CELMEC VIII

Roma, 5-9 September 2022

ABSTRACTS OF THE TALKS

Alain Albouy

Observatoire de Paris/CNRS, IMCCE, France

The Lambert problem rigorously: Number of solutions, simple and robust algorithm (with Antonio J. Ureña)

The Lambert problem is: given two positions A and B and a positive number T, what are the Keplerian arcs going from A to B in a time T? Many solvers were proposed, which "experimentally prove" some mathematical theorems. We rigorously give in all the cases (including multi-revolution) the number of the solutions. We give convexity results which allow to define a Newton method which always converges. Doing so, we slightly improve previous results by Carles Simó in 1973 and by us in 2020. We advertise the inverse of the angular momentum as a surprisingly good parameter for the arcs from A to B. References: Albouy, Alain, Antonio J. Urena, Some simple results about the Lambert problem, The European Physical Journal Special Topics 229.8 (2020), pp. 1405-1417 Simó, Carles, Solución del problema de Lambert mediante regularización, Collectanea Mathematica, 24 (1973), pp. 231-247

Vivina Barutello

Università di Torino, Dipartimento di Matematica, Italy

Symbolic dynamics in some models from Celestial Mechanics

In this talk we descrive some common techniques in the construction of a symbolic dynamic in the anisotropic N-center problem and in a keplerian refractive billiard. Symbols coincide in both cases with a suitable notion of central configurations and glueing strategies play a main role. Hight energies are required.

Konstantin Batygin

California Institute of Technology, USA

The Stability Boundary of the Distant Kuiper Belt

The distant scattered disk is a vast population of trans-Neptunian minor bodies that orbit the Sun on highly elongated, long-period orbits. The orbital stability of scattered-disk objects (SDOs) is primarily controlled by a single parameter - their perihelion distance. While the existence of a perihelion boundary that separates chaotic and regular motion of long-period orbits is well established through numerical experiments, its theoretical basis as well as its semimajor axis dependence remain poorly understood. In this talk, I will outline an analytical model for the dynamics of distant trans-Neptunian objects and show that the orbital architecture of the scattered disk is shaped by an infinite chain of exterior 2:j resonances with Neptune. The widths of these resonances increase as the perihelion distance approaches Neptune's semimajor axis, and their overlap drives chaotic motion. Within the context of this theoretical picture, we derive an analytic criterion for instability of long-period orbits, and demonstrate that rapid dynamical chaos ensues when the perihelion drops below a critical value. Additionally, I will discuss how within the stochastic layer, the Lyapunov time of SDOs approaches the orbital period, and present a corresponding semimajor axis diffusion coefficient.

Giulio Baù

University of Pisa, Italy

A generalization of the equinoctial orbital elements

A new set of non-singular elements is presented, which generalizes the classical equinoctial orbital elements when some or all the perturbing forces are derivable from a potential. The generalized equinoctial orbital elements (GEqOE) define at any time of the motion an ellipse that lies on the orbital plane and has a different shape and orientation with respect to the osculating conic defined by the Keplerian elements. After explaining how the GEqOE were derived, I will show some results on their application to uncertainty propagation of Earth satellites. Finally, I will address the derivation of the Lagrange planetary equations for the GEqOE, which is performed in a non-standard way since the new elements are not osculating. This presentation is based on the two papers: 1) "A generalization of the equinoctial orbital elements", Baù, Hernando-Ayuso, Bombardelli, CMDA, 2021, 2) "Near-Linear Orbit Uncertainty Propagation in the Perturbed Two-Body Problem", Hernando-Ayuso, Bombardelli, Baù, Martínez-Cacho, submitted on March 2022, and on ongoing work with C. Bombardelli.

Luis Benet

Universidad Nacional Autónoma de México (UNAM), Instituto de Ciencias Físicas, Mexico

Yarkovsky acceleration for Apophis exploiting automatic differentiation tools

We present an independent estimation of the Yarkovsky effect based on optical and radar astrometry for (99942) Apophis, which includes observations obtained during 2021 Apophis' fly-by, and a numerical approach which exploits automatic differentiation techniques. We find a non-zero Yarkovsky parameter, $A2 = (\[Minus]2.899 \[PlusMinus] 0.025)\times 10^{\[Minus]14}$~au~d${}^{\[Minus]2}$, with induced semi-major axis drift of $(\[Minus]199.0 \[PlusMinus] 1.5)$~m~yr${}^{\[Minus]1}$ for Apophis. Our results allow to estimate the collision probabilities for the close approaches in 2029, 2036 and 2068. This is joint work with Jorge A. Pérez-Hernández.

Jaime Burgos-García

Autonomous University of Coahuila. Faculty of Physics and Mathematics, Mexico

Some fresh periodic orbits for a restricted four-body problem

The circular restricted four-body problem (CRFBP) studies the dynamics of an infinitesimal mass under the gravitational force produced by three massive bodies, called primaries, that move in the Lagrangian solution of the three-body problem. In this talk, some new periodic orbits will be shown for the three-dimensional problem where a primary body is very small compared with the other two. These orbits were computed with techniques of rigorous numerics and regularization in three dimensions. We will see that they possess some properties of the so-called polar orbits for the classical restricted three-body problem. This is joint work with A. Bengochea y L. Franco.

Renato Calleja

IIMAS-UNAM, Mexico

Some choreographies of the n-body problem: connections between polygons and the figure eight

N-body choreographies are periodic solutions to the N-body equations in which equal masses chase each other around a fixed closed curve. In this talk I will present a systematic approach for continuing and proving the existence of choreographies in the gravitational body problem with the help of the digital computer. These arise from the polygonal system of bodies in a rotating frame of reference. In rotating coordinates, after exploiting the symmetries, the equation of a choreographic configuration is reduced to a delay differential equation (DDE) describing the position and velocity of a single body. For odd numbers of bodies between n = 3 and n = 15 we find numerically that the figure eight choreography can be reached starting from the regular n-gon. Based on these calculations we extend conjectures claiming the n-gon and the eight are in the same continuation class for all odd numbers of bodies. This is joint work with Carlos Garc a-

Azpeitia, Jason Mireles James, Jean-Philippe Lessard and is a continuation of work with Eusebius Doedel and C. Garc a-Azpeitia.

Maciej Capiński
AGH University of Science and Technology, Poland

Oscillatory motions and parabolic manifolds at infinity in the planar circular restricted three body problem

Consider the Restricted Planar Circular 3 Body Problem. If the trajectory of the body of zero mass is defined for all time, it can have the following four types of asymptotic motion when time tends to infinity forward or backward in time: bounded, parabolic (goes to infinity with asymptotic zero velocity), hyperbolic (goes to infinity with asymptotic positive velocity) or oscillatory (the position of the body is unbounded but goes back to a compact region of phase space for a sequence of arbitrarily large times). We will show a geometric mechanism that proves the existence of all possible combinations of past and future final motions, and apply it to the Jupiter-Sun system.

Chiara Caracciolo
University of Uppsala, Department of Mathematics, Sweden

Exploring KAM stability for extrasolar planetary systems

We study the long-term dynamics of several extrasolar planetary systems, focusing on the secular part of the planetary three-body problem, by constructing (sometimes librational) invariant KAM tori for initial conditions that are compatible with the observations. In particular, we discuss when it is convenient to introduce a resonant angle, i.e., the difference of the arguments of the pericenters. This allows us also to select ranges of initial conditions (mainly for the inclinations, that are usually unknown) which are consistent with KAM-stability.

Valerio Carruba
Unesp, Department of Mathematics, Brazil

Identifying the population of stable ${\nu}_6$ resonant asteroids using large databases.

Large observational surveys, like those that will be conducted at the Vera C. Rubin Observatory, are expected to discover up to one million new asteroids in the first year of operation. This will more than double the database of known asteroids in a very short time. New methods and techniques will be needed to handle the large influx of data. Here, we tested some of these new methods by studying the population of asteroids on stable orbits inside the ${\nu}_6$ secular resonance. This resonance is one of the strongest mechanisms for destabilizing the orbits of main-belt bodies and producing Near-Earth Asteroids (NEAs). Yet, stable orbital configurations where the asteroid pericenter is either aligned or anti-aligned with that of Saturn exist inside the resonance. The population of stable ${\nu}_6$ resonators, first discovered in the early 2010s, is now the largest population of asteroids in stable orbits inside a secular resonance. Here we obtained the largest sample of asteroids' proper elements for numbered and multi-opposition objects most likely to be affected by the resonance. Artificial Neural Networks (ANN) were then used to identify the images of resonant angles of asteroids in stable orbits, more than doubling their number. Clustering methods and the use of machine learning algorithms allowed the identification of the known asteroid families crossed by the ${\nu}_6$ resonance, the Tina, Euphrosyne, and Svea clusters, and of two entirely new groups: the Tiffanykapler and the 138605 QW177 families.

Irene Cavallari

University of Pisa, Italy

A dynamical definition of the Sphere of Influence of the Earth

The concept of planetary sphere of influence is central in the framework of close encounters: it is of interest in different contexts, from the impact monitoring to the design of the trajectories of spacecrafts undergoing planetary flybys. The concept of sphere of influence is still ambiguous as different definitions exist; the most classical ones are based on the distance between the planet and the small body of interest. We propose a new definition of the sphere of influence which depends on both the position and the velocity of the body with respect to the planet. For fixed values of the Jacobi constant, we compare the orbit obtained by patching two-body solutions at the boundary of the sphere with the trajectory computed in the circular restricted three-body problem. Our study is focused on the Earth and relies on an optimization process aimed at determining the most suitable radius for the sphere of influence. During the presentation, we will describe in detail the optimization process implemented and we will discuss the results obtained. This is a joint work with Clara Grassi, Giovanni F. Gronchi, Giulio Ba\`u and Giovanni B. Valsecchi.

Carolina Charalambous

University of Namur, Department of Mathematics, Belgium

Offsets in Laplace resonances: the case of Kepler-80

Systems with Tightly packed Inner Planets (STIPs) are believed to form farther from the star then migrate inward while the disc is present, to settle into their current orbits. Once the disc disperses, tidal interactions between STIPS and the star can provide a source of dissipation on very long timescales. Kepler-80 is a 6-planet STIP, and exhibits a particular dynamical configuration. The planets are in 2 and 3-planet mean-motion resonances (MMRs), not in the nominal positions, but slightly shifted exterior to it. Besides, the distance to the resonance (also called resonant offset) grows with the distance to the star. In this work, we propose a realistic scenario for the formation of this system and analyze how the tidal effects can transport from the inner planets to the outer ones through resonant interactions.

Alexandre Correia

University of Coimbra, Portugal

Tidal evolution of close-in planets

Close-in planets undergo strong tidal interactions with the parent star that modify their spins. Therefore, it is often believed that these planets are locked in synchronous rotation, as observed for the main satellites in the Solar System. In this talk, we revisit the two body problem with tides and present new approaches to the tidal theory. We show that for viscoelastic rheologies, which are suitable for rocky worlds, spin-orbit resonances arise naturally and delay the evolution towards the synchronous state. These equilibria are very important for Earth-like planets in the habitable zone of M-dwarf stars, as they help to sustain temperate environments and thus more favourable conditions for life.

Ariane Courtot

IMCCE, Observatoire de Paris, France

Chaos in Meteoroid Streams

Meteoroids have peculiar dynamics owing to their relativly high non-gravitational forces and their multiple close encounters. When a meteoroid stream meets with the Earth, a meteor shower is produced. Today more

than 900 meteor showers are listed by the IAU, meaning a large number of comet-like parent bodies existed in the Earth vicinity in the near past (1-100kyrs). This casts a doubt on methods used to find new meteor showers. Those methods could be improved by a better understanding of the dynamical evolution of meteoroids, which can be done by drawing chaos maps. Chaos maps are drawn using a chaos indicator. We studied four indicators : the Fast Lyapunov Indicator (FLI) and two of its derivative, as well as the Mean Exponential Growth factor of Nearby Orbits (MEGNO) and mean MEGNO. The choice of the Orthogonal Fast Lyapunov Indicator (oFLI) will be explained, as well as my firsts chaos maps on the Geminids meteoroid stream. Those maps show the influence of resonances on the meteoroids evolution, which will be discussed taking into account the objects radius. Some first constraint on the initial elements of the meteor shower will also be presented.

Jérémy Couturier

IMCCE, Observatoire de Paris, France

Dynamics of co-orbital planets in a first-order resonance chain

Co-orbital planets (in a 1:1 mean motion resonance) can be formed within a Laplace resonance chain. Here, we develop a secular model to study the dynamics of the resonance chain \$p:p:p+1\$, where the co-orbital pair is in a first-order mean motion resonance with the outermost third planet. Our model takes into account tidal dissipation through the use of a Hamiltonian version of the constant time-lag model, which extends the Hamiltonian formalism of the point-mass case. We show the existence of several families of equilibria, and how these equilibria extend to the complete system. In one family, which we call the \textit{main branch}, a secular resonance between the libration frequency of the co-orbitals and the precession frequency of the pericentres has unexpected dynamical consequences when tidal dissipation is added. We report the existence of two distinct mechanisms that make co-orbital planets much more stable within the \$p:p:p+1\$ resonance chain rather than outside it. The first one is due to negative real parts of the eigenvalues of the linearized system with tides, in the region of the secular resonance mentioned above. The second one comes from non-linear contributions of the vector field and it is due to eccentricity damping.

Irene De Blasi

University of Turin, Italy

Dynamical results for galactic billiards: linear stability, rotation numbers and symbolic dynamics

A new type of dynamical system, which can model the motion of a body in galaxies with a central mass, like a black hole, is considered. Here, a Keplerian attractive body sits in the inner region of a bounded domain, while a harmonic oscillator acts in the outside and a Snell-type refraction law holds on the interface. Treating the model as a generalization of the classical Birkhoff billiards, its orbits can be studied in terms of their rotation numbers and the linear stability of the equilibrium ones. Moreover, the existence of an associated symbolic dynamics can be proved. References: De Blasi I., Terracini S., Refraction periodic trajectories in central mass galaxies. Nonlinear Analysis, 218 (2022). De Blasi I., Terracini S., On some Refraction Billiards. Preprint, arXiv:2108.11159 (2021). Barutello V., De Blasi I., Terracini S., Symbolic dynamics for refraction billiards. In preparation (2022).

Rafael De La Llave

Georgia Inst. of Technology, USA

Global Melnikov theory for dissipative perturbations.

We present explicit formulars for the the change of invariant manifolds of normally hyperbolic invariant manifolds of Hamiltonian systems when subject to perturbations. The perturbations can be dissipative and time dependent. The existence of intersections for the manifolds shows heteroclinic connections between orbits of the NHIM, but does not need to assume that they are periodic or quasi-periodic. Joint work with M. Gidea and M. Musser.

Sara Di Ruzza

Department of Mathematics, University of Palermo, Italy

Chaos in two cases of the full three-body proble

In this talk we present two numerical studies of the three-body problem in the planar case. The first one concerns the motion of two bodies having equal mass (binary asteroids) and perturbed by a third one which is much heavier (a planet) and far away. The second one deals with 3 bodies with comparable masses focusing in the region of the unperturbed separatrix, which is complicated by a collision singularity; in both cases we define the model through a 4 degrees of freedom (dof) system, which turns out to be SO(2) invariant, then, introducing suitable coordinates in a rotating frame, we reduce this symmetry to get a 3 dof Hamiltonian. After a simply averaging, we reduce again the degrees of freedom to 2 and discuss in detail a 3-dimensional phase space through the use of first return mappings. We study the hyperbolic structure of the system and we show the existence of chaos, via the machinery of symbolic dynamics developed by Gidea, Zgliczynski. The work is joint with Jerome Daquin and Gabriella Pinzari.

Rudolf Dvorak

Vienna Observatory, Austria

Asteroids between Jupiter and Saturn

To clarify the question of asteroids between the two largest gas giants in our Solar System a numerical study of the stability of orbits of fictitious bodies in this region well outside Jupiter's and inside Saturn's orbits have been undertaken. There are in fact almost no minor planets moving there with the exception of some very faint asteroids with large eccentricities so they could be discovered in their perihelion positions. After determining the mean motion resonances and the three-body resonances we have studied in a first run the region of prograde orbits up to 90 degrees to find out stable regions from 6.8 to 8 au. This was done with a long term numerical integration in the dynamical model Sun - Jupiter - Saturn and mass-less fictitious asteroids. In a second, more detailed study with a finer grid in inclination from 1 to 30 degrees and semi-major axes from 6.88 to 7.3 au, we integrated ten thousands of orbits up to the moment of their escape. These results showed a region in semi-major axes with stable orbits up to 100 million years in the interval from a = 7.12 to 7.23 au for moderate inclinations up to about 10 degrees. Nevertheless of being a temporary capture there could be these kind of objects much larger than the tiny ones discovered up to now in the region between Jupiter and Saturn.

Antonio Elipe

Universidad de Zaragoza, Instituto Universitario de Matemáticas y Aplicaciones, Spain

Numerical Integration of Homologous Collapse

In a recent article, Slepian and Philcox derive an explicit solution of the homologous collapse from rest of a uniform density sphere under its self-gravity as a function of time. That solution is given in terms of two curvilinear integrals along a suitable Jordan contour; in practice it must be approximated by a quadrature rule. The aim of this presentation is to examine how the choice of the contour and the quadrature rule affects the accuracy and the efficiency of this integral solution approximation. More precisely, after a study of the complex roots of a transcendental equation that relates time with the variable, some alternative Jordan contours that turn out to be more convenient are proposed. Then, by using as quadrature rule the composite trapezoidal rule because of its reliability and spectral convergence accuracy, some numerical experiments are presented to show that the combination of contours and quadrature rule allows us to obtain numerical results with high accuracy and low computational cost. This method can be used for solving the classical Kepler equation.

Marco Fenucci

University of Belgrade, Serbia

Implementation of Yarkovsky/YORP effects in mercury and orbit9 N-body codes

The motion of small asteroids is influenced by the combined action of the Yarkovsky and The Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) effects. However, these effects are often either lacking or implemented in simplified versions in currently available N-body codes. In this talk, we introduce our publicly available implementation of the combined Yarkovksy/YORP effects in two popular N-body codes, i.e. the orbit9 and mercury packages. The Yarkovsky effect is produced by thermal radiation, and it causes a drift in the semi-major axis of the asteroid orbit. The YORP effect is still caused by thermal radiation, and it produces a non-zero net torque on asymmetrical objects, giving rise to a time evolution of the asteroid spin-axis. In addition, we included the effect of non-destructive collisions and rotational induced break-ups to properly model the spin state evolution. The codes include both a static and a stochastic modeling of the YORP effects, and they are suitable for statistical studies of the dynamics of asteroids. After introducing the codes, we present some applications and results. In particular, we show an example of the evolution of small members of the 288P asteroid family, and discuss possible implications for the NEO population. This is a joined work with Bojan Novaković and Mara Čutura.

Sylvio Ferraz-Mello

Universidade de São Paulo, Dept. Astronomy, Brazil

Tidal deformation of close-in exoplanets and host stars

Discussion of the similarities and differences of the creep tide theory and Darwin's tidal theory. These two theories lead to similar results and are in some respects equivalent. Indeed, if we consider Darwin's theory for viscous bodies (Darwin, 1880a,b) in the coplanar case (zero obliquity) and the initial versions of the creep tide theory (Ferraz-Mello, 2015), the equations are the same except for a difference in the adopted relaxation factor. Major differences however appear when we consider the parametric version of the creep tide theory (Folonier et al., 2018) or some contemporary versions of Darwin's theory adopting the so-called 'weak friction approximation (Gerstenkorn, 1955; Alexander, 1973). In the first versions of Darwin's and Ferraz-Mello's theories, the rotation is assumed to be uniform in short spans of time (only a slow secular variation is allowed). In the parametric version of the creep tide theory however, the equations for the shape and the rotation of the body are integrated simultaneously without the need of any ad hoc hypotheses and allow for the study of the variations on the tidal deformations of the body as thoroughly discussed in this presentation. Other important differences appear when the obliquity is not zero. In that case, the creep tide theory considers the fact that the vertex of the high tide is not permanently in the orbital plane, but oscillates around it (Folonier et al., 2022) with the same period as the orbital motion. The creep tide theory has also been extended to consider differentiated bodies (Folonier and Ferraz-Mello, 2018) and in theparticular case in which all layers are considered aligned and corotating, the results coincide with those of the contemporary constant lag time (CLT) theories where the fluid Love number appears multiplying the main effects (instead of the uniform factor 3/2 of the theories for homogeneous models).

Catalin Gales

Alexandru Ioan Cuza University of Iasi, Romania

Orbital evolution in the Earth's environment

The Keplerian motion of an object in the Earth's environment is perturbed by gravitational and non-gravitational forces. As a consequence, depending on the physical parameters of the body and its initial orbital elements, on long and secular times, the orbital dynamics reveals intriguing and complex phenomena, such as: resonances, overlapping of resonances and the onset of chaos, equilibria, bifurcations, etc. Modelling disturbing forces and their interactions, developing analytical and semi-analytical methods for stability analysis, modelling satellites breakup events and the orbital evolution of the resulted fragments are essential tools in estimating the resilience of the Earth's environment. This talk discusses some models, methods and tools developed to analyse the orbital evolution, on various time scales, in various regions of the circumterrestrial space.

Marian Gidea
Yeshiva University, USA

Melnikov method for non-conservative perturbations of the restricted three-body problem

We consider the planar circular restricted three-body problem (PCRTBP), as a model for the motion of a spacecraft relative to the Earth-Moon system. We focus on the collinear equilibrium points L1 and L2. There are families of Lyapunov periodic orbits around either L1 or L2, forming Lyapunov manifolds. There also exist homoclinic orbits to the Lyapunov manifolds around either L1 or L2, as well as heteroclinic orbits between the Lyapunov manifold around L1 and the one around L2. The motion along the homoclinic/heteroclinic orbits can be described via the scattering map, which gives the future asymptotic of a homoclinic orbit as a function of the past asymptotic. In contrast with the more customary Melnikov theory, we do not need to assume that the asymptotic orbits have a special nature (periodic, quasi-periodic, etc.). We add a non-conservative, time-dependent perturbation, as a model for a thrust applied to the spacecraft for some duration of time, or for some other effect, such as solar radiation pressure. We compute the first order approximation of the perturbed scattering map, in terms of fast convergent integrals of the perturbation along homoclinic/heteroclinic orbits of the unperturbed system. As a possible application, this result can be used to determine the trajectory of the spacecraft upon using the thrust.

Mar Giralt
Universitat Politècnica de Catalunya, Departament de Matemàtiques, Spain

Chaotic coorbital motions to L3 in the Restricted Planar Circular 3-Body Problem

We consider the Restricted Planar Circular 3-Body Problem (RPC3BP) with ratio between the masses of the primaries is small. This configuration can be modeled by a two degrees of freedom autonomous Hamiltonian. It has a saddle-center equilibrium point called L3 (collinear with the primaries and beyond the largest one) with a 1-dimensional stable and unstable manifold. Moreover, the modulus of the hyperbolic eigenvalues are weaker than the elliptic ones. In this work, we present an asymptotic formula for the distance between the stable and unstable manifolds of L3. Due to the rapidly rotating dynamics, this distance is exponentially small with respect to the mass ratio and, as a result, classical perturbative methods (i.e the Melnikov-Poincaré method) can not be applied. By means of this result, we prove the existence of Lyapunov periodic orbits exponentially close to L3 with stable and unstable invariant manifolds that intersect transversally. By the Smale-Birkhoff homoclinic theorem, this implies the existence of chaotic motions exponentially close to L3 and its invariant manifolds. This is a joint work with Inma Baldomá and Marcel Guardia.

Massimiliano Guzzo
University of Padova, Dipartimento di Matematica "Tullio Levi-Civita", Italy

Measures of slow diffusion in Hamiltonian Systems

The long-term diffusion in Hamiltonian systems with more than two degrees of freedom has been represented, since Arnold's example, as a slow diffusion within the so called Arnold web, an intricate web of the phase-space formed by chaotic trajectories. With modern computers it is possible on the one hand to perform numerical integrations which reveal the web of resonances as well as a slow diffusion occurring through it, on the other hand to compute resonant normal forms with an explicit representation of the remainders. A recent method developed by Guzzo, Efthymiopoulos and Paez, based on a stationary-phase approach to the analysis of the normal forms, provides quantitive predictions of the speed of the slow diffusion along the single resonances, with a good agreement with the numerically computed ones for a three-degrees of freedom Hamiltonian system.

Nader Haghighipour

Planetary Science Institute / Institute for Astronomy, University of Hawaii, USA

A Fully Comprehensive and Self-consistent Model of Terrestrial Planet Formation in the Solar System

We have developed the currently most comprehensive and self-consistent approach to realistically simulate the formation of terrestrial/habitable planets. Our approach begins with simulating the collisional growth of planetesimals and continues with resolving giant impacts and the full formation of terrestrial planets. It takes into account the dynamical friction due to the debris and planetesimal disks, migration of planetesimals and embryos, and the perturbation as well as possible migration of giant planets. As the most important step toward a fully comprehensive and realistic model, our approach incorporates SPH simulations into N-body integrations in real time allowing, for the first time, collisions to be simulated accurately as they occur. Results point to several important findings. For instance, in the context of our solar system, almost all simulations produced an Earth-analog. They also demonstrated that the similarities between the size and mass of Earth and Venus are a natural outcome of the formation process, and Mars-sized planets appear in systems where the mass distribution in the planetesimal disk is not uniform. When studying the effects of giant planets, results showed that secular resonances are the main reason that our solar system does not have Super-Earths. They are also the reason that terrestrial planets form interior to 2.1 AU. Simulations also show that the capture into resonance of migrating giant planets does not play a significant role on the formation of habitable planets, and while giant planets may affect the inventory of planet-forming material and water-carrying objects, especially when they migrate, they play no role in the mechanics of the formation of habitable planets and the transfer/transport of water to them. Formation and water delivery is merely due to the mutual interactions of planetary embryos, a process that occurs even when no giant planet exists. We will present the results of our study and discuss their applications to extrasolar habitable planets.

Matthew Hedman

University of Idaho Physics Department, USA

Resonances in Planetary Rings: A Survey of Structures in the Saturn System

Saturn's rings exhibit a complex array of structures that can be attributed to various mean-motion resonances, including noncircular edges, spiral waves and brightness variations. These features often have properties that are consistent with theoretical models. However, other aspects of these phenomena are still not fully understood. This talk will provide an overview of the currently available observational data and theoretical models of resonant ring structures, and discuss potentially productive areas for future work.

Xiyun Hou

Nanjing University, School of Astronomy and Space Science, China

Spin-orbit coupling of binary asteroids

Binary asteroids consitute a non-negligible fraction of small solar system bodies. Most binaries discovered till now have very close mutual orbits. Their irregular shapes, comparable sizes and close distance together lead

to strong spin-orbit coupling in the binaries. The assumption of an invariant mutual orbit is usually no longer valid when studying the spin-orbit coupling in the binaries. Inclusion of a time-varying mutual orbit in the spin-orbit coupling model brings some interesting new phenomena. For example, the center and the width of the spin-orbit resonances may be different. The so-called spin-spin and spin-orbit-spin resonances may appear. Also, the suspected long-term equlibrium state between the tidal torque and the BYORP torque in the synchronous or doubley synchronous binaries may be eventually broken by the secondary's free libration. For synchronous binaries, using the approach of periodic orbits, planar and spatial stability of the synchronous state can be studied. Chaotic tumbling or even the so-called barrel instability of the secondary may exist. These findings sometimes can be used to better constrain physical parameters of the binaries which are currently unavailable to ground observations. Using a simple planar model and carrying out the analytical spin-orbit coupling solutions to high orders, we are able to indentify secondary resonances that are responsible for the instability of the synchronous state.

Efsevia Karampotsiou

Aristotle University of Thessaloniki, Department of Physics, Greece

Computation and study of periodic orbits in the case of the Laplace resonance

The Laplace resonance is a major case of three-body resonances. A well-known example in our Solar system is observed in the case of the three inner Galilean satellites, Io, Europa and Ganymede and involves commensurability among the mean motions of the satellites. The aim of this work is to study the structure of the periodic orbits which is approached by implementing a model that includes the gravitational interaction with Jupiter, the mutual gravitational interactions of the satellites and the effects due to the oblateness of Jupiter. The families of periodic orbits are computed numerically and the linear stability of the families is studied. This study is crucial in order to understand the dynamical endstates allowed in the framework of the Laplace resonance and compare with the observed values of the Galilean satellites.

Zoran Knežević

Serbian Academy of Sciences and Arts, Serbia

Secular resonances in the asteroid belt: determination of positions and interaction with families

Briefly recalling the basic facts on secular resonances and on their role in the long term dynamics of asteroids, we discuss the importance of the precise determination of their positions in the asteroid Main Belt. An overview is given of different direct and indirect methods to pinpoint the locations of secular resonances, followed by a detailed presentation of a new method to compute, by means of the "synthetic theory", asteroid secular frequencies on a regular grid in the phase space of proper elements. The accurate polynomial fit to compute the frequencies enabled not only a subsequent determination of precise positions of resonances, but also a straightforward identification of the dynamical mechanisms affecting the computation and causing in some cases serious deterioration of the results (vicinity of the mean motion resonances, libration in secular resonances, ``cycle slips''). The rest of the talk is devoted to the results of the survey of secular resonances in 9 dynamically distinct zones of the asteroid belt: only resonances involving combinations with fundamental frequencies of Jupiter and Saturn were considered, including just a few special cases involving other planets and largest asteroids. Positions of all resonances up to order four, of a significant fraction of the order six resonances, and of several order eight ones were determined, plotted in the space of proper elements, and discussed in relation to the local dynamics and to the structure and shape of the nearby asteroid collisional families. Finally, we mention some possibilities for future work.

Giacomo Lari

Università di Pisa, Dipartimento di Matematica, Italy

Dynamical history of the Galilean satellites for a fast migration of Callisto

The three inner Galilean satellites (Io, Europa and Ganymede) are locked into a chain of mean motion resonances, known as Laplace resonance, which does not involve Callisto. Moreover, Galilean moons' dynamics is strongly influenced by tidal dissipation, which causes wide variations in their semi-major axes over large timescales. A recent tidal theory proposes that also outer moons can migrate fast because of large energy dissipation within their hosting planet. Such a theory, known as resonance locking, manages to explain the high migration rate of the Saturn's satellite Titan. In a similar scenario for the Jovian system, Callisto should experience (or have experienced) a phase of very fast orbital expansion. Because of mutual gravitational interactions, a fast migration for Callisto would have important consequences on the orbits of all four Galilean satellites. In particular, Callisto should have crossed the 2:1 mean-motion resonance with Ganymede, which could have affected the whole Laplace resonance as we know it today. In this talk, we investigate the possible outcomes of such a divergent resonance crossing and whether it is compatible with the current configuration of the Galilean satellites.

Jacques Laskar

IMCCE, Observatoire de Paris, France

The AstroGeo Project

Due to chaotic behaviour of the orbital motion of the planets in the solar system, it is not possible to predict the planetary motion beyond their horizon of predictability of about 60 Ma (Laskar, 1989, 1990, Laskar et al, 2011). This has special importance for geological studies and the construction of geological time scales.
Indeed, according to Milankovitch's theory (1941), part of the large climatic changes of the past is due to the variations of the insolation on the surface of the Earth resulting from the deformation of its orbit resulting from the gravitational disturbances of the other planets. These variations can be extracted from the stratigraphic records accumulated over several Ma. The correlation between the geological data and the calculations of celestial mechanics is now sufficiently established so that the geological time scales of the most recent periods are constructed using the astronomical solutions (Laskar et al, 2004).
The goal of the AstroGeo project is to use the cyclostratigraphic geological data in order to recover the history of the Earth, but also the history of the Earth-Moon system and the history of the Solar System beyond the horizon of predictability of 60 Ma.
In the present talk, I will concentrate on our recent results on the Earth-Moon system.
Due to tidal friction, the Earth's rotation slows down and the Moon goes away. This is measured with centimetre precision using Lunar Laser ranging. But up to now, there was no physical model that accounts for both this measured tidal dissipation and the age of the Moon. We have elaborated a physical model of the Earth-Moon orbital evolution that perfectly agrees with these constraints, and also to the available geological records, solving a fifty-year old paradox.

ref: M. Farhat, P. Auclair-Desrotour, G. Boué, J. Laskar, 2022, The resonant tidal evolution of the Earth-Moon distance. A&A letters, Forthcoming article
https://www.aanda.org/articles/aa/pdf/forth/aa43445-22.pdf [www.aanda.org]

Edoardo Legnaro

Academy of Athens - RCAAM, Aristotle University of Thessaloniki - Department of Physics, Greece

Analytic Theory for Secular Lunisolar Resonances

Inclination dependent lunisolar resonances occur whenever there is a commensurability relationship between the argument of perigee and the longitude of the nodes of a space debris and the perturbing bodies. They shape the dynamics of a MEO object (navigation satellite or space debris) over secular timescales (i.e. several decades). Exploiting such resonances has also been proposed as an efficient and cost-effective strategy for the End-of-Life (EoL) disposal of satellites. This is based on the eccentricity growth along the hyperbolic directions of the phase space, leading to fast re-entry. Our approach improves on the heuristic estimates of previous works. by providing a precise analytical calculation of the borders of the separatrices of the resonances. These results are then compared with a numerical cartography obtained by the FLI. We find that our analytical approach provides an excellent approximation for a wide range of values of the semi-major axis. However, this picture is disturbed when major crossings take place between the inclination-only dependent resonance considered and one of the two resonances $\Omega-\Omega_L$ and $2\Omega-\Omega_L$ that contain the precession of the lunar node. As these two resonances sweep the phase space (for increasing values of a) they cause large chaotic domains where our analytical estimates are no longer applicable. In these domains instead the secular evolution is governed by manifold dynamics. This is the dynamics of the stable and unstable manifolds emanating from the center manifold of circular orbits ($e=0$). In fact, the latter locally satisfies the properties of a normally hyperbolic invariant manifold (NHIM). Applications to the problem of EoL disposal are finally discussed.

Anne-Sophie Libert

naXys, University of Namur, Belgium

Dynamical constraints on extrasolar systems

The diversity of the hundreds of discovered extrasolar systems puzzles our understanding of the formation and long-term evolution of planetary systems. The detected planetary systems generally suffer from large observational uncertainties. In this talk, I will review recent results showing how dynamical studies can be useful to constrain the orbital parameters of tighly packed planetary systems which harbor two-body resonances and/or chains of resonances involving three or more planets. More precisely, I will show how i) periodic orbits can serve as dynamical clues to validate the parametrization of detected systems, ii) TTVs keep track of the migration history of planetary systems, and iii) TTVs provide signatures of three-body resonances accessible by future monitoring of the systems. Applications to K2-21, K2-24, Kepler-9, Kepler-108, and TRAPPIST-1 will be discussed. This talk aims at illustrating how the interplay between formation, dynamics, and stability can contribute to bridge the gap between observations and theoretical studies.

Birgit Loibnegger

University of Vienna, Department of Astrophysics, Austria

Effects of the passage of Gliese710

It is confirmed that the K-type star Gliese710 will encounter the Solar system in approximately 1.36 Myr and possibly come as close as 4300 au. At this fly-by distance it will pass right through the inner part of the Sun's Oort cloud. Motivated by this we investigate the influence of this special stellar passage on trajectories of the small bodies orbiting there. In a former study we presented our newly developed GPU-based N-body code which is able to handle hundreds of thousands of massless objects and have shown the influence of the passage of Gliese710 on Oort cloud objects. We have been able to increase the number of simulated objects even more and can now show an update of our study. We concentrate on the motion of about 70 million testparticles distributed in the Oort cloud. In order to simplify the problem the Oort cloud was divided into two parts namely the disk like inner part (flat and flared disk) and the spherical outer part. Each part was divided into shells for our computations. We investigate the influence of the passage of Gliese 710 at the closest and farthest of the predicted fly-by distances: 4300 and 12000 au. Additionally, we compare the results with an even closer distance of 1200 au (1/10th of the predicted farthest passing distance) in order to investigate the influence on the number of scattered objects. Results show that the passage of Gliese710 will lead to a non negligible number of comets scattered towards the planetary region around the Sun. Some of these objects are even scattered to orbits with q < 5 au (observable region).

Ezequiel Maderna

Universidad de la República - IMERL, Uruguay

Green bundles in the n-body problem

Green bundles on hyperbolic orbits are well defined. This is a consequence of the fact that each hyperbolic motion becomes, after some time, a gradient curve of a Buesmann function, hence a geodesic rays. For the geodesic flow of a compact manifold, the transversality of these bundles is equivalent to the flow being Anosov. We will see how, for the classical n-body problem, the transversality implies the possibility of freely perturb the limit shapes, in the past and the future, of the orbit to get a new orbit close.

Marcelo Marchesin

FEDERAL UNIVERSITY OF MINAS GERAIS- UFMG, Brazil

A family of linear stable equilibria in the Sun-Earth-Sail problem

The collinear libration point of the Sun-Earth Circular Restricted Three-Body Problem (CR3BP), L3 is located opposite to the Earth with respect to the Sun. Whereas several space missions have been launched to the other two collinear equilibrium points, i.e., L1 and L2, the region around L3 is so far unexploited essentially because of thesevere communication limitations caused by Sun's blocking location. By using an adequate size, location and attitude of a solar sail, the equilibrium point can be displaced from its original location to allow direct communication between the satellite and Earth. This paper presents several families of artificial equilibria located on the semi-space which is permanently opposite to Earth in relation to the Sun, but which allows direct communication with Earth. We present a family of such equilibria which are linearly stable and therefore veryuseful for space mission.

Josep J. Masdemont

Universitat Politècnica de Catalunya; Mathematics Dept., Spain

Dynamical reshaping techniques for low-thrust station-keeping of unstable orbits

In this talk, we present a geometric method based on the reshaping of Floquet modes for the station-keeping of unstable orbits by means of a continuous impulse. The methodology is applied to libration point problems, including the solar sailing control of spacecraft in the Earth-Moon system. Not just restricted to the particular cases considered in the talk, the techniques presented provide a general framework to analyze the geometric behavior of any continuous impulsive control law and open the possibility to compact on-board control procedures.
This is a joint work in collaboration with Chen Gao and Gerard Gomez.

Rita Mastroianni

University of Padova, Department of Mathematics "Tullio Levi-Civita", Italy

Secular orbital dynamics of the innermost exoplanet of the upsilon-Andromedae system in the framework of a quasi-periodic restricted model

We introduce a quasi-periodic restricted Hamiltonian to describe the secular motion of a small-mass planet in a multi-planetary system. In particular, we refer to the motion of upsilon-And b which is the innermost planet among those discovered in the extrasolar system orbiting around the upsilon-Andromedae A star. We preassign the orbits of the Super-Jupiter exoplanets upsilon-And c and upsilon-And d in a stable configuration. The Fourier decompositions of their secular motions are reconstructed by using the well known technique of the (so called) frequency analysis and are injected in the equations describing the orbital dynamics of upsilon-And b under the gravitational effects exerted by those two external exoplanets (that are expected to be major ones in such an extrasolar system). Therefore, we end up with an Hamiltonian model having 2+3/2 degrees of freedom; its validity is confirmed by the comparison with several numerical integrations of the complete 4-body problem. Furthermore, the model is enriched by taking into account also the effects due to the relativistic correction on the secular motion of the innermost exoplanet. We focus on the problem of the stability of upsilon-And b as a function of the parameters that mostly impact on its orbit, that are the initial values of its inclination and the longitude of its node (as they are measured with respect to the plane of the sky). In particular, we study the evolution of its eccentricity, which is crucial to exclude orbital configurations with high probability of (quasi)collision with the central star in the long-time evolution of the system. Moreover, we also introduce a normal form approach, that is based on the complete average of our restricted model with respect to the angles describing the secular motions of the major exoplanets. Therefore, our Hamiltonian model is further reduced to a system with 2 degrees of freedom. This allows us to very quickly preselect the domains of stability for upsilon-And b, with respect to the set of the initial orbital configurations that are compatible with the observations. This work is made in joint collaboration with U. Locatelli.

Federico Mogavero
IMCCE, Observatoire de Paris, France

The origin of chaos in the Solar System through computer algebra

The discovery of the chaotic motion of the planets in the Solar System goes back to more than thirty years ago. Still, no analytical theory has satisfactorily addressed the origin of chaos so far. Implementing canonical perturbation theory in the computer algebra system TRIP, we systematically retrieve the secular resonances at work along the orbital solution of a forced long-term dynamics of the inner planets. We compare the time statistic of their half-widths to the ensemble distribution of the maximum Lyapunov exponent, and establish dynamical sources of chaoticity in an unbiased way. New resonances are predicted by the theory and checked against direct integrations of the Solar System. The image of an entangled dynamics of the inner planets emerges. Joint work with J. Laskar. Accepted for publication in Astronomy & Astrophysics.

Jose Manuel Montilla
Universidad de Sevilla, Applied Mathematics, Spain

Continuation and Bifurcations of Quasi Satellite Orbits in the Mars Phobos RTBP

We present continuation of families of periodic Quasi Satellite orbits in the RTBP in the Mars Phobos system. We have adapted our analysis of the Lagrangian families in the classical RTBP with well proven pseudo arclength continuation methods for periodic orbits in conservative systems. We include a close monitoring of the Floquet Multipliers to detect and continue bifurcation as well as a calculation of the invariant manifolds. A general skeleton of the bifurcation diagram will display the relation and connection of the different families. We will discuss the evolution from planar to spatial solutions as well as the effect of including eccentricity in the model.

Tere M-Seara
Universitat Politècnica de Catalunya, Dpt. Matematics, Spain

Chaos in the three body problem

Consider the planar three body problem with masses m0,m1,m2 > 0, and assume that all three are not equal. We consider a Poincaré map defined on a section of the phase space. After reduction of the problem by its first integrals this is a \$4\$-dimensional map. We construct a hyperbolic invariant set of the Poincaré map where its dynamics is conjugated to the (infinite symbols) Bernouilli shift. As a consequence we prove the existence of chaotic motions and positive topological entropy for the planar three body problem. The chaotic behaviour also provides the existence of oscillatory motions for the planar three body problem.

Begoña Nicolás
Universitat de Barcelona, Spain

Invariant tori around the Moon in an Earth-Moon system under solar gravitation

In the RTBP there are many families of periodic orbits. In particular, there is a family of retrograde periodic orbits around the Moon, contained in the horizontal plane. This family is stable, what makes it suitable for many purposes like performing physical measurements or just to be employed as lunar parking orbits where to capture an asteroid or to keep materials for some mission in the Moon. In the BCP, these periodic orbits become 2D invariant tori, under generic non-resonance and non-degeneracy conditions are satisfied. We compute the invariant tori making use of a temporal Poincaré map, P, in which they are seen as one-dimensional invariant curves. In this process, we find difficulties due to the closeness of these quasi-periodic solutions to a resonance with the natural frequency of the problem, what gives rise to the appearance of chaos. To show this chaos, we analyse the Lyapunov exponent in a neighbourhood of these quasi-periodic solutions: we take a point in one of the invariant curves of the map P , define a mesh of initial conditions around it and colour them according to their Lyapunov exponent.

Roberto Paoli
Universitate Alexandru Ioan Cuza din Iasi, Romania

Resonances arising from the interaction of the Earth's oblateness with the Solar third-body and radiation pressure effects

The aim of this work is to study the resonances resulting from the coupling of the effect due to the oblateness of the Earth with the third body perturbations and the Solar Radiation Pressure (SRP) effects. In particular we are going to focus on resonant terms whose associated resonant angle involves a linear combination of the argument of perigee, right ascension of the ascending node and mean anomaly of the Sun. Resonances involving semi-fast angles such as the Sun's mean anomaly are called semi-secular in [1]. The approximate location of the Solar and SRP semi-secular resonances is computed using the J2-induced rates of changes of the orbital elements. We first study the Solar and SRP resonances separately using Hamiltonian toy models, with a focus on the case of an object with high area-to-mass ratio. Then we present some numerical tests using Fast Lyapunov Indicators, [2], to draw the phase space near resonances as a consequence of the two perturbations. Finally, we describe how it is possible, for orbits with small eccentricity, to model the two perturbations using a first or second degree Extended Fundamental Model of resonances, [3], and we describe the phase space resulting from their interaction using Poincaré-like variables. [1] Celletti, Alessandra & Gales, Catalin & Lhotka, Christoph. (2019). Resonances in the Earth's Space Environment. [2] Breiter, S.. (2001). Lunisolar Resonances Revisited. Celestial Mechanics and Dynamical Astronomy. 81. 10.1023/A:1013363221377. [3] Guzzo M., Lega E. and Froeschlé C. :"On the numerical detection of the effective stability of chaotic motions in quasi-integrable systems ''. Physica D, Volume 163, Issues 1-2, 1-25, 2002.

Antoine Petit
Globe Institute, Copenhagen University, Denmark

An integrable model for first-order three-planet mean motion resonances

Recent works on three-planet mean motion resonances (MMRs) have highlighted their importance for understanding the details of the dynamics of planet formation and evolution. While the dynamics of two-planet MMRs are well understood and approximately described by a one-degree-of-freedom Hamiltonian, little is known of the exact dynamics of three-body resonances besides the cases of zeroth-order MMRs or when one of the bodies is a test particle. In this work, I propose the first general integrable model for first-order three-planet mean motion resonances. I show that one can generalize the strategy proposed in the two-planet case to obtain a one-degree-of-freedom Hamiltonian. The dynamics of these resonances are governed by the second fundamental model of resonance. The model is valid for any mass ratio between the planets and for every first-order resonance. I show the agreement of the analytical model with numerical simulations. As examples of application, I show how this model could improve our understanding of the capture into MMRs as well as their role in the stability of planetary systems.

Elke Pilat-Lohinger

University of Vienna, Department of Astrophysics, Austria

Exo-comets caused by the stellar fly-by of Gliese 710

In this study we discuss the effect of the stellar fly-by of Gliese 710 when passing through the Oort cloud and causing a scattering of comets into the inter-stellar space. Numerical simulations are performed for two fly-by distances (12000 and 4000 au). Splitting the region outside Neptune (from 35 au to 100000 au) into subspaces which allows an investigation of some 100 million objects. For the numerical simulations we use our recently developed GPU N-body code. A stellar fly-by causes a "V-type" structure in the sub-regions of the cometary reservoir.The branch pointing towards the Sun is steeper (indicating higher eccentricites) than the outward directed arm. Moreover, the closer the comets move to Gliese710's orbit the stronger the perturbations are and the higher is the probability that these comets are either ejected from the system or scattered towards the Sun.

Juan Pons

Instituto de Física de la Facultad de Ciencias, UdelaR. Departamento de Astronomía, Uruguay

Secular evolution of resonant small bodies: semi-analytical approach for arbitrary eccentricities in the coplanar case

We study the secular evolution of a particle in deep mean motion resonance (MMR) with a planet in the planar elliptic restricted three body problem. We do not consider any restriction neither in the planet's eccentricity ep nor in the particle's eccentricity e. The methodology used is based on a semi-analytical model that consists on calculating the averaged resonant disturbing function numerically, assuming for this that in the resonant scale of time all the orbital elements of the particle are constant. In order to obtain the secular evolution inside the MMR, we make use of the adiabatic invariance principle, assuming a zero-amplitude resonant libration. We construct two-dimensional surfaces (called H surfaces) in the three-dimensional space (σ, e, ϖ) that allow us to predict the secular evolution of these three variables. The 2:1 MMR is used as example to show some results. We found four apsidal corotation resonance (ACR) families, two symmetric and two asymmetric. One of the symmetric families exists for almost any ep value. The other one for ep > 0.3 and the asymmetric ones for ep > 0.44. We corroborate the secular variations in e and ϖ predicted by the model through numerical integrations even when the initial conditions are displaced from those ACR. Some peculiar examples are presented for the 3:1 and 3:2 MMR showing large excursions in eccentricity. As an application, the Planet 9 is investigated as a possible responsible of high eccentric distant TNOs.

Alessandro Portaluri

Università di Torino, DISAFA, Italy

Spectral stability, spectral flow and circular relative equilibria for the Newtonian n-body problem

For the gravitational N-body problem in the Euclidean space, the simplest periodic solution is provided by the (circular) relative equilibrium (RE) associated to a central configuration . A classical problem aims to relate the instability properties of a (RE) to the index properties of the central configuration generating the (RE). In this talk we provide a sufficient condition for the spectral and linear instability in terms of the parity of the inertia indices of the corresponding central configuration. This talk is based on a recent joint work with Prof. Dr. Luca Asselle (Ruhr Universitat Bochum, Germany) and Prof. Dr. Li Wu (Shandong University (Jinan), China).

Alexandre Pousse

CNR-IMATI, Italy

About the horseshoe motion: a discussion on the limit of the averaged problem

A classical approach of the restricted three-body problem is to analyze the dynamics of the massless body in the synodic reference frame: families of periodic orbits as well as their neighborhood are computed with the help of Poincaré maps and continuation methods. Perturbative treatments provide another one. In particular, the averaged problem of a mean-motion resonance allows to investigate the long-term behavior of the dynamics through a suitable approximation that focuses on a particular region of the phase space of the restricted three-body problem. In the framework of the dynamics in 1:1 mean-motion resonance, generally known as co-orbital motion, I will discuss about how to bridge a gap between these two approaches and exploit the dynamical properties given by the averaged problem in order to understand the complete dynamics and compute co-orbital solutions in the synodic reference frame. I will focus especially on the horseshoe motion and talk about the role of the distance with the secondary on the "distance" between the averaged approximation and the "complete" problem. Some practical applications on asteroids and mission design (disposal orbit) will also be sketched.

Giuseppe Pucacco

Dept Physics - University Tor Vergata, Italy

Normal forms for Laplace-like resonances

We describe the generalisation of the de Sitter equilibria in multi-resonant 1+3 body systems. The analysis of stability provides hints for the structure of multi-resonant planetary systems.

Alice C Quillen

University of Rochester, Department of Physics and Astronomy, USA

Soft Astronomy

Compact mass-spring models can measure remarkably small deformations while conserving angular momentum. We use them within an N-body code to simulate tidal evolution of viscoelastic spinning objects, tying simulated rheology to spin dynamics, orbital drift and the internal distribution of tidally generated heat. Discoveries made using this simulation technique include a resonant mechanism to tilt Pluto and Charon's minor satellites and a way to shut down Bi-YORP effect drift in binary asteroids with spin-synchronous secondaries.

Adrian Rodriguez

Observatorio do Valongo, Universidade Federal do Rio de Janeiro, Brazil

The migration of Mimas and the implications for the resonant motion of small Saturnian moons

Mimas is currently evolving in resonance motion with three tiny Saturnian moons, namely, Aegaeon, Methone and Anthe. The tidal interaction with Saturn results in the orbital expansion of Mimas in timescales which depend on physical properties of both the satellite and its host planet. In this work we investigate the dynamical environment of the resonant moons during the Mimas' tidal expansion. Through the construction of dynamical maps, varying the initial Mimas' semi-major axis and eccentricity, we show that Aegaeon, Methone and Anthe may have experienced multiple crossings of first-order mean motion resonances with Mimas, depending onthe strength of the satellites' orbital expansion, revealing a complex dynamical history. The tidal interaction is modeled by assuming internal dissipation in Mimas and Saturn. For Mimas we use the creep tidal theory, whereas for Saturn we assume a two-layer body (rocky/ice core and gas envelope) with viscoelastic dissipation in the core.

Óscar Rodríguez
Università di Pisa, Dipartimento di Matematica, Italy

The Keplerian Integral methods for initial orbit determination

In this talk we explore two recent methods for the computation of preliminary orbits. These two methods are based on the conservation laws of Kepler's problem, and enable the linkage of very short arcs of optical observations even when they are separated in time by a few years. For our analysis we have used both synthetic and real data belonging to 822 main belt asteroids. The differences between computed and true orbital elements have been analysed for the true linkages, as well as the occurrence of alternative solutions. In order to quantify the results we have introduced different metrics. In addition, the aim of these metrics is to discard as many of the false linkages as possible and to keep the vast majority of true ones. These numerical experiments provide thresholds for the metrics which take advantage of the knowledge of the \emph{ground truth}: the values of these thresholds can be used in normal operation mode, when we do not know the correct values of the orbital elements and whether the linkages are true or false. We will also present a strategy to explore large data sets of unlinked detections. This is a joint work with Giovanni F. Gronchi, Giulio Baù and Robert Jedicke.

Aaron J. Rosengren
University of California San Diego, Department of Mechanical and Aerospace Engineering, USA

Space manifold dynamics of transitory Centaurs

Centaurs are a prominent group of small Solar-System bodies in highly dispersed orbits between the two main belts. They are a short-lived transient phase connecting their source reservoir in the Kuiper Belt to the Jupiter-family comets (JFCs) and Halley-type comets (HTCs), but the precise dynamical hand-off process between the Centaurs and these short-period comet (SPC) populations remains unclear. The recent finding of an orbital gateway between Jupiter and Saturn, funneling Centaurs into the inner Solar System, has cast some light on the orbital migration of these enigmatic bodies. Yet, no dynamical mechanism to explain the existence of this apparent conduit has hitherto been offered. The talk will link seminal research conducted over two decades ago on Jovian-induced space manifolds with our recent understanding of the orbital architecture of these gravitational structures to elucidate the nature of this and other dynamical channels for the rapid transport of planet-crossing small bodies of the outer Solar System.

Aspects of this research have been done in collaboration with J. D. Castro-Cisneros, J. Fitzgerald, R. Malhotra, J. D. Ross, N. Todorovic, and D. Wu.

Mattia Rossi
University of Padova - Department of Mathematics "Tullio Levi-Civita", Italy

Manifolds in the Solar System and application to L4/L5 asymmetry of Trojan asteroids

The "arches of chaos" (Todorović et al. 2020) consist in an "ornamental structure" of manifolds generated by all planets connected in a series of arches that spread throughout the whole Solar System. In this work we propose to investigate the nature of the strongest ones, stemming from Jupiter, and their implications for small body dynamics in the various regions of the phase-space. As a pivotal case, we consider the well-known open problem in our solar system concerning the observed asymmetry in the number and phase-space distribution of Trojan asteroids around Jupiter's equilateral equilibrium points L4 and L5. We discuss possible links of this problem to an observed dynamical asymmetry affecting the heteroclinic intersections (with various other periodic orbits) of the two opposite branches of the stable and unstable manifolds of the short-period family of unstable periodic orbits emanating from the Lagrangian point L3 for different energy and inclination regimes.

Melaine Saillenfest

IMCCE, Observatoire de Paris, France

Coevolution of moons and the spin axis of their host planet

Over the last decades, astrometric observations of the moons of Jupiter and Saturn have revealed that their orbits expand over time much faster than previously envisioned. This significant orbital "migration" implies that efficient mechanisms of tidal energy dissipation are at play in the interiors of gaseous planets. These findings profoundly affect our understanding of the formation and evolution of moons, but also of the spin-axis dynamics of their host planet. In this talk, I will review this intricate dynamical system, and show that it involves a variety of different phenomena with timescales ranging from a few days to billions of years. I will explain how the coupling between Cassini states and Laplace plane dynamics can gradually tilt the planet and make the system converge to a highly unstable configuration. This mechanism has beautiful applications in the solar system: Jupiter today is about to begin the tilting phase, Saturn is probably halfway in, and Uranus may have completed the final stage.

Patricia Sanchez-Martin

CUD, IUMA-Universidad de Zaragoza, Spain

Dynamic origins of asymmetry of arms in barred galaxies

Analyzing the sample of barred galaxies in the Sloan Digital Sky Survey catalog, many of them exhibit asymmetric arms: one of the arms is denser than the other. In this work we explore, employing dynamical system tools, how this asymmetry can be caused by an asymmetry in the distribution of mass in the central part of the galaxy, which is observed frequently among these galaxies. The equations of motion of a particle in a rotating system allow the study of the trajectories of stars under the potential of the system. In our case, the potential is described by the analytical equations that reproduce the morphological components of the galaxy. The invariant manifolds associated with the periodic orbits around the unstable equilibrium points placed at each end of the bar of the galaxy sketch the arms and rings of barred galaxies, transporting matter between the interior and exterior parts of the galaxy. Orbits trapped in the interior of these manifolds are in fact in charge of this transport of matter through the Lyapunov periodic orbits around the equilibrium points, which act as gates between the regions of the galaxy delimited by the zero velocity curves. The asymmetry in the central part of the galaxy manifests itself as an off-center bulge potential, whose position induces a difference in behavior between both ends of the bar. The unstable equilibrium point on the side of the bar with less mass has an associated family of Lyapunov orbits which are narrower than those of the equilibrium point on the other side of the bar. This smaller diameter leads to a lower amount of orbits that cross from the interior to the exterior part of the galaxy on the side of the bar with less mass. The result is an asymmetric galaxy, where the two arms have different sections and carry different amounts of matter. This is a joint work with C. García-Gómez, J. Masdemont and M. Romero-Gómez.

Erica Scantamburlo

Università degli Studi di Pisa, Italy

Interplanetary transfers in the elliptic restricted three-body problems using the manifold tubes originating at L1 and L2

The Lagrangian points and the solutions originating from them in the Sun-Earth circular restricted three-body problem have been widely investigated in connection to space flight dynamics. But if we aim to perform an interplanetary transfer between L1 and L2 of different Sun-planet systems, such as from the Earth to Mercury or from the Earth to Mars, the circular restricted three-body problem provides only a rough approximation of the spacecraft dynamics, since the eccentricities of both Mercury and Mars are not negligible. During the talk we describe the orbital elements associated to the manifold tubes originating at L1 and L2 in different elliptic restricted three-body problems defined by the Sun and the inner planets of the Solar System; we conclude with a discussion of the computation of the interplanetary transfers through Hohmann transfer between the manifold tubes. The talk is based on the joint work with Massimiliano Guzzo and Rocio Isabel Paez.

Daniel Scheeres

University of Colorado, USA

Limits on Energy and Angular Momentum for Escape and Collapse in the Full N-Body Problem

The range of energies and angular momentum that exist for the full N-Body problem when components are resting on each other is studied. Using a simple model of equal sized spheres, lower and upper bounds on a system?s energy are found for the collection to remain connected as a single body, when fission into multiple components will occur, and what the implications are when individual components escape from the system.

Vladislav Sidorenko

Keldysh Institute of Applied Mathematics RAS, Russia

Adiabatic approximation in dynamical studies of exoplanetary systems in mean-motion resonance

The available observational information indicates that in exoplanetary systems mean motion resonances (MMR) often occur. An effective approach to the study of secular effects caused by MMR was proposed by J. Wisdom (1985), but so far it has been used only in the framework of the restricted three-body problem (R3BP). We demonstrate how Wisdom's approach should be modified so that it can be applied to the general three-body problem. As an example, we consider the dynamics of a system consisting of a star and two planets in co-orbital motion, interpreting this situation as a 1:1 MMR.

Winston Sweatman

Massey University, School of Mathematical and Computational Sciences, New Zealand

Symmetrical Schubart orbits and related periodic motions

Since Schubart's 1956 discovery of a periodic collinear three-body orbit, a number of related orbits have been found. These include both collinear and planar trajectories. Here, we consider some families of such motions. They contain periodic orbits with four, five and six bodies that have a rotational symmetry.

Paolo Teofilatto

Sapienza- School of Aerospace Engineering, Italy

Hamiltonian normal forms for the coorbital motion in the presence of zonal harmonic perturbations , joint work with Stefano Carletta, Mauro Pontani

In recent years, the possibility of replacing large satellites with distributed systems of smaller spacecraft, operating in formations, has been investigated with growing interest. To operate properly, the satellites of a formation must maintain a bounded relative coorbital configuration. In the presence of gravitational perturbations, not negligible in a variety of actual scenarios, the bounded configurations defined using the popular Hill-Clohessy-Wiltshire equations are known to be unstable in time and can evolve into impact or drifting trajectories, eventually jeopardizing the mission. Techniques aimed at designing stable configurations in the presence of the dominant J2 zonal harmonic term were proposed by Schweighart and Sedwick [1], who considered its time averaged effect, and by Kasdin, Gurfil and Kolemen [2], who introduced a variation of parameters procedure developed using the Hamiltonian formalism. The method proposed in this work, can evaluate the effect of all the desired zonal terms, therefore avoiding errors introduced by the averaging procedure. The problem of relative coorbital motion is modelled using the Hamiltonian formalism and a canonical transformation is introduced to set the dynamic equations of the perturbed relative coorbital motion, including the zonal harmonic terms, to a normal form equivalent to the Hill-Clohessy-Wiltshire equations, plus negligible higher order terms. This method allows absorbing by the canonical change of coordinates the complexity introduced in the equations by the zonal harmonic terms [3]. From a practical perspective, the main advantage introduced is the possibility of defining the desired bounded configuration from the well-known analytical solutions of the Hill-Clohessy-Wiltshire equations, valid for the unperturbed case, and determining the corresponding ones for the perturbed case by applying the inverse of the canonical transformation. The effectiveness of the method is verified through numerical analysis on some cases of interest.

Francesco Topputo
Politecnico di Milano - Department of Aerospace Science and Technology, Italy

The ERC-Funded EXTREMA Project: Achieving Self-Driving Interplanetary CubeSats

Since the beginning of the space era, interplanetary probes have commonly been operated from ground. Operations are conducted by flight control and involve per- forming a number of routine tasks, mainly of scientific, systems engineering, and flight-related nature. Governing the space flight consists of determining the space- craft position, planning its trajectory, and controlling its motion. Accordingly, these activities are known as a whole as Guidance, Navigation and Control (GNC). The EXTREMA project (Engineering Extremely Rare Events in Astrodynamics for Deep-Space Missions in Autonomy) aims towards a paradigm shift on how deep-space GNC is performed, enabling CubeSats with autonomous capabilities. The project has received a consolidator grant from the European Research Council (ERC), a prestigious acknowledgment that funds cutting-edge research in Europe. This presentation is intended to give an overview of EXTREMA, highlighting the approaches, methodologies and objectives; moreover, the expected results, outcomes, and impact on future space exploration scenarios are also discussed.

Eva Tresaco
Instituto Universitario de Matemáticas y Aplicaciones - Universidad de Zaragoza, Spain

Heteroclinic connections in the Dipole-segment model for unequal masses

Understanding the gravitatory dynamics around asteroids is crucial for approaching and orbiting close to them. Because of their small size and irregular shape, the description of their gravitational field is challenging. A simplified mathematical model provides qualitative information of the environment of the asteroid that is useful for predicting the orbital motion of particles and for asteroid observation mission planning. In particular, this work is derived from previous researches for modelling an elongated asteroid by a massive finite segment potential and more recently by a dipole-segment model. In the present work we explore the dynamics of the dipole-segment model with unequal endpoint masses. In particular, we study the stable and unstable invariant manifolds associated to the Lyapunov orbits around the unstable equilibrium points.

Invariant manifolds can be used to identify low-energy trajectories approaching the surface of the body, orbiting around it or transferring from an equilibrium point to another using homoclinic and heteroclinic connections. In this work we focus on the evolution of heteroclinic connections depending on the parameter that rules the asymmetry of the end masses in the dipole-segment.

Giovanni Valsecchi

IAPS-INAF, Italy

Capture of interstellar objects at planetary close encounters

The recent discovery of the first two interstellar objects has renewed the interest in the possibility for some of them to be captured in elliptic heliocentric orbits as a consequence of close encounters with one of the giant planets. We treat this problem in the framework of the analytical theory of close encounters, that has the advantage of allowing the exploration of the range of initial conditions for which this type of capture is possible, and of the main features of the resulting orbits.

Tudor Vartolomei

Dipartimento di Matematica, Università di Roma "Tor Vergata", Italy

Computation of Proper Elements for the Space Debris problem

The computation of the proper elements of a Hamiltonian system can be done by using perturbation theory techniques. For the space debris problem, the normalization procedure is an iterative method of reducing the initial Hamiltonian system to a simplified form, by removing the fast and semi-fast angles of the model which describes the dynamics of the system. It is worth mentioning that in the space debris problem, the perturbations to be included in the model depend on the altitude of the space debris. The new elements obtained after the normalization process define the proper elements, which are quasi-invariants of motion, hence quantities nearly constant in time. We show that these elements are particularly useful in the classification and back-tracing of space debris, as well as in the possible recognition of their origin. The results are supported by appropriate simulations and data analysis. Work in collaboration with A. Celletti and G. Pucacco.

Massimiliano Vasile

Mechanical and Aerospace Engineering, University of Strathclyde, UK

Long-term Attitude Motion Induced by Space Debris Impacts

The talk will present some results on the long-term attitude dynamics of a rigid body subject to impacts with debris fragments. Impacts are modelled as impulses following a random-walk stochastic process. Impacts act as a small random perturbation to the natural dynamics. The magnitude of each impact and the time at which the impact occurs are drawn from current distribution models representing the flux of debris around the Earth. We will start from a simple spin-orbit theoretical model [1] and study how impacts induce a diffusive dynamics. We will quantify the degree of diffusion for different initial conditions and moments of inertia with a recently-introduced pseudo-diffusion exponent [2][3]. The pseudo-diffusion exponent is built from a polynomial chaos expansion of the dynamics as a function of the time and magnitude of the impacts. Further to this initial study of the spin-orbit problem, results will be presented for a problem with more degrees of freedom, assuming an extended three-dimensional object such as a rocket body. In this case the interest is to study the long term evolution of otherwise conserved quantities, that under the effect of continuous impacts are subject to a slow diffusion. [1] Celletti, A., Gales, C., Rodriguez-Fernandez, V. , Vasile, M. Classification of regular and chaotic motions in Hamiltonian systems with deep learning. Sci Rep 12, 1890 (2022). https://doi.org/10.1038/s41598-022-05696-9. [2] Vasile, M. (2021) Fast chaos expansions of diffusive and sub-diffusive processes in orbital mechanics. In: 72nd International Astronautical Congress, 2021-10-25

- 2021-10-29. [3] Vasile, M., Manzi , M. Polynomial Stochastic Dynamic Indicators, Communications in Nonlinear Science and Numerical Simulations, under review. Preprint available at https://www.researchgate.net/publication/359017219_Polynomial_Stochastic_Dynamic_Indicators

Mara Volpi

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Determination of KAM-stable 3D configurations for exoplanetary systems: a parametric study

We study the KAM stability of several two-planet non-resonant extrasolar systems. It is likely that the observed exoplanets are the most massive of the system considered. Therefore, their robust stability is a crucial and necessary condition for the long-term survival of the system when considering potential additional planets yet-to-be seen. Our study is based on the construction of a combination of lower-dimensional elliptic and KAM tori, so as to better approximate the system secular dynamics. For each system, we explore the parameter space of both inclinations: the one with respect to the line of sight and the mutual inclination between the planets. Our approach shows that significant inclinations, resulting in three-dimensional architectures that are far from being co-planar, can be compatible with the stability of the system. We find that the highest values of the mutual inclinations are comparable to those of the few systems for which said inclinations could be determined by the observations. Joint work with U. Locatelli, C. Caracciolo, M. Sansottera.

Patricia Yanguas

Universidad Publica de Navarra, Departamento de Estadistica, Informatica y Matematicas, Spain

Invariant 4-tori in the co-orbital motion of three bodies

We provide a qualitative explanation of the co-orbital motion of two small moons orbiting a planet. The two small bodies revolve about the planet in nearly circular orbits with almost equal radii. The system is modelled as a planar three-body problem whose Hamiltonian is expanded as a perturbation of two uncoupled Kepler problems. A combination of averaging, normal form, symplectic scaling and Hamiltonian reduction theories and the application of a KAM theorem for high-order degenerate systems allows us to establish the existence of quasi-periodic motions and KAM 4-tori related to the co-orbital motion of the moons. By conveniently selecting a suitable region of the reduced phase space (which is the Cartesian product of a two-dimensional sphere and one sheet of a two-sheet hyperboloid of revolution), we are able to establish the existence of these quasi-periodic motions that are valid for any value of an action variable, related to the angular momenta of the two moons. The present work is a step forward of the article [On co-orbital quasi-periodic motion in the three-body problem, [SIAM Journal on Applied Dynamical Systems, {\bf 18}(1), 334--353, 2019.]

Max Zimmermann

University of Vienna, Department of Astrophysics, Austria

Planetesimal and planetary embryos interactions in the presence of a secondary star

We investigate the mutual interactions of planetesimals and planetary embryos in a S-type binary star configuration. In our work the gas has already dissipated hence only gravitational interactions are taken into account. The protoplanetary disk contains some thousands planetesimals and a few tens planetary embryos. They are distributed between 1 and 4 au around the host star and are initially dynamically cold. The planetary embryos are Moon to Mars-sized, while the planetesimals are smaller than Moon-sized and have initially the same mass. We vary the secondary stars' semi-major axis (30, 60, 100 and 150 au), eccentricity (0.0, 0.2 and 0.4) and inclination (0°, 20°, 45° and 60°). Both stars have a mass of 1 M_sun. For the given binary parameters the disk objects are within the area of stable motion according to [1]. Due to the O(N^2) complexity of the n-

body problem we use a self-developed highly parallelized GPU n-body integrator [2] to simulate our systems. It uses the Bulirsch-Stoer method and can handle up to some thousand gravitational interacting objects. Collisions are handled via the so-called "perfect merging". Most systems have been simulated up to 1 Myr. In some cases we simulated up to 10 Myr. In the inclined cases the results show a distribution of the disk objects between 0° and ~ 2i_b. For systems with a higher inclination (45° and 60°) the inner planetary embryos tend to migrate inwards. Contrary in planar systems planetesimals residing at the outer part of the disk tend to migrate outwards. Additionally, systems with inlined binary stars show a lower number of mutual collisions of disk objects. [1] Pilat-Lohinger, E. and Dvorak, R.: Stability of S-type orbits in binaries. Celestial Mechanics and Dynamical Astronomy 82.2, pp. 143-153, 2002 [2] Zimmermann, M.: The influence of binary systems on planetesimal disks. Master thesis, University of Vienna, 2021