

Emanuele N. Spadaro

## Variational Calculus and Applications

Advanced topics in Analysis

The course runs bi-weekly (4 hours per week) from 1 April to 31 May 2024 (in case you need an introduction to the course from 28 February to April, please contact the lecturer for details).

### PROGRAM:

The course introduces the variational methods underlying numerous problems in mathematical analysis and applied mathematics.

Beginning with classical methods of Variational Calculus developed since the 18th century, it will lead to a discussion of more recent results, such as the solution of Hilbert's 19th problem, notions of variational convergence and phase separation models in mathematical physics. - Classical Problems of the Calculus of Variations and Examples of Applications

- Examples of existence and non-existence
- Euler Lagrange equations and differential equations in weak form
- Direct method of the Calculus of Variations
- Necessary and sufficient conditions for the semi-continuity of integral functionals
- Vector problems of the Calculus of Variations Convexity and quasi-convexity
- Regularity of minima and solution of Hilbert's 19th problem
- Variational convergence. Gamma convergence
- Application to asymptotic problems of the calculus of variations:
  - o Caccioppoli sets
  - o Phase transitions and the Modica Mortola functional