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Referee Report on the Ph. D. Thesis “Fine Properties of Correlations in Hyperbolic Dynamics” by Giovanni Canestrari.

This thesis explores the behavior of correlations, mixing, and related spectral properties in various classes of dynamical systems, with a particular emphasis on systems that are hyperbolic or exhibit expanding behavior—sometimes complicated by discontinuities or the presence of an indifferent fixed point. The scope of the thesis is remarkably broad, it is well-written, and it integrates multiple key threads in modern ergodic theory and smooth dynamical systems, including:

1. **Spectral Methods for Discontinuous and Non-Markov Maps**
   The thesis addresses the challenge of understanding Ruelle–Pollicott resonances when the map exhibits discontinuities. Through careful construction of Banach spaces and analysis of transfer operators, the author demonstrates, under mild assumptions (e.g., existence of certain types of discontinuities), that the essential spectral radius satisfies some natural lower bounds, thus signaling intrinsic obstacles to purely “nice” spectral decompositions. There are results for one-dimensional maps and maps on surfaces.
2. **Global–Local Mixing in Infinite Measure**
   For one-dimensional expanding maps with an indifferent fixed point and preserving an infinite measure the thesis studies the concept of *global–local* mixing. By introducing specific classes of “global” observables that possess a uniform infinite volume average, and pairing them with local observables in L^1, the author shows that certain families of maps exhibit a precise mixing property under appropriate assumptions. This is a significant step forward in clarifying the conditions under which global–local mixing holds in infinite-measure contexts.
3. **Standard Pairs and Hyperbolic Dynamics**
   A significant portion of this thesis relies on the powerful concept of standard pairs to study coupling arguments for hyperbolic maps. This framework is applied to establish new mixing results and, crucially, to address questions about the differentiability of SRB measures (linear response). Standard pairs have proven to be instrumental in elucidating the behavior of these measures under the iteration of strongly hyperbolic maps. The author makes extensive use of this approach to develop advanced results, such as those described below.
4. **Linear Response for Discontinuous Perturbations**
   One of the more striking results is that certain discontinuous perturbations of hyperbolic systems, even if they alter the topological class of the original map, can still exhibit linear response (differentiability of the physical/SRB measure) near the unperturbed system. This partially contrasts with the one-dimensional expanding case, where discontinuities typically obstruct linear response. The thesis thus opens interesting avenues for further

investigation, especially for higher-dimensional systems or those with fewer smoothness assumptions.

5. **Hydrodynamic Limits in Deterministic Systems**
Finally, the thesis explores the derivation of macroscopic equations from microscopic deterministic models—an assembly of harmonic oscillators coupled to a rapidly evolving external field. The author shows that, in the limit of infinitely many particles and appropriate time rescalings, the energy distribution converges (in a suitable sense) to the solution of a heat-like PDE. This result is significant both mathematically—demonstrating convergence to a diffusive process from a purely deterministic system—and physically, underscoring how collective chaotic effects mimic stochastic behavior under certain conditions.

Strengths and Contributions

Despite the potentially disparate nature of the topics—transfer operators for discontinuous maps, infinite-measure mixing, coupling and standard pairs, linear response, and hydrodynamic limits—the thesis manages to tie these strands together cohesively, rooted in the unifying theme of studying statistical properties of nonlinear and, often, non-smooth dynamical systems.

The work displays strong command of modern techniques in ergodic theory, including analytic methods (Banach spaces, spectral decomposition, essential spectral radius arguments), coupling methods, and geometric considerations for hyperbolic maps.

The extension of linear response theory to certain classes of discontinuously perturbed hyperbolic maps is original and clarifies which hypotheses truly obstruct differentiability of SRB measures in higher-dimensional systems. Likewise, the convergence results for deterministic chains under fast-evolving external fields to a heat equation fill a gap in the literature on hydrodynamic limits.

The thesis is largely well-structured. It provides motivation, states main results carefully, and offers a roadmap for subsequent chapters. The mathematical arguments appear detailed and rigorous, featuring appropriate references to related literature.

Overall Assessment

Based on the introduction and the core ideas outlined, this thesis represents a thorough and valuable contribution to contemporary ergodic theory and dynamical systems. The author not only addresses multiple open problems—such as linear response in higher dimensions with discontinuities—but also applies sophisticated methods (standard pairs, coupling, Banach-space spectral theory) to unify these results under a coherent line of inquiry. The final chapter's success in deriving a macroscopic PDE from a deterministic, high-dimensional system is particularly noteworthy, as it contributes to the mathematical understanding of how chaotic, non-stochastic dynamics can exhibit diffusive behavior in the thermodynamic limit.

Recommendation

I find the thesis both mathematically sound and innovative. The candidate demonstrates deep knowledge of the field and independent problem-solving skills, which are hallmarks of doctoral-level research. Thus, I recommend the thesis for the award of a PhD degree.

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