Exploiting Retrograde Mean-Motion Resonances for Spacecraft Trajectory Design in the Bicircular Restricted Four-Body Problem

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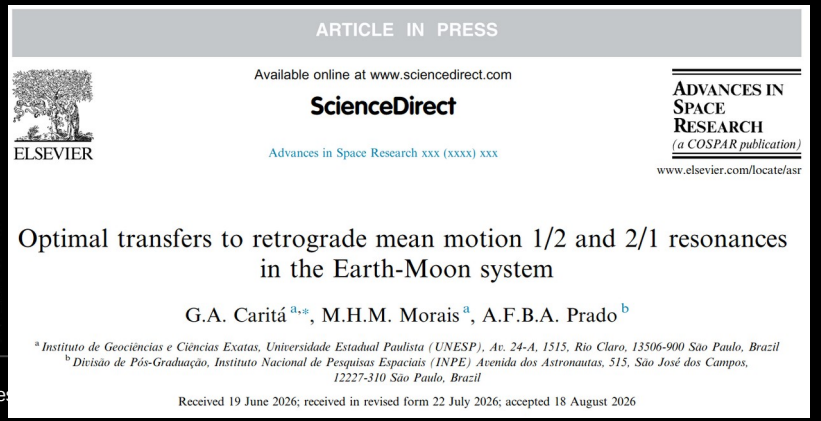
Introduction

- Orbital Transfers are a fundamental aspect of space mission design;
- In complex scenarios, additional considerations are required;
- Optimization Problem / Optimization methods;
- Several missions have utilized orbital resonances for space exploration (JUICE);
- Retrograde mean motion resonances for space exploration (periodicity and close encounters);
- Refueling hubs, space stations, Lunar logistics and Lunar exploration;

Introduction

- Oshima (2021,a,b, 2022a) proposed the use of retrograde resonances;
- Oshima presents a novel approach to link Earth and interplanetary space;
- Previous studies mainly on coorbital retrograde resonances:

  Oshima ((2021,a,b), (2022a), (2025b)); Caritá (2025a ,2026a, 2026b);
- This study focuses on 2/-1 and 1/-2 MMR resonances;



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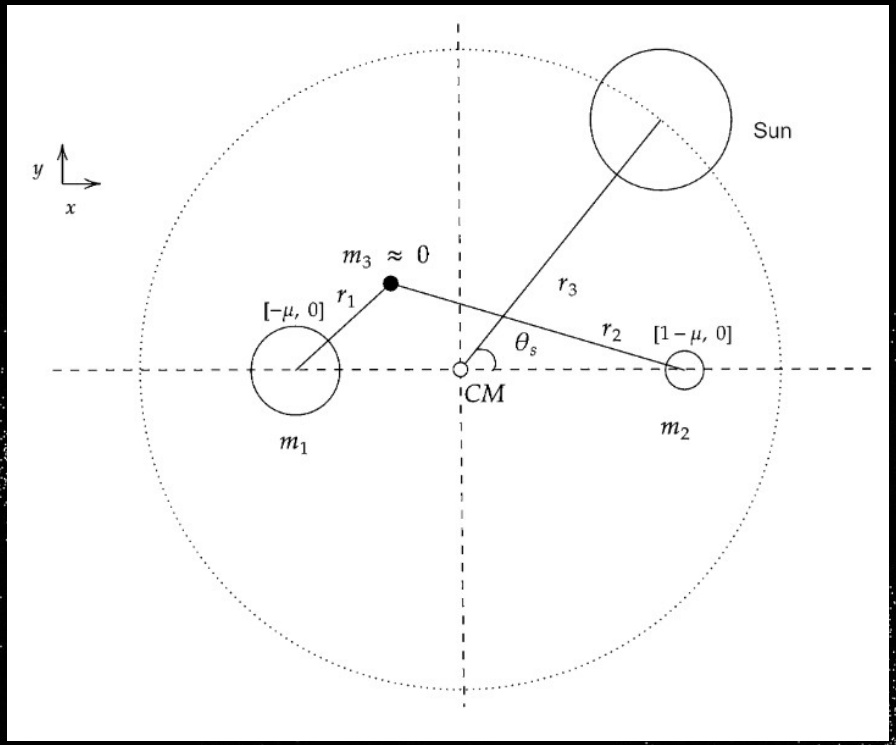
Optimal transfers to retrograde mean motion 1/2 and 2/1 resonances in the Earth-Moon system

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BCR4BP



$$\bar{U}_{4BP} = \bar{U} - \frac{m_s}{r_3} + \frac{m_s}{a_s^2}(x\cos(\theta_s) + y\sin(\theta_s))$$

$$r_3 = \sqrt{(x - a_s cos(\theta_s))^2 + (y - a_s sin(\theta_s))^2}$$

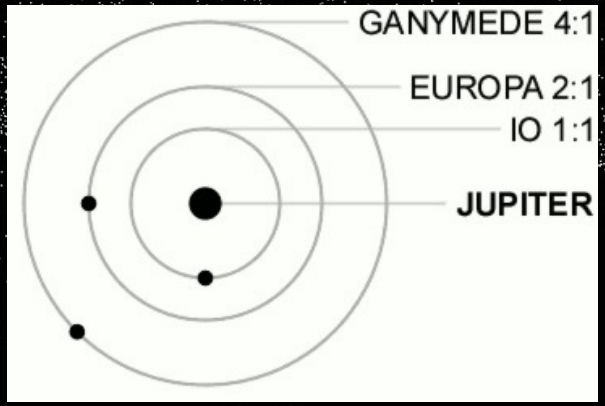
$$\theta_s = \theta_{s_0} + \omega_s t$$

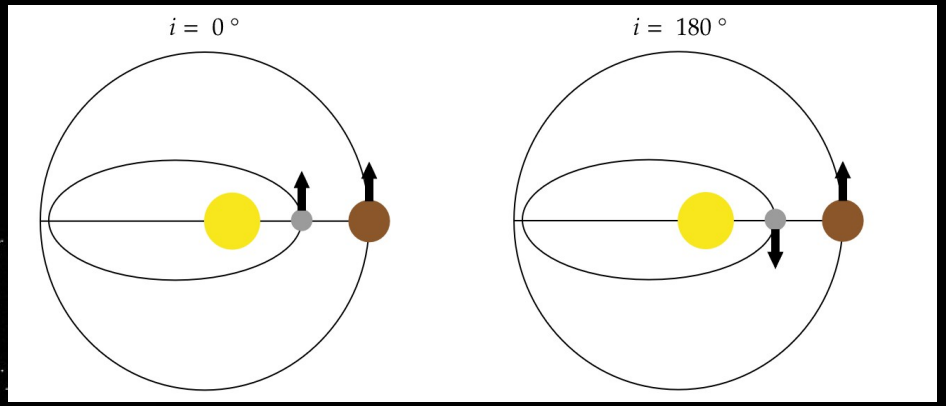
$$\ddot{x} = 2\dot{y} - \frac{\partial U_{4\mathrm{BP}}}{\partial x},$$

$$\ddot{y} = -2\dot{x} - \frac{\partial U_{4\mathrm{BP}}}{\partial y},$$

Retrograde Mean Motion Resonance

- Mean Motion Resonances (MMR);
- Orbital periods forms simple integer ratio
- Retrograde orbit (inc > 90 deg);

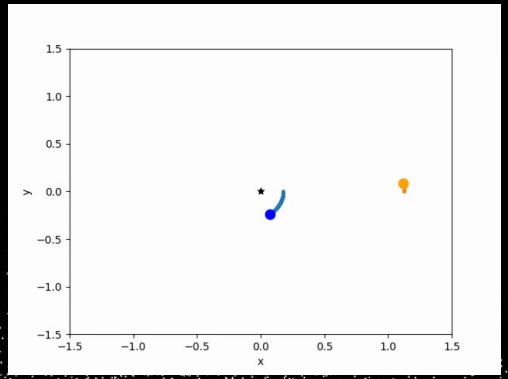


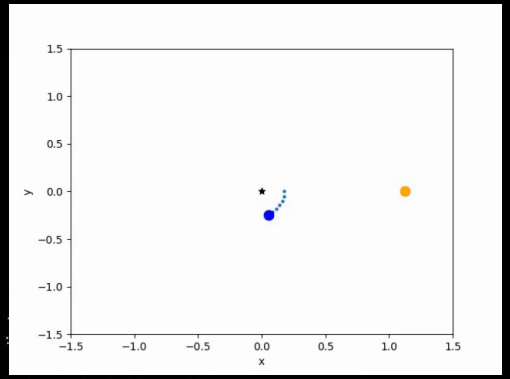


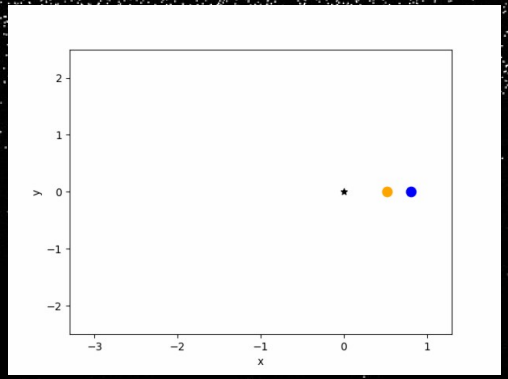
$$\hat{\phi_{p/q}} = q\,\hat{\lambda} + p\,\hat{\lambda}' - (p+q)\,\hat{\varpi}$$

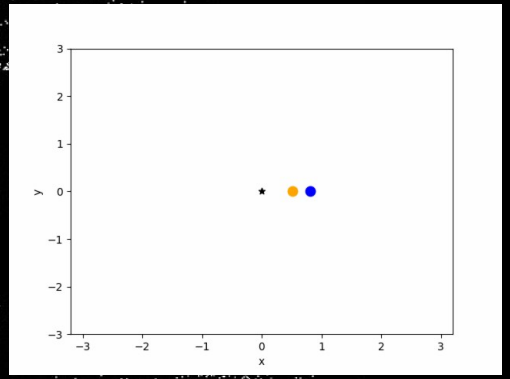
Morais & Namouni, CMDA 2013

2/-1 and 1/-2 MMR









JOURNAL ARTICLE

A numerical study of the 1/2, 2/1, and 1/1 retrograde mean motion resonances in planetary systems FREE

Gabriel Antonio Caritá ✉, Alan Cefali Signor, Maria Helena Moreira Morais ✉

Monthly Notices of the Royal Astronomical Society, Volume 515, Issue 2, September 2022, Pages 2280–2292, https://doi.org/10.1093/mnras/stac1606

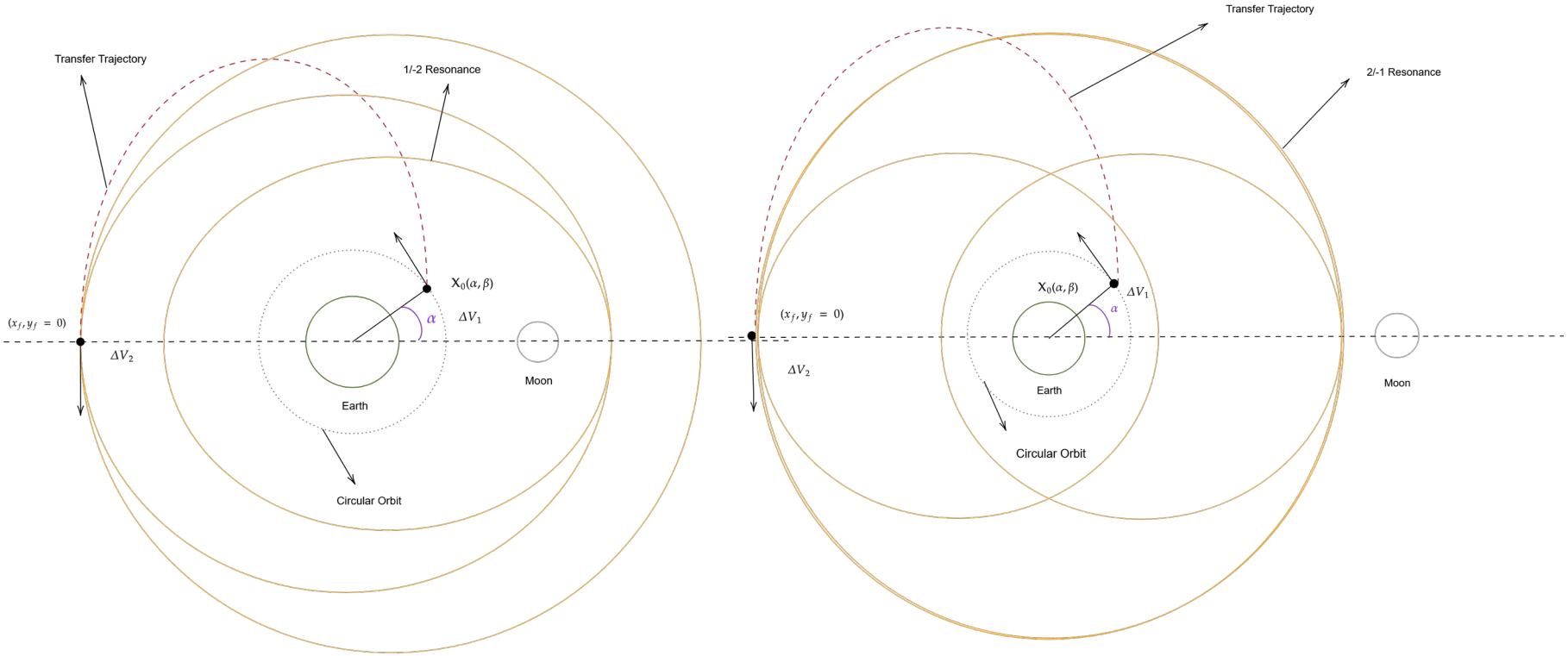
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Bi-impulsive Maneuvers and Transfers

Transfers to 1/-2 and 2/-1

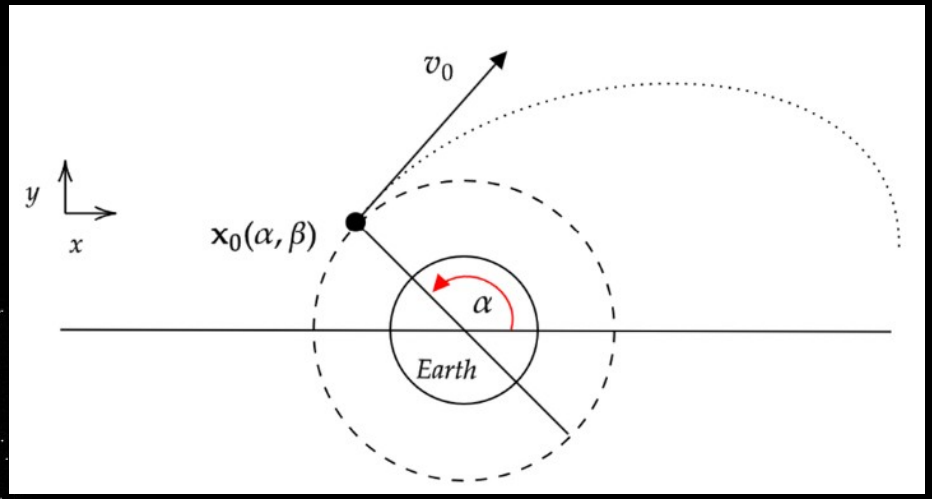


Bi-impulsive Maneuvers and Transfers

- Circular orbit Initial guess;

$$x = r_0 \cos\alpha - \mu$$
$$y = r_0 \sin\alpha$$
$$\dot{x} = -(v_0 - r_0)\sin\alpha$$
$$\dot{y} = (v_0 - r_0)\cos\alpha$$

(Topputo CMDA 2013)



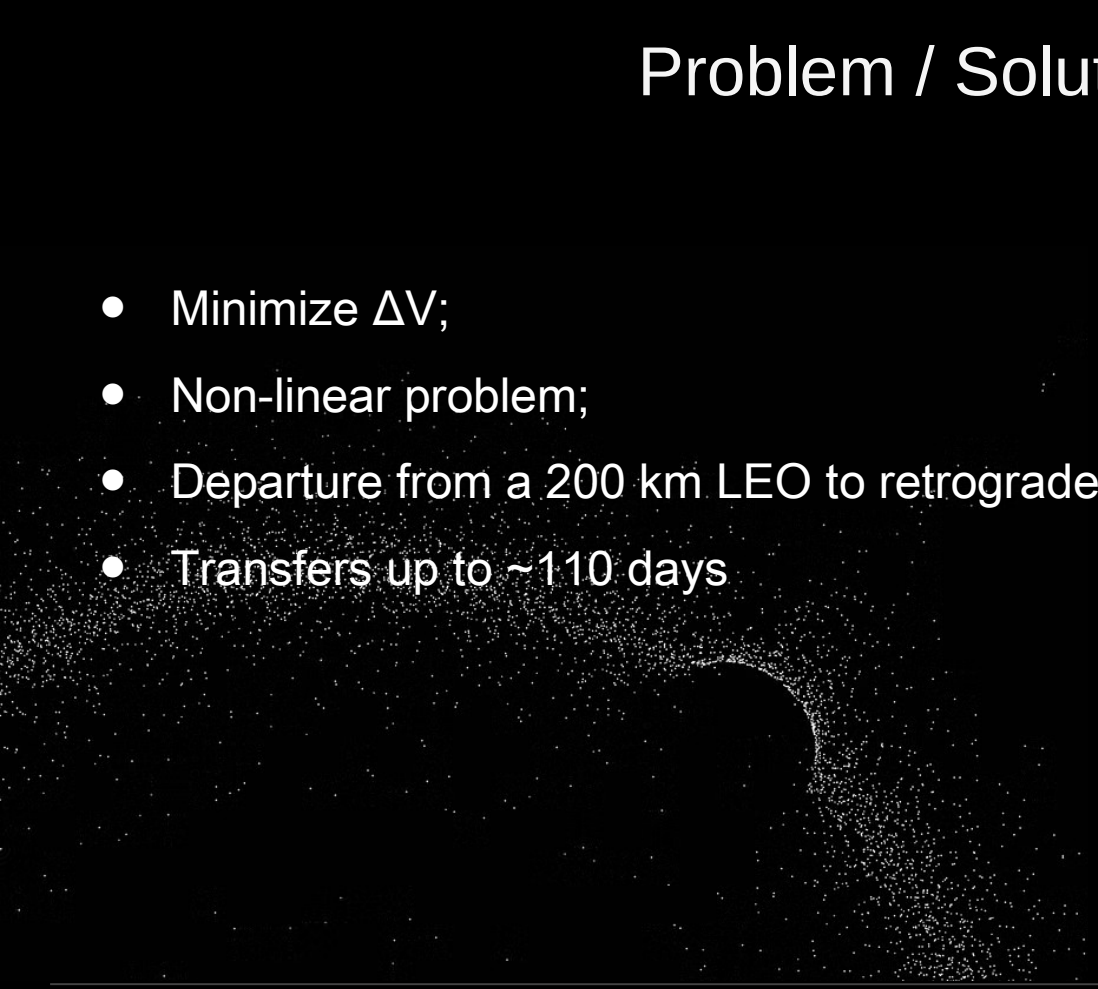
$$v_0 = \beta\sqrt{(1-\mu)/r_0}$$

$$|\beta| \in [1, \sqrt{2}]$$

Circular orbit and escape velocity

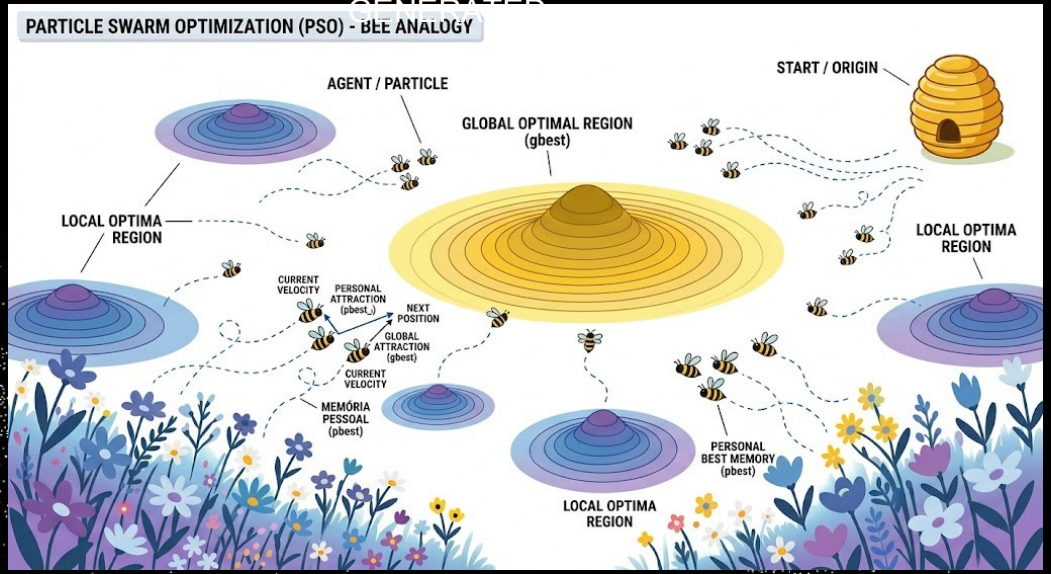
Problem / Solution

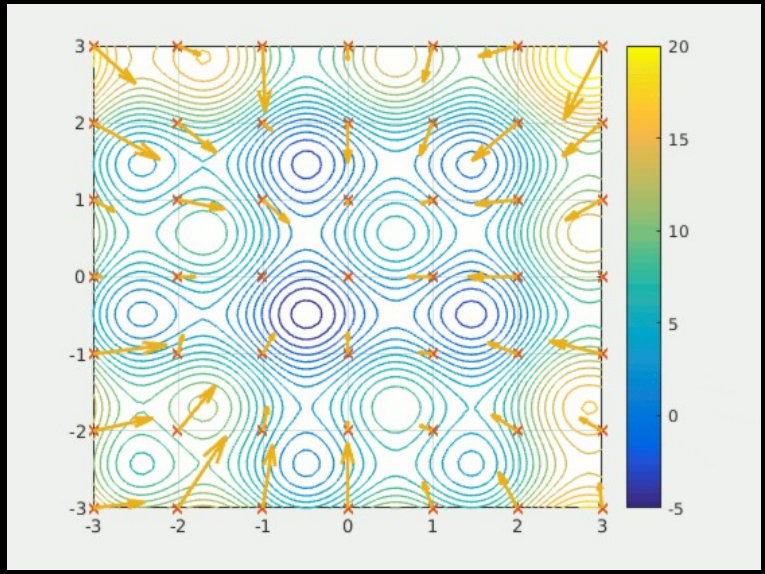
- Minimize ΔV;
- Non-linear problem;
- Departure from a 200 km LEO to retrograde mean motion resonant orbits;
- Transfers up to ~110 days



Metaheuristics methods: Nature-inspired methods

SOURCE: AI GENERATED





This study we use Differential Evolution Variants (Caritá et al, 2026b) related with Evolution

Optimization Strategy

$$J(\mathbf{v}) = \Delta V + \rho(|\mathbf{d}|) \cdot |\mathbf{d}|$$

$$\Delta V = \Delta V_\beta + \Delta V_0 + \Delta V_f.$$

$$\Delta V_\beta = ||\beta| - 1| V_{circ}$$

$$\mathbf{v} = [\delta\dot{x}, \delta\dot{y}, \beta, \alpha]$$

$$\rho(\|\mathbf{d}\|) = \begin{cases} 100 & \text{if} \|\mathbf{d}\| > 10^{-5} \\ 1 & \text{otherwise} \end{cases}$$

- Optimization variables and constraints;
- Convergence parameter (1e-5);
- Collision constraints;

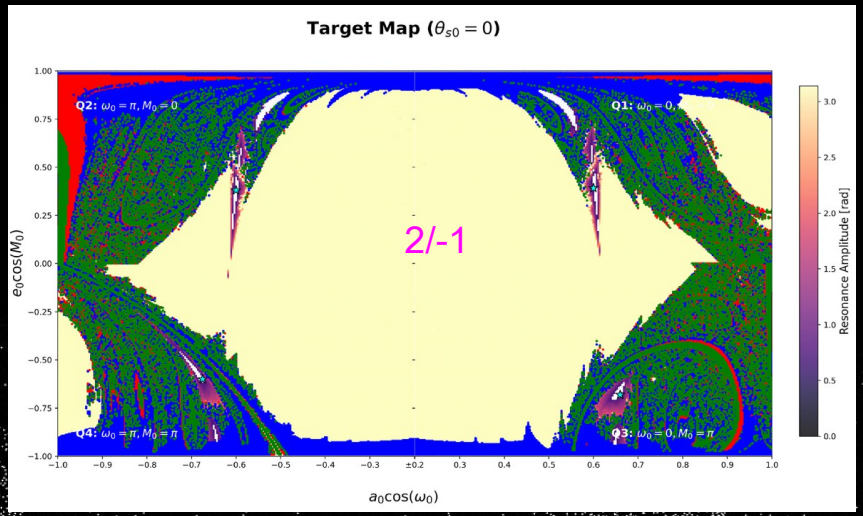
$$-2.5 \leq \delta\dot{x} \leq 2.5$$

$$-2.5 \leq \delta\dot{y} \leq 2.5$$

$$-\sqrt{2} \leq \beta \leq \sqrt{2}$$

$$0 \leq \alpha \leq 2\pi$$

Targets and Optimization



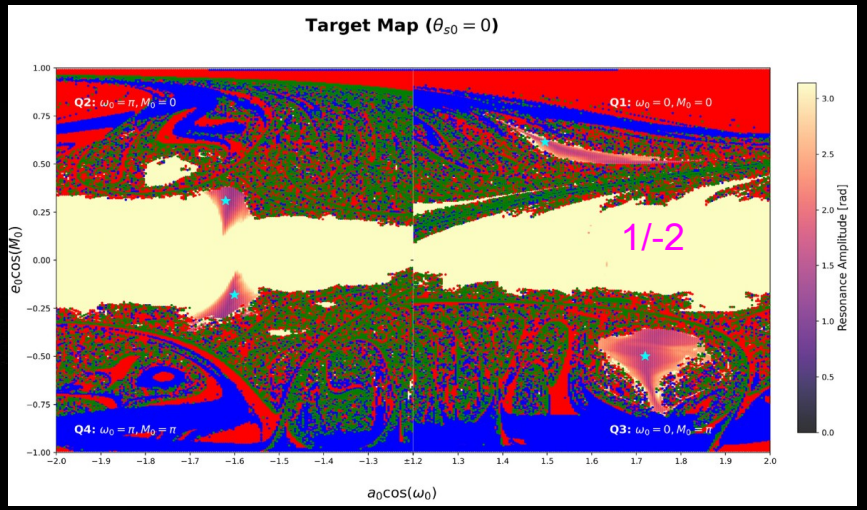


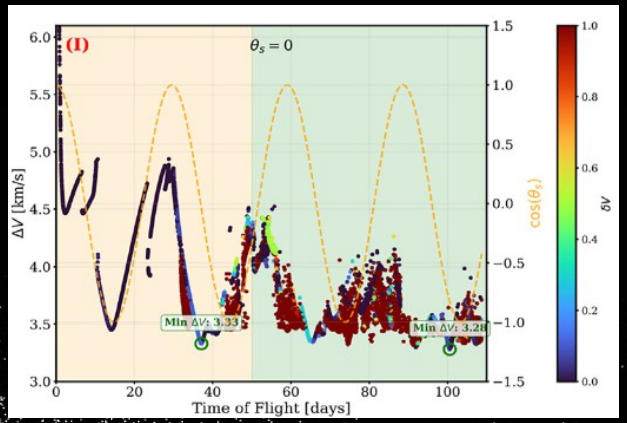
Table 2 Configurations for M_0 e ω_0 for Q_i

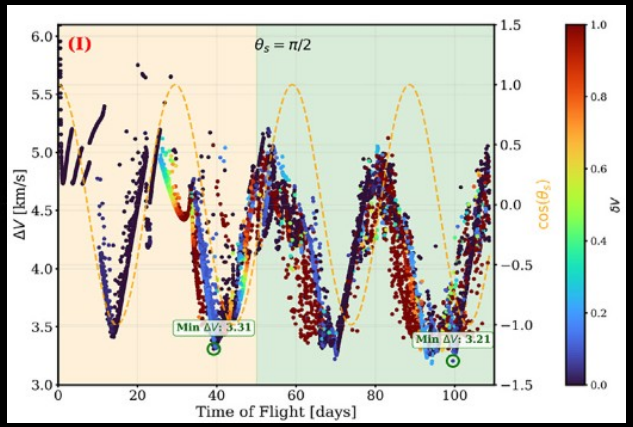
Configurations (Q_i)	M_0	ϖ_0
Q_1	0	0
Q_2	π	0
Q_3	0	π
Q_4	π	π

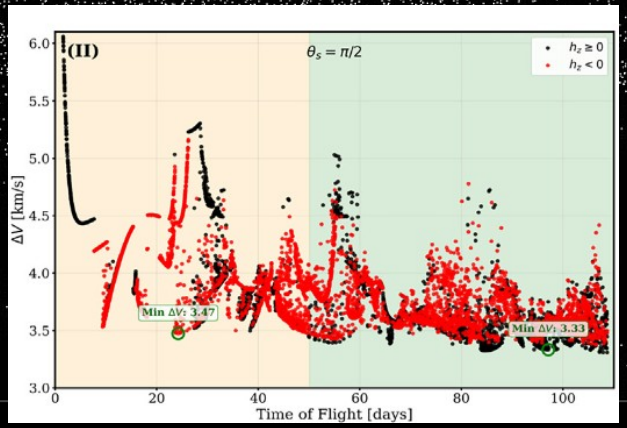
Solar Sensitivity

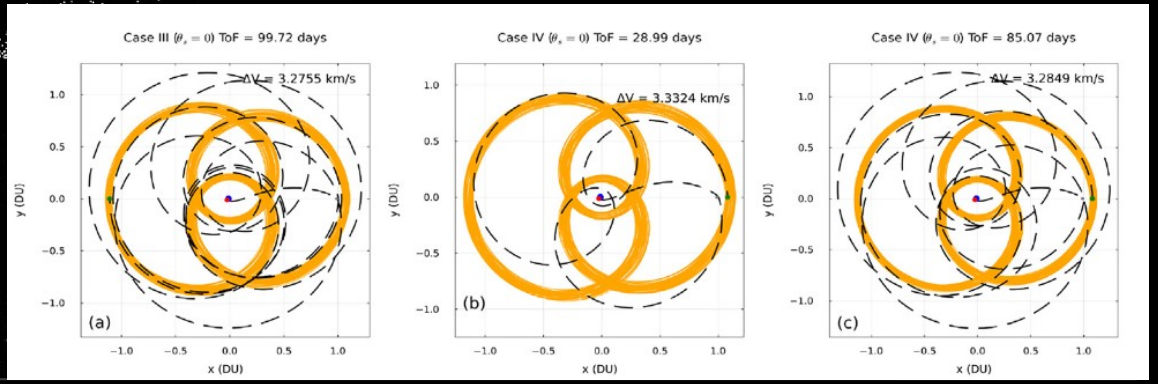
- Optimization Scheme:
- DE-Adaptative (60,000 individuals);
- 105,000,000 function calls;
- Two stages.

Transfers to 2/-1 and 1/-2









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Energy Integral Index

To isolate the influence of each body, we define the Energy Integral Index (I_i) as the cumulative work performed by a specific body i over the flight time t_f[4]:

$$I_i = \int_0^{t_f} (\mathbf{a}_i \cdot \mathbf{v}) dt \qquad (13)$$

- Work Contribution
- Moon Swing-by Contribution

1/-2

% Moon	% Sun	Gain (%)
7.07%	0.24%	37.51
0.09%	2.31%	0.01
9.44%	0.11%	38.30
8.20%	1.10%	34.56
8.51%	0.35%	34.67
8.75%	0.78%	35.54
8.53%	0.13%	36.28
10.46%	0.76%	37.73
4.97%	0.32%	42.08
0.22%	2.40%	6.00
9.01%	0.10%	38.52
3.35%	1.76%	23.93
8.55%	0.37%	38.15
1.98%	0.62%	37.75
6.85%	0.18%	37.29
3.13%	1.61%	35.09

2/-1

% Moon	% Sun	Gain (%)
8.05%	0.04%	46.19
6.63%	0.24%	43.87
5.53%	0.08%	46.22
8.55%	0.13%	47.35
7.26%	0.11%	42.29
6.51%	0.21%	48.75
2.02%	0.04%	70.62
6.05%	0.18%	49.76
7.68%	0.04%	46.71
5.15%	0.16%	48.66
8.13%	0.07%	46.43
7.74%	0.13%	49.00
6.60%	0.10%	42.86
5.46%	0.15%	39.42
7.19%	0.07%	39.80
5.64%	0.11%	43.93

Conclusions and Further steps

- 1/-2 and 2/-1 could be relevant for logistical gateways, refueling hubs or resupply stations (periodicity/close encounters);
- 2/-1 is promising due transfer stability and efficient transfers (ranging from 3.25 km/s to 3.37 km/s) ;
- Lunar close flybys are the main mechanism for reducing the propellant;
- Further steps : Higher resonant orders, 3D dynamics, low-thrust propulsion and escape trajectories.
- Optimization Methods - Social Elastic Centroid Collision (SECC) (under review in springer journal Computing)

Few References

Anderson, R.L.: Low thrust trajectory design for resonant flybys and captures using invariant manifolds. PhD thesis, University of Colorado at Boulder (2005)

Anderson, R.L., Lo, M.W.: A dynamical systems analysis of resonant flybys: ballistic case. J. Astronaut. Sci. **58**(2), 167–194 (2011)

Caraffini, F., Neri, F., Passow, B.N., Iacca, G.: Re-sampled inheritance search: high performance despite the simplicity. Soft Comput. **17**(12), 2235–2256 (2013)

Caritá, G.A., Cefali Signor, A., Morais, M.H.M.: A numerical study of the 1/2, 2/1, and 1/1 retrograde mean motion resonances in planetary systems. Mon. Not. R. Astron. Soc. **515**(2), 2280–2292 (2022)

Conn, A.R., Scheinberg, K., Vicente, L.N.: Introduction to Derivative-Free Optimization, vol. 8. SIAM, Philadelphia (2009)

Han, D., Young, B.: Aerobraking the exomars tgo: the jpl navigation experience (2019). https://doi.org/2014/50471

Lancaster, E.R., C., B.R.: A Unified Form of Lambert's Theorem. National Aeronautics and Space Administration, Washington (1969)

McAdams, J.V., Farquhar, R.W., Taylor, A.H., Williams, B.G.: Messenger mission design and navigation. Space Sci. Rev. **131**(1), 219–246 (2007)

Mejía-de-Dios, J.-A., Mezura-Montes, E.: A new evolutionary optimization method based on center of mass. In: Decision Science in Action: Theory and Applications of Modern Decision Analytic Optimisation pp. 65–74 (2019)

Morais, M., Giuppone, C.: Stability of prograde and retrograde planets in circular binary systems. Mon. Not. R. Astron. Soc. **424**(1), 52–64 (2012)

Morais, M., Namouni, F.: Retrograde resonance in the planar three-body problem. Celest. Mech. Dyn. Astron. **117**(4), 405–421 (2013)

Morais, M., Namouni, F.: On retrograde orbits, resonances and stability. Comput. Appl. Math. **35**(3), 881–891 (2016)

Murray, C.D., Dermott, S.F.: Solar System Dynamics. Cambridge University Press, Cambridge (2000)

Oshima, K.: Capture and escape analyses on planar retrograde periodic orbit around the Earth. Adv. Space Res. **68**(9), 3891–3902 (2021a)

Oshima, K.: Retrograde co-orbital orbits in the Earth–Moon system: planar stability region under solar gravitational perturbation. Astrophys. Space Sci. **366**(9), 88 (2021b)

Oshima, K.: Continuation and stationkeeping analyses on planar retrograde periodic orbits around the Earth. Adv. Space Res. **69**(5), 2210–2222 (2022a)

Oshima, K.: 3d stable and weakly unstable periodic orbits around the Earth near the retrograde co-orbital resonance with the Moon. Astrophys. Space Sci. **367**(4), 42 (2022b)

Prado, A.: Bi-impulsive control to build a satellite constellation. Nonlinear Dyn. Syst. Theory **5**(2), 169–175 (2005)

Rackauckas, C., Nie, Q.: DifferentialEquations.jl–a performant and feature-rich ecosystem for solving differential equations in Julia. J. Open Res. Softw. **5**(1), 15 (2017)

Robič, T., Filipič, B.: Differential evolution for multiobjective optimization. In: International Conference on Evolutionary Multi-Criterion Optimization, pp. 520–533. Springer, Berlin (2005)

Rocco, E., Prado, A., Souza, M.: Bi-impulsive orbital transfers between coplanar orbits with time limit. In: AIAA/AAS Astrodynamics Specialist Conference and Exhibit, p. 4549 (1998)

Sengupta, S., Basak, S., Peters, R.A.: Particle swarm optimization: a survey of historical and recent developments with hybridization perspectives. Mach. Learn. Knowl. Extr. **1**(1), 157–191 (2018)

Simó, C., Gómez, G., Jorba, À., Masdemont, J.: The bicircular model near the triangular libration points of the rtbp. In: From Newton to Chaos: Modern Techniques for Understanding and Coping with Chaos in N-Body Dynamical Systems, pp. 343–370 (1995)

Szebehely, V.: Theory of Orbit: The Restricted Problem of Three Bodies. Academic Press, New York (1967)

Topputo, F.: On optimal two-impulse Earth–Moon transfers in a four-body model. Celest. Mech. Dyn. Astron. **117**, 279–313 (2013)

Thank you!



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