The equation between 3-body mean motion resonances and Yarkovsky drift speeds on eccentricities higher than 0.1

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Abstract

We studied the motion of asteroids across the 3-body mean motion resonances (MMRs) with Jupiter and Saturn and with the Yarkovsky drift speed in the semimajor axis of the asteroids. The research was conducted using numerical integrations performed using the Orbit9 integrator with 72,000 test asteroids. We calculated time delays, $\mathrm{d}tr$, caused by the six 3-body MMRs on the mobility of test asteroids with 10 positive and 10 negative Yarkovsky drift speeds, which are reliable for Main Belt asteroids. Our final results considered only test asteroids that successfully crossed over the MMRs without close approaches to the planets. We devised equations that approximately describe the functional relation between the average time $\langle \mathrm{d}tr \rangle$ spent in the resonance, the strength of the resonance SR, and the semimajor axis drift speed $\mathrm{d}a/\mathrm{d}t$ (positive and negative) with the orbital eccentricities of asteroids in the range (0.1, 0.2). Comparing the values of $\langle \mathrm{d}tr \rangle$ obtained from the numerical integrations and from the derived functional relations, we analysed average values of $\langle \mathrm{d}tr \rangle$ in all 3-body MMRs for every $\mathrm{d}a/\mathrm{d}t$. The main conclusion is that the analytical and numerical estimates of the average time $\langle \mathrm{d}tr \rangle$ are in very good agreement, for both positive and negative $\mathrm{d}a/\mathrm{d}t$. Finally, this study shows that the functional relation we obtain for 3-body MMRs for orbital eccentricities of asteroids in the range (0.1, 0.2) is analogous to that previously obtained for orbital eccentricities of asteroids in the range (0, 0.1) in Milic Žitnik 2021.

Motivation

- Gravitational and non-gravitational forces
- Mean motion resonances (MMRs)
- The Yarkovsky effect (Yarkovsky 1901)
- The collisions among the asteroids and the YORP effect are bound to introduce uncertainties in asteroids' orbital motion, but averagely and statistically, the calculations in these studies are reliable
- Semi-major axis drift under the influences of the Yarkovsky effect (Milić Žitnik and Novaković 2015, 2016 ; Milić Žitnik 2016)
- Asteroids with very small Yarkovsky drift speeds move extremely rapidly across MMRs (especially across strong MMRs) : absolute values less than 7×10^{-5} au/Myr (Milić Žitnik 2018a, 2018b, 2019, 2020, 2021). Therefore, we excluded that asteroids.

Numerical integrations for $e \in (0.1, 0.2)$

- Numerical integrations *ORBIT9* (Milani and Nobili 1988)
- 72,000 test asteroids
- Orbital motion was tracked depending on the strength of the resonance and on the value of the Yarkovsky drift speed
- Numerical method (Milić Žitnik and Novaković 2015, 2016)
- 6 isolated 3MMRs with Jupiter and Saturn
- 10 equidistant $\mathrm{d}a/\mathrm{d}t \in (-4 \times 10^{-5}, -2 \times 10^{-3})$ au/Myr and 10 equidistant $\mathrm{d}a/\mathrm{d}t \in (4 \times 10^{-5}, 2 \times 10^{-3})$
- Moment of entering (t_1) and exiting (t_2) from MMR
- Time lead/lag in MMRs : $\mathrm{d}tr = (t_2 - t_1) - \Delta a/(\mathrm{d}a/\mathrm{d}t)$
  $\Delta a = a_2 - a_1$ for moments t_2, t_1
  $\mathrm{d}a/\mathrm{d}t$ – the Yarkovsky drift speed

Results

Table: The properties of the 6 selected 3-body resonances. In the first column there are names of 3-body MMRs. In the second column it is showed their nominal semimajor axis. The widths of the 6 selected 3-body resonances for $e = 0.2$ are in the third column, propagated by the numerical method with the Orbit9. The last two columns contain their strengths SR, in cases of negative and positive $\mathrm{d}a/\mathrm{d}t$, calculated with the numerical method given by Gallardo (2014).

MMR	a_{res} [au]	width($e = 0.2$) [au]	SR $\mathrm{d}a/\mathrm{d}t < 0$	SR $\mathrm{d}a/\mathrm{d}t > 0$
1 :-3J :1S	2.75180	0.00350	0.00137283	0.00130213
2 :-7J :3S	2.55896	0.00338	0.00099704	0.00086637
2 :-7J :2S	2.44715	0.00200	0.00006268	0.00009325
1 :-5J :6S	2.75761	0.00158	0.00000826	0.00000679
3 :-6J :-1S	3.13787	0.00156	0.00001396	0.00003654
3 :-8J :1S	2.79909	0.00092	0.00000877	0.00001229

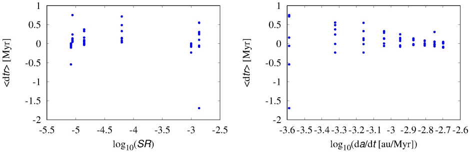

Figure: Relation between $\langle \mathrm{d}tr \rangle$ and $\log_{10}(SR)$ (left panel), and $\log_{10}(\mathrm{d}a/\mathrm{d}t)$ (right panel) without -4×10^{-5} au/Myr, for negative Yarkovsky drift speeds.

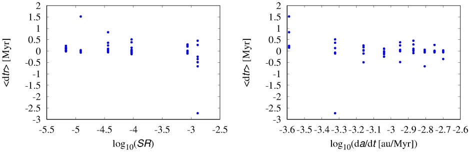

Figure: Relation between $\langle \mathrm{d}tr \rangle$ and $\log_{10}(SR)$ (left panel), and $\log_{10}(\mathrm{d}a/\mathrm{d}t)$ (right panel) without 4×10^{-5} au/Myr, for positive Yarkovsky drift speeds.

The equation of $\langle \mathrm{d}tr \rangle$, SR, $\mathrm{d}a/\mathrm{d}t$ for 3-body MMRs for e in the interval (0.1, 0.2)

$$\langle \mathrm{d}tr \rangle = (0.5 + \mathrm{d}a) \log_{10}(SR) + (\mathrm{d}b - 1.0) \log_{10}(\frac{\mathrm{d}a}{\mathrm{d}t}) + (\mathrm{d}c + 5.0) \quad (1)$$

for negative Yarkovsky drift speeds :
$\mathrm{d}a = -0.543 \pm 0.049$, $\mathrm{d}b = 0.990 \pm 0.166$, $\mathrm{d}c = -5.147 \pm 0.545$

for positive Yarkovsky drift speeds :
$\mathrm{d}a = -0.682 \pm 0.076$, $\mathrm{d}b = 0.729 \pm 0.244$, $\mathrm{d}c = -6.545 \pm 0.784$

Average time was expressed in Myr and the Yarkovsky drift speed was expressed in au/Myr.

The range of Equation 1 :

$$\sigma(\langle \mathrm{d}tr \rangle) = \sigma(\mathrm{d}a) \log_{10}(SR) + \sigma(\mathrm{d}b) \log_{10}(\frac{\mathrm{d}a}{\mathrm{d}t}) + \sigma(\mathrm{d}c) \quad (2)$$

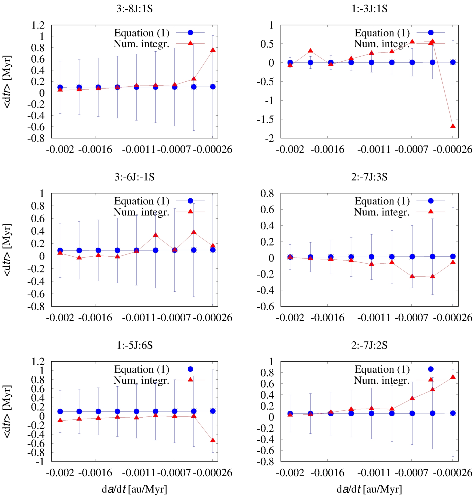

Figure: Values of $\langle \mathrm{d}tr \rangle$ obtained from Equation 1 in the 6 three-body resonances for the smallest 9 negative $\mathrm{d}a/\mathrm{d}t$ for eccentricity in the interval (0.1, 0.2). Outcomes from Equation 1 are showed with 3σ interval error of $\langle \mathrm{d}tr \rangle$ from Equation 2.

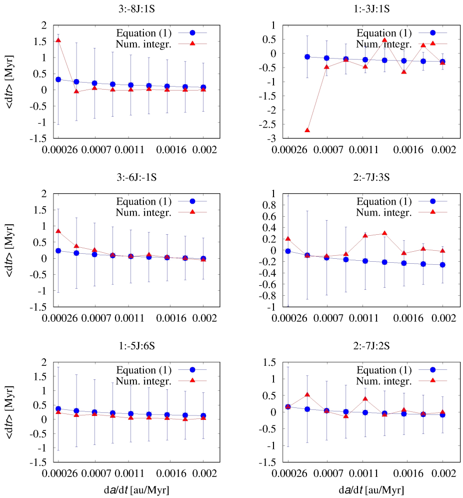

Figure: Values of $\langle \mathrm{d}tr \rangle$ obtained from Equation 1 in the 6 three-body resonances for the largest 9 positive $\mathrm{d}a/\mathrm{d}t$ for eccentricity in the interval (0.1, 0.2). Outcomes from Equation 1 are showed with 3σ interval error of $\langle \mathrm{d}tr \rangle$ from Equation 2.

Conclusions

- Here it is presented a portrayal of the orbital properties of asteroids in 3-body resonances due to the impact of the Yarkovsky force for orbital eccentricity in the interval (0.1, 0.2).
- We derived Equation 1 that joined the $\langle \mathrm{d}tr \rangle$ that an asteroid spent inside a three-body MMR, the SR and the $\mathrm{d}a/\mathrm{d}t$ for eccentricity in the interval (0.1, 0.2).
- The equation enables quick propagation of the $\langle \mathrm{d}tr \rangle$ in a three-body resonance with known SR in the interval $[6.79 \times 10^{-6}, 1.37 \times 10^{-3}]$, with the negative and positive Yarkovsky drift speeds in the interval $[2.6 \times 10^{-4}, 2 \times 10^{-3}]$ au/Myr and with an asteroid's orbital eccentricity in the interval (0.1, 0.2).

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THANK YOU FOR YOUR ATTENTION !