CORIOLIS COUPLING IN A HÉNON-HEILES SYSTEM

V. Lanchares
Universidad de La Rioja

In collaboration with
M. Iñarrea, J.P. Salas (Universidad de La Rioja)
and D. Farrelly (Utah State University)

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THE SYSTEM

Two-degrees of freedom rotating system with Hamiltonian

$$\mathcal{H} = \frac{1}{2}(X^2 + Y^2) - \omega(xY - yX) + \frac{1}{2}(x^2 + y^2) + yx^2 - \frac{1}{3}y^3$$

Coriolis term

- (x, y) cartesian coordinates and (X, Y) conjugate canonical momenta
- We use a **co-rotating reference** frame
- ω parameter: the rotational angular velocity of the system
- We study the effect of the angular velocity ω in the escape dynamics

Hamiltonians with Coriolis term appear in contexts such as:

- Atomic and molecular Physics
- Celestial mechanics
- Galactic dynamics

EQUILIBRIUM POINTS

Four equilibrium points: (x_e, y_e, X_e, Y_e)

$$E_0 \equiv (0, 0, 0, 0)$$

$$E_1 \equiv \big(0,\ 1-\omega^2,\ \omega(\omega^2-1),\ 0\big)$$

$$E_2 \equiv \left(\tfrac{\sqrt{3}}{2}(1-\omega^2),\ \tfrac{1}{2}(\omega^2-1),\ \tfrac{1}{2}\omega(1-\omega^2),\ \tfrac{\sqrt{3}}{2}\omega(1-\omega^2)\right)$$

$$E_3 \equiv \left(\tfrac{\sqrt{3}}{2}(\omega^2-1),\ \tfrac{1}{2}(\omega^2-1),\ \tfrac{1}{2}\omega(1-\omega^2),\ \tfrac{\sqrt{3}}{2}\omega(\omega^2-1)\right)$$

E_1 located in the $x = Y = 0$ manifold

$E_{2,3}$ located symmetrical respect to that manifold

Stability and energy of the equilibria:

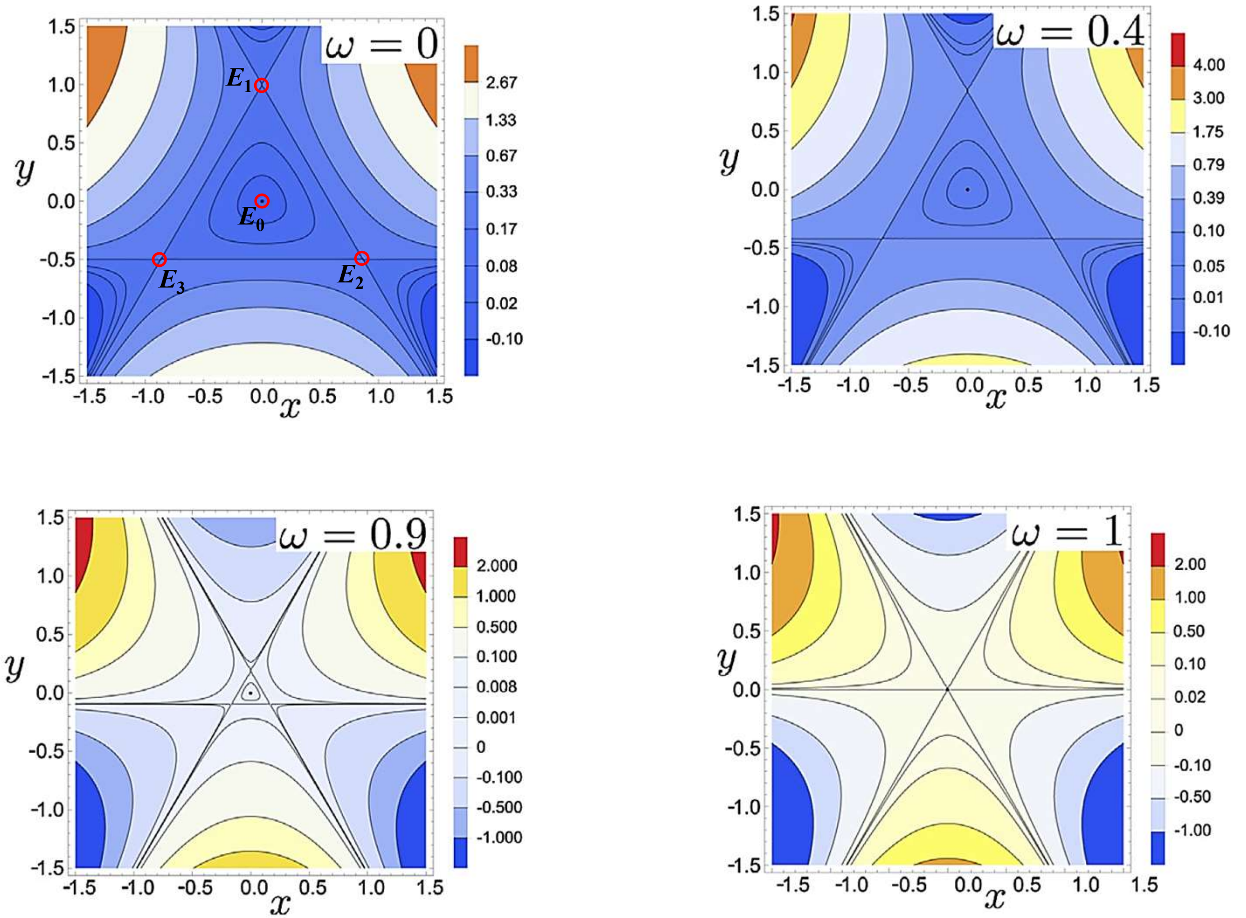
E_0 Stable equilibrium with energy $\varepsilon_0 = 0$

$E_{1,2,3}$ Unstable center $\times$ saddle equilibrium points

with the same energy $\varepsilon_{1,2,3} = \dfrac{(1-\omega)^3}{6}$

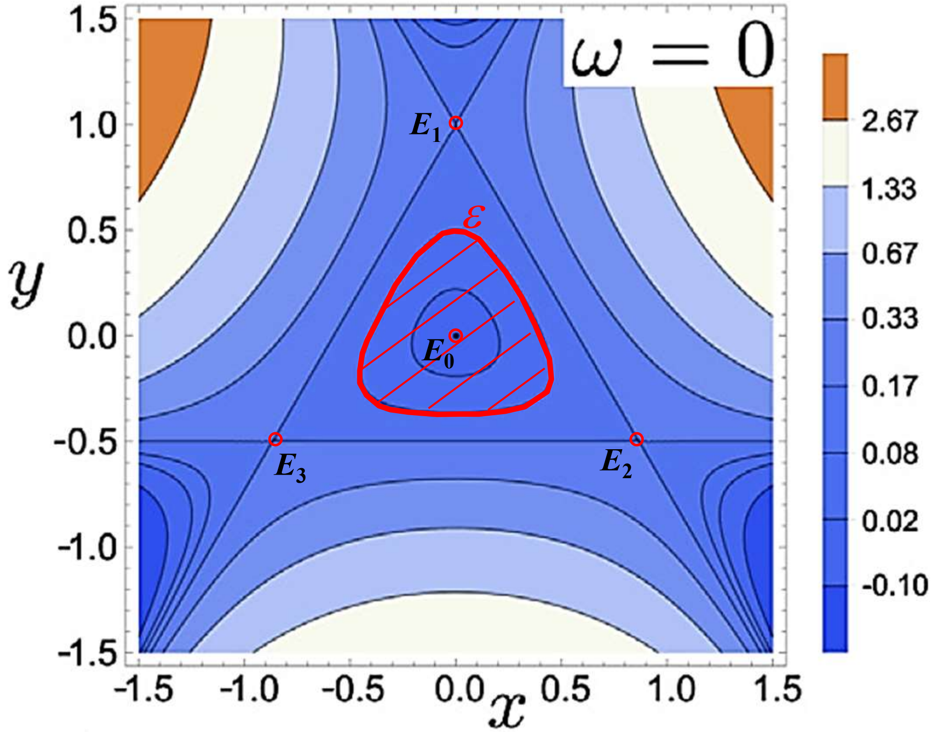
EFFECTIVE POTENTIAL $U(x, y)$. EVOLUTION WITH ω

$$U(x,y) = \mathcal{H} - \frac{1}{2}(\dot{x}^2 + \dot{y}^2) = \frac{1-\omega^2}{2}(x^2 + y^2) + x^2 y - \frac{1}{3}y^3$$



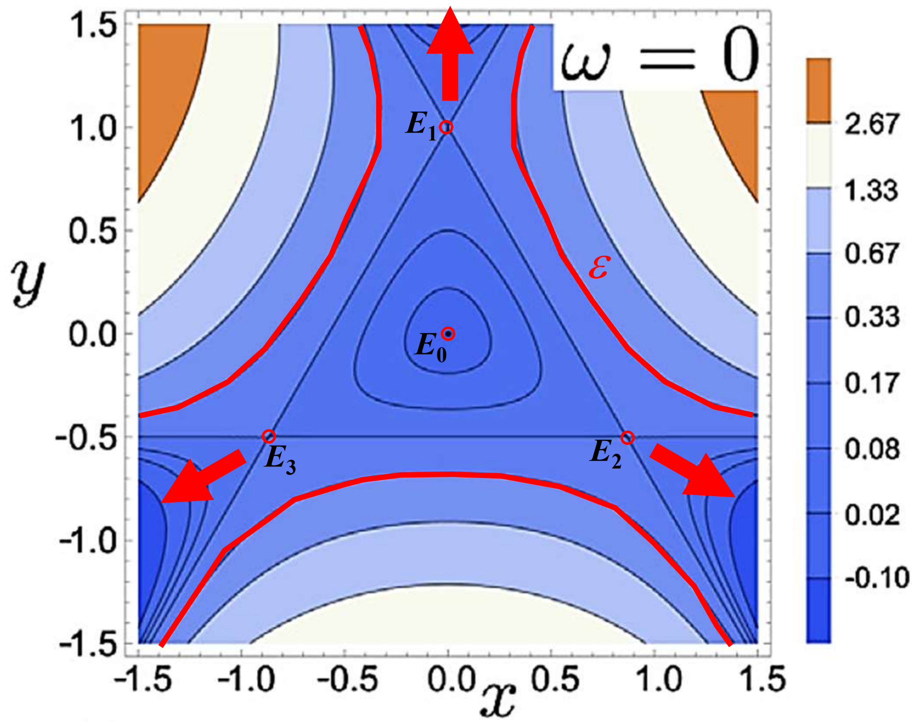
INNER TRAPPING REGION AND ESCAPE CHANNELS

For energies $\varepsilon < \varepsilon_{\text{saddles}}$



The orbits are **confined** in a inner trapping region around the origin

For energies $\varepsilon > \varepsilon_{\text{saddles}}$



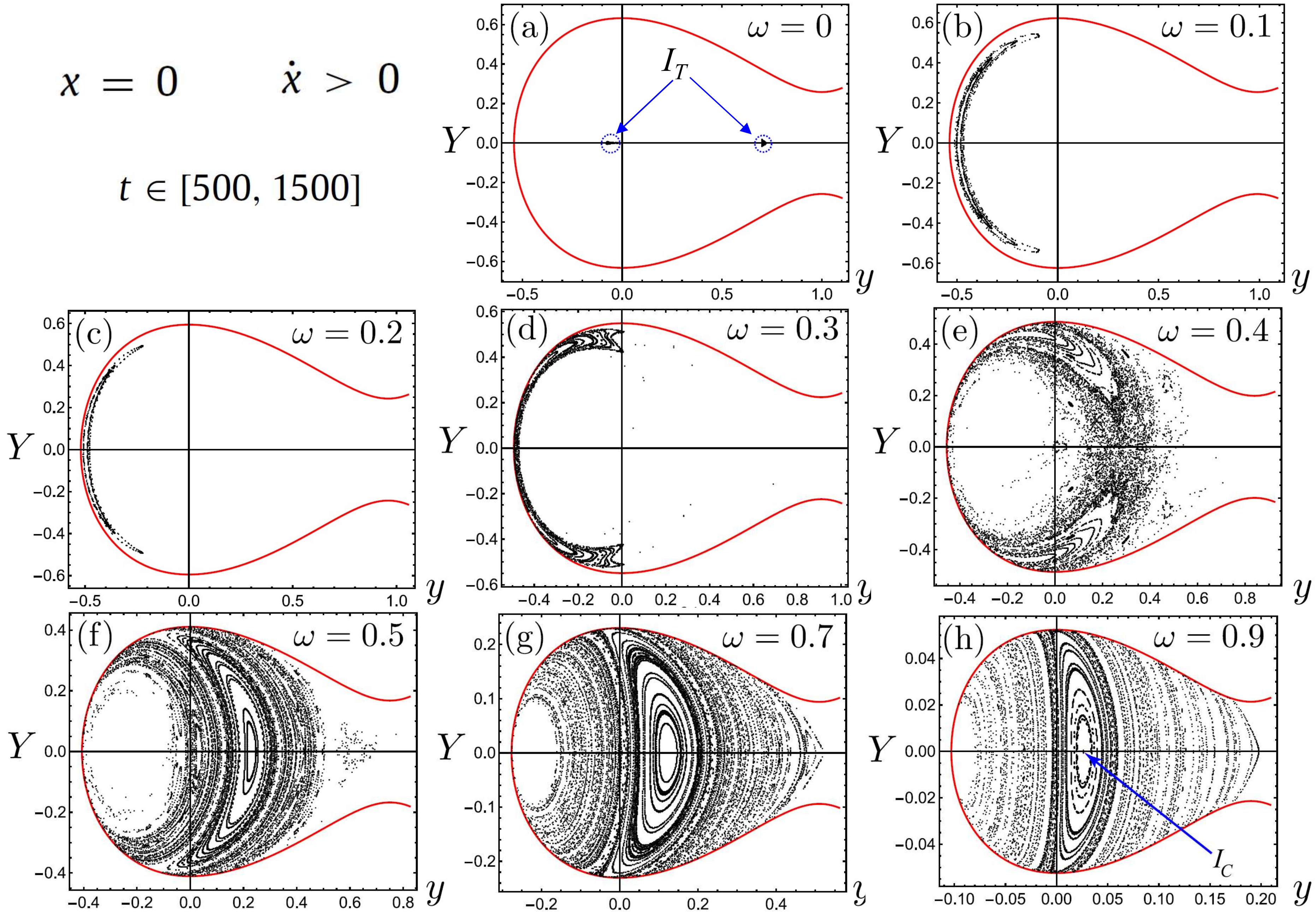
The orbits can **escape** from the inner región through **3 different channels.**

We study the escape dynamics for **an energy** $\varepsilon = \mathbf{1.2}\ \varepsilon_{\text{saddles}}$

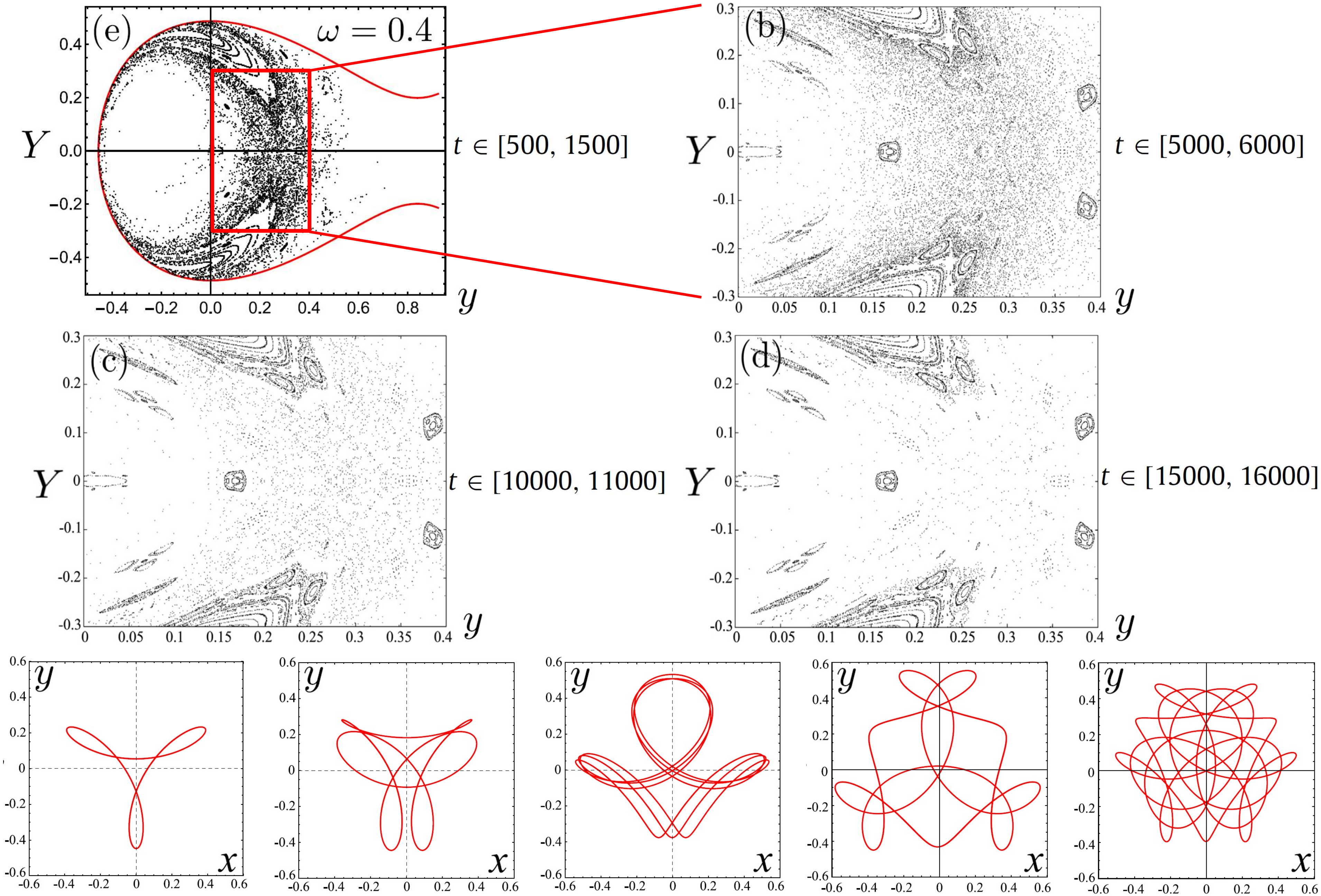
SURFACES OF SECTION. EVOLUTION WITH ω

$x = 0 \qquad \dot{x} > 0$

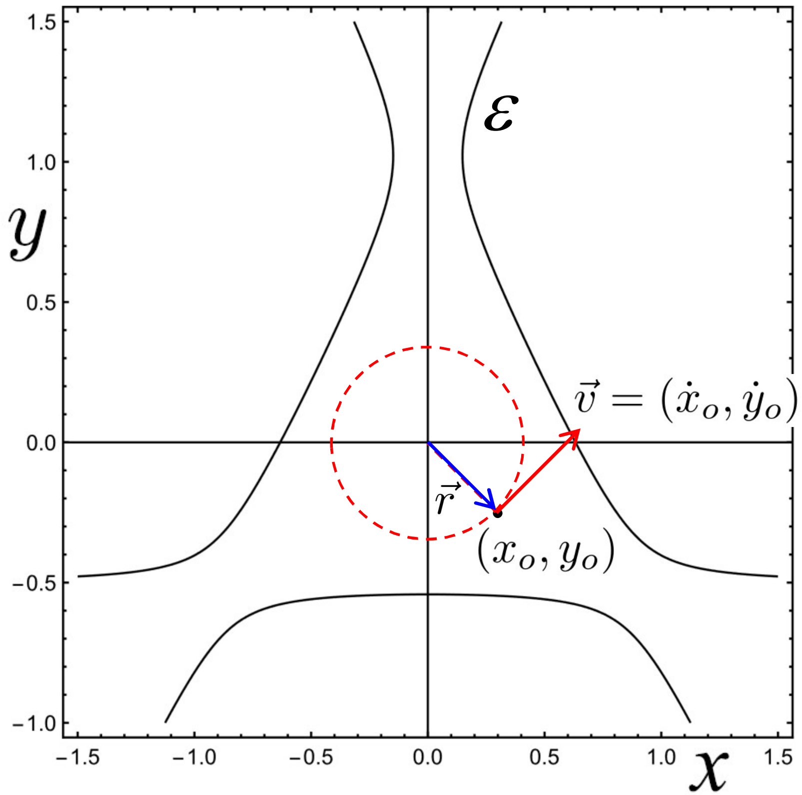
$t \in [500, 1500]$



LONG LIVING STICKY ORBITS



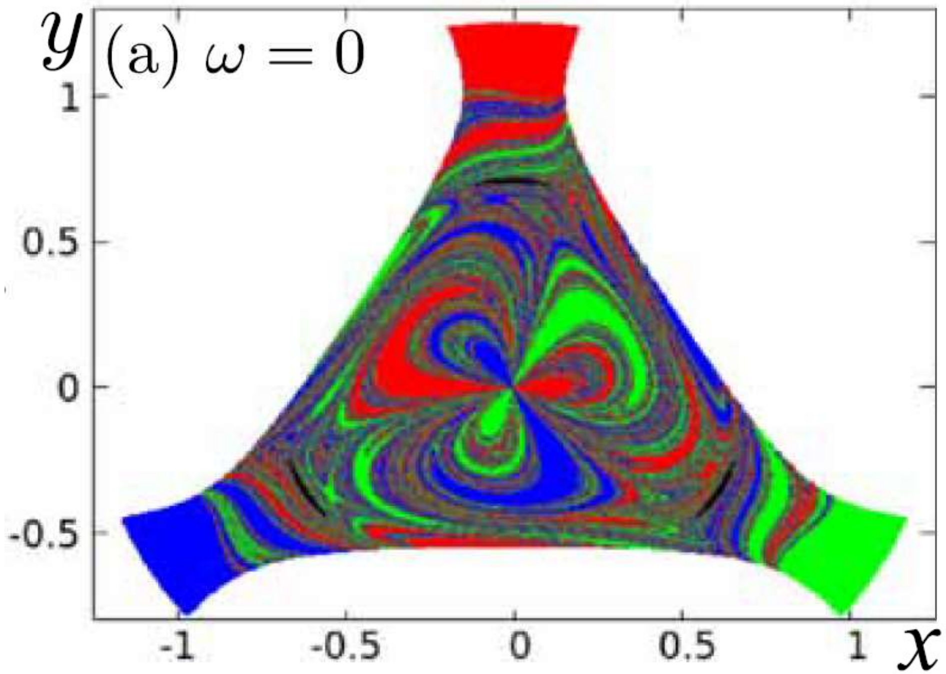
ESCAPE BASINS



$$\vec{r} \cdot \vec{v} = 0$$

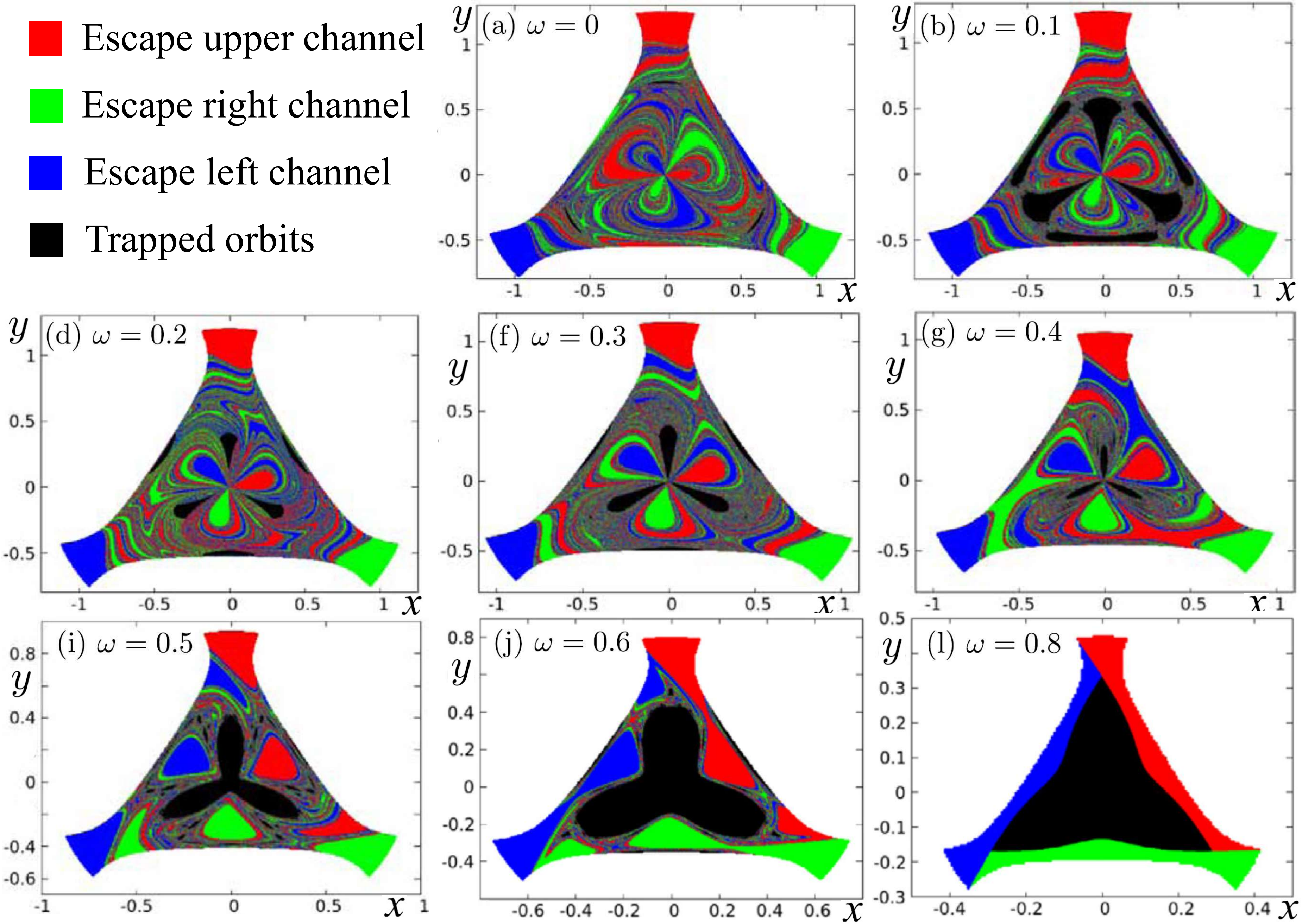
$\vec{r} \times \vec{v}$ Pointing in the positive sense of the *z* axis

$$t_f = 2 \times 10^4$$

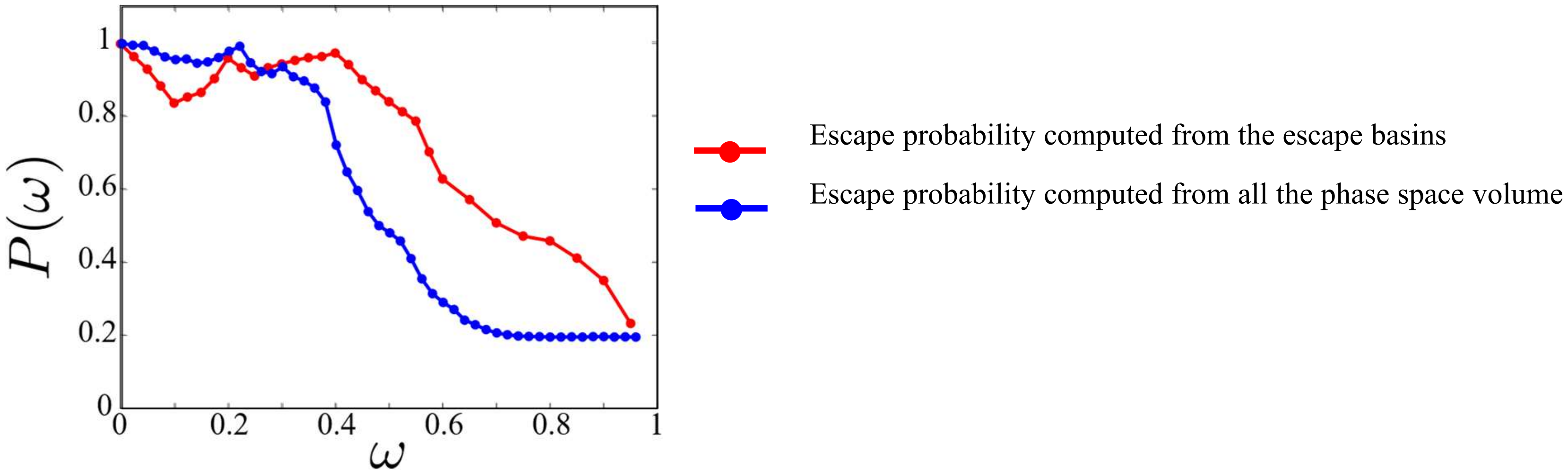


- Escape upper channel (red)
- Escape right channel (green)
- Escape left channel (blue)
- Trapped orbits (black)

ESCAPE BASINS. EVOLUTION WITH ω



ESCAPE PROBABILITY. EVOLUTION WITH ω



CONCLUSIONS

Effects of the rotational angular velocity ω on the escape dynamics as the angular velocity ω increases from 0 to 1:

- The size of the inner trapping region shrinks.
- The chaoticity and uncertainty of the escape dynamics drecreases.
- The escape probability also drecreases non-monotonically.