

**Updates for
Some open problems on Coxeter groups and unimodality**

1. Unknown to me (and to John Stembridge, and to the author) Problem 1.1 had been solved in the affirmative in [R. De Man, *The generating function for the number of roots of a Coxeter group*, J. Symbolic Computation, **27**(1999), 535–541].
2. Conjecture 1.4, for the R -polynomials, has been proved for elementary intervals of the symmetric group in [G. Barkley, C. Gaetz, *Combinatorial invariance for Kazhdan-Lusztig R -polynomials of elementary intervals*, Math. Ann. **392**(2025), 3299–3317].
3. Conjectures 1.4 and 1.6 have been proved to be equivalent in [G. Barkley, C. Gaetz, *On combinatorial invariance of parabolic Kazhdan-Lusztig polynomials*, Selecta Math. (N.S.) **31**(2025), Paper No. 51, 9 pp.].
4. Conjectures which are analogous to Conjecture 1.7, but for the R -polynomials (i.e., explicit algorithms for computing the Kazhdan-Lusztig R -polynomial of a pair of elements u, v in a symmetric group starting from the Bruhat interval $[u, v]$ as an abstract poset) have been proposed in [F. Brenti, M. Marietti, *Kazhdan-Lusztig R -polynomials, combinatorial invariance, and hypercube decompositions*, Math. Zeitschrift **309**(2025), Paper No. 25, 19 pp., Conj. 6.1] and [G. Barkley, C. Gaetz, *Combinatorial invariance for Kazhdan-Lusztig R -polynomials of elementary intervals*, Math. Ann. **392**(2025), 3299–3317, Conj. 1.3]. No logical implications between these two conjectures, or between any of them and Conjecture 1.7, are known.
5. Conjecture 1.7 has been proved for lower intervals (i.e., intervals $[e, v]$ where e is the identity) in [G. Barkley, C. Gaetz, *The BBDVW Conjecture for Kazhdan-Lusztig polynomials of lower intervals*, Int. Math. Res. Not., to appear]