Analysis of Resonance Transition Periodic Orbits in the CRTBP

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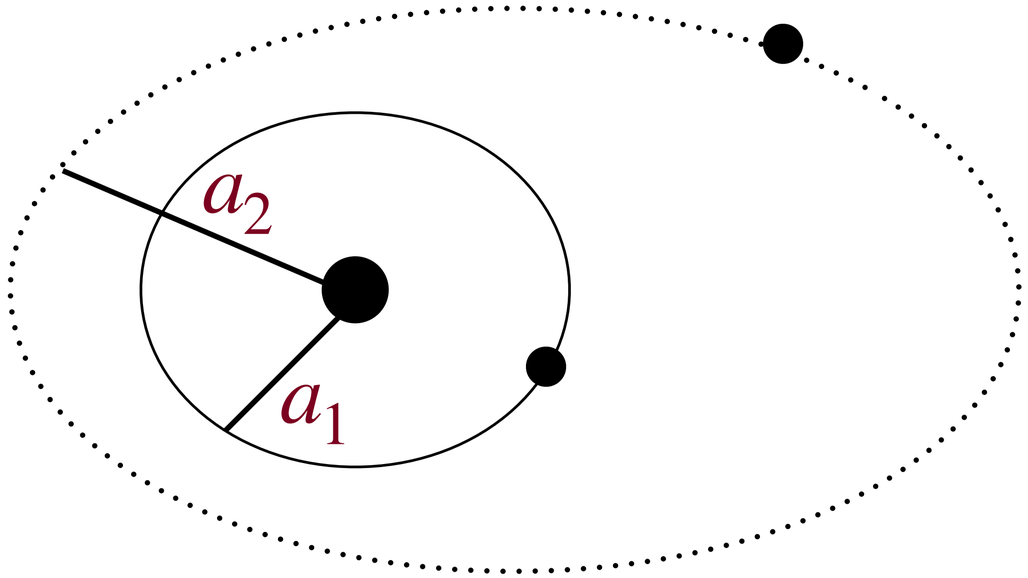
Orbital Resonances



Two satellites orbiting a planet

$$n_1 = \frac{2\pi}{T_1} = (\frac{GM}{a_1^3})^{1/2}$$

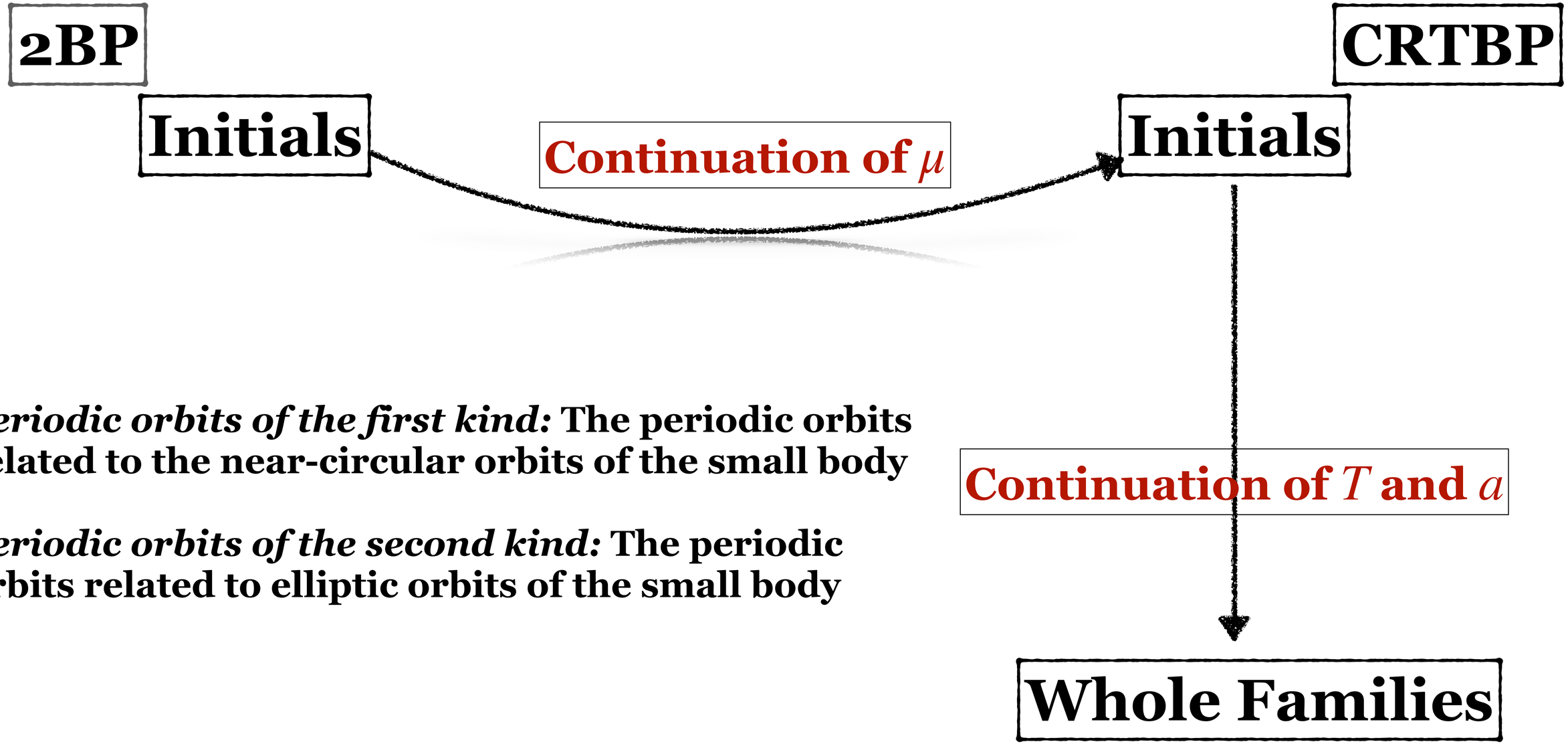
$$n_2 = \frac{2\pi}{T_2} = (\frac{GM}{a_2^3})^{1/2} < n_1$$

$k = |n_1 - n_2|$ $\quad$ $k \geq 0,$ $\quad$ k is the resonant order.

Motivation

- **To analyse the first-order resonance structure by increasing the mass parameter μ**
- **To investigate how the 1:1 resonance overlaps with nearby first-order resonance with increasing μ**
- **To seek the natural transition between the 1:1 resonance and interior/exterior first-order resonances**

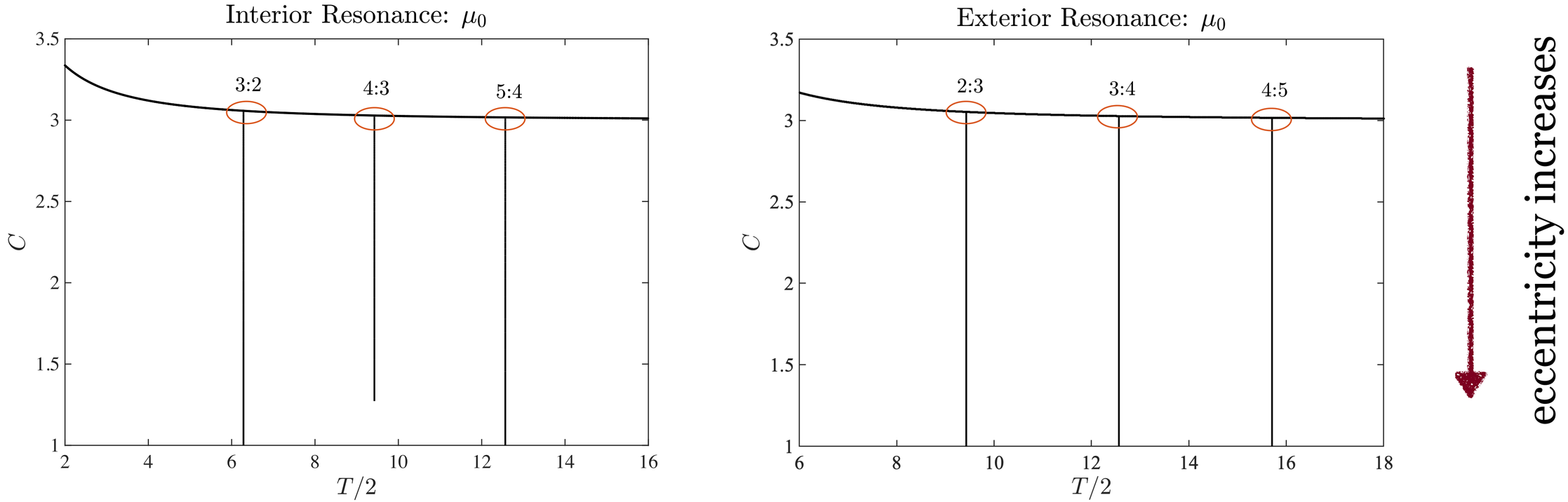
Methods



Periodic orbits of the first kind: **The periodic orbits related to the near-circular orbits of the small body**

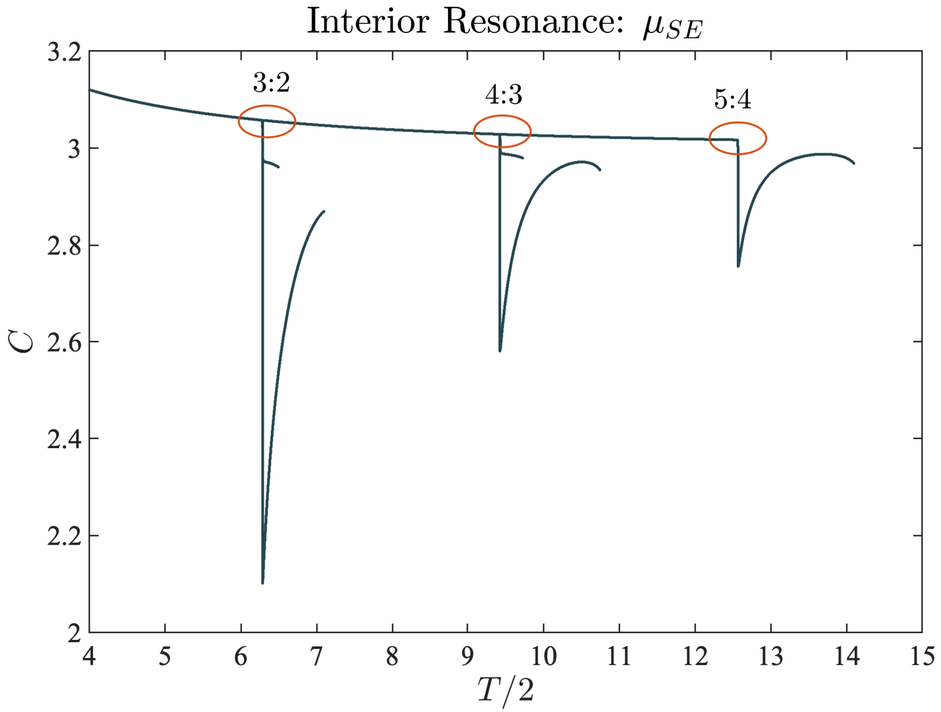
Periodic orbits of the second kind: **The periodic orbits related to elliptic orbits of the small body**

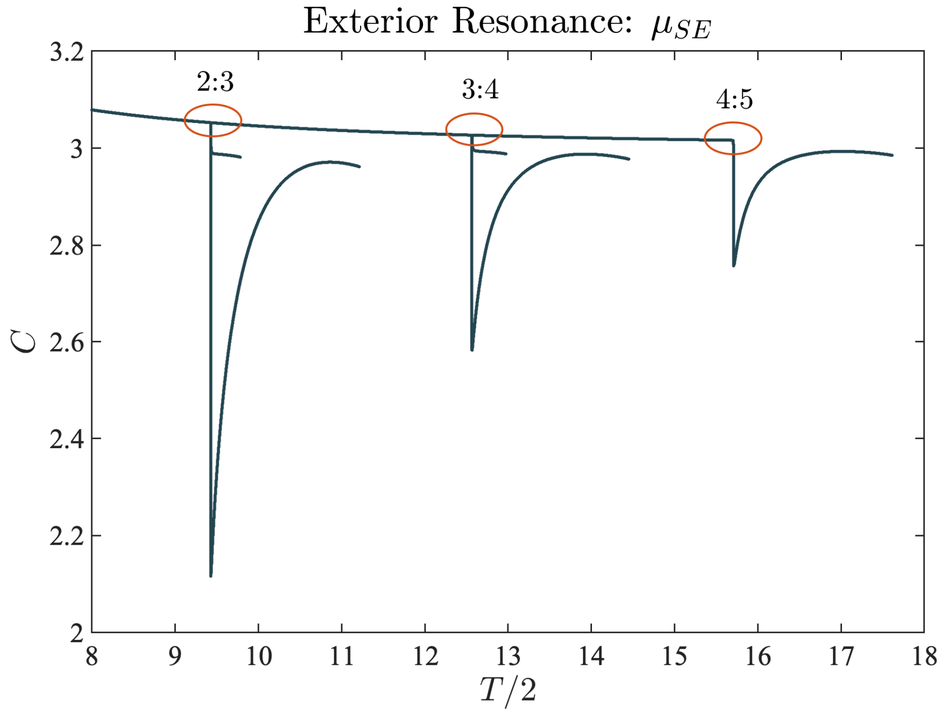
Two-body Problem



- **For each first-order resonance, two branches exist. They differ only in phase.**
  - **Branch-I: the perigee on the positive x-axis**
  - **Branch-II: the apogee on the positive x-axis**
- **First-order resonance separate with near-circular family at low eccentricities.**

Increase mass parameter μ

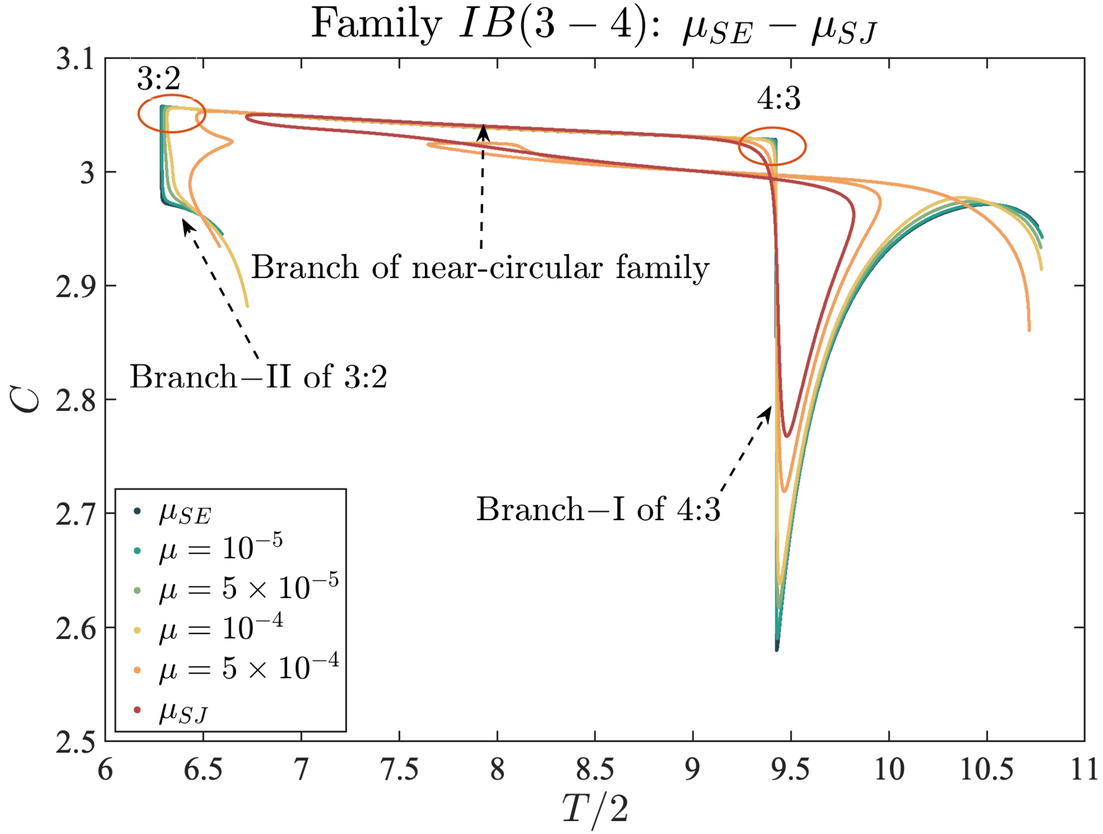




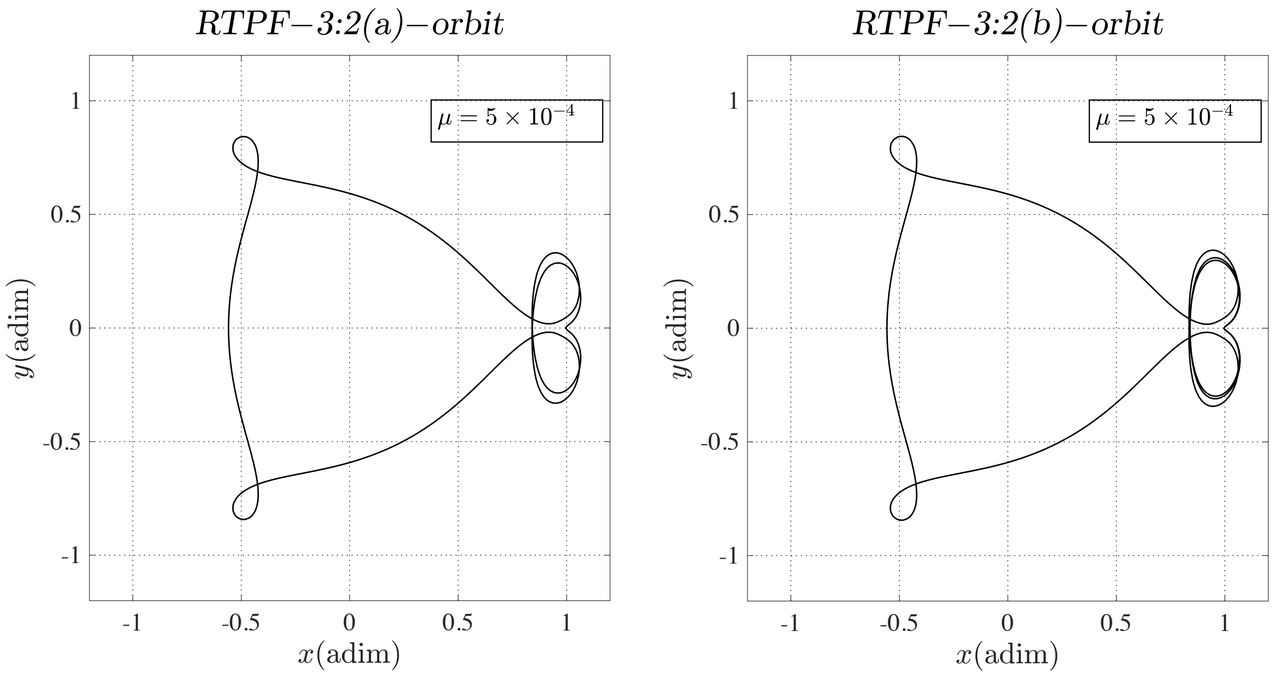
- **Divergence:** two-branches of each first-order resonant periodic family split
- **Combination:** first-order resonance combine with near-circular family at low eccentricities

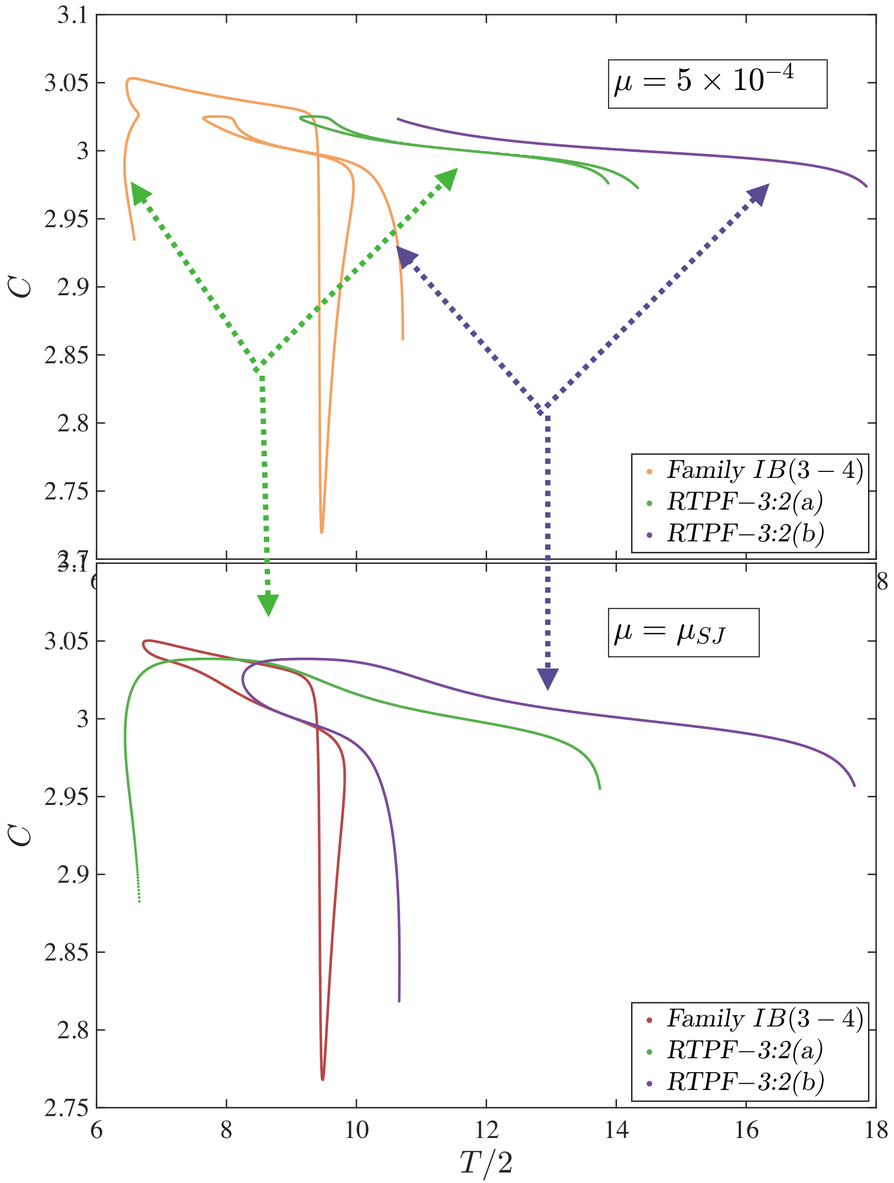
Continuation of μ

- **Near-circular family as a bridge**
- ***Family IB(3–4)*: family connect "Branch–II" of 3:2 resonance and "Branch–I" of 4:3 resonance.**
- **Break up and recombination happen in the process of increasing μ .**



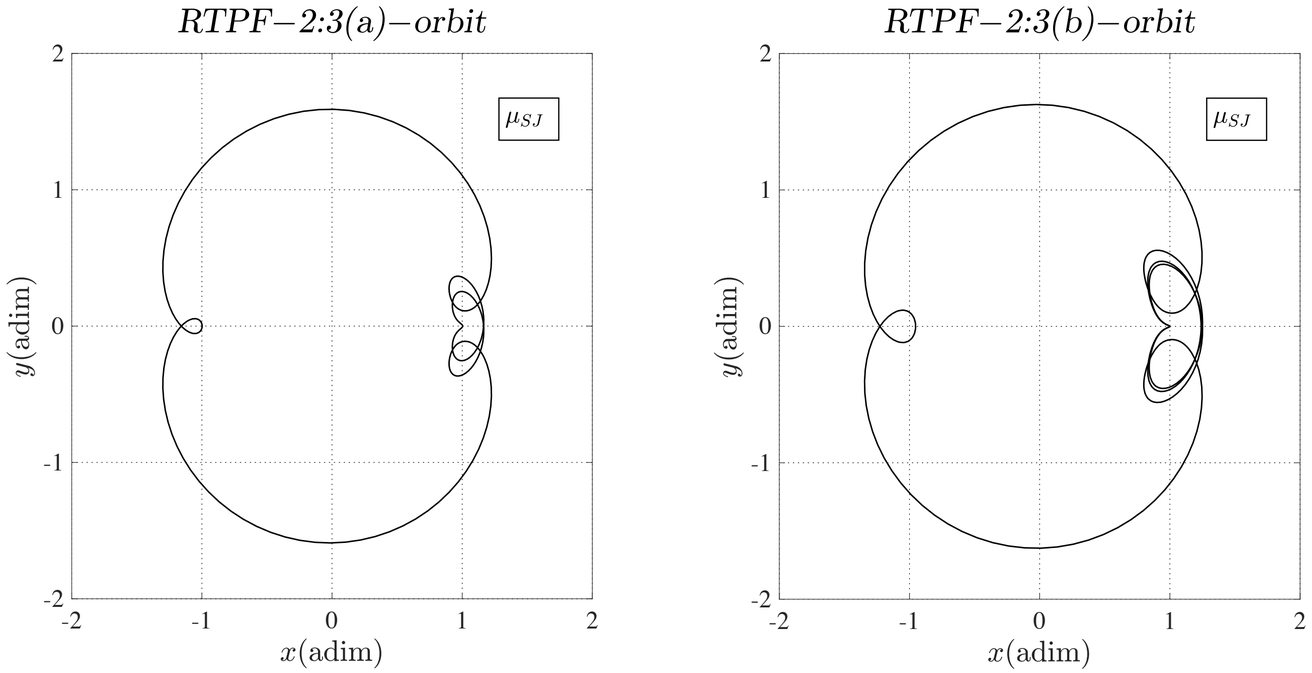
Recombination (1)

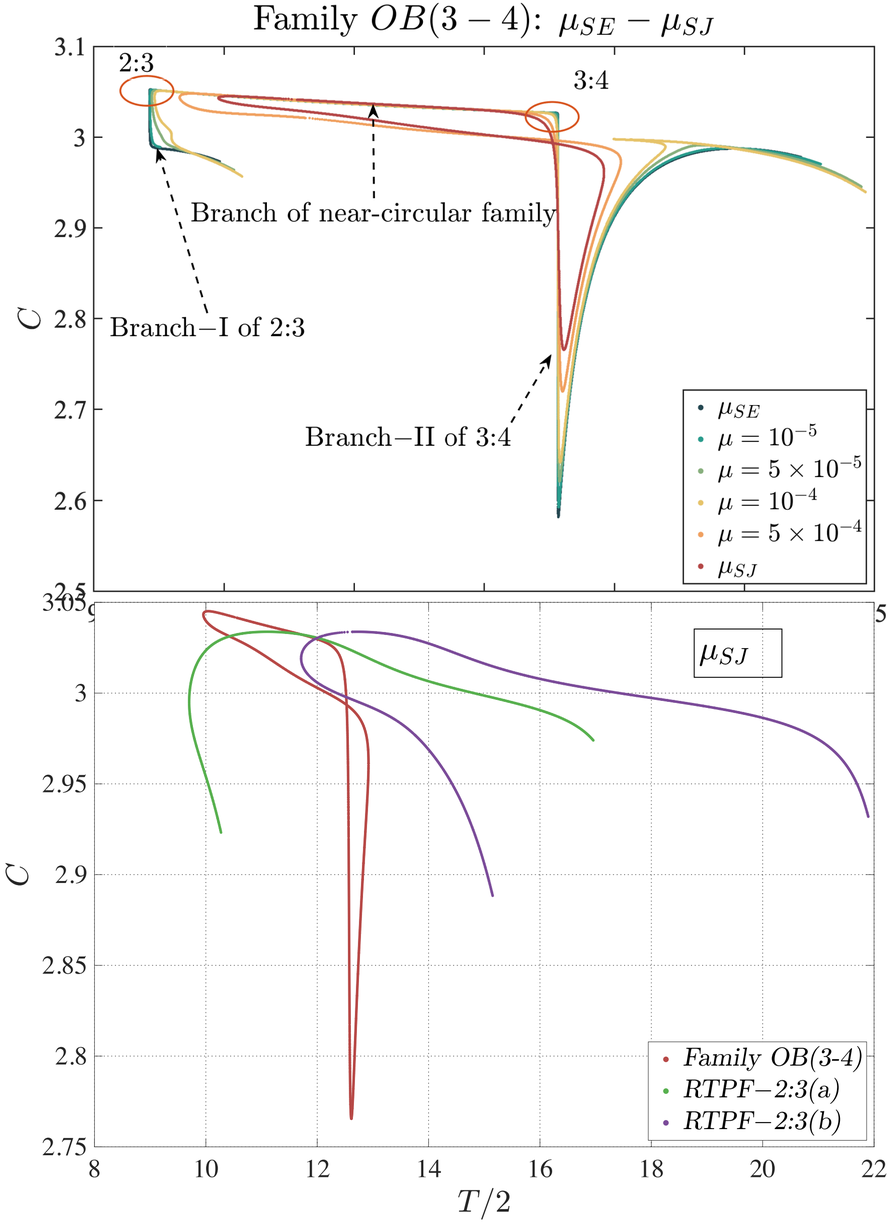




- ***RTPF−3:2*: resonance transition periodic family near 3:2 resonance**
- **Recombination of interior first-order resonance branches and *RTPF−3:2***
- **Orbital period difference related to the difference in numbers of revolution near the libration points**

Recombination (2)





- **Family OB(3-4): family connect "Branch-I" of 2:3 resonance and "Branch-II" of 3:4 resonance.**
- **Recombination of exterior first-order resonance branches and *RTPF-2:3***

Conclusion

- **The structure of the interior/exterior phase space from 1:1 resonance gradually deviates from that of 2BP model with progressively increasing μ.**

- **Resonance transition periodic orbits connecting the 1:1 resonance and the interior/exterior resonances appear with increasing μ.**

- **The obtained resonance transition periodic orbits have potential applications for the missions requiring long-term continuous observation of the secondary and tour missions in a multi-body environment.**