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ABSTRACTS OF THE POSTERS

Md Chand Asique
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The study of the equilateral restricted restricted four-body problem with non-spherical test particle

The dynamics of the non-spherical test particle in the equilateral restricted four-body problem is illustrated. The movement of the position of the libration points is depicted when the value of oblateness/prolateness parameters changes. Further, the effect of these parameters on the stability of the libration points and the zero velocity curves are shown. In addition, the Newton-Raphson iterative scheme is used to depict the basins of convergence connected to the libration points. The outcomes of numerical calculation unveils that the oblateness/prolateness parameters is indeed a very influential factors in this dynamical model.

Nelson Callegari Júnior
Sao Paulo State University (Unesp), Institute of Geosciences and Exact Sciences (IGCE), Brazil

Resonant dynamics of inner small satellites of Saturn

The Cassini spacecraft discovered many close-in small satellites in Saturnian system, and several of them exhibit exotic orbital states due to interactions with Mimas and the oblateness of the planet. This work is devoted to Anthe, Methone and Aegaeon, which are currently involved in the 11:10, 15:14 and 6:7 Mean-Motion Resonance with Mimas. We give an in deep study of the current orbits of the small satellites by analyzing and identifying the short, resonant and long-term gravitational perturbations on their orbit. In addition, we perform numerical integrations of full equations of motion of ensembles of close-in small bodies orbiting the non-central field of Saturn. Spectral analyses of the orbits and interpretation of them in dynamical maps allow us to describe the orbits and the dynamics of Anthe, Methone and Aegaeon in view of resonant and long-term dynamics. We show that the current geometric orbit of Methone is aligned with Mimas' due to a forced resonant component in eccentricity, leading to simultaneous oscillations of several critical angles of the expanded disturbing function. Thus, we explain the simultaneous oscillations of four critical arguments associated to the resonance. The mapping of the Mimas-Methone resonance shows that the domains of the 15:14 Mimas-Methone resonance are dominated by regular motions associated to the Corotation resonance located at osculating eccentricities lower than ~0.015 and osculating semi-major axis in the interval 194,660-194,730 km. Methone is currently located deeply within this site (Callegari, Rodríguez & Ceccatto 2021, CMDA - https://rdcu.be/cA7KO). In the case of Anthe, it is shown that the current resonant state of the pair Mimas-Anthe is characterized uniquely by the stable libration of the corotation angle, C. The component C is identified in the time variations of the orbital elements and in the other arguments of the disturbing function, all of them which circulate in prograde sense driven by secular components, despite showing episodic oscillations in short time intervals (Callegari and Yokoyama 2021, Icarus). The mapping of the 11:10 Mimas-Anthe resonance in the eccentricity-semi-major axis phase space shows the non-chaotic regions around both the Corotation and Lindblad resonances, which are separated by chaotic zones. The orbit of Anthe is located deeply inside the stable regions of the Corotation resonance. The current resonant dynamics of Aegaeon is also dominated by Corotation resonance with Mimas (Hedman et al. 2010, Icarus). The phase space of Aegaeon shows many differentiated structures, like secular-like resonances (Callegari & Rodríguez 2022 \[Dash] work in progress). Acknowledgements We are grateful to Fapesp (Sao Paulo State Research Funding Agency), through the processes 2019/15162-2, 2020/06807-7.

Irene Cavallari
University of Pisa, Italy

On the Sun-shadow dynamics

The patched conics approximation is one of the most classical methods adopted to design spacecraft trajectories in multi-body environments. We have investigated the possibility to exploit a similar technique, consisting in patching different basic dynamics to model a more complex dynamical system: the planar motion of a particle in a force field defined by patching Kepler's and Stark's dynamics is studied. The model is called {\em Sun-shadow} dynamics, referring to the motion of an Earth satellite perturbed by the solar radiation pressure and considering the Earth shadow effect. The existence of periodic orbits of brake type is proved, and the Sun-shadow dynamics is investigated by means of a Poincar\'e-like map defined by a quantity that is not conserved along the flow. This is a joint work with Giovanni F. Gronchi and Giulio Ba\`u.

Shipra Chauhan

Department of Mathematics, Deshbandhu College, India

Combined effect of viscosity and finite straight segment in the circular Robe's restricted three-body problem

In this paper, the effect of viscous force on the linear stability of equilibrium points of the circular Robe's restricted three-body problem (CRR3BP) with smaller primary as a finite straight segment is studied. The present model comprises of a more massive primary which is a rigid spherical shell filled with a homogeneous incompressible fluid of density and the less massive primary lies outside the shell. The infinitesimal mass is a small solid sphere moving inside the more massive primary. The pertinent equations of motion of the infinitesimal body are derived and solved for the equilibrium points. Routh-Hurwitz criterion is used to detect the stability of the obtained equilibrium points. The stability of the collinear equilibrium points has been studied systematically in the different regions for the various values of the parameters involved. These points are found to be conditionally stable, whereas the non-collinear and out-of-plane equilibrium points are always unstable for all the values of the parameters. We observed that viscosity has no effect on the location of equilibrium points. However, its effect along with the length parameter is evident on the stability of equilibrium points.

Veronica Danesi

University of Rome Tor Vergata, Department of Mathematics, Italy

Construction of the Kolmogorov normal form for three-body planetary problems with double Mean-Motion Resonance: the case study of the HD60532 extrasolar system

We investigate the dynamics of the two giant planets in the HD60532 system, which gives an example of extrasolar system in mean-motion resonance. We consider the secular approximation at order one in the masses which results (after the reduction of the constant of motions) in a resonant Hamiltonian with two libration angles. In this framework, the usual algorithms constructing the Kolmogorov normal form approach do not easily apply and we need to perform some untrivial preliminary operations, in order to adapt the method to this kind of problems. In practice, we start from an averaging procedure over the fast angle of libration which provides an integrable approximation of the Hamiltonian. Then we introduce action-angle variables that are adapted to such an integrable approximation. This sequence of preliminary operations brings the Hamiltonian in a suitable form to successfully start the Kolmogorov normalization scheme. This allows us to reconstruct the quasi-periodic motion of the system, with initial conditions that are compatible with the observations. This work is made in joint collaboration with U. Locatelli and M. Sansottera.

Fredy Leonardo Dubeibe

Universidad de los Llanos, Colombia

Multiple moment expansion for spinning astrophysical bodies

In this poster, we present a new technique for the analytical derivation of approximate gravitational potentials including the rotation (spin) of the astrophysical object. The series expansion is made using the Ernst potentials of axisymmetric relativistic distributions of mass for well-known compact objects, such that in the limit of weak-fields and low velocities the resulting expressions conserve not only the mass, quadrupole moment, octupole moment, charge, or magnetic dipole, but also the spin of the body. An example using the Kerr metric is presented, giving place to the approximate gravitational potential of a non-spherical spinning object. Some applications to n-body systems composed of rotating and non-spherical primaries are introduced.

Hugo Alberto Folonier

Universidade de São Paulo, Dept. Astronomy, Brazil

Ellipsoidal equilibrium figure and Cassini states of rotating planets and satellites deformed by a tidal potential in the spatial case

The equilibrium figure of a body, assumed as a perfect fluid without viscosity, deformed by the tide and the rotation is the starting point for the construction of many theories of dynamic tide, such as Darwinian theories (Darwin, 1880; Kaula, 1964; Mignard, 1979; Efroimsky and Lainey, 2007; Ferraz-Mello et al., 2008) or the hydrodynamic creep tide theory (Ferraz-Mello, 2013; Folonier et al. 2018). In this work we present the ellipsoidal figure of equilibrium when the angular velocity vector of the deformed body is not perpendicular to the orbital plane of the companion. The equatorial and polar flattenings are obtained as functions of the Jeans and Maclaurin flattenings, and the angle between the angular velocity vector and the radius vector. The equatorial vertex of the resulting equilibrium ellipsoid does not point towards the orbital companion, which produces a torque perpendicular to the rotation vector, which introduces secular terms of precession and nutation. Finally, we also study the so-called Cassini states of this problem. Ignoring the short period terms in the differential equation for the spin direction and the secular variation of the orbital elements, and assuming a uniform precession of the ascending orbital node line, we obtain the same differential equation found by Colombo (1966). That is, a tidally deformed inviscid body has exactly the same Cassini states as a rotating symmetric rigid body, and the tidal bulge has no secular effect to the first order.

Yeva Gevorgyan

Lunar and Planetary Laboratory, University of Arizona, USA

Comparison of Stratified and Effective Rheological Models For Icy Worlds

We compare multilayered and effective rheological models for the dynamics of extended deformable bodies evolving under the influence of gravitational forces. A typical case is that of a satellite with icy crusts, subsurface oceans, molten mantles and solid cores orbiting a giant planet in either our solar system or in exoplanetary systems. The goal is to explore the limits of applicability of effective rheological models to stratified icy satellites.

Anna Gierzkiewicz

Jagiellonian University, Poland

Chaotic tumbling of Hyperion

Hyperion is a moon of Saturn, known of its non-round shape and visibly chaotic rotation. In our previous work with Piotr Zgliczyński, we studied the 3-dimensional system of one-axis rotation of an oblate satellite to find rigorously (by a computer-assisted proof) the symbolic dynamics, which could explain the chaotic rotation of the moon. However, from observations, it is clear that the actual tumbling of Hyperion cannot be well modeled by this simple system. Therefore we try to use the full Euler equations of triaxial rotation (tumbling) of a rigid ellipsoidal body to study the problem.

Clara Grassi

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Canonical formalism for resonant returns

Close encounters between planets and asteroids can result in orbital resonance between the two bodies which will lead to a resonant return. Although approximate analytical models already exist, our aim is to introduce a Hamiltonian description of resonant returns. For a suitable subspace of the phase space, we build a chain of canonical transformations linking the encounter state before the first encounter to the state at the resonant return. The sequence of transformations is based on a two-body patched-conics approach. We check that our model gives good approximations by a comparison with a three-body evolution. A set of canonical coordinates that gives us the position of the small body on the target plane is also introduced. Finally, we describe the domain of our canonical transformation and its image at the second encounter.

Bhavneet Kaur

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Stability analysis in the Perturbed Robe's- Finite straight segment model under the effect of viscosity

In this Article, Robe-finite straight segment model is analysed under the effects of viscosity and perturbations in the Coriolis and centrifugal forces. We have taken the first primary as a rigid spherical shell filled with viscous, homogeneous incompressible fluid , and the second primary as a finite straight segment of length 2l. A third body, moving inside the bigger primary is a small solid sphere. We prove how the locations of equilibrium points are affected by the presence of perturbation in the centrifugal force. However, these remain unaffected by the viscosity and perturbation in the Coriolis force. The stability criteria for them are investigated and it has been observed that the stability of the collinear equilibrium points is affected by the viscosity and perturbation of the centrifugal force. It is prominently observed that the viscosity changes their nature of stability from being stable to asymptotically stable. The non collinear equilibrium points are unstable irrespective of the perturbation and viscosity.

Aiken Kosherbayeva

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Equations of secular perturbations of exoplanetary systems with variable masses

The study of dynamical evolution of exoplanetary systems is actual topic in astrodynamics and in celestial mechanics. For today, more than 5,000 confirmed exoplanets are known [1], and this list is growing rapidly. Researching of dynamics of exoplanets in the non-stationary stage of its formation gives us the opportunity to determine further evolutionary tracks. The influence of the variability of the masses of celestial bodies is explored on the dynamic evolution of planetary systems, considering that the masses of bodies change isotropically with different velocities. The problem of many bodies is considered in a relative coordinate system, with assuming that the most massive body - the parent star is located at the origin of this coordinate system. All n bodies in the system will interact with each other according to Newton's law. Orbits of n planets around the parent star are quasi-elliptical and we believe that they do not intersect. Bodies are considered spherically symmetrical with isotropically varying masses. We consider the laws of the masses to be known and arbitrary functions of time. Differential equations of motion of n bodies in the relative coordinate system are given in the works [2-3]. The methods of canonical perturbation theory are used here, which developed on the basis of aperiodic motion over a quasi- canonical section [4] in analogues of the second Poincaré system of variables. The obtained canonical equations of perturbed motion [5] are most convenient for describing the dynamic evolution of planetary systems in the case when analogues of eccentricities and analogues of inclinations of the orbital plane are small enough. The non-resonant case is researched. The Wolfram Mathematica package is used in the expansion of perturbing functions into series. Since we are interested in the evolution of orbital parameters over long periods of time, short-period perturbations associated with the orbital motion of bodies should be eliminated by averaging the perturbation functions by mean longitudes. As a result, we get the secular parts of perturbing functions. Secular perturbations of eccentric and oblique elements are defined as solutions of a system of 4n linear differential equations. As an example, we consider the four-planet exosystem V1298 Tau (spectral type K0) [6] in the non-stationary stage of its evolution. To find secular perturbations, it will be necessary to solve a system of 16 linear non-autonomous differential equations. The obtained equations of secular perturbations are studied by the numerical method. Bibliography [1] https://exoplanets.nasa.gov/ [2] Minglibayev M.Zh., Kosherbayeva A.B. Differential equations of planetary systems // Reports of the National Academy of Sciences of the Republic of Kazakhstan, - 2020, -Vol.2(330). -P. 14-20, https://doi.org/10.32014/2020.2518-1483.26 [3] Minglibayev M.Zh., Kosherbayeva A.B. Equations of planetary systems motion // News of The National Academy of Sciences of the Republic of Kazakhstan. Physico-Mathematical Series, -2020, -Vol.6(334). -P. 53 – 60, https://doi.org/10.32014/2020.2518-1726.97 . [4] Minglibayev M.Zh. Dynamics of gravitating bodies with variable masses and sizes [Dinamika gravitiruyushchikh tel s peremennymi massami i razmerami]. LAP LAMBERT Academic Publishing. –2012. –P. 224. Germany.ISBN:978-3-659-29945-2 [5] Prokopenya A. N., Minglibayev M. Zh., Kosherbaeva A. B. Derivation of Evolutionary Equations in the Many-Body Problem with Isotropically Varying Masses Using Computer Algebra // Programming and Computer Software. –2022. –Vol.48(2). –P. 107–115. DOI:10.1134/S0361768822020098 [6] David Trevor J., Petigura Erik A., Luger Rodrigo, Foreman-Mackey Daniel, Livingston John H., Mamajek Eric E., Hillenbrand Lynne A. Four Newborn Planets Transiting the Young Solar Analog V1298 Tau // The Astrophysical Journal Letters, - 2019, -Vol. 885(1) :L12, 10pp. DOI: 10.3847/2041-8213/ab4c99

Sumit Kumar

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Effects of a finite straight segment on the non-linear stability of the equilibrium point in the planar Robe's problem

The Arnold-Moser theorem (Kolmogorov-Arnold-Moser theory) has been used to study the non-linear stability of the equilibrium point (-\mu,0) in the planar Robe's restricted three-body problem when the second primary is a finite straight segment of length 2l. The density parameter k is considered zero. The equilibrium point has been found to be unstable whenever \mu does not belong to the interval \left(8\left(1-l^2\right)/9,1-l^2\right). However, in the interval \left(8\left(1-l^2\right)/9,1-l^2\right), it has been found that the Arnold-Moser theorem is not applicable when the mass ratio \mu is equal to one of \mu_i,I = 1,2,3, \ldots, 6, where \mu_1 = 0.9371108601 \left(1 - l^2\right), \mu_2 = 0.967292242\left(1-l^2\right), \mu_3 = 8/9, and \mu_4, \mu_5, \mu_6 are found graphically. Therefore, no conclusion about the non-linear stability has been drawn for these values of mass ratio \mu. It has been found that the equilibrium point is stable in the non-linear sense for all the values of mass ratio \mu in the linear stability interval \left(8\left(1-l^2\right)/9,1-l^2\right) except ~{\mu_i,i=1,2,3,\ldots, 6}.

DInesh Kumar

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Effect of straight segment and oblateness on the restricted problem of 2+2 bodies

In this paper, we investigate the combined effects of the oblateness and straight segment on the positions and linear stability of the equilibrium points in the restricted problem of 2+2 bodies. The present model holds fourteen equilibrium points, out of which six are collinear with the centers of the primaries and rest are non-collinear. It is observed that the positions of all the equilibrium points are subsequently affected by the oblateness and length of the primary bodies. The linear stability of the equilibrium points is also presented by slightly perturbing the position of the equilibrium points. It is observed that for a considered set of parameters, all the fourteen equilibrium points are unstable. An application of the present model is also studied, for which the position and stability of the equilibrium points are investigated for Earth-22 Kalliope-dual satellite system.

Eduard Kuznetsov

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The search for resonant chains on four-planet system K2-72

We studied the long-term orbital stability of the four-planet system K2-72. We varied the semi-major axes of planetary orbits within standard deviations. It is a dual process which breaks down in two steps. Firstly, construction of a sequence of convergent fractions for each ratio of planetary mean motions and search for possible resonant chains. Secondary, simulation of dynamic evolution taking into account tides in the Posidonius software. We demonstrate possible scenarios safeguarding K2-72 in the resonant chains 3:2, 5:3, 2:1, 3:2, 3:2, 2:1, 4:3, 2:1, 3:2, and 7:5, 3:2, 2:1 depends on the semi-major axes of the planet orbits. In addition, we carried out a simulation of K2-72 orbital evolution by resonant semi-analytical motion theory, which is constructed by authors. The work was supported by the Ministry of Science and Higher Education of the Russian Federation, project FEUZ-2020-0038.

Jose Lamas Rodriguez

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Parabolic ejection and collision orbits for the restricted planar circular three body problem

We consider the restricted planar circular three body problem (RPCTBP), which describes the motion of a massless body under the attraction of other two bodies, the primaries, which describe circular orbits around their common center of mass located at the origin. In a suitable system of coordinates, this system has two degrees of freedom and the conserved energy is usually called the Jacobi constant. We are interested in solutions of the RPCTBP that collide with the big primary at some instant t_0. We will distinguish between an \textit{ejection orbit} (that is, the particle kicks out from collision with the big primary at some instant t_0) and a \textit{collision orbit} (the particle goes to collision with the big primary at some instant t_1). In particular, we will study the case when both ejection and collision orbits travel close to infinity for small values of the mass ratio. To obtain such orbits, we show that, for small values of the mass ratio and the Jacobi constant, there exist transversal intersections between the stable (unstable) manifold of infinity and the unstable (stable) manifold of the collision.

Victor Lanchares

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Coriolis coupling in a Hénon-Heiles system

We study the impact of a Coriolis term in the dynamics of a Hénon-Heiles Hamiltonian. The strength of the Coriolis coupling is a frequency ω that regulates two different regimes. If $\omega < 1$, there is a trapping region around a minimum of the effective potential, and orbits can escape through three equiprobable channels. This escape region shrinks as the frequency approaches one. For $\omega > 1$, the minimum becomes a stable maximum, and an open region of stable motion around that maximum appears. We restrict our study to ω \[Element] [0, 1). We use Poincaré surfaces of section to show how the strength of the Coriolis coupling controls the size of the trapping area. While, for $\omega = 0$, most of the orbits escape, for ω \[TildeTilde] 1 most of the orbits remain trapped. The transition from one situation to the other one reveals complex resonant structures giving rise to a chaotic sticky region of long living orbits. The study of the attraction basins also evolve from a complex structure, with fractal boundaries, to basins with smooth boundaries. The computation of the evolution of the basin entropy confirms this fact. The escape probability as a function of ω is also calculated. Both the evolution of the escape probability and the entropy is not monotonous, revealing the rich and complex dynamics for intermediate values of ω.

Xiaodong Lu

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A Gauss pseudo-spectral method for end-of-life disposal of spacecraft leveraging lunisolar perturbation

The international interest towards end-of-life disposal strategies is growing rapidly because of the increasing number of space debris and the guidelines not to leave inoperative spacecraft in orbit. The two typical solutions for end-of-life are either a re-entry or moving to a stable graveyard orbit. The eccentricity of an elliptic orbit can be considerably affected by gravitational attraction from a third body, which is an advantage when dealing with end-of-life disposal aiming at a re-entry. Past research at Politecnico di Milano has demonstrated optimal manoeuvre designed in the phase space of orbital elements employing global search with genetic algorithm, particle-swarm algorithm. Lidov-Kozai's analytical theory and the Hamiltonian function of the system was applied on secular evolution of orbits and analysing the results of the global search. However, the Hamiltonian lines are not fully exploited when solving the optimal control problem. In this work we apply orthogonal collocation approach to discretise the optimal control problem and transform it into a Nonlinear Programming (NLP) problem. A preliminary investigation of using Lidov-Kozai's theory coupled with orthogonal collocation approaches to design the end-of-life disposal manoeuvre is carried out. Regarding the approach of using Kozai's theory, the problem is transformed into the orbital elements phase space and the manoeuvre is considered as a deviation of the original phase space. An accelerated gradient method is applied to solve the optimal control problem using Hamiltonian function of the system.

Matteo Manzi

MOD S.R.L.S.

Interplay between chaos and stochasticity in celestial mechanics

Chaotic behavior is omnipresent in celestial mechanics dynamical systems and it is relevant for both the understanding and leveraging the stability of planetary systems, the inner solar system in particular. The quantification of the probability of impacts of near-Earth objects after close encounters with celestial bodies; the possibility of designing robust low energy transfer trajectories, not limited to invariant manifolds but also leveraging the weak stability boundary for the design of the ballistic captures trajectories in time-dependent dynamical systems; the characterization of diffusion processes in Nearly-Integrable Hamiltonian systems in celestial mechanics. In order to have a robust description of chaos, therefore being able to describe chaotic motion in the context of dynamical systems characterized by parametric uncertainties, and in parallel being able to investigate the effect of random perturbations (e.g. Langevin equation, jump-diffusion processes) this work builds on "Polynomial Stochastic Dynamic Indicators" (Vasile, Manzi) in which tools from functional analysis, such as orthogonal polynomials (e.g. PolyChaos.jl) and more in general feature maps coming from the theory of support vector machine and kernel methods are used to approximate the functional describing a positive measure defining the state of the system. These probabilistic generalizations of existing chaos indicators will be computed for a number of dynamical systems (e.g. Duffing oscillator, circular and elliptic restricted three-body problem, etc.) and the relevance of uncertainty quantification for robust trajectory design will be discussed. This framework will be used to understand the effect of uncertainty and stochasticity, on the behavior of both individual trajectories and ensembles of trajectories coming from the sampling of the probabilistic space; the influence of this in the overall goal of predicting chaotic dynamical systems characterized by parametric uncertainties will be assessed. Bifurcating phenomena and invariant sets in time-dependant dynamical systems will be discussed, particularly in the context of Lagrangian coherent structures. Moreover, the relation between memory effects in non-Markovian processes, fractional calculus, and time-delay embedding will be outlined using the aforementioned tools. The computational efficiency of numerical integration schemes of Ordinary and Stochastic Differential Equations will be exploited to produce animations describing bifurcating phenomena and the chaotic nature of dynamical systems.

Vitor Martins de Oliveira

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Spin-orbit coupling: the effect of rheology

The tidal evolution of planets and satellites may lead to spin-orbit metastable equilibria at half-integers of the mean motion (e.g., Mercury's 2-3 spin-orbit resonance). The term metastable refers to attracting transient states that are long-lived and hence behave as if they were stable: to detect their instability it may be necessary to observe the system over a large time window. The goal of this work is to evaluate the effect of complex rheological models, e.g. bodies with several layers, onto the time-duration of the metastable states. The results to be presented are for planar systems, although the models to be used are fully three-dimensional. This work is part of a project from the São Paulo Research Foundation (FAPESP - Grant 2021/11306-0) coordinated by C. Ragazzo with the collaboration of A. Correia.

Om Prakash Meena

University of Delhi and Department of Mathematics, India

Periodic orbits around the non-collinear equilibrium points in the CR4BP

In this paper, we determined the long and short periodic orbits in the restricted four-Body problem. The fourth body of infinitesimal mass moves under the gravitational law due to three primaries whose motions are not affected by this body. We have illustrated the equilibrium points in CR4BP. To calculate Fourier series for the infinitesimal periodic orbits around the equilibrium points, valid for all values of the mass-product, for which the periodic orbits exist, terms up to the third order with regard to the size of the orbit being retained. We have also shown that the period of periodic orbits depends on the size of the orbit by retaining the second and third-order terms in the variational equations.

Poonam Meena

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EFFECT OF RADIATION AND OBLATENESS ON THE LINEAR STABILITY OF EQUILIBRIUM POINTS IN THE ERFBP

In this manuscript, we studied the linear stability of equilibrium points under the frame of a photo-gravitational elliptic restricted four-body problem with oblateness, in which masses of two primaries are equal. In particular, we have determined the equilibrium points and examined the effect of radiation pressure force and oblateness on the existence and positions of equilibrium points. Positions of equilibrium points show a deviation from the classical results. Again, the linear stability of equilibrium points in the presence of radiation and oblateness is also examined. Furthermore, we have noticed some variations in the stability range of equilibrium points from the classical results.

Amit Mittal

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Periodic orbits around non-collinear libration points with zero abscissa in the SCR4BP

Amit Mittal, Md Sanam Suraj, Rajiv Aggarwal, Om Prakash Meena In this manuscript, we shall explore the

short and long periodic orbits in the spatial collinear restricted four-body problem (SCR4BP) when the two primaries are non-spherical bodies. The fourth body of infinitesimal mass moves under the Newtonian gravitational law due to three primaries whose motions are not affected by this body. We have used the Fourier series expansion method to obtain the periodic orbits around the non-collinear (lies only on the y\[Minus]axis) libration points. We have also shown that the period of periodic orbits depends on the size of the orbit by retaining the second and third-order terms in the variational equations. Moreover, we have unveiled how oblateness and mass parameters influence the shape, size, and period of periodic orbits.

Shanshan Pan

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Stability and bifurcation analyses for the first-, second-, and third-order resonant families in the Earth-Moon system

This work analyses the stability and bifurcation for the first-, second-, and third-order resonant families in the ERTBP model of the Earth-Moon system, including the interior and exterior cases. The initials are obtained for certain resonance ratio orbits in the CRTBP model. The continuation method is applied to compute the resonant families in the planar elliptic restricted three-body problem. Utilizing the numerical approach, the interior and exterior resonant families are studied. The distribution of eigenvalues and stability curves are presented. In particular, some critical points, corresponding to period-doubling and tangent bifurcations, are found in the stability curves. The bifurcation and stability analyses show overall features of the resonant orbits in the Earth-Moon system. This study may provide alternative orbital design ideas using the resonant orbits and their invariant manifolds in the Earth-Moon system.

Luke Peterson

N/A

Birkhoff Orbital Elements for Restricted 3- and 4-Body Problems

In the two-body problem, the Keplerian orbital elements are a set of parameters that uniquely describe the time history of motion of a celestial body. The Keplerian orbital elements are used to describe the integrals of motion of the (integrable Hamiltonian) dynamical system. Once perturbations are added, the system is no longer integrable, and the Keplerian orbital elements are not well-defined. However, when we consider nearly integrable Hamiltonian dynamical systems, such as Restricted 3- and 4-Body Problems, we can define a new set of orbital elements locally, using normal form theory. In this work, we introduce a set of Birkhoff orbital elements which are defined locally about any bounded special solution of a nearly integrable Hamiltonian system. Birkhoff orbital elements are defined using action-angle coordinates in the Birkhoff normal form about the bounded invariant manifold. As an example, and for simplicity, we study the regions about the five equilibria in the R3BP, though these elements exist generically in nearly integrable Hamiltonian systems. Using Birkhoff elements, we state conjectures relating arbitrary points in phase space with bounded, invariant dynamical structures of the R3BP. In doing so, we relate the local and global dynamics of the system.

Antonio Prado

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Close proximity motion relative to (99942) Apophis

Tumbling asteroids belong to a small group of objects, whose angular velocity vector is unaligned with any of its principal axes of inertia. This leads to challenging efforts to model the trajectory of any spacecraft designed to orbit these bodies. In this work, we provide a preliminary realistic analysis of the orbital dynamics around the asteroid (99942) Apophis, one of the most interesting Near-Earth Asteroids due to its Earth close approach on April 13th.,2029. We used the dynamical model developed in previous works, for which we represented the gravitational field of the asteroid by a cloud of 3996 point masses system distributed inside a polyhedral shape. In a first step, we explored the impact of the close approach with our planet on the dynamics of a spacecraft orbiting around Apophis, considering the gravitational perturbations of the planets on the Solar System and the Solar radiation pressure. We carried out a 60-days integration ranging 43 days before and 16 days after the encounter and studied the dependence of the stability of orbits with respect to the initial value of the semi-major axis. We found that in a very large majority of cases, the spacecraft undergoes a collision or escape due to the perturbation caused by the close encounter, whereas it shows in all cases a very stable orbit before. We applied the sliding mode control theory in order to solve the stabilization problem for the system. With a total \[Laplacian]V of 0.495 m/s, we successfully stabilized an orbit with an initial semimajor axis of 0.5 km. Then, we included the effects of the changes of Apophis' spin state due to the terrestrial torques during the close encounter, corresponding respectively to the minimum and maximum values of the spin variations. We applied a time series prediction with Neural Networks in Python with Keras to classify orbits based on a relationship between the difficulty in the prediction and the stability. This method can isolate the most regular orbits in the system. A good correlation was found between the Time-Series prediction approach and MEGNO or the Perturbation Map of type II.

Alexander V. Rodnikov

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On Keeping of a Light Spacecraft near an Unstable Libration Point of a Binary Asteroid by the Solar Radiation

If a binary asteroid moves at 2-4 AU from the Sun and the distance between its components is about 100-200 km then a light spacecraft equipped with a controlled reflecting plane can be kept near a Lagrangian libration point of the binary using the solar radiation only, even if this relative equilibrium is unstable. Basing on a fact that the solar radiation force is the main force that influences the spacecraft motion with respect to the libration point in the case under consideration, one might realize such keeping choosing the direction of the normal to the reflecting plane invariably with respect to the solar rays direction and the plane of the binary mass center orbit. Such chose is possible for initial conditions from some six-dimensional manifold with the libration point in its boundary. To prolong the keeping time one can improve the algorithm of choosing taking, for instance, the solar radiation force projection onto the plane of the binary asteroid spin motion as a specially rotating vector of fixed length. In this case the spacecraft motion with respect to the libration point may remain bounded indefinitely. Note also that the spacecraft motion can be stabilized if one realize the algorithm of choosing of the normal direction at any moment of time, i.e. as a control law.

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Secular dynamics of circumbinary coorbital motion

Until now, the Kepler mission has detected a dozen planets evolving in circumbinary orbits around binary star systems. Most of them are single-planet systems located near the inner boundary of the circumbinary stability region. The formation, evolution and stability of these planets is still an open problem, in particular the lack of planetary companions. In particular, recent hydrodynamical simulations, have shown the possibility of forming pairs of coorbital planets around binaries. In this work, we study the resonant and secular dynamics of trojan planet configurations around binary stars. We construct dynamical maps of the pahse space, and we investigate the structure and stability of the regions of coorbital libration, as well as the existence of asymmetrical corotation resonancesand anti-lagrangean solutions. We also construct surfaces of sections from an analytical averaged model, which is valid for small eccentricities of the trojan pair and high eccentricities of the binary. We show that the location of the coorbital libration centers is independent of orbital configuration of the binary, but it depends only on the masses of the system. The study is extended to larger eccentricities of the trojan pair through numerical integrations. We also verified that the evection resonance, which relates the pericenter precession of the binary with any fast or semi-fast frequency of the trojan motion, cannot exist in these systems because it occurs in the unstable circumbinary region. We apply these findings to understand the dynamics of potential torjan companions of the Kepler-413 system.

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Photogravitational Axisymmetric Restricted Five-body Problem

This paper deals with the photogravitational axisymmetric restricted five-body problem. The four radiating primaries having masses m1, m2, m3, and m4 (m3=m4). We have derived the equations of motion of the mentioned problem and find the liberation points for a fixed value of parameters Alpha, and Beta and various values of radiation parameters. Further, we derived the zero velocity surfaces and Basins of attraction for fixed values of parameters Alpha, Beta and various values of radiation parameters.

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The study of periodic orbits around the collinear libration points in the perturbed spatial collinear restricted four-body problem

In this study, a the long and short periodic orbits in the spatial collinear restricted four-body problem with non-spherical primaries are generated. The fourth body of infinitesimal mass orbits under the gravitational effects of three primaries which are situated in collinear configuration whose motions are not affected by this body. We have deployed the Fourier series method, by evaluating terms up to the third order, to draw the periodic orbit around the collinear libration points. However, the method is valid for all values of the mass-product, for which the periodic orbits exist. Moreover, we have also studied that how oblateness and mass parameter influence the geometry and period of the periodic orbits.

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Zero velocity surfaces in general three body problem

The zero velocity surfaces of general planar three body problem are considered in shape space. The regularization by Lemaitre is used. There are five different type of surfaces.

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Stability of orbits in the dynamical environment of the binary system Didymos-Dimorphos

We study the orbital evolution of a small body (spacecraft, rock, debris, etc) in the dynamical environment of the binary asteroid system Didymos-Dimorphos which is the target of the space missions DART and Hera. In the first model Didymos and Dimorphos are approximated by an oblate and an ellipsoid body, respectively. In the second model the Didymos is simulated by a set of mascons derived from the current estimations of its shape provided by the observations. The effect of solar radiation pressure is taken into account, too. In the oblate-ellipsoid model we compute families of planar periodic orbits, their stability and collisions. Also,

we compute dynamical maps of initial conditions where the orbits are classified as bounded, collisional or escape orbits. The effect of the solar radiation or/and the shape-asymmetry and rotation of Didymos is studied and evaluated. The families of periodic orbits and dynamical maps of the oblate-ellipsoid model are used as guidelines to search for stable orbits in the realistic model by using relatively long term numerical integrations and a particle swarm optimization algorithm. The presentation is limited to almost planar orbits.

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Celestial viscoelastic bodies shapes

It is considered the motion of two viscoelastic bodies in the field of attraction of a massive homogeneous solid ball. We use the theory of small deformations, as well as the Kelvin-Voigt viscosity model, in order to describe the deformed state of bodies. Due to deformations, celestial bodies surfaces are changed, energy dissipation occurs and celestial bodies evolve. Previously elastic, viscous and viscoelastic displacements of points of bodies, caused by tides, were found. Due to the linearity of the theory, these displacements were obtained at first approximation in small parameter that is inversely proportional to Young's modulus and they were presented as the sum of two terms. The first term is caused only by the action of elastic forces and the second term is caused by dissipative forces. All displacements are given in the rotating coordinate frame rigidly connected to the body. We found such coordinate frames, in which the equations of the surfaces of the bodies have the canonical form. For elastic displacements the body surface is the ellipsoid of rotation elongated along the symmetry axis directed to the attracting body. For viscous displacements the body surface is the triaxial ellipsoid and its semi-major axis forms the angle of 45 degrees with the direction to the attracting body. Under the combined action of these displacements the body surface is the triaxial ellipsoid. Its semi-major axis forms the tidal lag angle with the direction to the attracting body, the value of which depends on viscosity, angular velocity of rotation of the body and orbital angular velocity of the attracting body. These results were applied for calculating the heights of the tidal bulges of the Earth, the Moon and Mimas, moon of Saturn. The obtained height of the tidal bulge of the Earth was compared with results of other authors. The height of the tidal bulge of the Moon was compared with NASA mission observations.
Keywords: elastic, viscous, displacements; Young's modulus; Poisson's ratio; surface shape; tidal bulge; triaxial ellipsoid; the Earth; the Moon; Mimas.

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Orbital evolution of circumbinary planets due to creep tides

Most of the confirmed circumbinary planets are located very close to their host binary. At such distances, tidal forces are expected to play an important role in their dynamics. Here, we consider the orbital evolution of a circumbinary planet with arbitrary viscosity, subjected to tides due to both central stars. We adopt the creep tide theory and assume that the orbital evolution occurs after the planet acquires its pseudo-synchronous stationary rotational state. We consider the case in which the planet is the only extended body in the system, which is shown to be a very accurate approach of the all-extended full problem. We perform a set of numerical integrations of the creep tidal equations in the case with two perturbers, using a Kepler 38-like systems as a working example. Curiously, we find that for this system not only the amount but also the direction of the planetary tidal migration depends on its viscosity. The eccentricity and pericenter evolution are little affected by the tides, but mainly dominated by the pure gravitational interactions. We present an analytical secular model for the planetary semimajor axis and eccentricity evolution, expanded up to low order in the eccentricities but up to high order in the semimajor axes ratio. This model reproduces very well the mean behaviour of the full tidal equations and provides a simple criterion to determine the directions of migration of the planets. We apply this criterion to the confirmed circumbinary planets and find

that according to our model, some of them are expected to be tidally migrating inwards but others are expected to migrate outwards. However, the typical timescales are predicted to be very long. Thus, not much orbital tidal evolution is expected in the currently observed circumbinary planets. Finally, we revisit the orbital evolution of a circumbinary planet in the framework of the Constant Time Lag model. We find that the cross tides have a non-negligible net secular contribution in the orbit and must be taken into in the general N-body problem. However, the results predicted by this model are identical to the ones predicted by the creep formalism, in the limit of gaseous bodies