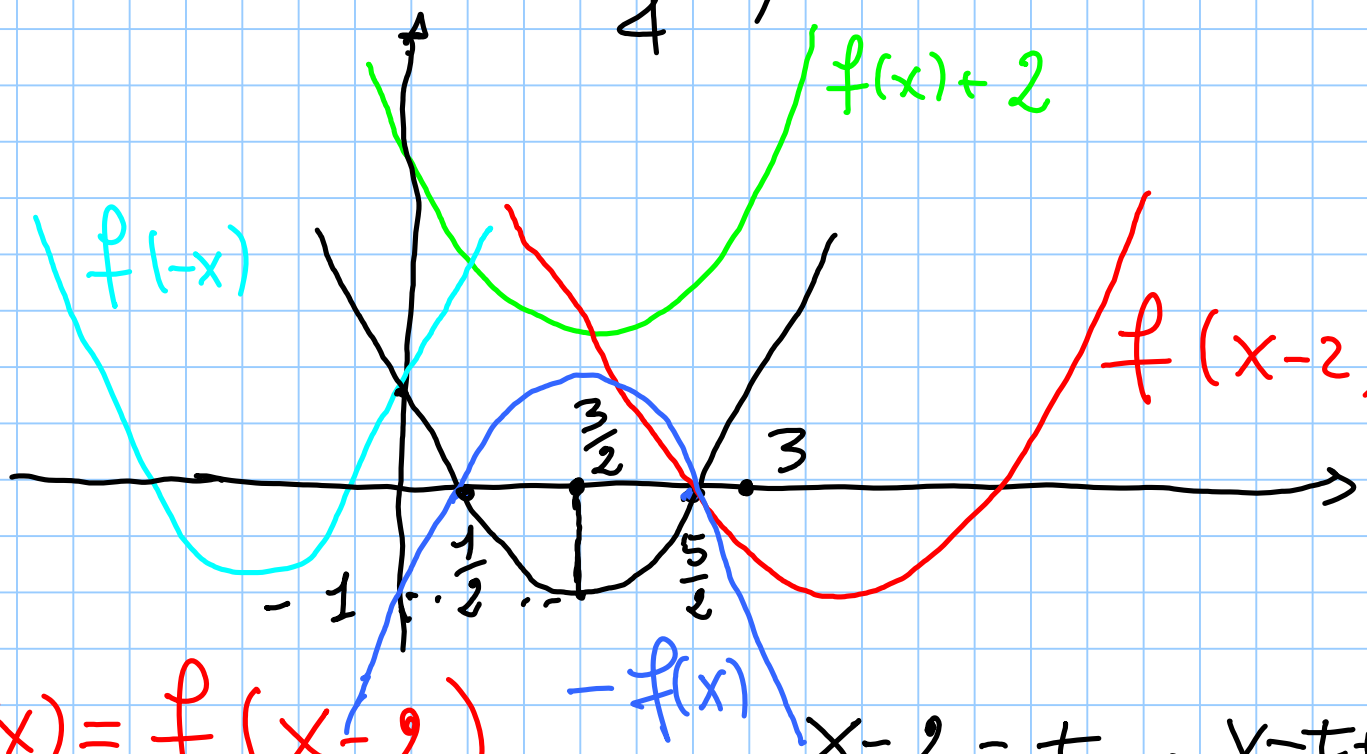
$$f(x)=x^2-3x+\frac{5}{4}\ ,\ x\in\mathbb{R}$$



$$g(x)=f(x-2)$$

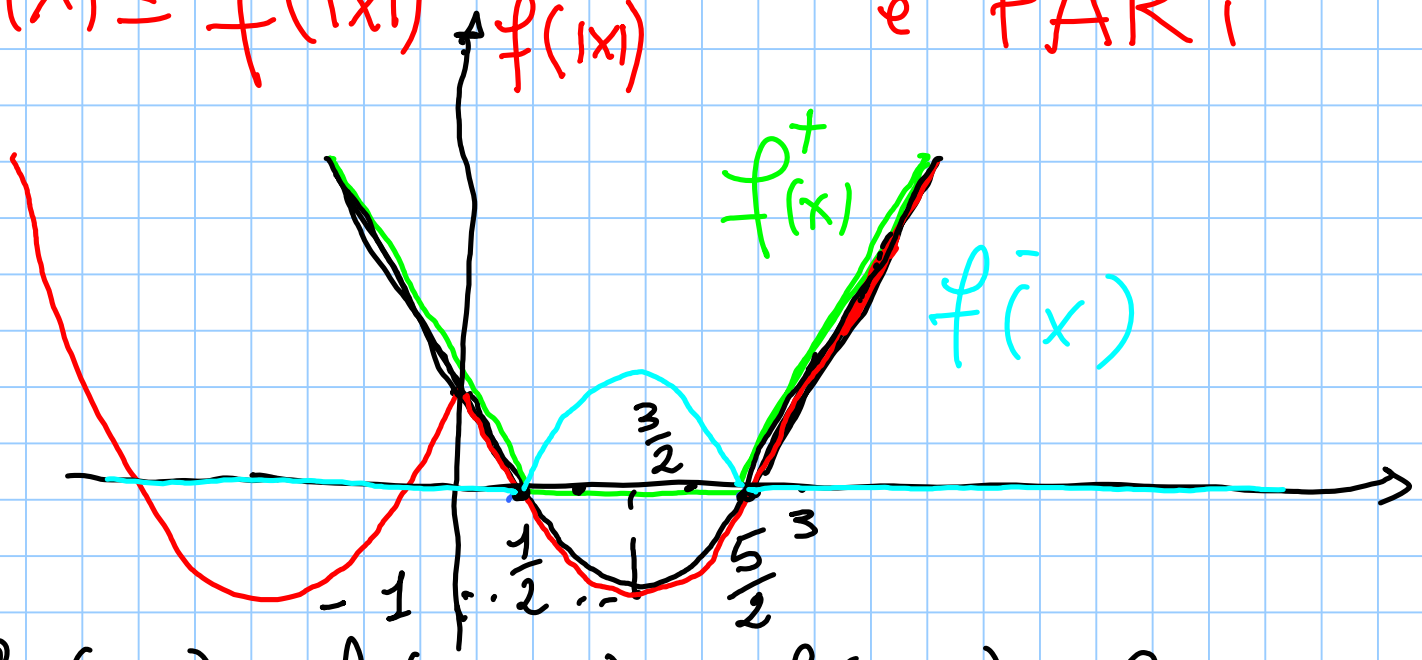
$$x-2=t\ ,\ \underline{x=t+2}$$

$$f(x-2)=f(t)$$

$$g(x)=f(x)+2\ ,\ g(x)=-f(x)$$

$$g(x)=f(-x)$$

$g(x) = f(|x|)$ è PARI



$$g(-x) = f(|-x|) = f(|x|) = g(x)$$

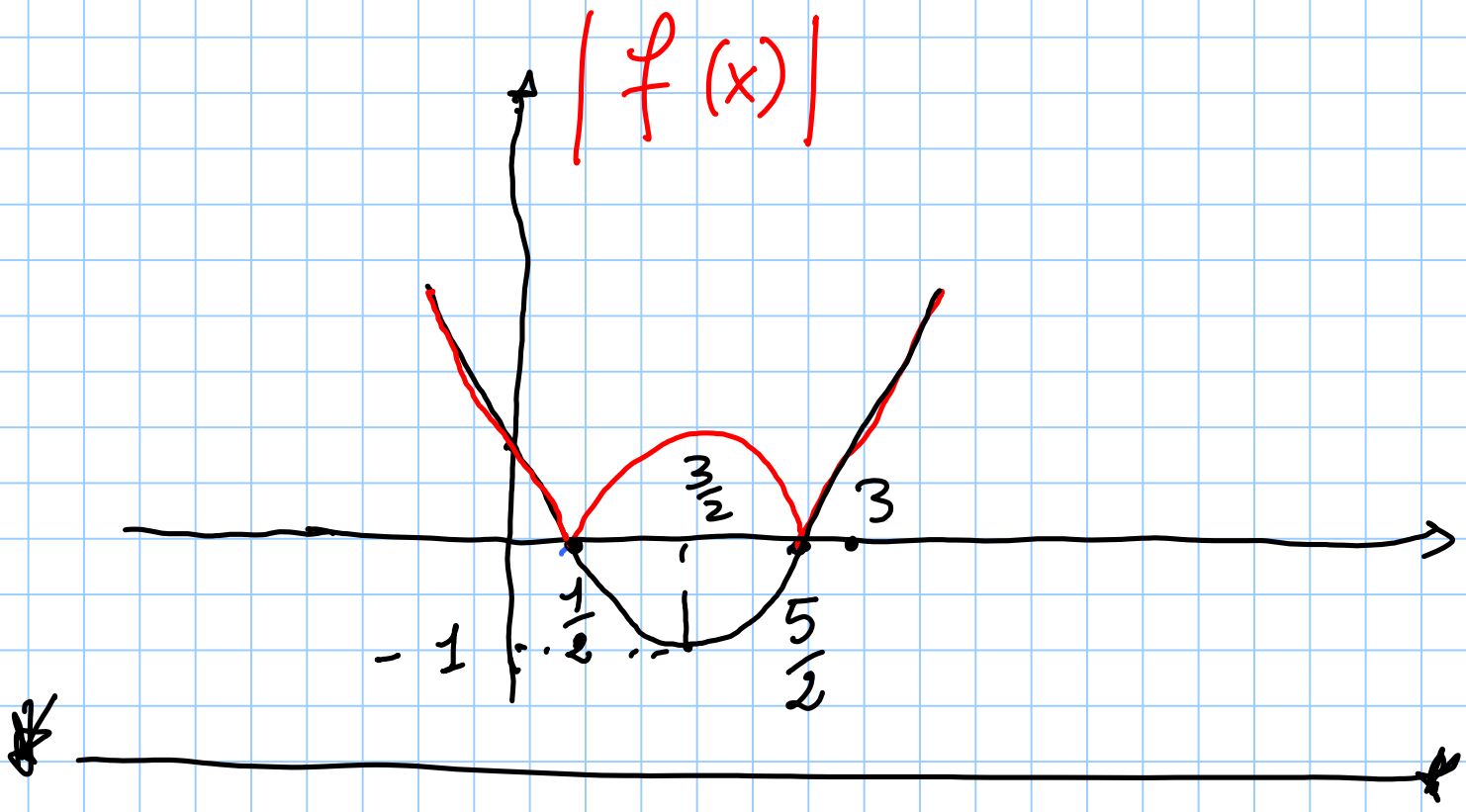
$f(x^2)$ è PARI

$f(x^{2m})$ $\quad m \in \mathbb{Z} \setminus \{0\}$

$g(x) = f^+(x)$ PARTE POSITIVA di f

$g(x) = f^-(x)$ PARTE NEGATIVA di f

$$g(x) = |f(x)| = f^+(x) + f^-(x)$$



$g(x) = x^2 - x \quad , x \in \mathbb{R}$

$h(y) = 3^y \quad , y \in \mathbb{R}$

Posso sostituire $x^2 - x$ al posto di y?

$$x \in dom(g) \overset{?}{\Longrightarrow} g(x) \in dom(h)$$

RISPOSTA:
SI PERCHE' $Im(g) \subseteq dom(h)$

$$Im(g) = \left[-\frac{1}{4}, +\infty\right) \subseteq \mathbb{R}$$

FUNZIONE COMPOSTA

$$f = h \circ g$$

$$f(x) = h\left(g(x)\right), \quad x \in V$$

$$V = dom(g) = \mathbb{R}$$

$$f(x) = 3^{x^2 - x}, \quad x \in \mathbb{R}$$

$$g(x) = x^2 - x, \quad x \in dom(g) = \mathbb{R}$$

$h(y) = \log_2(y)$, $y \in \text{dom}(h) = (0, +\infty)$

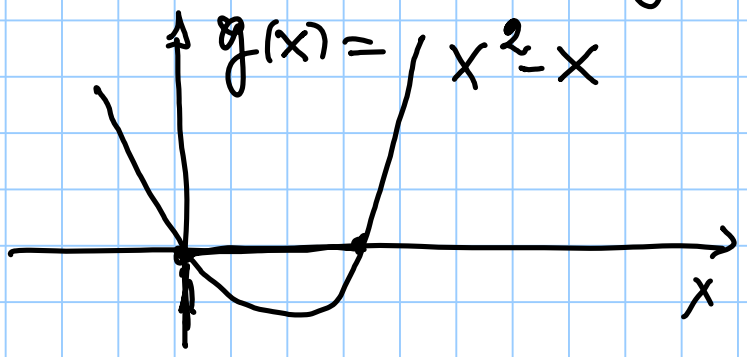
È VERO CHE

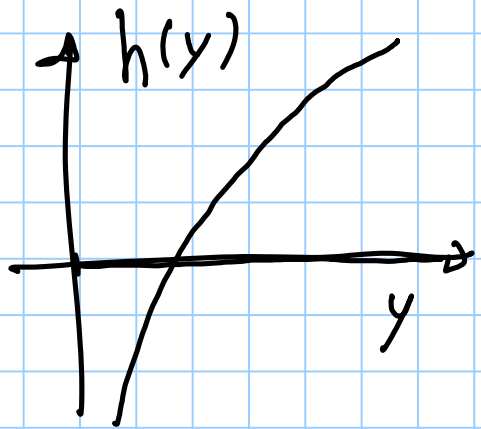
$$\text{Im}(g) \subseteq \text{dom}(h) ?$$

$$\text{Im}(g) = \left[-\frac{1}{4}, +\infty\right)$$

$$x^2 - x > 0 \iff x < 0 \text{ e } x > 1$$

$$x \in [0,1] \Rightarrow g(x) \notin \text{dom}(h)$$





$$f(x) = h(g(x)), \quad x \in V$$

$$f = h \circ g$$

$$V = \{x \in \text{dom}(g) : g(x) \in \text{dom}(h)\}$$

$$= \{x \in \mathbb{R} : x^2 - x > 0\} =$$

$$= (-\infty, 0) \cup (1, +\infty)$$

DEFINIZIONE

Sia $g : X \longrightarrow Z$, $h : W \longrightarrow Y$. Si definisce la **FUNZIONE COMPOSTA**

$$f = h \circ g$$

$$f(x) = h(g(x)) \quad , x \in V$$

$$V = \{x \in X : g(x) \in W\}$$

! $X = \text{dom}(g)$, $W = \text{dom}(h)$!

$\Longrightarrow V = \text{dom}(f)$

$g : X \to \mathbb{R}$ monotona

CRESCENTE in X

$g \uparrow$ .... $(\downarrow)$

TEOREMA [MONOTONIA FUNZIONE COMPOSTA]

Siano $g : X \to \mathbb{R}$, $h : W \to \mathbb{R}$

e $f = h \circ g : V \to \mathbb{R}$

ALLORA:

$g \uparrow \circ h \uparrow \Rightarrow f \uparrow$

$g \uparrow \circ h \downarrow \Rightarrow f \downarrow$

$g \downarrow \circ h \downarrow \Rightarrow f \uparrow$

$g \downarrow \circ h \uparrow \Rightarrow f \downarrow$

DIMOSTRAZIONE

$$f(x) = h(g(x))$$

$g \uparrow$ e $h \uparrow$ $\Rightarrow$ $f \uparrow$

$x_1 < x_2$, $x_i \in V$, $i = 1, 2$

$g \uparrow \Rightarrow g(x_1) \leq g(x_2)$

$\| \qquad \| $

$y_1 \leq y_2$

$h \uparrow \Rightarrow h(y_1) \leq h(y_2)$

$f(x_1) = h(g(x_1)) \leq h(g(x_2)) = f(x_2)$

$x_1 < x_2 \Rightarrow f \uparrow$

$g \uparrow$ e $h \downarrow$ $\Rightarrow$ $f \downarrow$

$x_1 < x_2$, $x_i \in V$, $i = 1, 2$,

$g \uparrow \Rightarrow g(x_1) \leq g(x_2)$

$\| \qquad \| $

$y_1 \leq y_2$

$h \downarrow \Rightarrow h(y_1) \geq h(y_2)$

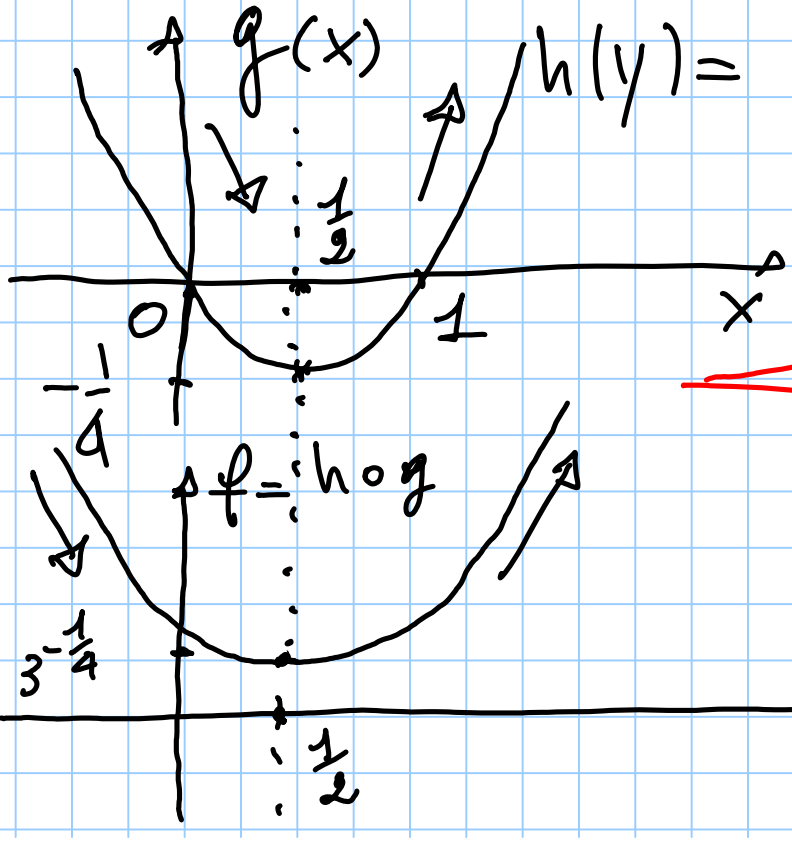
$f(x_1) = h(g(x_1)) \geq h(g(x_2)) = f(x_2)$

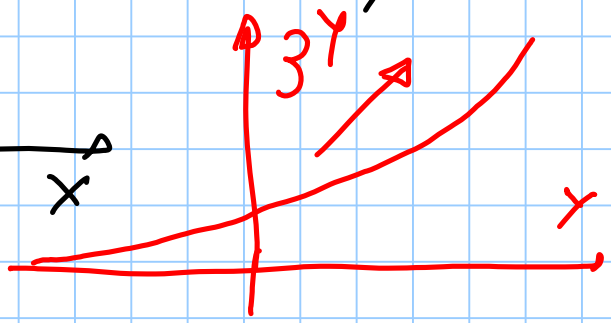
ES. COMPLETARE LA DIM. $\Rightarrow f \downarrow$ ■

$f(x) = 3^{x^2 - x}$, $x \in \mathbb{R}$

$f = h \circ g$, $g(x) = x^2 - x$, $x \in \mathbb{R}$

$h(y) = 3^y$, $y \in \mathbb{R}$





h∘g, g↓, h↑, f↓

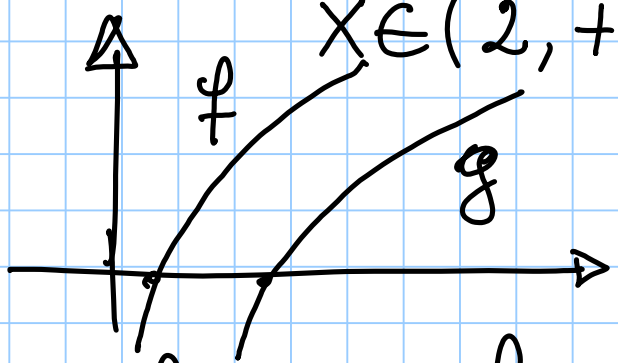
h∘g, g↑, h↑, f↑

$$3^{-\frac{1}{4}} = \frac{1}{3^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{3}} < 1$$

$$f(x) = \log_2(x), \quad x \in (0, +\infty)$$

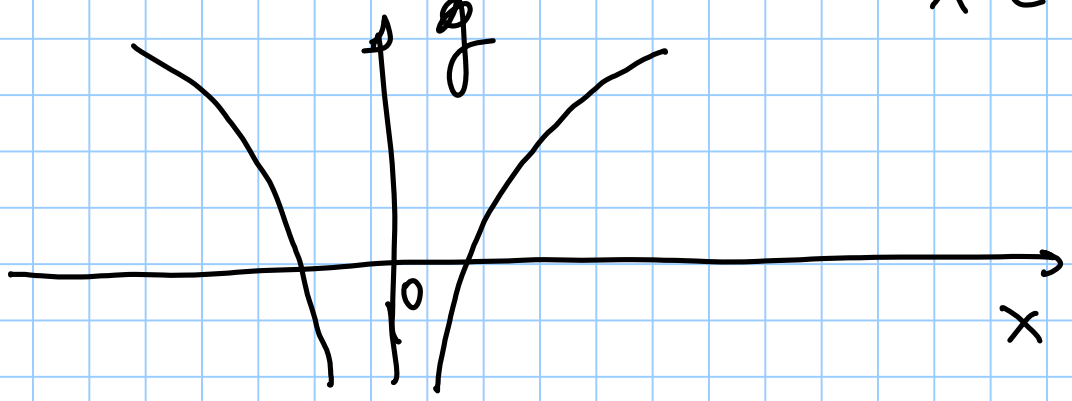
$$g(x) = f(x-2) = \log_2(x-2)$$

$x \in (2, +\infty)$



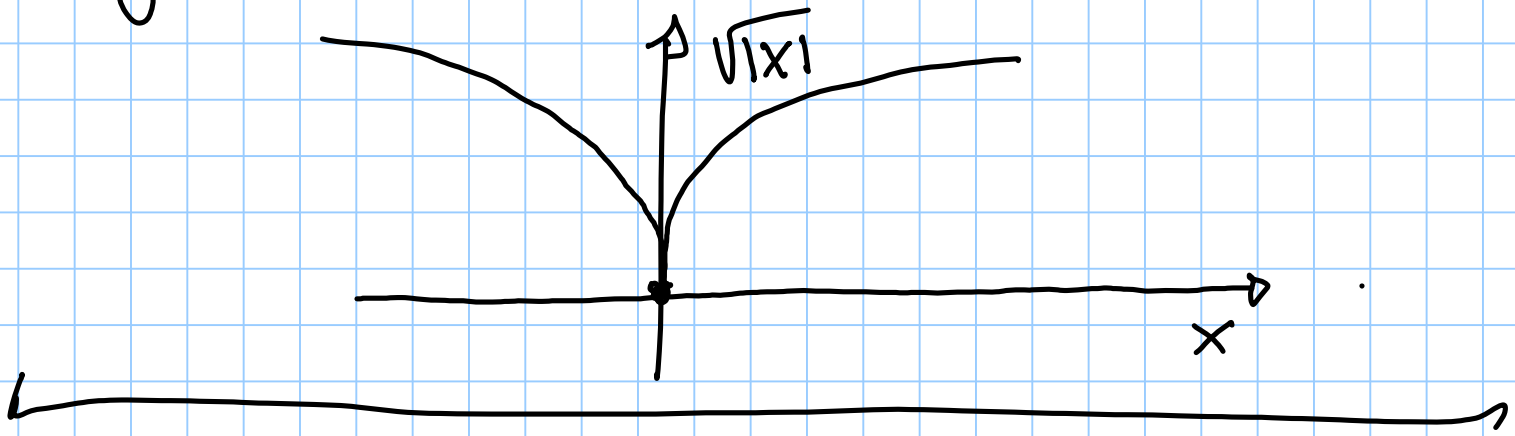
$$g(x) = f(|x|) = \log_2(|x|)$$

$x \in \mathbb{R} \setminus \{0\}$



$$f(x)=\sqrt{x}\,,\quad x\in[0,+\infty)$$

$$g(x)=f(|x|)=\sqrt{|x|}\,,\quad x\in\mathbb{R}$$

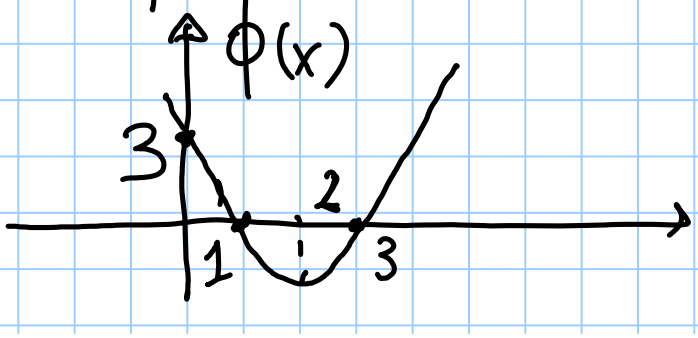


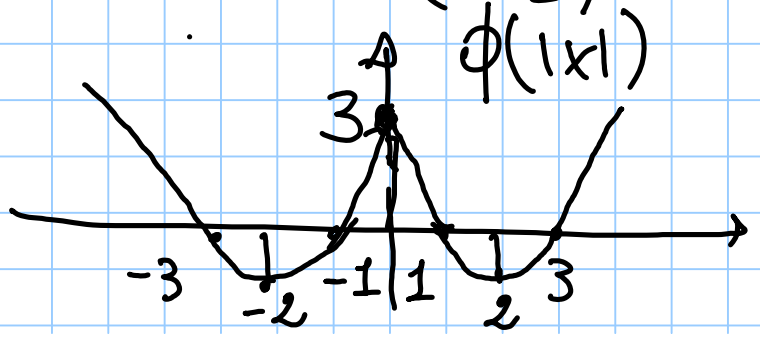
$$f(x)=\log_2\left(|x^2-4|x|+3|\right)$$

$$h(t)=\log_2(t)\,,\quad t\in(0,+\infty)$$

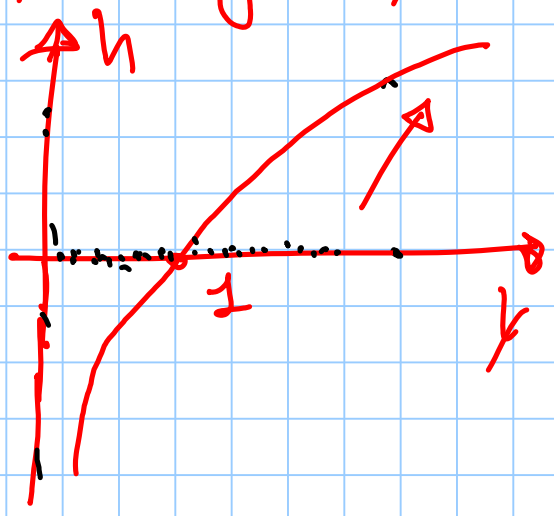
$$g(x)=|x^2-4|x|+3|$$

$$\phi(x)=x^2-4x+3=(x-1)(x-3)$$



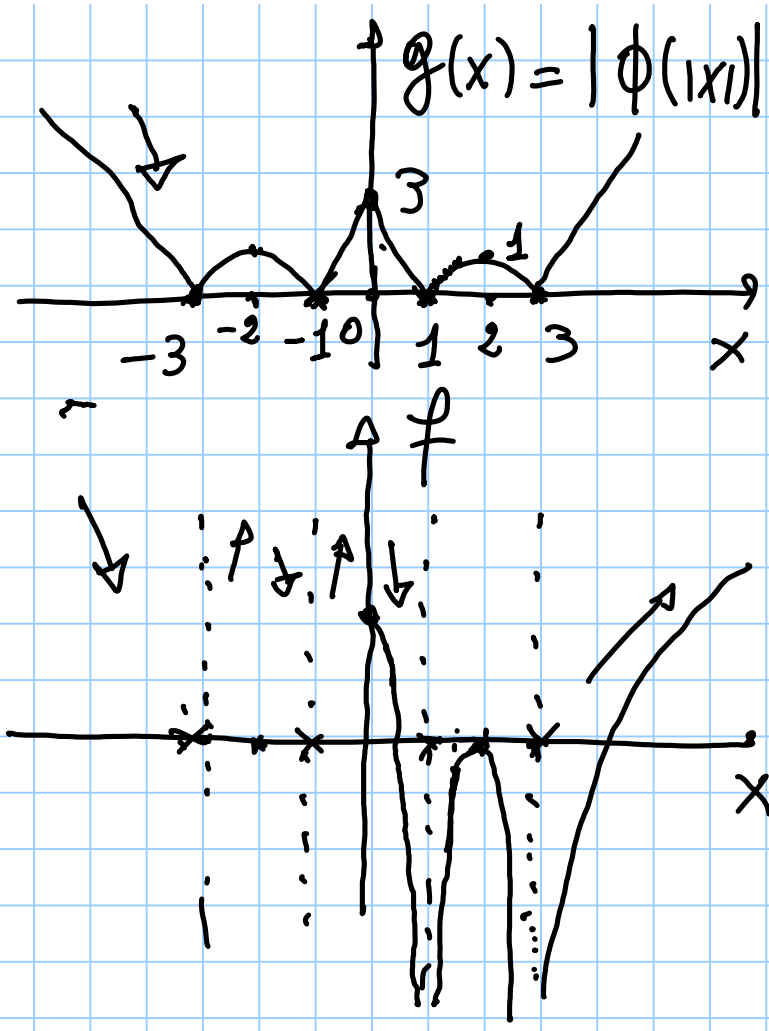


$h(y) = \log_2(y)$



$y \in (0, +\infty)$

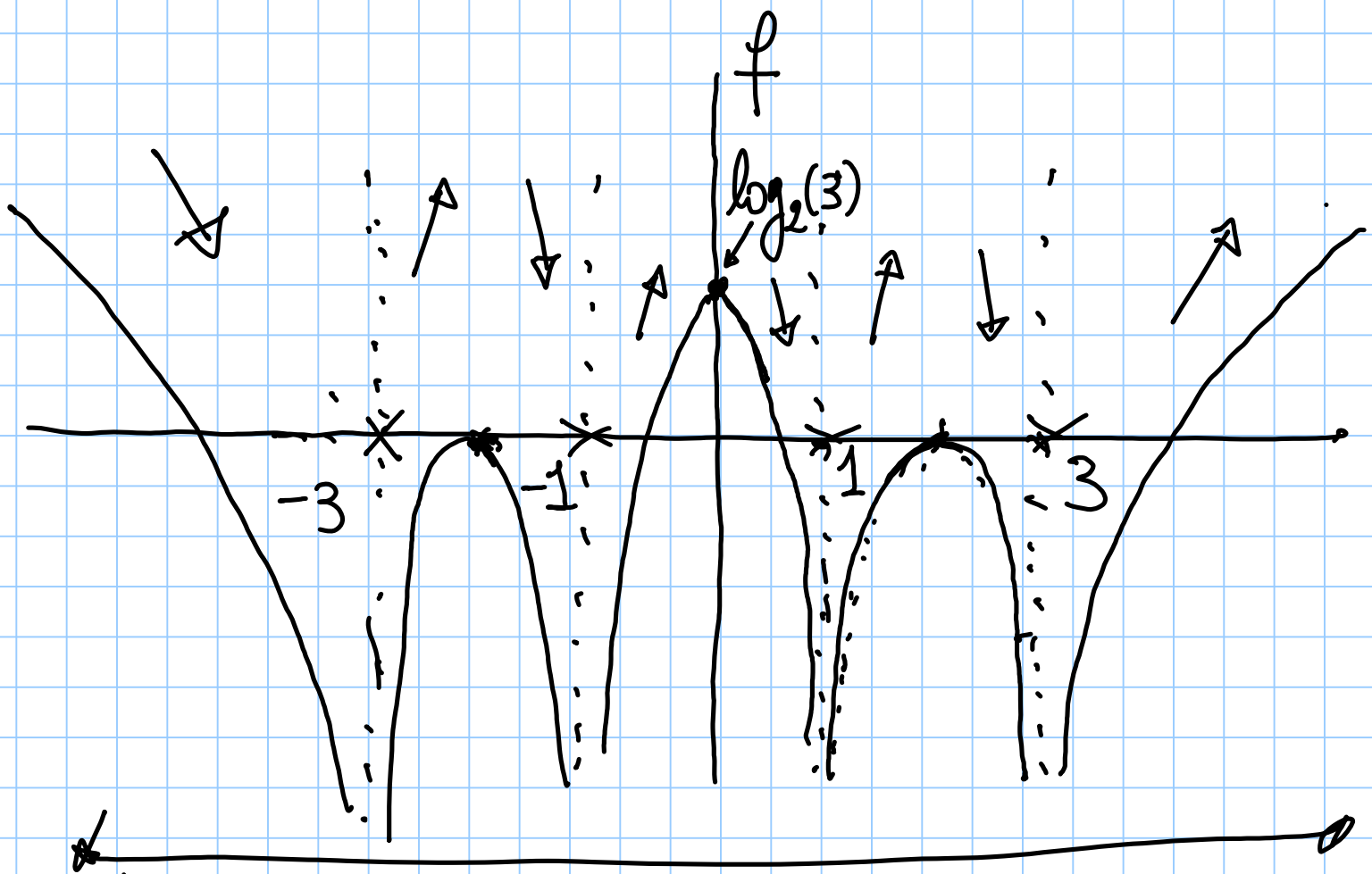
$f(x) = h(|\phi(|x|)|)$



$g(x) = |\phi(|x|)|$

$\text{Im}(g) = [0, +\infty)$

$\text{dom}(f) = \mathbb{R} \setminus \{-3, -1, 1, 3\}$

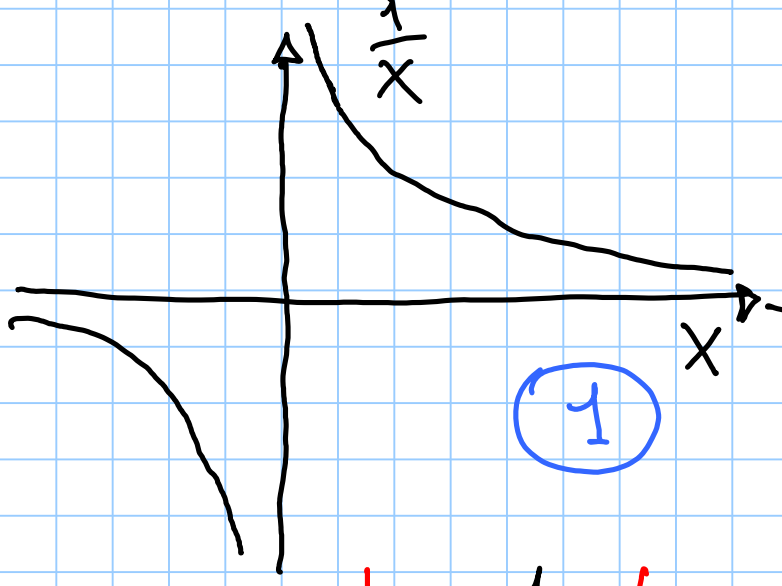


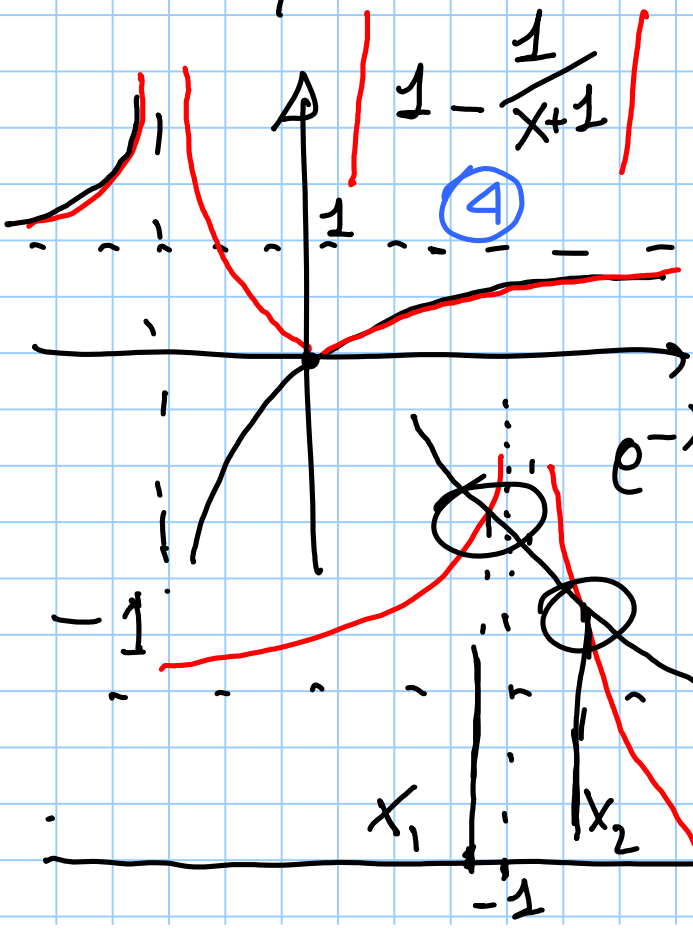
$e \cong 2{,}7$

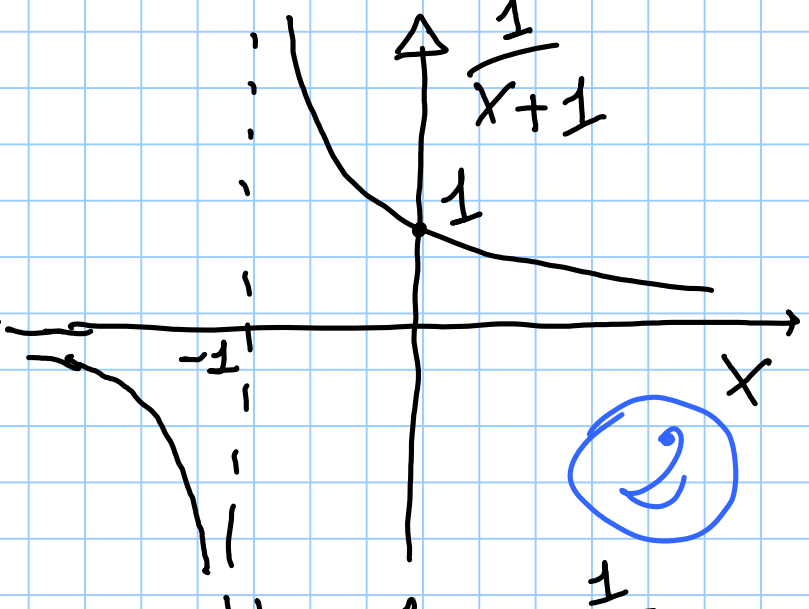
$$e^{-x} > \left| \frac{x}{x+1} \right|$$

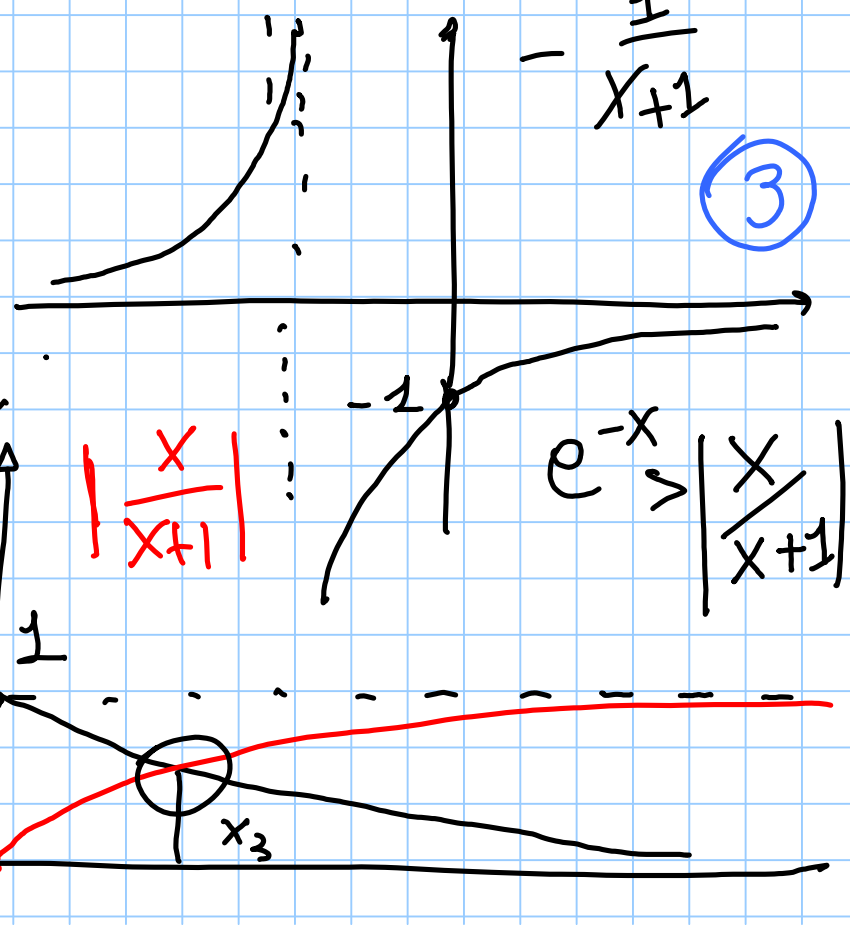
$$e^{-x} = \left| \frac{x}{x+1} \right|$$

$$\frac{x}{x+1} = \frac{x+1-1}{x+1} = 1 - \frac{1}{x+1}$$









$x \in (-\infty, x_1) \cup (x_2, x_3)$

$x_1 < -1 \quad , \quad -1 < x_2 < 0 \quad , \quad x_3 > 0$

$f(x) = x e^{-x}$

MONOTONIA di f?