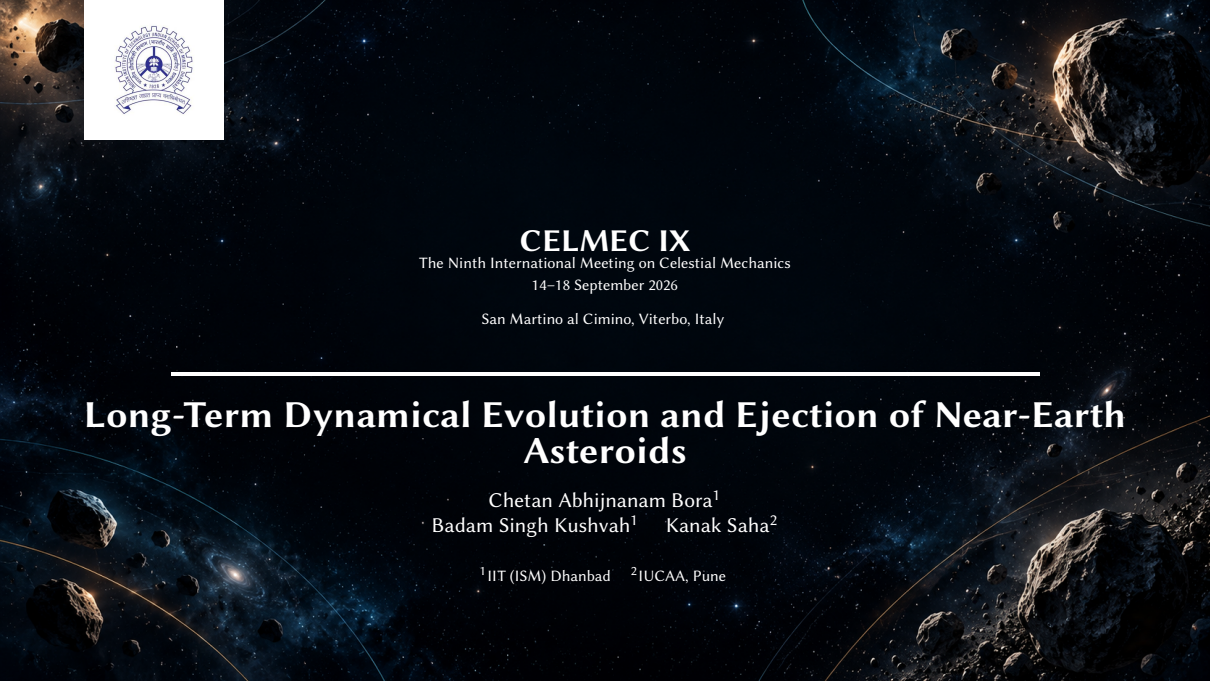


CELMEC IX

The Ninth International Meeting on Celestial Mechanics

14–18 September 2026

San Martino al Cimino, Viterbo, Italy

Long-Term Dynamical Evolution and Ejection of Near-Earth Asteroids

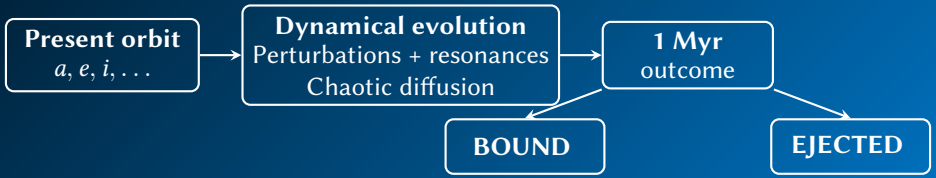
Chetan Abhijnanam Bora[1]

Badam Singh Kushvah[1] Kanak Saha[2]

[1] IIT (ISM) Dhanbad [2] IUCAA, Pune

A 1 Myr Question Hidden Inside a Chaotic Dynamical System

Which NEAs remain bound — and which are ejected?



Computational challenge

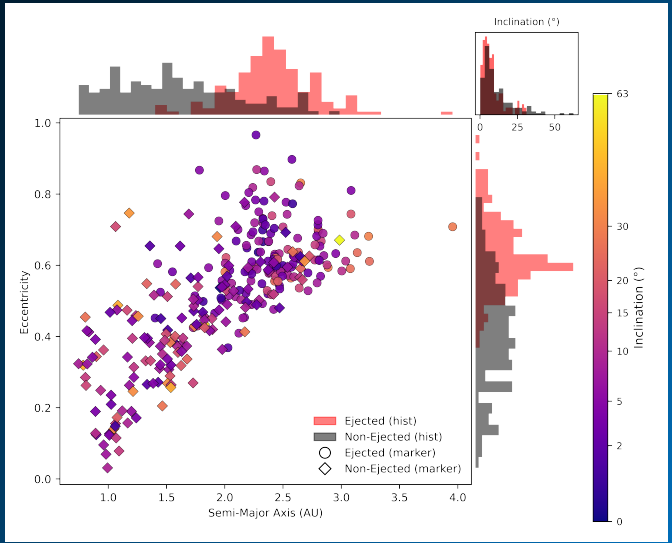
Long-term high-accuracy N-body integrations are **computationally expensive** for large populations.

Dynamical challenge

$$\tau_L \sim 10^2 \text{ yr} \ll 1 \text{ Myr}$$

We classify **dynamical outcomes**, not exact trajectories.

Ejection Is Structured — But Not Simply Separable



Where do ejections concentrate?

$$a \sim 1.5 - 3.5 \text{ AU}$$

$$e \sim 0.4 - 0.8$$

a, e: **strongest separation**

i: **weak separation**

Key observation

The classes still overlap strongly.

What Information Is Sufficient to Anticipate Ejection?

STATIC

Initial orbital elements

$(a, e, i, \omega, \Omega, M, q, \text{class})$

↓

Tree-based ML

↓

1 Myr ejection outcome

DYNAMIC

Short-term orbital evolution

$a(t),\ e(t) \qquad [0.2\ \text{Myr}]$

↓

Recurrence Plot

↓

CNN

↓

1 Myr ejection outcome

Building the Dynamical Ground Truth

JPL data 35,455 NEAs → **MERCURY** Bulirsch–Stoer → **1 Myr** integration → **Ejection** criterion → **EJECTED** / NON-EJECTED

Dynamical model



Sun + 8 planets + Pluto

Massless test particles

Numerical setup

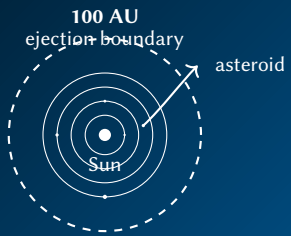
$\Delta t = 8$ d

$\Delta t_{\text{output}} = 10$ yr

Tolerance $= 10^{-12}$

$\left|\frac{\Delta E}{E}\right| \lesssim 10^{-10}$

Defining the Target



Sun + planets + Pluto; schematic, not to scale.

EJECTED

heliocentric distance threshold of **100 AU** or attains **hyperbolic escape velocity**

at any point during the **1 Myr** integration

⇓

NON-EJECTED

neither criterion satisfied

5,232 / 35,455 nominal integrations were ejected $\Rightarrow \quad \approx 14.8\%$

A Chaotic System Also Means Uncertain Labels

Nominal orbit

JPL covariance

⇓

50 Monte-Carlo clones

⇓

Independent 1 Myr integrations

⇓

EJECTED / **NON-EJECTED**

Outcome robustness

$> 75\%$

of clones share the same **macroscopic dynamical outcome** on average

Interpretation

Nearby initial conditions can diverge, **but the broad dynamical outcome is often preserved.**

We assess robustness of the classification, not exact long-term orbital histories.

Constructing the Recurrence Matrix

1. Prepare the dynamical signal

$$\boxed{a(t),\ e(t)}$$

0.2 Myr evolution

$\Downarrow$

Independent normalization to $[0, 1]$

$\Downarrow$

$$\boxed{x = [a_1, \ldots, a_{2000}, e_1, \ldots, e_{2000}]}$$

Downsampled + concatenated $L = 4000$

2. Define recurrence

For every pair of states x_i, x_j

$$\boxed{R_{ij} = \Theta(\epsilon - \|x_i - x_j\|)}$$

Euclidean distance in normalized state space

ϵ: 20% recurrence rate

$\Downarrow$

Binary RP

resized to 150×150

What Does the Recurrence Plot Tell Us?

RP structure

self	cross
$a - a$	$a - e$
$e - a$	$e - e$

Diagonal blocks: self-recurrence

Off-diagonal blocks: cross-recurrence

Key idea

RP morphology provides a dynamical signature of the orbital evolution.

Dynamical morphology

Ordered

Grid-like / diagonal structures
$\rightarrow$ regular or quasi-periodic motion

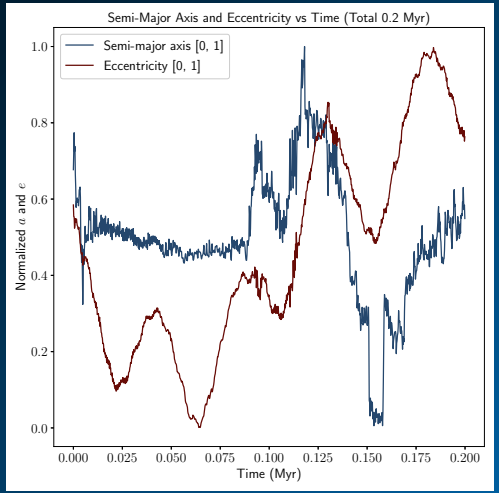
Fragmented

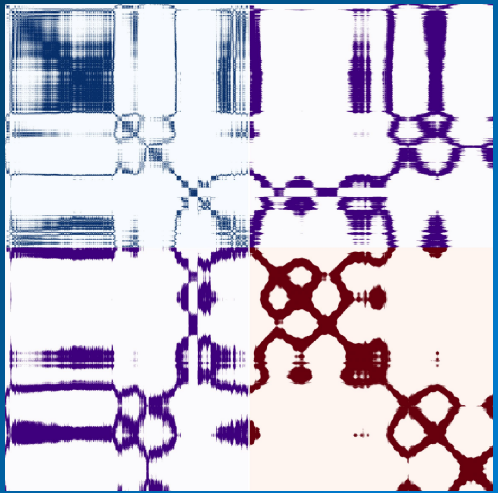
Scattered / irregular structures
$\rightarrow$ chaotic evolution

Trapping / transitions

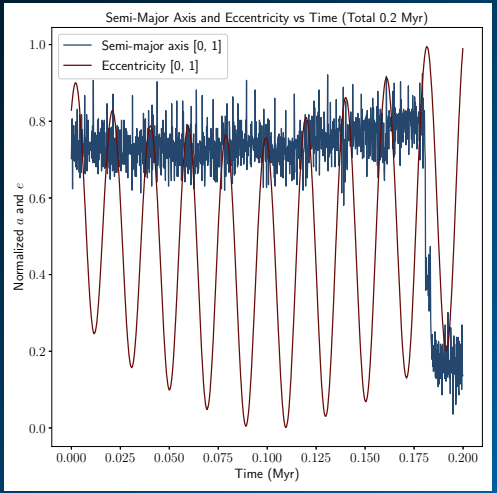
Vertical–horizontal lines / clustered motifs
$\rightarrow$ intermittent trapping or dynamical transitions

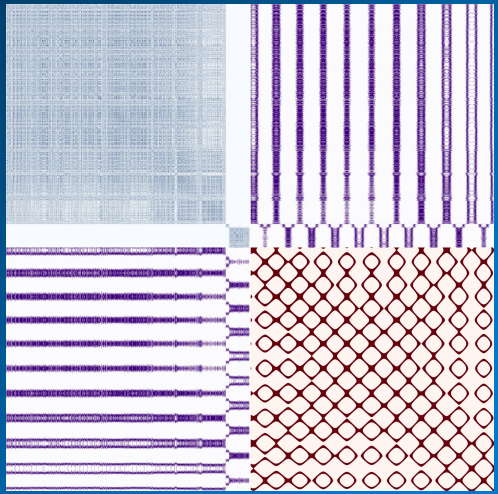
2007 VP243 — Dynamically Unstable





2005 WE55 — Long-Lived





Learning from Initial Orbital Geometry

Initial orbital elements

$$(a, e, i, \omega, \Omega, M, q, \text{class})$$

$\longrightarrow$

KNN | Decision Tree | Random Forest | ExtraTrees

MLP | AdaBoost | Gradient Boosting

$\longrightarrow$

1 Myr ejection outcome

Ejected / Non-ejected

Learning from Short-Term Dynamical Evolution

Short-term orbital evolution

$a(t),\ e(t) \quad [0.2\ \mathrm{Myr}]$

↓

Recurrence Plot

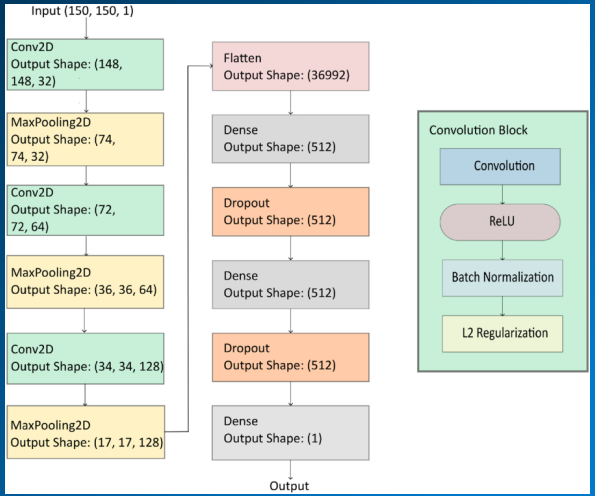
↓

CNN

↓

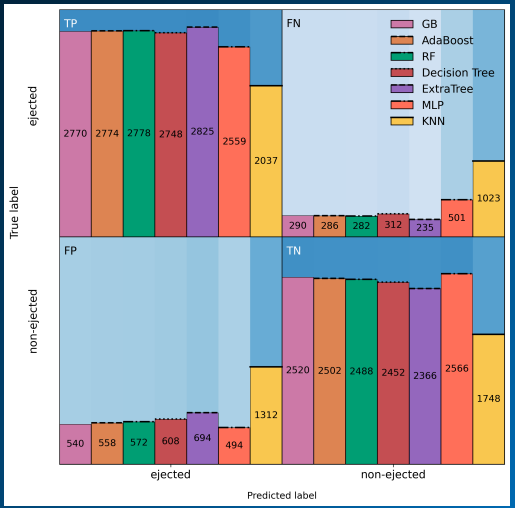
1 Myr ejection outcome

Ejected / Non-ejected



CNN architecture for recurrence-plot classification

Static Orbital Geometry Already Predicts Ejection Well



Test accuracy

KNN	61.9%
MLP	83.7%
ExtraTrees	84.8%
Decision Tree	85.0%
RF	**86.1%**
AdaBoost	**86.2%**
GB	**86.4%**

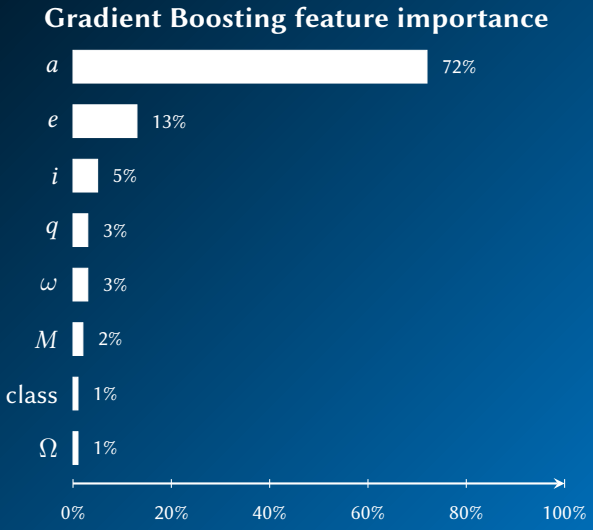
Best model: GB

Accuracy $= 86.4\%$

Recall $= 90.5\%$

$F_1 = 87.0\%$

The Model Points Back to Orbital Geometry



Dominant signal

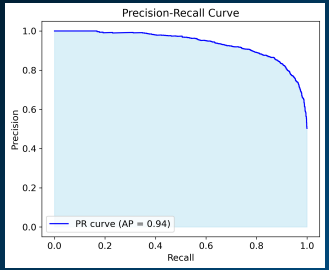
$$a + e \approx 85\%$$

The prediction is driven mainly by **orbital size and shape**.

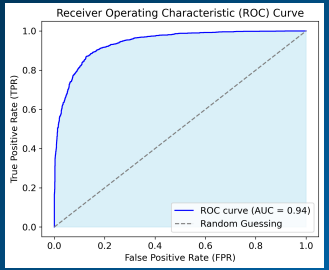
Interpretation

Initial orbital geometry contains most of the discriminative signal.

Short-Term Dynamics Carry Comparable Predictive Information



Precision–Recall



ROC

CNN test performance

0.869

Test accuracy

0.866

F_1 score

0.94

Average precision

0.94

ROC–AUC

A 0.2 Myr dynamical snapshot contains enough recurrence structure to approach the performance of static orbital-geometry models.

Static Geometry vs. Dynamical Morphology

STATIC

Initial orbital geometry

Orbital elements $\longrightarrow$ Tree ensembles

Accuracy ≈ 0.86

$F_1 \approx 0.87$

Interpretable | Computationally efficient

DYNAMIC

Short-term dynamical morphology

$a(t), e(t)$ $\longrightarrow$ Recurrence plot + CNN

Accuracy ≈ 0.87

$F_1 \approx 0.87$

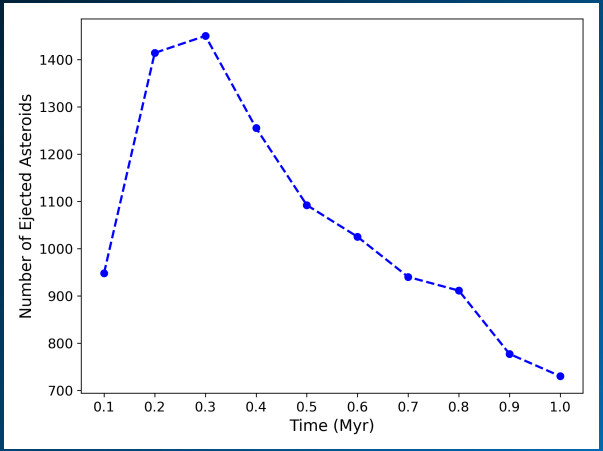
Richer temporal information | Higher cost

Key result

Comparable predictive performance.

The CNN adds temporal information, but only marginally improves upon what is already encoded in orbital geometry.

Ejection Is Strongly Front-Loaded



$\sim 15\%$

ejected within 1 Myr

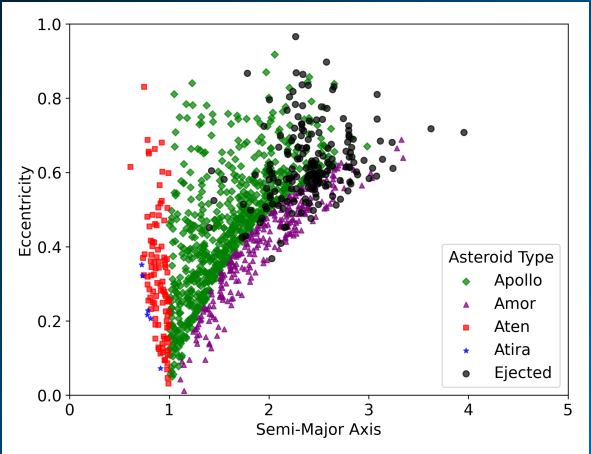
$t \sim 0.2\text{–}0.4$ Myr

peak ejection activity

Rapid early depletion

$\rightarrow$ progressively more stable survivors

Where Are the Vulnerable NEAs?



$a \sim 1.5\text{–}3.5$ AU

$e \sim 0.4\text{–}0.8$

Apollo + Amor dominate ejections

Atens + Atiras: comparatively few ejections

What Controls Long-Term Ejection?

1. Ejection is structured

Vulnerable objects concentrate in specific orbital regions, with **Apollo and Amor** populations dominating ejections.

2. Initial geometry carries most of the signal

Tree ensembles reach $\approx 86\%$ accuracy; orbital size and shape (a, e) provide the dominant information.

3. Early dynamics add predictive information

A **0.2 Myr recurrence-plot snapshot** reaches $\approx 87\%$ accuracy, comparable to static geometry.

4. ML/DL enables scalable screening

Use ML/DL to **prioritize objects** for expensive integrations.

Long-term ejection

is difficult to predict trajectory-by-trajectory,

but its macroscopic structure is learnable.

ML/DL complement high-accuracy dynamics rather than replace it.

Thank You

For Listening.