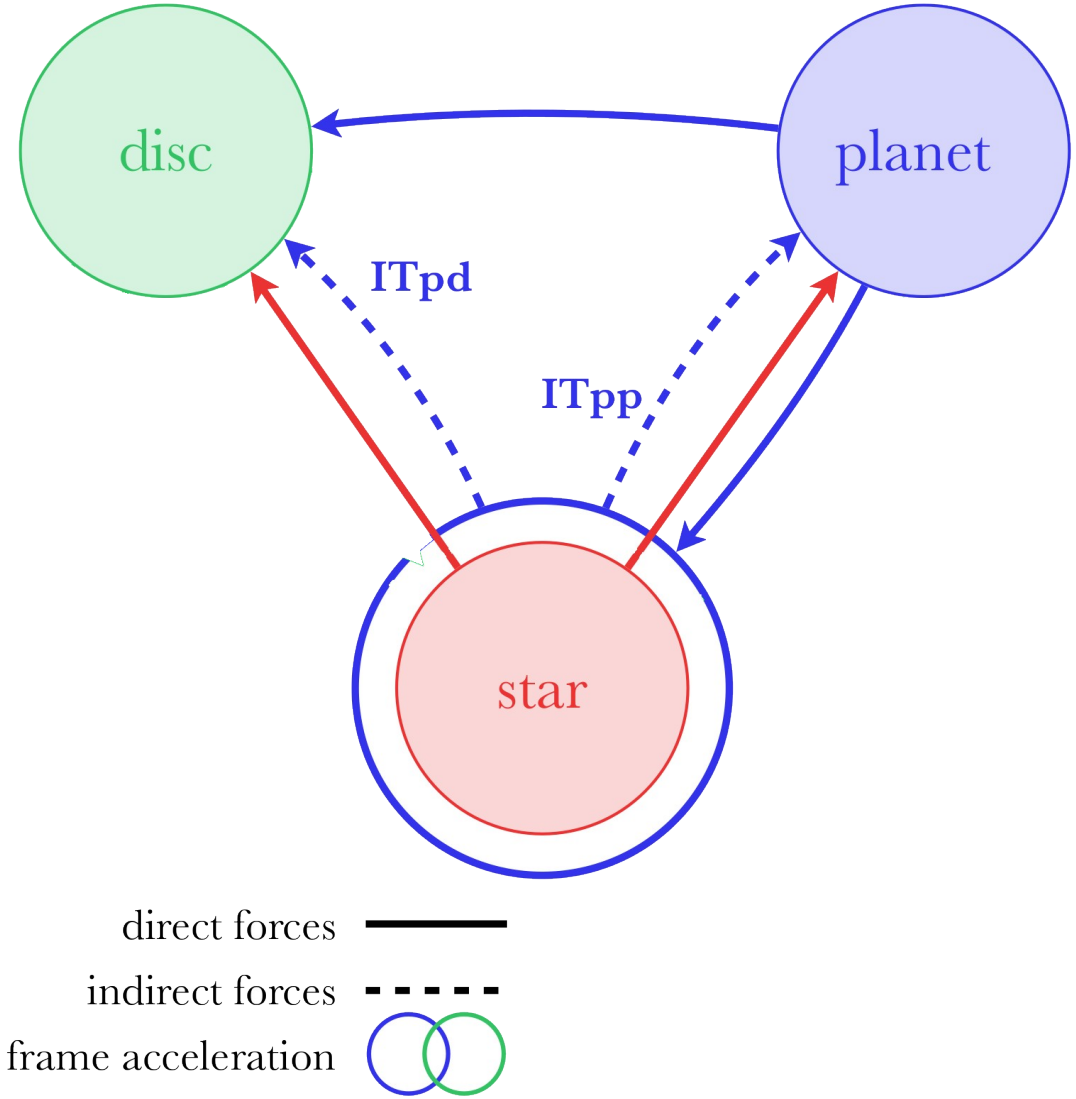
is applied to all the elements of the simulations.



Gravity forces

Direct gravity from star : Kepler's law.

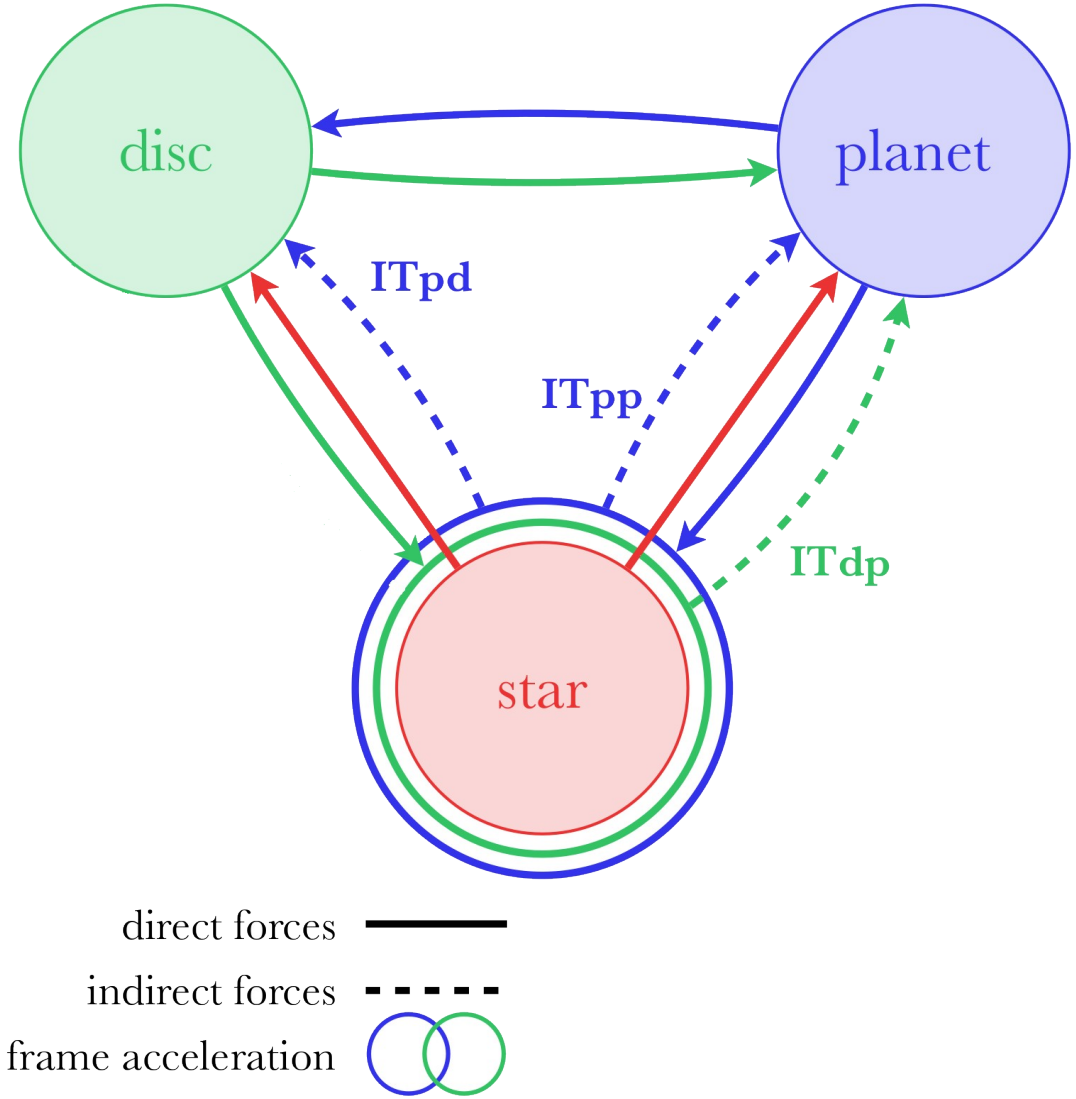
Gravity forces



Direct gravity from star : Kepler's law.

Gravity from planet :
- direct effect on the disc
- accelerates the star → indirect term on pl & disc. (Lagrange points)

Gravity forces

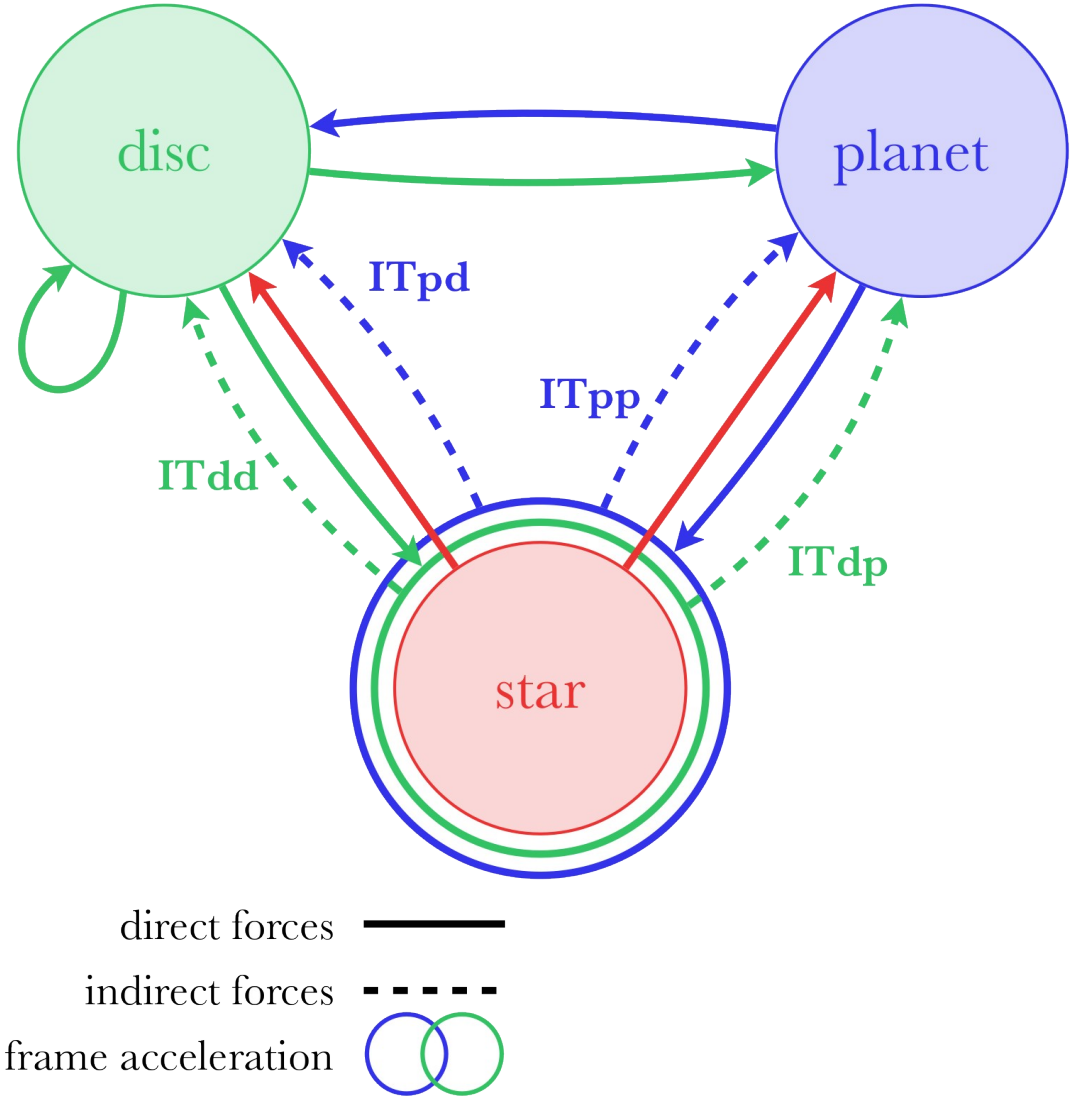


Direct gravity from star : Kepler's law.

Gravity from planet :
- direct effect on the disc
- accelerates the star → indirect term on pl & disc.

Gravity from disc :
- direct onto planet
- indirect onto planet

Gravity forces

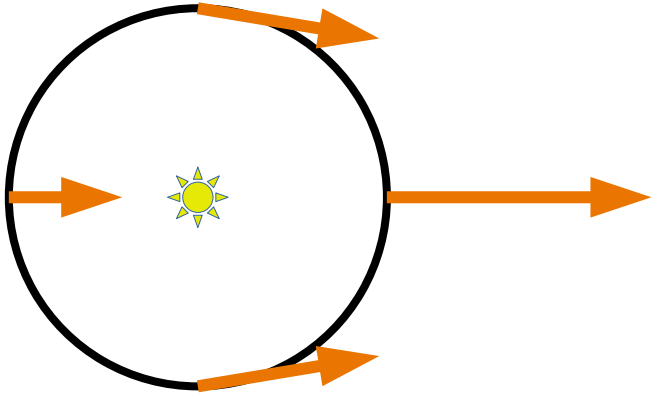


Direct gravity from star : Kepler's law.

Gravity from planet :
- direct effect on the disc
- accelerates the star → indirect term on pl & disc.

Gravity from disc :
- direct onto planet & disc
- indirect onto planet & disc

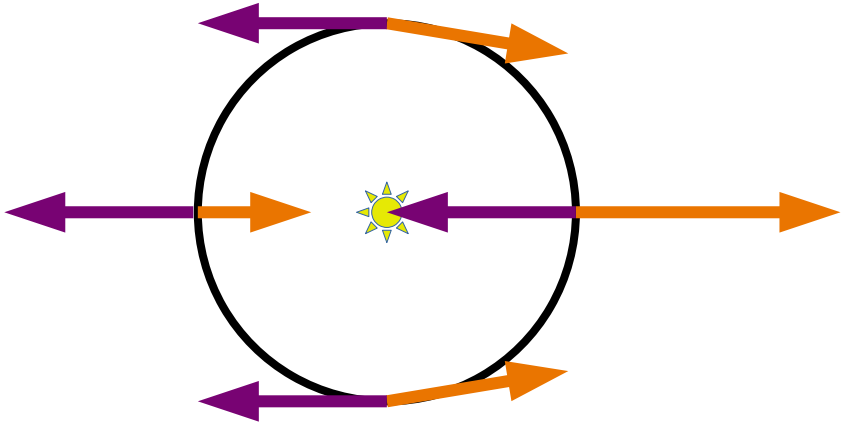
Tidal acceleration





Direct acceleration from smiley face.

Tidal acceleration

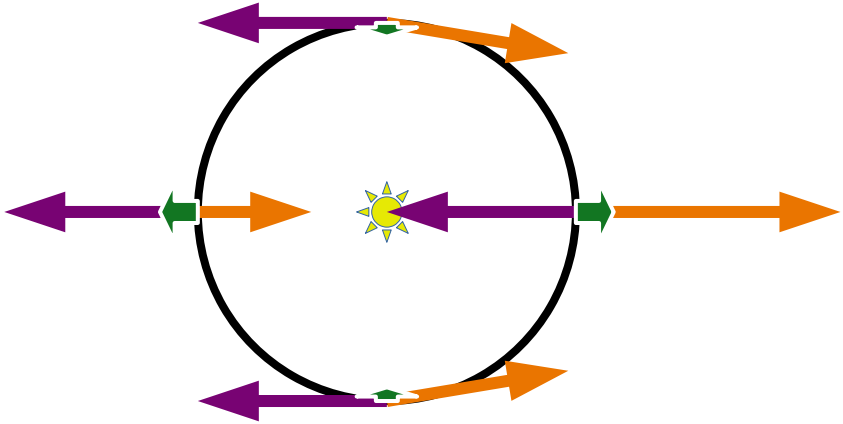




Direct acceleration from smiley face.

Indirect term = Opposite of the acceleration of the star (uniform).

Tidal acceleration



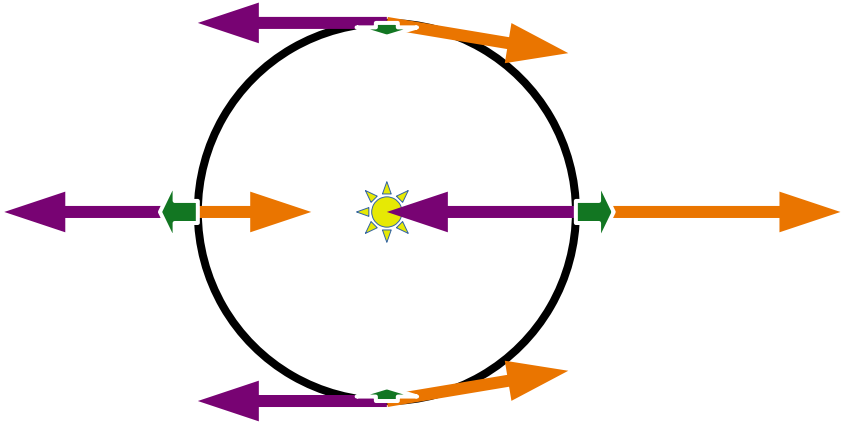


Direct acceleration from smiley face.

Indirect term = Opposite of the acceleration of the star (uniform).

Sum = tidal acceleration. Differential effect, ~zero on average.

Tidal acceleration





Direct acceleration from smiley face.

Indirect term = Opposite of the acceleration of the star (uniform).

Sum = tidal acceleration. Differential effect, ~zero on average.

Forget the indirect term, you are dramatically wrong.
Take **only** the indirect term, you are just as wrong...

Summary on the Indirect TermS

« THE Indirect Term » = mistake.
- Comes from various objects
- Applied to different elements.

→ ITpp, ITpd, ITdp, ITdd

	planet 1	planet 2	disc
planet 1: fixed	$\mathbf{ITp_1p_1}$: ✓	$\mathbf{ITp_2p_1}$: ×	**ITdp** : ×
planet 1: migrating	$\mathbf{ITp_1p_1}$: ✓	$\mathbf{ITp_2p_1}$: □	**ITdp** : ✓
disc element	**ITpd** : ✓		**ITdd** : □*

TABLE 2
TABLE SHOWING AS COLUMNS THE CONSTITUENTS OF A PROTOPLANETARY DISC FROM WHICH AN ELEMENT IN A ROW SHOULD FEEL THE INDIRECT TERM. CAPTION: ✓: YES, ALWAYS. ×: NO, NEVER. □: IF AND ONLY IF THE DIRECT TERM (E.G. SELF GRAVITY FOR THE DISC) IS ALSO TAKEN INTO ACCOUNT. * SEE PAPER II ABOUT THE INSTABILITY IT CAN TRIGGER.

Each object could feel its own indirect term,

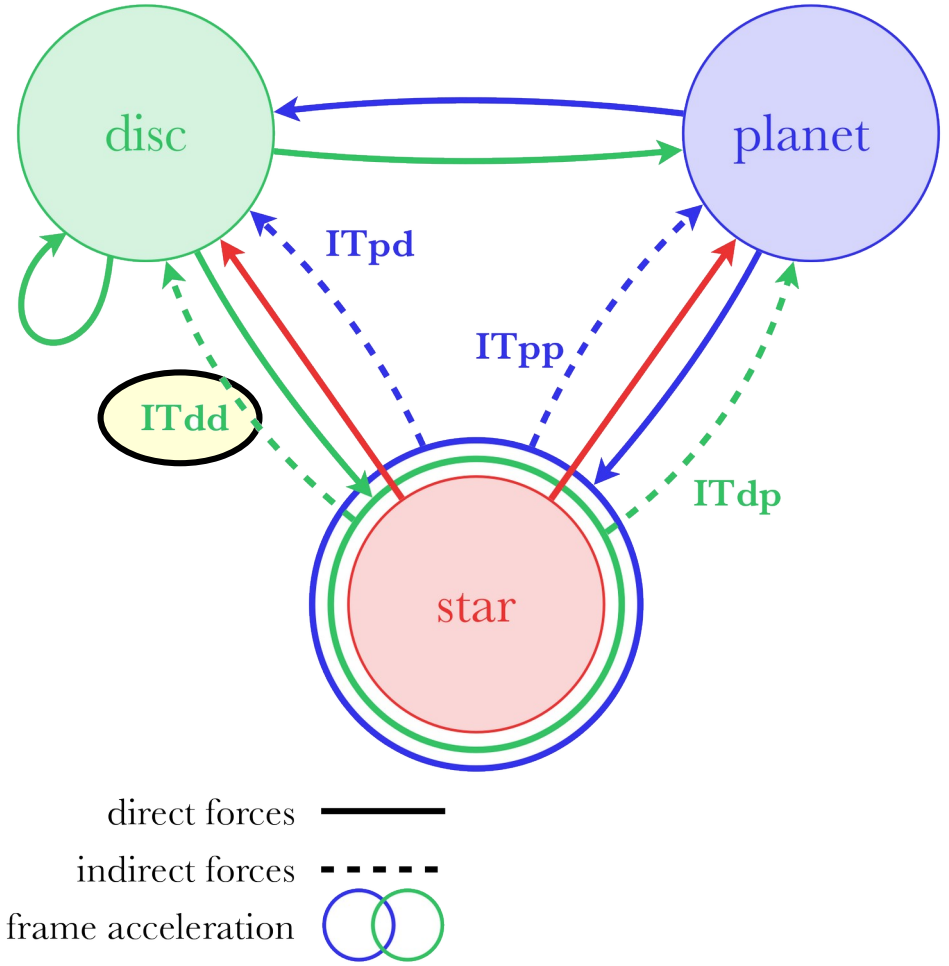
= – the acceleration of the star due to
{all the elements from which the said object feels the gravity}.

See : ***On inertial forces (indirect terms) in problems with a central body***

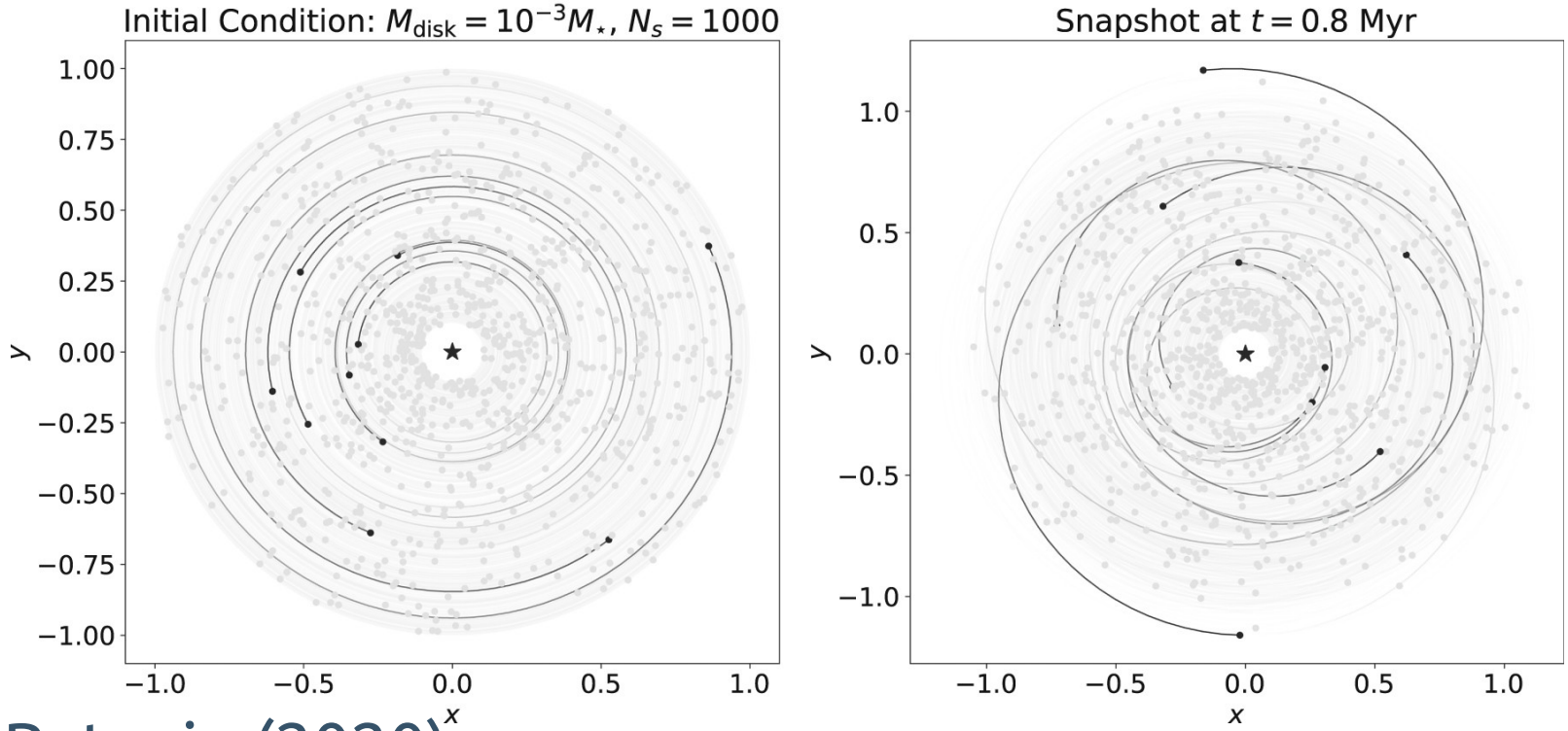
Crida, Baruteau, Griveaud, Lega, Masset, Béthune, Gonzalez, Méheut, Morbidelli, Gero-sa, Kloster, Marques, Miniussi, Minker, Pichierri, Segrétain. **2025, TOJA, vol 8, id 84**

The reflex instability

A dramatic application of ITdd...



Example in N-body

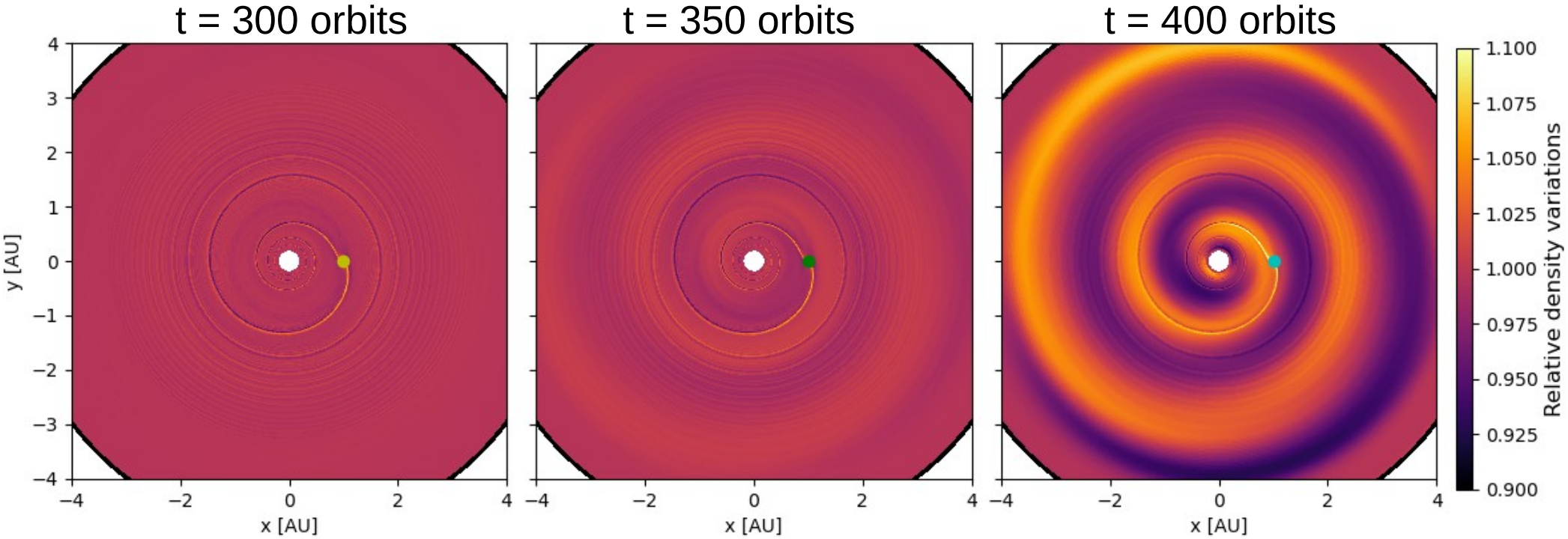


Peng & Batygin (2020):

e growth in N-body simulations of *« small particles »*, that is particles not feeling each other directly, but through indirect term.

ITdd → Instability

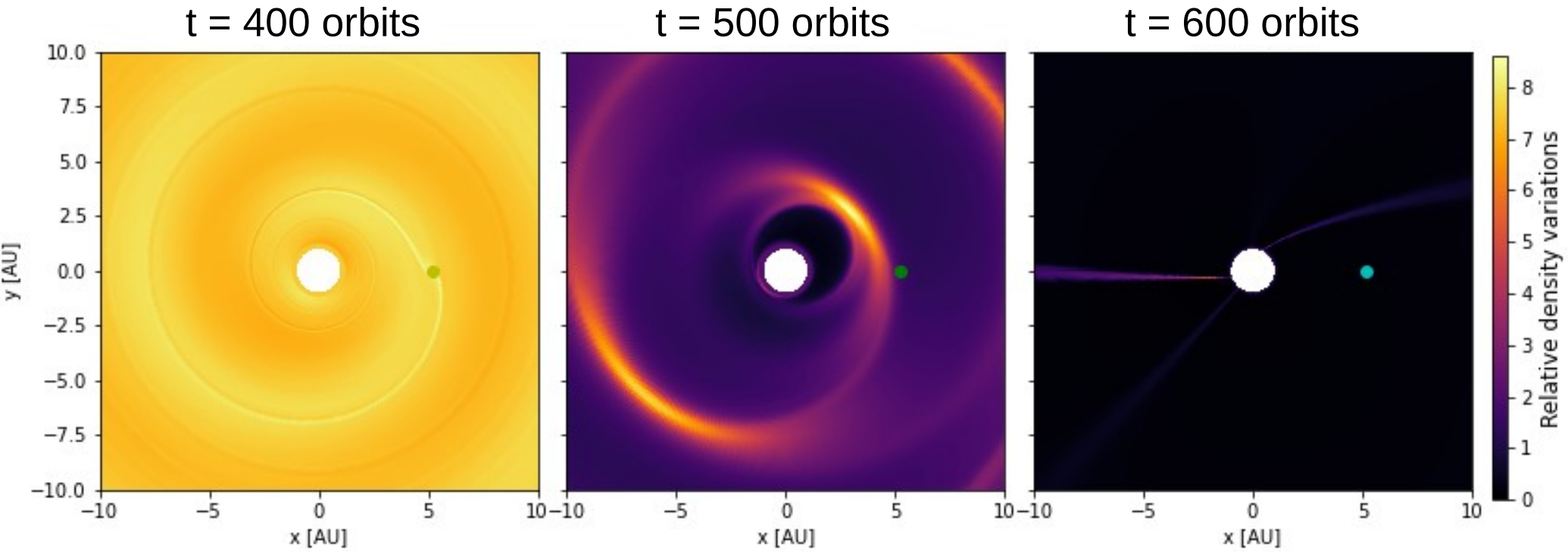
Numerical simulation with ITdd, $M_p/M_* = 10^{-5}$, $\Sigma = 10^{-2}\ M_*/r_0^2$:



An instability develops. Spiral mode ≠ planet wake.

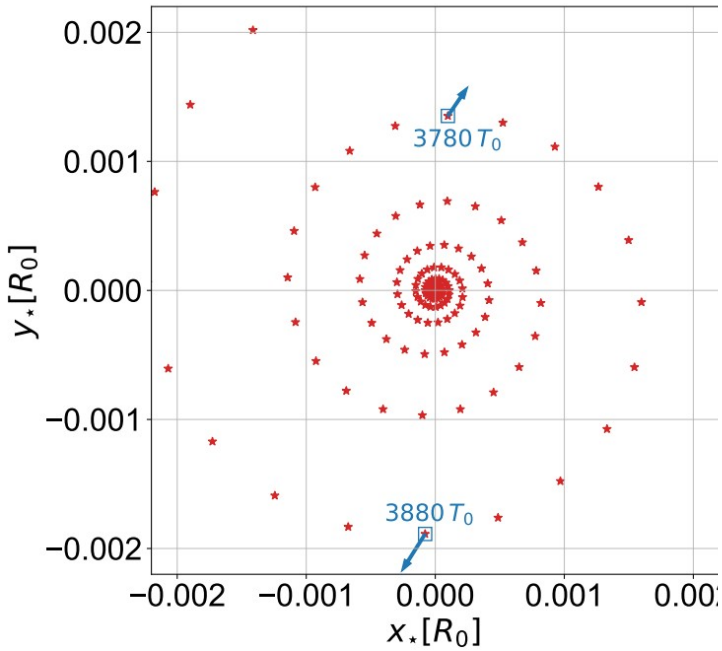
ITdd → Instability

Numerical simulation with ITdd, $M_p/M_* = 10^{-5}$, $\Sigma = 10^{-2}\ M_*/r_0^2$: crash !

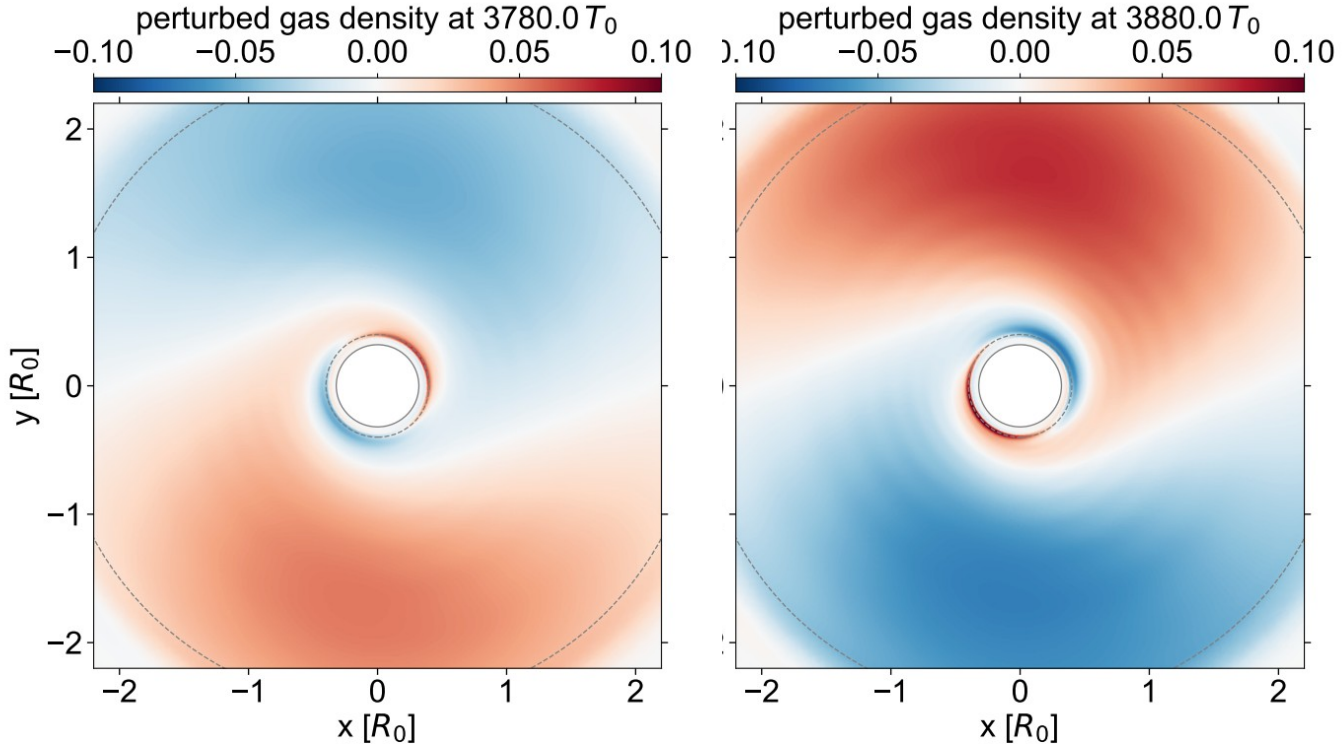


An instability develops. Spiral mode ≠ planet wake.

Barycentric simulation, no planet

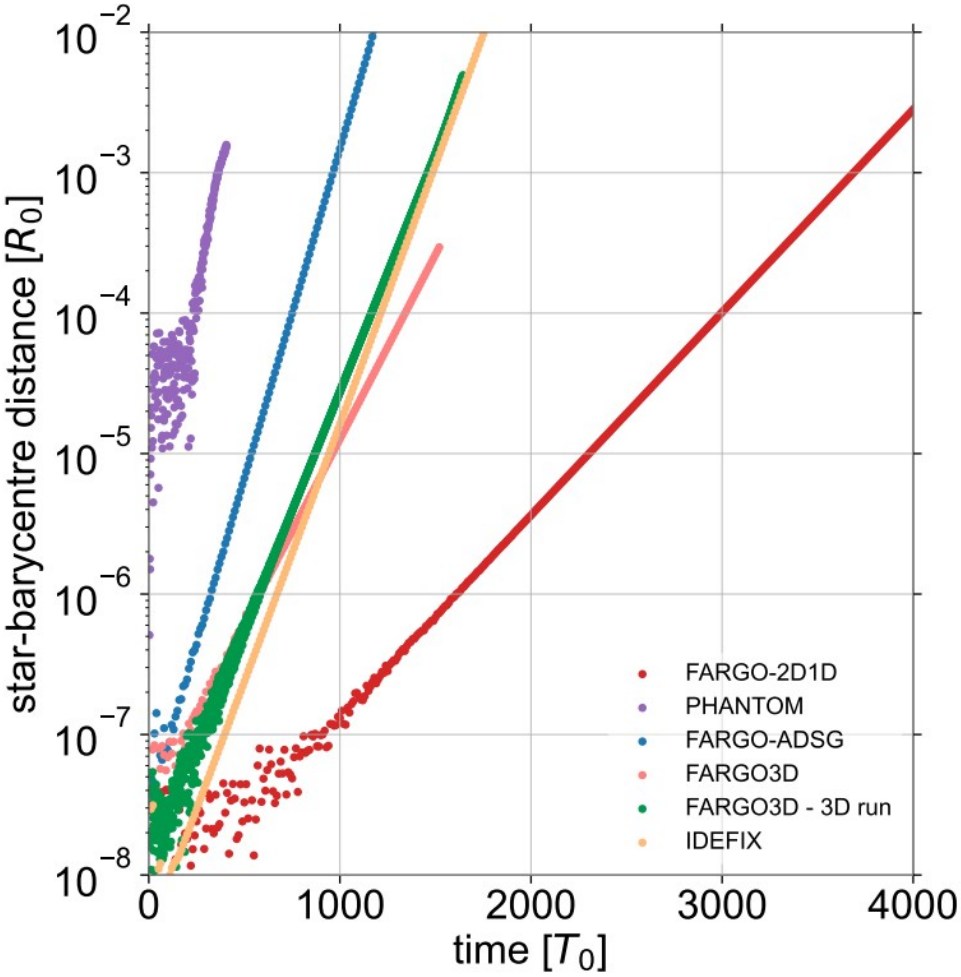


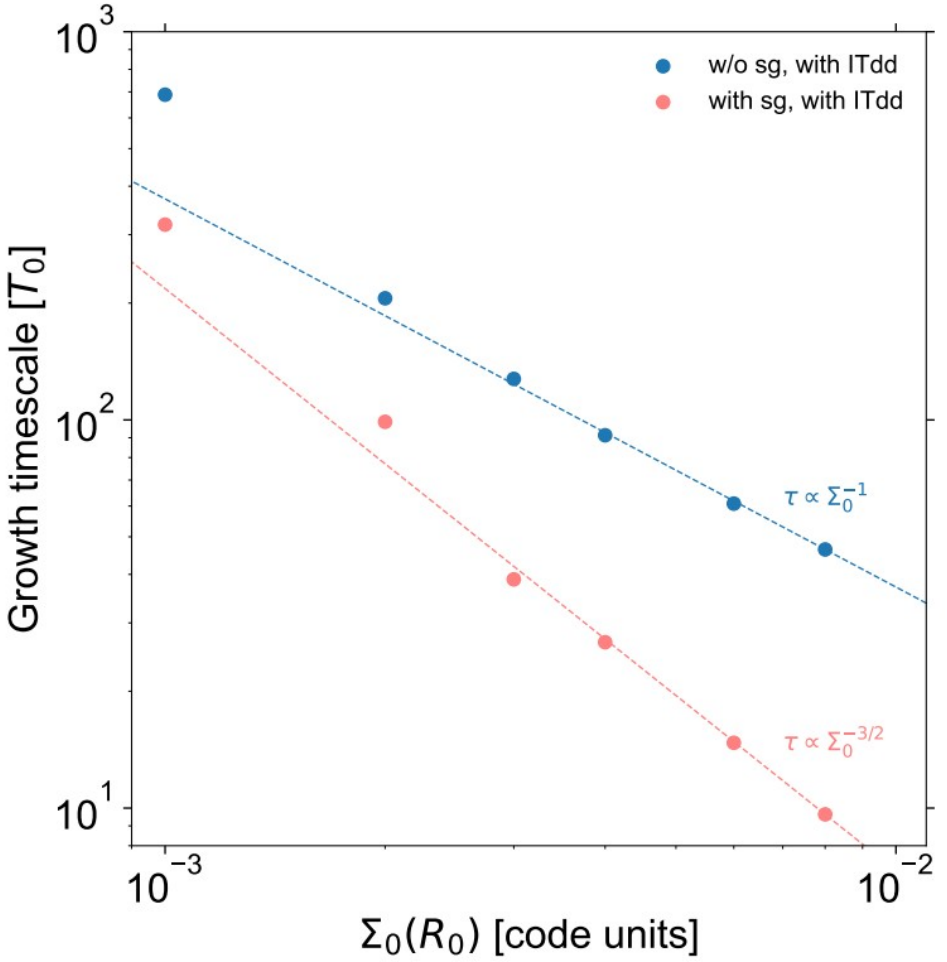
Star position + acceleration at t=3780 and t=3880.



Normalised perturbation of the gas density after 3780 and 3880 orbits @r=1

A linear instability





Reflex Instability

This phenomenon is found **if and only if ITdd is implemented**

- in grid codes and SPH codes
- in barycentric and star-centric simulations
- with Godunov or Van Leer shemes
- independant of resolution and timestep
- with various (or no) boundary conditions

→ real, physical phenomenon, which we call ***reflex instability***.

The reflex instability :
Exponential growth of a large-scale m = 1 mode in astrophysical discs

Crida, Baruteau, Gonzalez, Masset, Segrétain, Griveaud, Méheut, Lega.

2025, TOJA, vol 8, id115

Bracket mode

Sanchez et al. (2025, Appendix 3) and Magnan et al. (submitted)
- surface density perturbation in $\Sigma = \exp(i\omega - \varphi)$
- dispersion relation in the barycentric frame:

$$\omega^2 = [\Sigma/r](R_{max}) - [\Sigma/r](R_{min}),$$

Generally < 0 if grid inside the disk → complex ω → exp. growth.

Magnan et al. (subm): in a *stellocentric* frame, however, ω is *real* and the bracket mode is marginally stable !

Different physics in different frames because different physical inner edge: circle centred on the star or the barycentre.

Note : Reflex instability ≠ Bracket mode.

Reflex instabilities

Work in progress...

Magnan et al. (in prep): At least 7 types of instability identified.

→ All our numerical tools may not be appropriate.
→ Some of these modes should affect real protoplanetary discs.

In simulations where the disk is shattered, it ends more extended, and eccentric. Can this explain some observations ???

Conclusion and summary

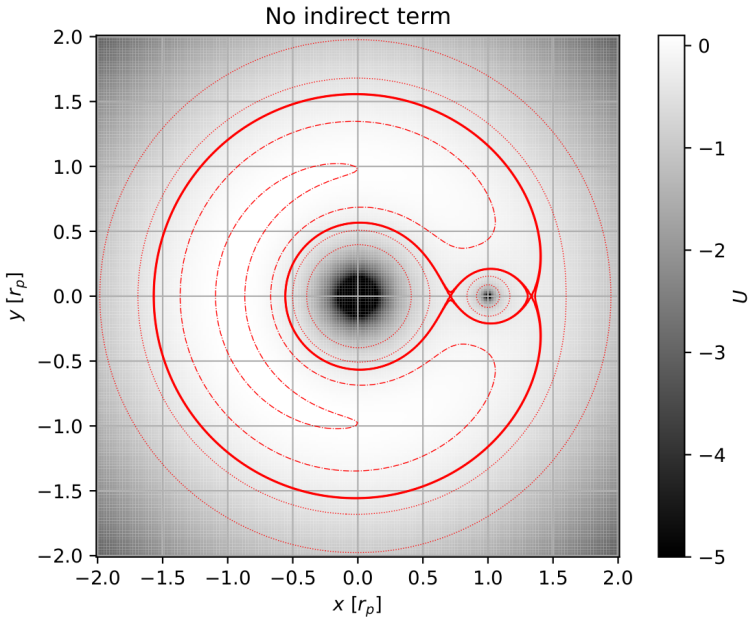
- Planet-disk interactions → migration.
- Planet-planet interactions → resonances.
- In the Solar System : no Grand Tack, resonant chain of 5 giant planets → early Nice-model instability.
  (Griveaud et al. 2023, A&A 672 ; 2024, A&A 688)
- Say explicitely how the Indirect Term**s** are handled in your simulations (Crida et al. 2025a *The Open Journal of Astrophysics, vol 8, id84*) **.**
- Beware of the reflex instabilities ! (ex: even if invisible, the m=1 mode perturbs planet migration)
  (Crida et al. 2025b *The Open Journal of Astrophysics, vol 8, id115*)

Extra-slides

Restricted 3-body problem

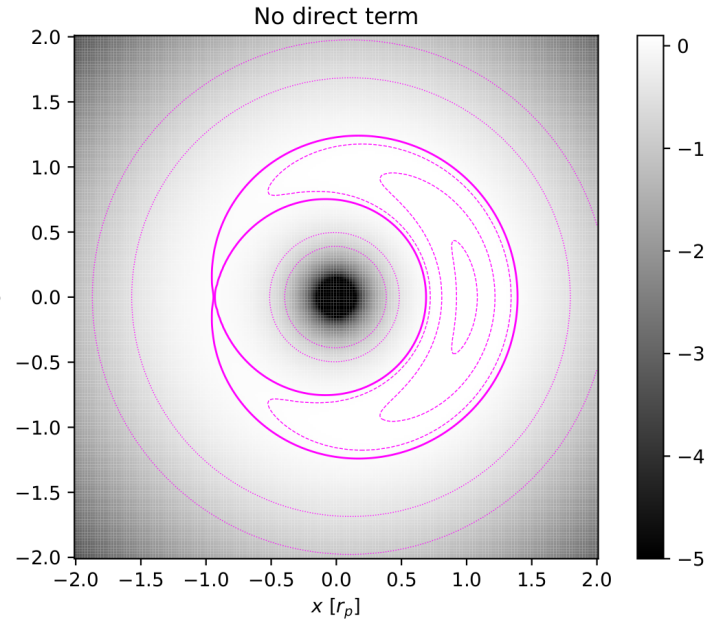
Potential energy from stellar gravity + centrifugal

+ planet gravity



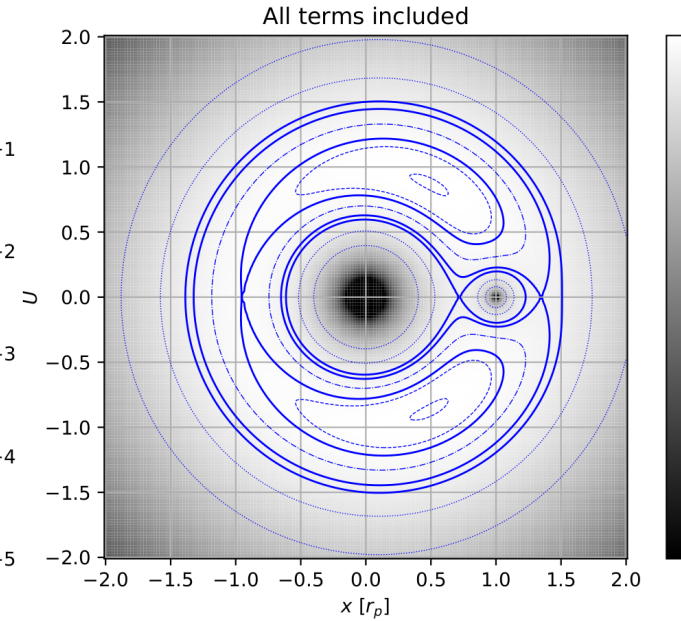
3 equilibirum points along y=0

+ Indirect Term



2 equilibrium points along y=0

+ both



5 Lagrange points

Acceleration and eccentricity

1 test particle in orbit around a star.

$\mathbf{a} = -(GM_*/r^2)\,\mathbf{u_r}$

→ circular orbit.

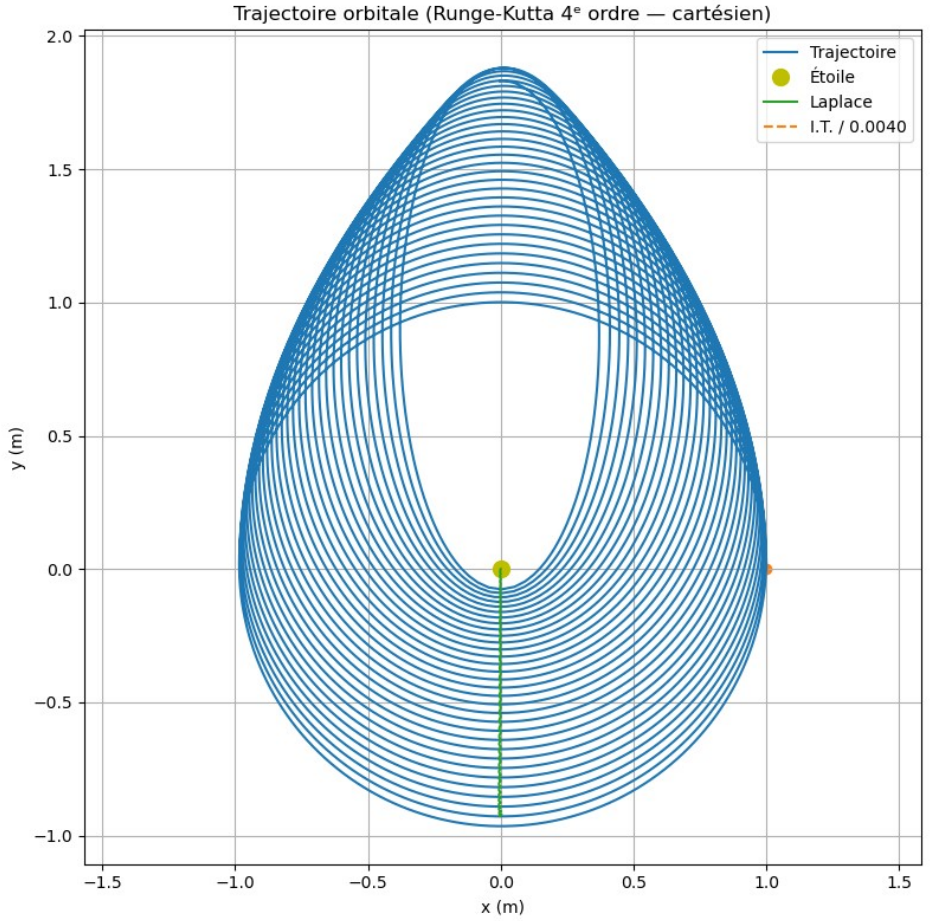
$\mathbf{a} = -(GM_*/r^2)\,\mathbf{u_r} + \varepsilon\,\mathbf{u_x}$

→ linear increase of *e*, pericenter in the $-\mathbf{u_y}$ direction.

<u>Laplace vector:</u>

$|\mathbf{P}| = e$, towards pericenter.

Here, $\mathbf{P} = -\varepsilon \times t \times \mathbf{u_y}$.

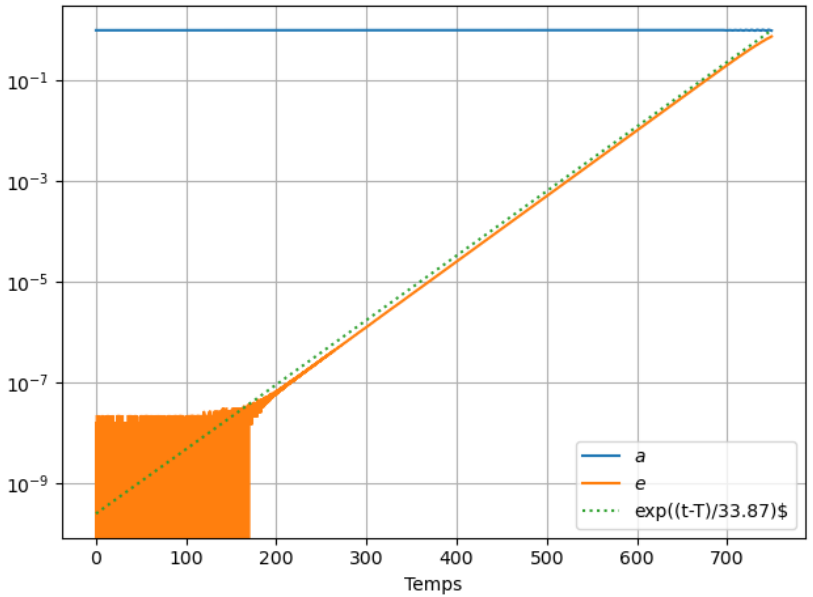


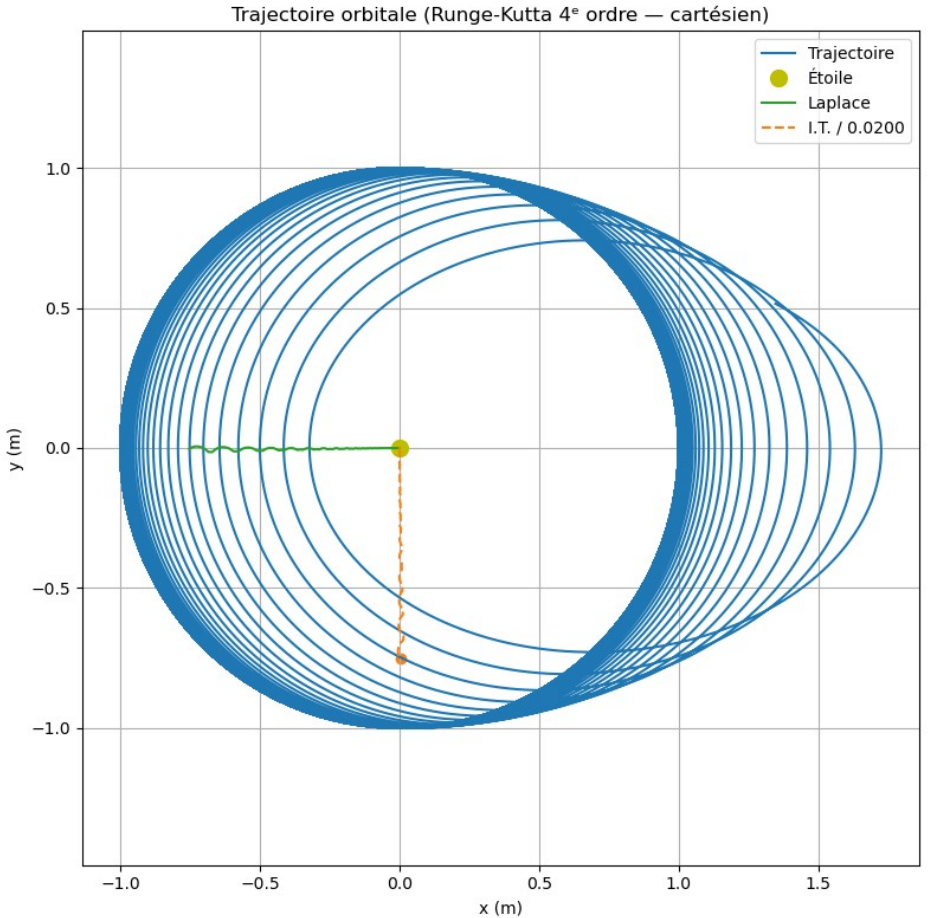
Acceleration and eccentricity

1 test particle in orbit around a star.

$$\mathbf{a} = -(GM_*/r^2)\,\mathbf{u_r} + \varepsilon\,\mathbf{u_z}\times\mathbf{P}$$

→ exponential increase of e .



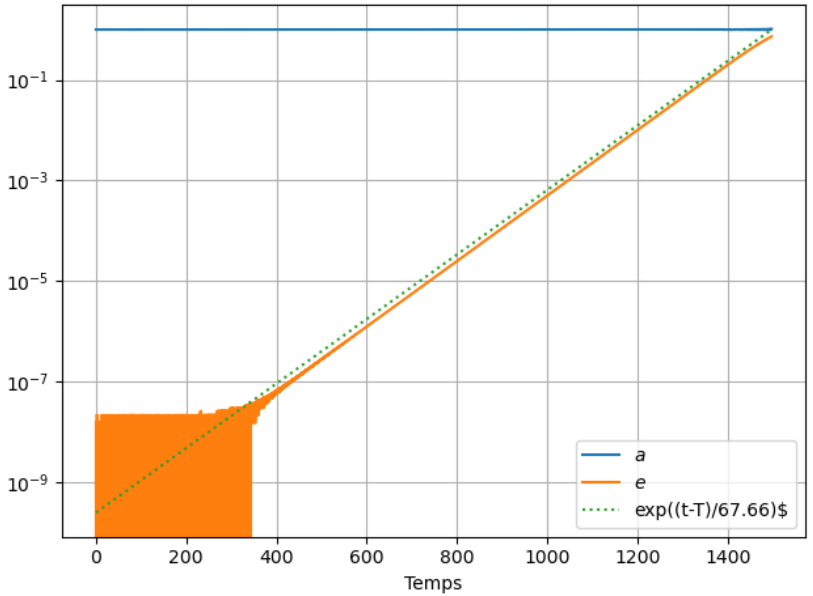


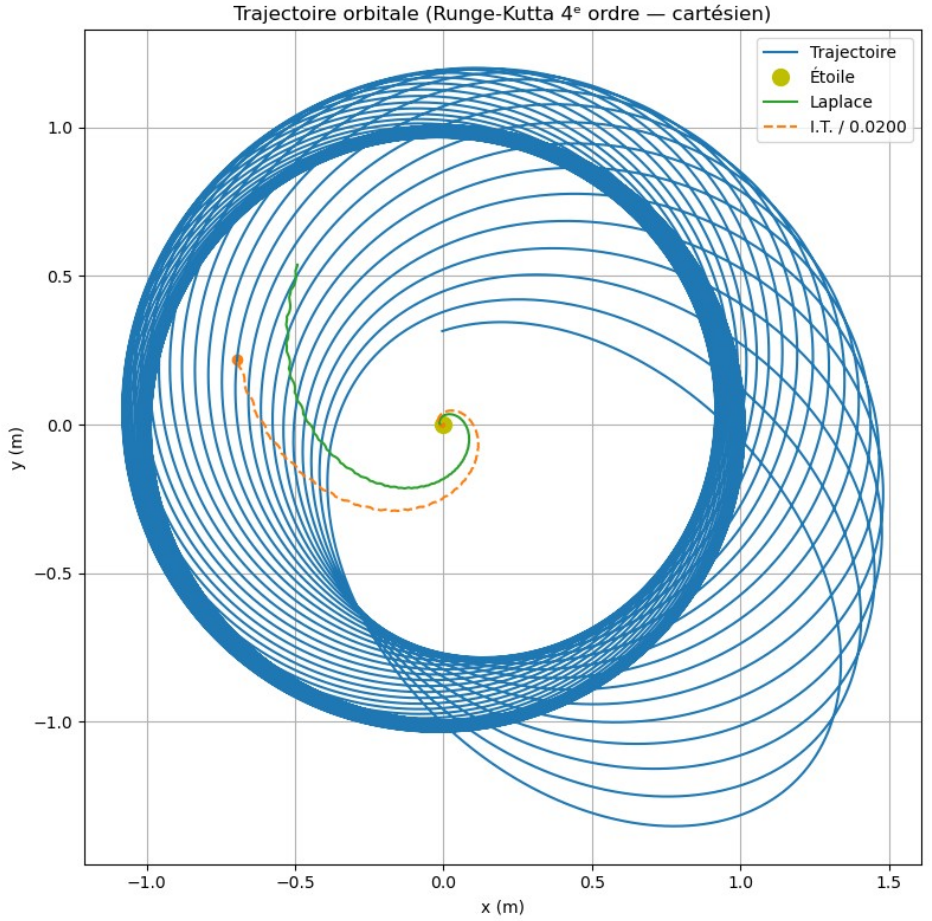
Acceleration and eccentricity

1 test particle in orbit around a star.

$$\mathbf{a} = -(GM_*/r^2)\,\mathbf{u_r} + \varepsilon\,\mathrm{Rot}_{\pi/6}(\mathbf{P})$$

→ exponential increase of *e*
+ linear roation





In the gas disk

Mode m=1 in the gas distribution → Indirect term.

Acceleration (indirect term) → growth of *e*, mode m=1.

If the Indirect term is pointing between 0 and 180° from the pericenter, exponential growth.
Pressure effects → development of a spiral → the acceleration felt by the star is offset with respect to the pericenter in most of the disk.

