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Spin-orbit coupling: the effect of rheology

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- This work is part of a postdoctoral project from the São Paulo Research Foundation (FAPESP - Grant 2021/11306-0)
  - Title: Spin-orbit coupling: the effect of the rheology
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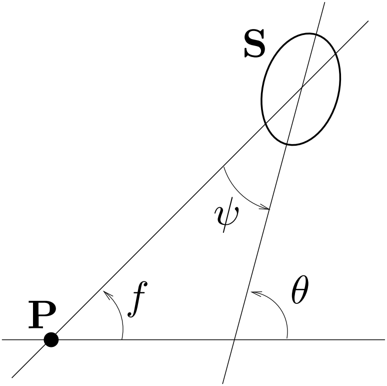
  - Collaboration: Alexandre Correia (U. Coimbra)

- Our main objective is to study the scenario for which spin-orbit coupling occurs, eventually considering the effect of the body's rheology on this phenomenon

- This research is in its early stages. We will discuss a few preliminary results regarding the rotational dynamics of Hyperion, a moon of Saturn that is currently rotating chaotically, without considering the body's rheology

- Open-source software being developed at `https://github.com/vitor-de-oliveira/spin-orbit/tree/dev`

Physical model



$\mathbf{S}$: Satellite
$\mathbf{P}$: Planet
f: true anomaly
θ: rotation angle

Assumptions

- $\mathbf{S}$ orbits $\mathbf{P}$ in a fixed Keplerian ellipse with semi-major axis a, eccentricity e, and instantaneous radius r
- spin axis parallel to the largest principal moment of inertia and perpendicular to the orbit plane
- the only forces that act on $\mathbf{S}$ are the ones generated by the gravitational field of $\mathbf{P}$

Spin-orbit resonance (SOR)

If T is the body's orbital period, then a spin-orbit resonance of type p/q, for two relatively prime integers p, q, with $q > 0$, is a solution for the system such that

$$\theta(t + Tq) = \theta(t) + 2\pi p \qquad \forall t \in \mathbb{R},$$

i.e., after q revolutions the orbiting body has made p rotations about its spin axis

SOR in the Solar System

- Earth's Moon
  - 1/1 resonance
- Pluto and Charon
  - 1/1 resonance (both)
- Mercury
  - 3/2 resonance
- Many other moons (Phobos, Deimos, Io, Europa, Ganymede, Callisto, ...)
  - 1/1 resonance

- $I_1 < I_2 < I_3$: principal moments of inertia of $\mathbf{S}$
- I_3: moment of inertia about the spin axis

Rotational dynamics of an almost rigid body

$$I_3\ddot{\theta} = -\frac{3}{2}(I_2 - I_1)\frac{Gm_P}{r^3}\sin 2(\theta - f) - 3k_2\frac{Gm_P^2R^5}{a^6}\tau\left(\overline{L}\dot{\theta} - \overline{N}\right)$$

where

$$\bar{L}(e) = \frac{1}{(1-e^2)^{9/2}}\left(1 + 3e^2 + \frac{3}{8}e^4\right)$$

$$\bar{N}(e) = \frac{1}{(1-e^2)^6}\left(1 + \frac{15}{2}e^2 + \frac{45}{8}e^4 + \frac{5}{16}e^6\right)$$

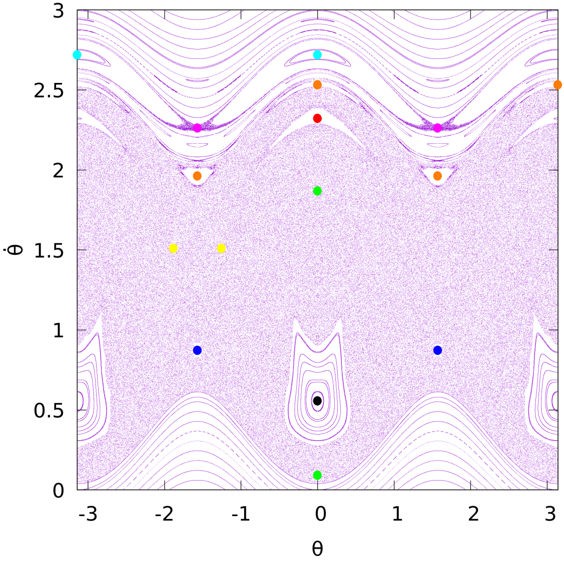
Physically relevant parameters

orbital eccentricity	equatorial flattening	dissipation constant
e	$\gamma := \frac{I_2 - I_1}{I_3}$	$K := 3k_2\frac{Gm_P^2R^5}{a^6}\tau$

Hyperion

- Moon of Saturn
- Chaotic rotation
- Very aspherical shape, being nearly twice as long as it is across (Voyager 2)
- Physical parameters: $e \approx 0.1$ and $\gamma \approx 0.264$
- Main references: Wisdom and Peale (1984) and Wisdom (1987)

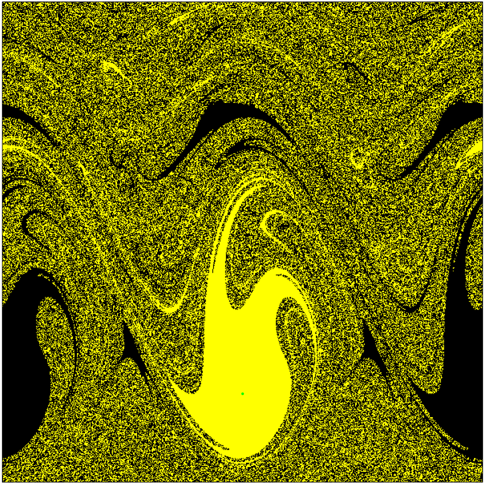
Phase space – $\theta \times \dot{\theta}$



Resonances:

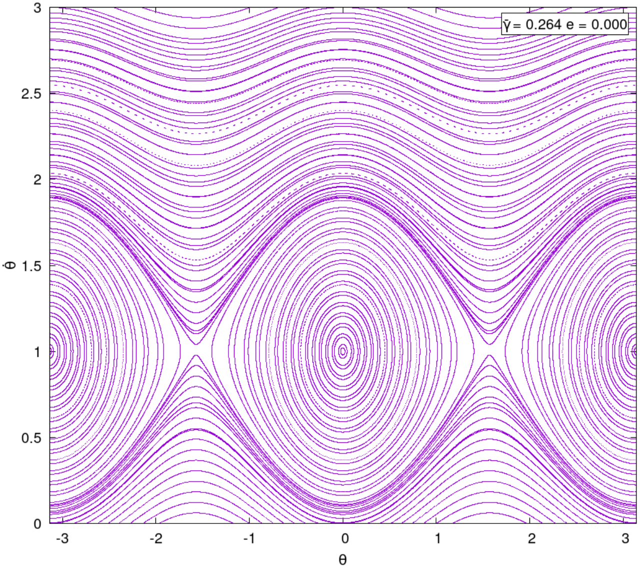
- 1/1 stable (black)
- 2/1 stable (red)
- 1/2 stable (blue)
- 2/2 unstable (green)
- 3/2 unstable (yellow)
- 5/2 stable (light blue)
- 5/2 unstable (pink)
- 9/4 stable (orange)

Basin of attraction for resonance 1/1 with $K = 10^{-2}$

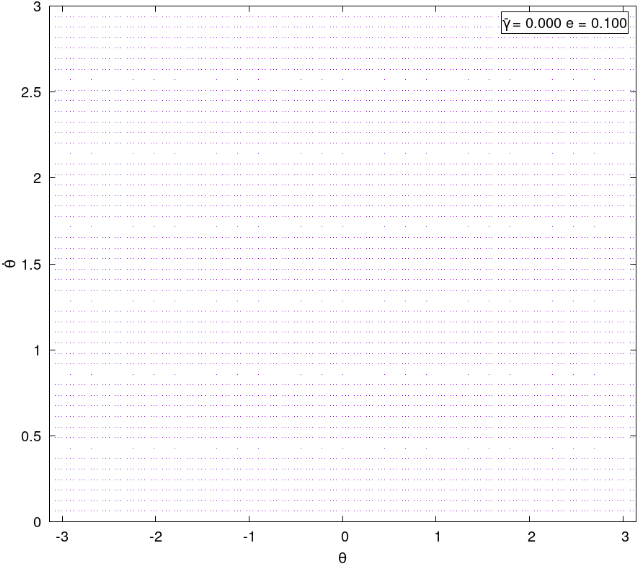


The basin of attraction of the stable $1/1$ SOR presents a very complex structure

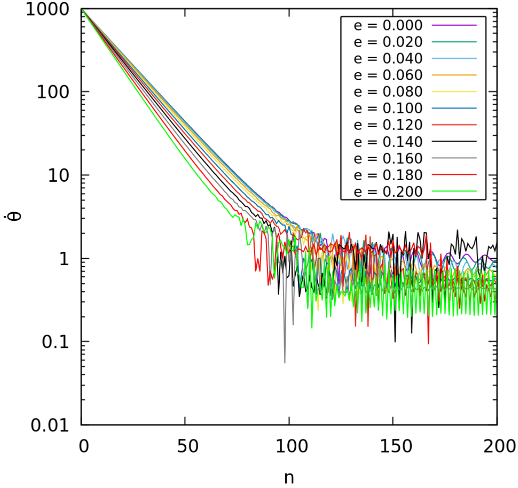
Hyperion - Phase space varying e (gif)



Hyperion - Phase space varying γ (gif)

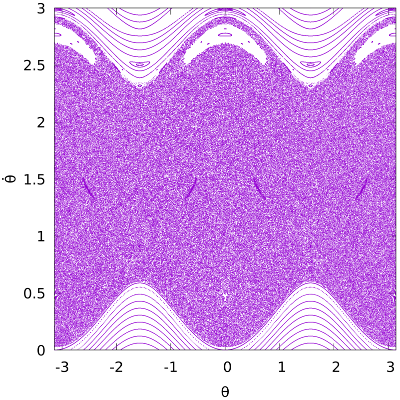


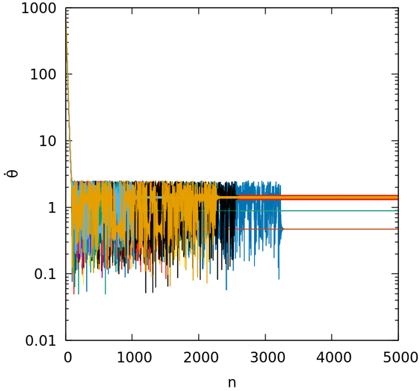
Time series of $\dot{\theta}(n)$



Same initial condition for $K = 10^{-2}$ and different values of the eccentricity e. In all cases, the angular velocity decays exponentially at first, with slope depending on e, before experimenting the chaotic region and finally converging to a SOR, which also depends on e.

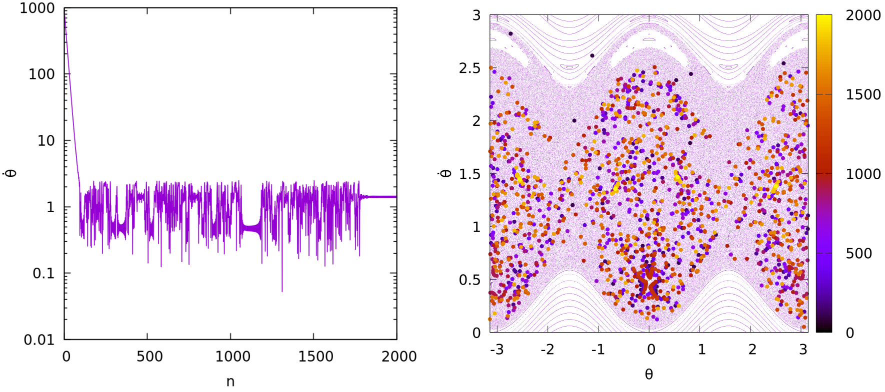
Hyperion for $e = 0.14$





(left panel) The phase space for $e = 0.14$ presents a large chaotic region. (right panel) 20 initial conditions that are very close to each other converge to 4 different final states.

Metastable state



(left panel) Time series of $\dot{\theta}$ for a solution with $e = 0.14$. (right panel) Same orbit on phase space. The colorbox represent the orbit cycle n. We observe that the trajectory spends some time around the $1/1$ SOR before converging to a higher-order resonance.

Next steps

- Measure the size of the basin of attraction for different spin-orbit resonances and different values of γ and e
- Account for changes in the equatorial flattening by considering a linear viscoelastic rheology model for the body, and analyze how these changes affect the entrapment into spin-orbit resonances and the timescale of metastable states

Main references

- Celletti, A. and Chierchia, L., *CM&DA*, 2008
- Celletti, A., *ZAMP*, 1990
- Goldreich P. and Peale S., *The Astronomical Journal*, 1966
- Ragazzo, C. and Ruiz, L. S., *CM&DA*, 2017