Numerical behavior of the Keplerian Integrals methods for initial orbit determination

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Structure

1. Introduction
2. The Keplerian Integrals methods
3. The data
4. Numerical results on synthetic data
5. Numerical results on real data
6. Applications

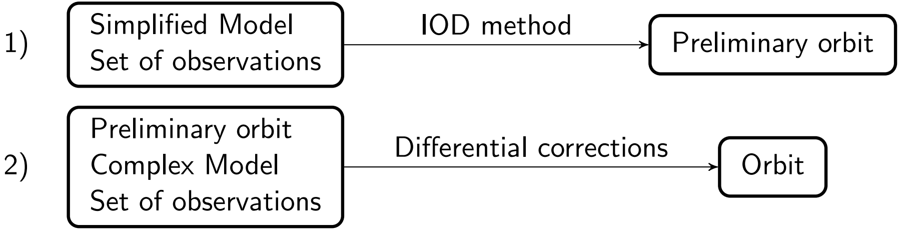
Introduction

Introduction

Initial orbit determination problem

Attempt to determine a preliminary orbit of an object from a set of observations.

- **Orbit determination**



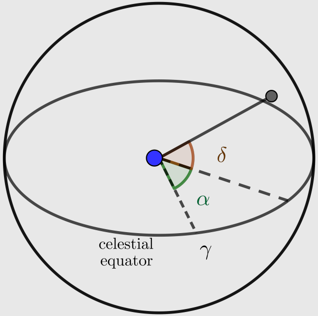
Question:

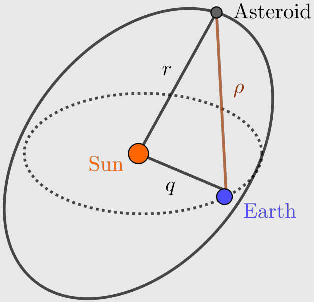
How is defined each set of observations?

Arc of observations

Optical observation

- Two angles (α, δ) giving a point on the celestial sphere.
- The topocentric distance of the asteroid ρ is unknown.





Arc of observations II

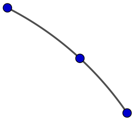
Arc of observations

- Set of consecutive observations of the same object.

$t = t_1$

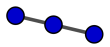
Computing a preliminary orbit

- If the length of the arc is sufficiently large:
  - Gauss
  - Laplace
  - ...



- If the length of the arc is too short (TSA)

⚠ We can not deal with only one TSA



The linkage problem

The linkage problem consists in trying to join two (or more) TSAs to determine an orbit.

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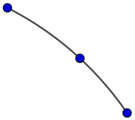

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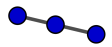

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Aim

Goal

Obtain linkages in a very large database.

Requirements for the method:

- Fast
- Recover a high percentage of solutions

We will study two methods:

- Link2 method (Gronchi et al, 2015) for linking two TSAs
- Link3 method (Gronchi et al, 2016) for linking three TSAs

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The Keplerian Integrals methods

Preliminaries:

Given a set of $m \geq 2$ optical observation, also called **tracklet**, $\{(\alpha_i, \delta_i) \,|\, i = 1, .., m\}$ obtained at different times t_i, $i = 1, ..., m$, it is possible to compute the attributable vector

$$\boldsymbol{\mathcal{A}} = (\alpha, \delta, \dot{\alpha}, \dot{\delta}),$$

at the mean time $\bar{t} = \frac{1}{n} \sum_{i=1}^{m} t_i$.

Goal

Determine ρ and $\dot{\rho}$ joining two or more tracklets and therefore a preliminary orbit.

The KI methods

The idea of the Keplerian integrals (KI) methods is to exploit the conservation laws of Kepler's dynamics to write down equations for the linkage.

The conserved quantities are

$$\boldsymbol{c} = \boldsymbol{r} \times \dot{\boldsymbol{r}}, \qquad \text{(angular momentum)}$$

$$\mathcal{E} = \frac{1}{2}|\dot{\boldsymbol{r}}|^2 - \frac{\mu}{|\boldsymbol{r}|}, \qquad \text{(energy)}$$

$$\boldsymbol{L} = \frac{1}{\mu}\dot{\boldsymbol{r}} \times \boldsymbol{c} - \frac{\boldsymbol{r}}{|\boldsymbol{r}|}, \quad \text{(Laplace-Lenz vector)}$$

Idea of the methods

Link2

Given two attributables $\mathcal{A}_1, \mathcal{A}_2$ we consider the system

$$\boldsymbol{c}_1 = \boldsymbol{c}_2, \qquad [\mu(\boldsymbol{c}_1 - \boldsymbol{c}_2) - (\mathcal{E}_1 \boldsymbol{r}_1 - \mathcal{E}_2 \boldsymbol{r}_2)] \times (\boldsymbol{r}_1 - \boldsymbol{r}_2) = \boldsymbol{0},$$

where the subscripts refer to the two epochs.

By elimination of variables we obtain a **polynomial equation of degree 9** in ρ_1 or ρ_2.

Link3

Given three attibutables $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_3$ we consider the system

$$\boldsymbol{c}_1 = \boldsymbol{c}_2, \qquad \boldsymbol{c}_2 = \boldsymbol{c}_3,$$

that is by using the conservation law of angular momentum only.

By elimination of variables we obtain a **polynomial equation of degree 8** in ρ_2.

Indicators: χ_4, $\Delta_\star$ and rms

The preliminary orbits computed with the both algorithms have an associated covariance matrix. From this information we use:

- The χ_4 norm for the `Link2` algorithm.
- The star norm ($\Delta_\star$) for the `Link3` algorithm.

Another metric that can be used to measure the quality of preliminary orbits is the rms of the observations

$$rms = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \Delta_{\alpha_i}^2 \cos^2\delta_i + \Delta_{\delta_i}^2}$$

with $\Delta_{\alpha_i} = \alpha_i - \alpha(\bar{t}_i), \Delta_{\delta_i} = \delta_i - \delta(\bar{t}_i)$, where the computed values $\alpha(\bar{t}_i), \delta(\bar{t}_i)$ in the residuals comes from a two-body propagation.

The Test dataset

Test dataset

The datasets that we consider are structured as follows:

- N different physical objetcs
- 3 TSAs per object
- different number of observations per TSA.

TSA 1	TSA 2	TSA 3	TSA 4	TSA 5	TSA 6	TSA 7	TSA 8	TSA 9	TSA 10	...
3 obs	3 obs	4 obs	3 obs	5 obs	3 obs	3 obs	4 obs	3 obs	3 obs	

Different datasets:

- Real observations
- Simulated 2-body propagation $\longleftarrow$ with $0''/0.1''/0.2''/0.5''/1.0''$ astrometric error
- Simulated n-body propagation $\longleftarrow$ with $0''/0.1''/0.2''/0.5''/1.0''$ astrometric error

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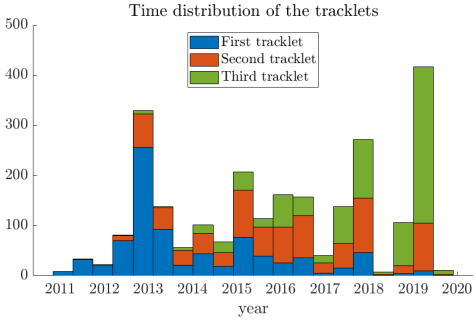
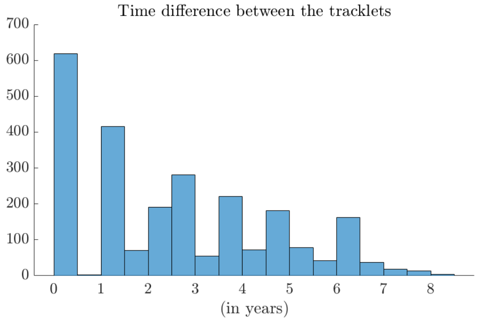
Test dataset

- We will consider the different datasets described previously with $N = 822$





Time distribution of the tracklets in the data sample (left) and time difference between pairs of tracklets for the same object (right).

Procedure

- **LK2**: For each dataset we will select 2 TSAs per object.

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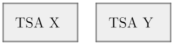
LK2



Procedure

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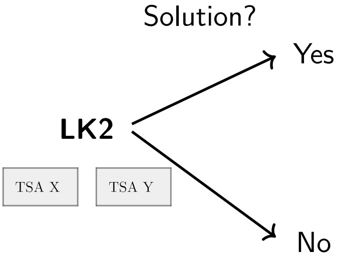

Procedure

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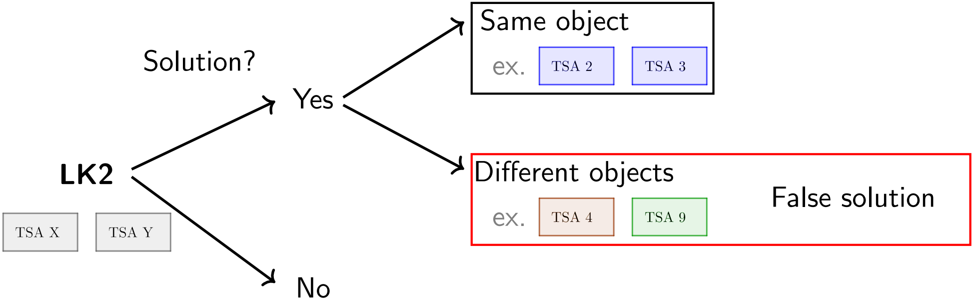

Procedure

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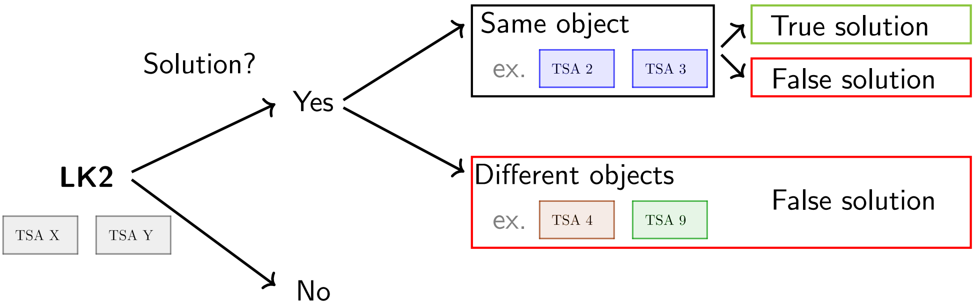


Questions

Two natural questions:

Question 1:

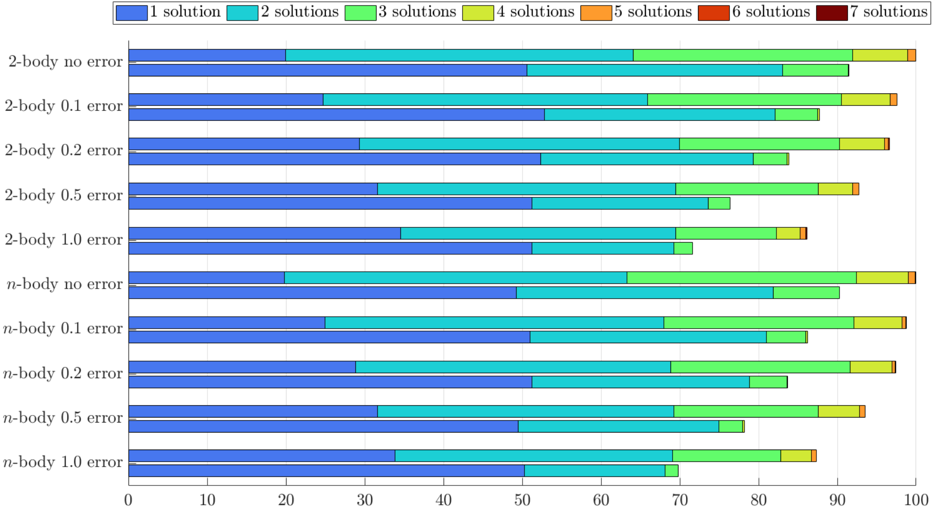
What % of true solutions have we recovered?

Question 2:

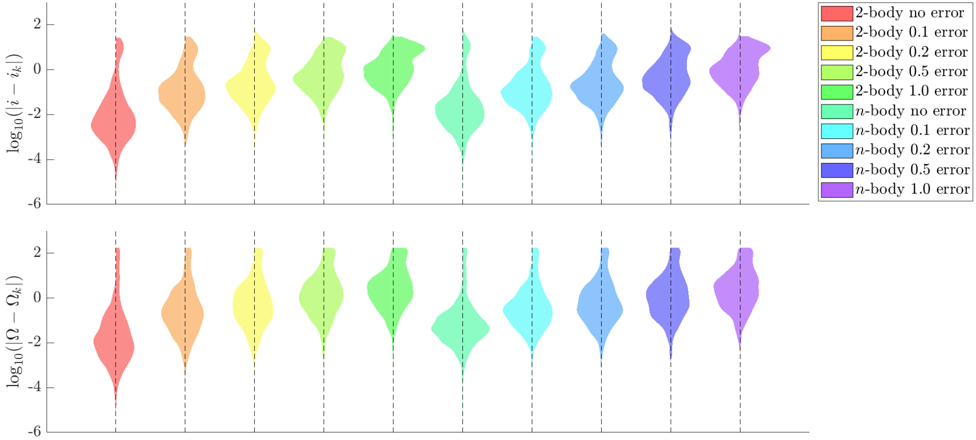
Which is the number of false solutions?

Synthetic data - Numerical results

Percentage of solutions recovered with each method

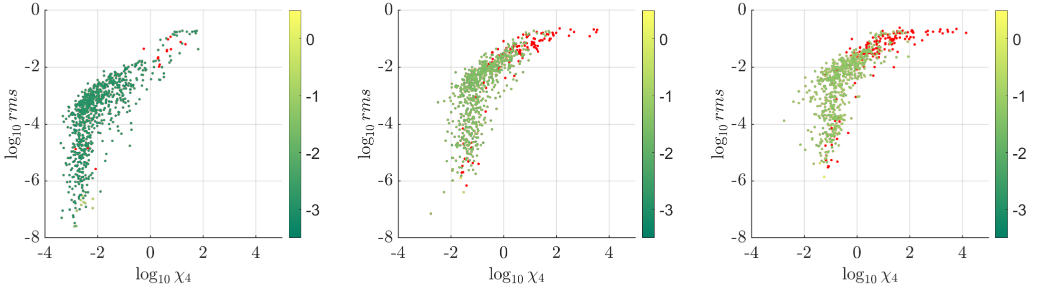


Quality of the solutions (ex. plane of motion)



Least square orbit

Values of the logarithm of the residual R_{LS} of the least squares orbits obtained from the preliminary orbits computed with `Link2` as a function of the logarithms of the χ_4 and the rms with the synthetic data using a n-body propagation without error, $0.1''$ error and $0.2''$ error (from left to right). In red the preliminary orbits that do not converge in the differential correction scheme.



Comments

Question 1: What % of true solutions have we recovered?

High! We recover a high percentage of solutions and the quality is good if we consider datasets with lower astrometric error.

But... How to deal with multiple solutions?

Answer: Selecting the best one using the χ_4, $\Delta_\star$, rms...

Number of false solutions

Question 2: Which is the number of false solutions?

We obtain a large number of false solutions

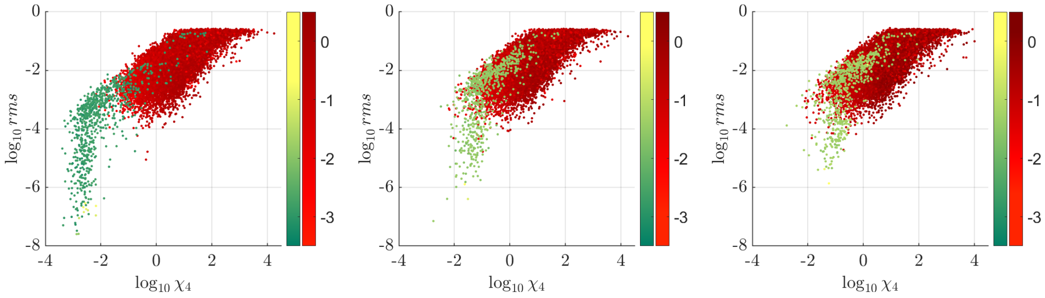
Example: With the dataset n-body $0.2''$ we obtain more than 1 million of false solutions.

How to deal with it?

Same answer: Using the χ_4, $\Delta_\star$, rms...

`Link2`: All vs all least squares orbits

Plot of the residual of the least square orbits of the true linkages (green scale) and false linkages (hot colors) with the synthetic data and a n-body propagation without error, 0.1″ error and 0.2″ error (from left to right).



Real data - Numerical results

Distribution of the astrometric error

In the synthetic data we assume that the error is independently distributed following a 2D Gaussian distribution.

In this way, the astrometric error in the observation (α_i, δ_i) is given by

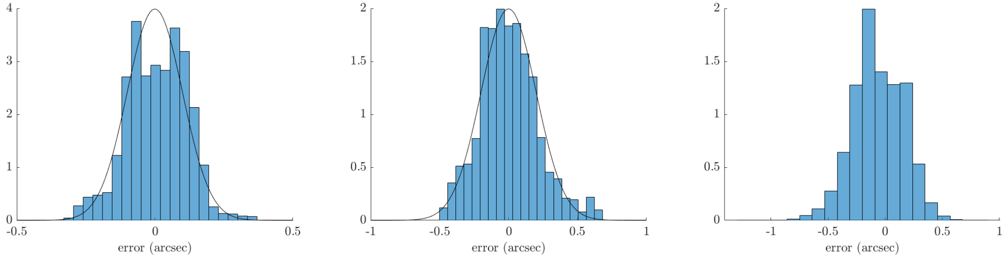
$$e_i = \mathsf{sign}(\Delta_{\alpha_i})\sqrt{\Delta_{\alpha_i}^2 \cos^2 \delta_i + \Delta_{\delta_i}^2} \sim N(0, \sigma^2),$$

where $\Delta_{\alpha_i} = \alpha_i^* - \alpha_i$ and $\Delta_{\delta_i} = \delta_i^* - \delta_i$ being (α_i^*, δ_i^*) the perfect observation.

And in the real data?

Distribution of the astrometric error

Normalized histogram of the astrometric error for the n-body data sets with $0.1''$ and $0.2''$ error and the real data (from left to right).



Distribution of the astrometric error

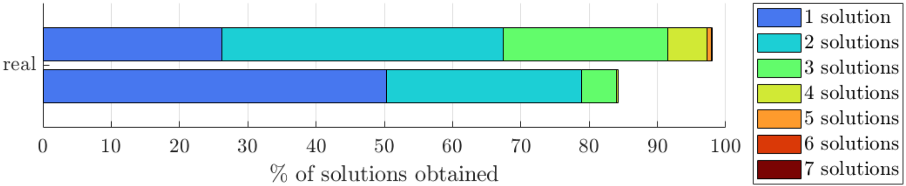
The distribution of the error is more or less normally distributed.

Dataset	Expected μ	Computed μ	Expected σ	Computed σ
n-body 0.1	0	0.0073	0.1	0.1088
n-body 0.2	0	0.0037	0.2	0.2099
real	-	0.0593	-	0.2227

But if we randomly select two observations per tracklet and compute the correlation between the errors of these observations we obtain a significant correlation (> 0.5) in the case of real data:

	n-body 0.1	n-body 0.2	real
Correlation	0.0365	0.0458	0.5206

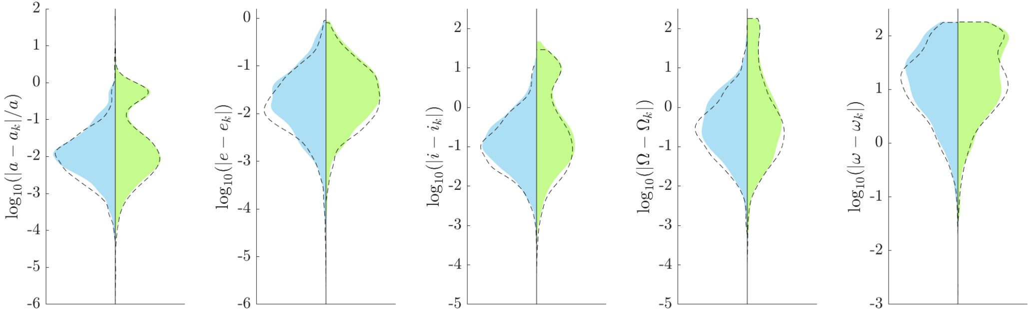
Percentage of solutions recovered with each method



Similar results

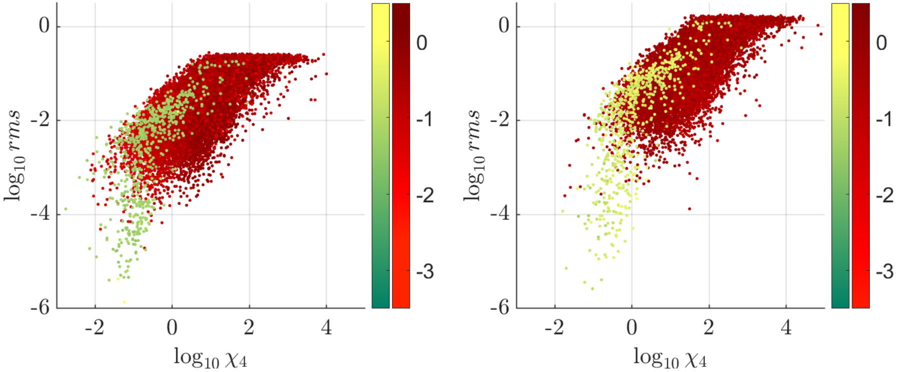
The percentage of recovered linkages with both methods is quite similar to the one obtained for the synthetic population (with 2-body or n-body propagation) with astrometric error of $0.2''$.

Quality of the solutions



`Link2`: All vs all least squares orbits

Plot of the residual of the least square orbits of the true linkages (green scale) and false linkages (hot colors) with the synthetic data and a n-body propagation with $0.2''$ error (left) and the real data (right).



Applications

The Isolated Tracklet File

ITF

The Isolated Tracklet File (ITF) from the Minor Planet Center (MPC) is a repository of more than 9 million unlinked detections.

What does the ITF look like?

```
CA1962I KC2019 11 24.24031 11 04 01.03 -07 33 14.7          17.9 R      954
CA1962I KC2019 11 24.24531 11 04 01.54 -07 33 28.9          17.8 R      954
CA1962I*KC2019 11 24.24719 11 04 01.68 -07 33 32.7          17.8 R      954
P10Wsaz  C2020 01 22.64206911 37 01.692+40 07 17.25         22.08wU     F51
P10Wsaz  C2020 01 22.64848811 37 02.125+40 07 32.60         22.18wU     F51
P10Wsaz  C2020 01 22.65490711 37 02.558+40 07 47.93         22.08wU     F51
P10Wsaz  C2020 01 22.66136711 37 03.001+40 08 03.35         21.88wU     F51
P20WZxd  C2020 01 25.49526 09 09 10.701+03 48 38.89         20.6 wU     F52
P20WZxd  C2020 01 25.50853 09 09 05.373+03 50 25.15         20.8 wU     F52
P20WZxd  C2020 01 25.52166 09 09 00.100+03 52 10.19         20.9 wU     F52
```

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Pan-STARRS1 (F51)

Pan-STARRS1:

The Panoramic Survey Telescope and Rapid Response System 1 (Pan-STARRS1).

https://panstarrs.stsci.edu/

Some data:

- > 4 million observations in the ITF
- > 1 million different tracklets in the ITF.

Pan-STARRS1 (F51)

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The Panoramic Survey Telescope and Rapid Response System 1 (Pan-STARRS1).

https://panstarrs.stsci.edu/

Some data:

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Applications and work in progress:

- We used `Link2` as a first step to do a complete exploration of the ITF F51.

And also...

- Application to space debris using the data provided by the TAROT Network (joint work with Carlos Yañez).

Thank you!

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