$$\frac{5}{4} i z^{10} + \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) z^5 - \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right) = 0$$

$$\frac{\sqrt{3}}{2} + \frac{i}{2} = e^{i\pi/6}$$

$$\frac{\sqrt{3}}{2} - \frac{i}{2} = e^{-i\pi/6}$$

$$\frac{5}{4} i z^{10} + e^{i\pi/6} z^5 - e^{-i\pi/6} = 0$$

$$W = z^5$$

$$\frac{5}{4} i W^2 + e^{i\pi/6} W - e^{-i\pi/6} = 0$$

$$W_{\pm} = \frac{-e^{i\pi/6} \pm \sqrt{|\Delta|}\, e^{i\theta/2}}{\frac{5}{2} i}$$

$$\Delta = e^{i\frac{\pi}{3}} - 5i\left(-e^{-i\frac{\pi}{6}}\right)$$

$$\theta = \arg(\Delta)$$

$$\Delta = e^{i\frac{\pi}{3}} + 5i\, e^{-i\pi/6} =$$

$$= e^{i\frac{\pi}{3}} + 5\, e^{i\pi/2 - i\pi/6} = \left\{\theta = \frac{\pi}{3}\right\}$$

$$= e^{i\pi/3} + 5\, e^{i\pi/3} = 6\, e^{i\frac{\pi}{3}} = |\Delta| e^{i\theta}$$

$$\sqrt{|\Delta|}\, e^{i\theta/2} = \sqrt{6}\, e^{i\pi/6}$$

$$W_{\pm} = \frac{-e^{i\pi/6} \pm \sqrt{|\Delta|}\, e^{i\theta/2}}{\frac{5}{2} i}$$

$$\Delta = |\Delta| e^{i\theta}, \quad \eta^2 = \Delta \longrightarrow \eta = \pm \sqrt{|\Delta|}\, e^{i\frac{\theta}{2}}$$

$$W_{\pm} = \frac{-e^{i\pi/6} \pm \sqrt{6}\, e^{i\pi/6}}{5/2\, i} =$$

$$= \frac{2}{5} \frac{\left(-1 \pm \sqrt{6}\right) e^{i\frac{\pi}{6}}}{i} =$$

$$= \begin{cases} \frac{2}{5}\left(\sqrt{6}-1\right) e^{i\pi/6 - i\pi/2} = \frac{2}{5}\left(\sqrt{6}-1\right) e^{-i\frac{\pi}{3}} \\ -\frac{2}{5}\left(\sqrt{6}+1\right) e^{i\pi/6 - i\frac{\pi}{2}} = \end{cases}$$

$$= \frac{2}{5}\left(\sqrt{6}+1\right) e^{i\pi/6 - i\pi/2 + i\pi} = \frac{2}{5}\left(\sqrt{6}+1\right) e^{i\frac{2}{3}\pi}$$

$$\begin{cases} W_+ = \frac{2}{5}\left(\sqrt{6}-1\right) e^{-i\pi/3} \\ W_- = \frac{2}{5}\left(\sqrt{6}+1\right) e^{i\frac{2}{3}\pi} \end{cases}, \quad W = z^5$$

$$z^5 = \frac{2}{5}\left(\sqrt{6}-1\right) e^{-i\pi/3}$$

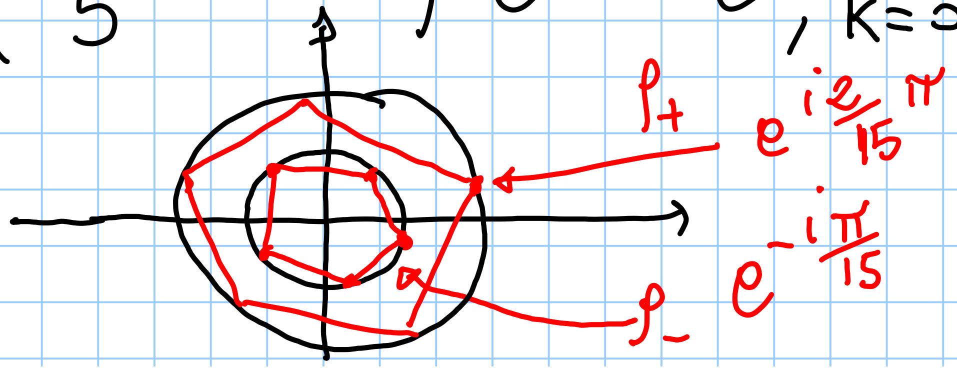
$$z^5 = \frac{2}{5}\left(\sqrt{6}+1\right) e^{i\frac{2}{3}\pi}$$

$$z_k = \left(\frac{2}{5}\left(\sqrt{6}-1\right)\right)^{\frac{1}{5}} e^{-i\pi/15} e^{i\frac{2k\pi}{5}}, \quad k=0,1,\dots,4$$

$\rho_-$

$$z_k = \left(\frac{2}{5}\left(\sqrt{6}+1\right)\right)^{1/5} e^{i\frac{2}{15}\pi} e^{i\frac{2k\pi}{5}}, \quad k=0,1,\dots,4$$

$\rho_+$



$$az^2+bz+c=0$$

$$z_\pm = \frac{-b \pm \sqrt{|\Delta|}\, e^{i\theta/2}}{2a}$$

$$\Delta = |\Delta|\, e^{i\theta}$$

Se $\Delta = w^2 \longrightarrow \sqrt{|\Delta|}\, e^{i\theta/2} = w$

DIMOSTRAZIONE

$w = |w|\, e^{i\phi}$, $w^2 = |w|^2 e^{i2\phi}$

$\Delta = w^2 \longrightarrow |\Delta| = |w|^2$, $\arg(\Delta) = 2\phi$

$$\sqrt{|\Delta|}\, e^{i\frac{\arg(\Delta)}{2}} = |w|\, e^{i\phi} = w$$

$\blacksquare$

$$P_3(z) = z^3 - (2+i)z^2 + 2(1+i)z - 2i$$

(1) VERIFICARE CHE $P_3(i) = 0$

(2) CALCOLARE $\dfrac{P_3(z)}{z-i}$

(3) DETERMINARE TUTTE LE SOL. di $P_3(z) = 0$

$$P_3(i) = (i)^3 - (2+i)(i)^2 + 2(1+i)i - 2i$$

$$= -i + 2 + i + 2i - 2 - 2i = 0$$

(2)

$$
\begin{array}{llll|l}
z^3 & -(2+i)z^2 & +2(1+i)z & -2i & z-i \\
\cline{5-5}
z^3 & -iz^2 & +0 & +0 & z^2-2z+2 \\
\cline{1-4}
0 & -2z^2 & +2(1+i)z & -2i & \\
 & -2z^2 & +2iz & +0 & \\
\cline{2-4}
 & 0 & 2z & -2i & \\
 & & 2z & -2i & \\
\cline{3-4}
 & & 0 & 0 &
\end{array}
$$

$$\frac{P_3(z)}{z-i} = z^2 - 2z + 2$$

$$P_3(z) = (z-i)(z^2 - 2z + 2)$$

(3) $z^2 - 2z + 2 = 0$ , $\Delta = -4$

$$z_{\pm} = \frac{+2 \pm \sqrt{|\Delta|}\, e^{i\frac{\theta}{2}}}{2} =$$

$$= \frac{2 \pm \sqrt{|4|}\, e^{i\frac{\pi}{2}}}{2} = \frac{2 \pm 2 e^{i\frac{\pi}{2}}}{2} =$$

$$= 1 \pm i$$

$z_1 = i$, $\quad z_2 = 1 + i$, $\quad z_3 = 1 - i$

# ESERCIZI DI RIEPILOGO

$$\int_0^{+\infty} \underbrace{\frac{e^{-x}\left[\log(x+\cos(x)) - x + x^2\right]}{x^\beta}}_{f_\beta(x)}\,dx$$

DISCUTERE LA CONVERGENZA DELL'INTEGRALE IMPROPRIO

$$\int_0^{+\infty} f_\beta(x)\,dx = \int_0^1 f_\beta(x)\,dx + \int_1^{+\infty} f_\beta(x)\,dx$$

$$x + \cos(x) \geqslant 1 \quad \forall\, x \in [1, +\infty)$$

$$\int_1^{+\infty} f_\beta(x)\,dx \quad ; \quad g(x) = \frac{1}{x^2}, \quad x \in [1, +\infty)$$

$$\lim_{x \to +\infty} \frac{f_\beta(x)}{g(x)} = \lim_{x \to +\infty} x^2 \frac{e^{-x}}{x^\beta} x^2 (1 + o(1)) =$$

$$= \lim_{x \to +\infty} e^{-x} x^{4-\beta} (1 + o(1)) = 0$$

T. CONFRONTO ASINTOTICO

CON $L = 0 \implies \int_1^{+\infty} g(x)\,dx$ CONVERGE

$$\Downarrow$$

$\int_1^{+\infty} f_\beta(x)\,dx$ CONVERGE $\forall \beta \in \mathbb{R}$

$$\int_0^1 f_\beta(x)\,dx$$

$\beta \le 0$,

$f_\beta \in \mathcal{R}(0,1)$

$\boxed{\beta > 0}$

$$f_\beta(x) = \frac{e^{-x}}{x^\beta}\Big[\underbrace{\log(x+\cos(x)) - x + x^2}_{h(x)}\Big]$$

$$\log(x+\cos(x)) - x + x^2 =$$

$$\log\left(x + 1 - \frac{x^2}{2} + o(x^2)\right) - x + x^2 =$$

$$= \log\left(1 + x - \frac{x^2}{2} + o(x^2)\right) - x + x^2$$

$$= \left(x - \frac{x^2}{2} + o(x^2)\right) + o\left(x - \frac{x^2}{2} + o(x^2)\right) - x + x^2$$

$$\log(1+y) = y + o(y), \quad y = x - \frac{x^2}{2} + o(x^2)$$

$$= \cancel{x} - \frac{x^2}{2} + o(x^2) + o(x) \cancel{- x} + x^2 =$$

$$= -\frac{x^2}{2} + o(x) + x^2 = o(x)$$

$$\log(1+y) = y - \frac{y^2}{2} + o(y^2)$$

$$= \left(x - \frac{x^2}{2} + o(x^2)\right) - \frac{1}{2}\left(x - \frac{x^2}{2} + o(x^2)\right)^2 + o\left(x - \frac{x^2}{2} + o(x^2)\right)^2$$

$$- x + x^2$$

$$= \cancel{x} - \cancel{\frac{x^2}{2}} + o(x^2) - \cancel{\frac{x^2}{2}} + o(x^2) + o(x^2) \cancel{-x} + \cancel{x^2}$$

$$= o(x^2)$$

$$\log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3)$$

$$\left(x - \frac{x^2}{2} + \underline{o(x^3)}\right) - \frac{1}{2}\left(x - \frac{x^2}{2} + o(x^3)\right)^2 +$$

$$+ \frac{1}{3}\left(x - \frac{x^2}{2} + o(x^3)\right)^3 +$$

$$\boxed{\cos(x) = 1 - \frac{x^2}{2} + o(x^3)}$$

$$o\left(\left(x - \frac{x^2}{2} + o(x^3)\right)^3\right) - x + x^2 =$$

$$\cancel{x} - \cancel{\frac{x^2}{2}} + o(x^3) - \cancel{\frac{1}{2}x^2} + \frac{1}{2}x^3 + o(x^3) + \frac{1}{3}x^3 + o(x^3)$$

$+o(x^3) \quad \cancel{-x} + \cancel{x^2} = \frac{5}{6}x^3 + o(x^3)$

$$f_\beta(x) \underset{x\to 0}{=} \frac{(1+o(1))}{x^\beta}\, x^3 \left(\frac{5}{6}+o(1)\right) =$$

$$= \frac{1}{x^{\beta-3}}\left(\frac{5}{6}+o(1)\right)$$

T. del CONFRONTO ASINTOTICO CON $L = \frac{5}{6}$ $\Longrightarrow$ L'INTEGRALE IMPROPRIO CONVERGE SE E SOLO SE

$$\beta - 3 < 1$$

$$\boxed{\beta < 4}$$