



PhD Course 2025

Titolo: Riemann zeta function and connections

SPEAKER/LECTURER: Efthymios Sofos

Joint with Roma 1, La Sapienza and Roma Tre

Period: November-January (once a week in Tuesday)

Schedule: Morning 11.00-13.00

Room: 1101 “C. D’Antoni”

ABSTRACT

Summary:

In the first half of the course we will cover the theory of the famous paper of Riemann on the zeta function and the prime numbers. This will include the Riemann hypothesis and the proof of the Explicit Formula and the prime number theorem. In the second half of the course we will see how this theory is used in research today in 2 topics in Arithmetic Geometry. The first topic is Serre's problem on the probability that a random Diophantine equation has a solution. The second topic is Goldfeld's conjecture on the probability that a randomly chosen elliptic curve with integer coefficients has infinitely many rational solutions.

Prerequisites: Undergraduate complex analysis



Reading material:

- H. Davenport, Multiplicative Number Theory, Graduate Texts in Mathematics (GTM, volume 74). (We will go through 1/3 of the chapters.)
- (2) J.-P. Serre, Spécialisation des éléments de $\text{Br}_2(\mathbb{Q}(T_1, \dots, T_n))$, C. R. Acad. Sci. Paris Ser. I Math. (1990), pages 397–402.
- (3) D.R. Heath-Brown. The size of Selmer groups for the congruent number problem. Invent. Math. 111(1) (1993), pages 171–196. (We will cover a small part of the proofs, namely the one that is related to the first moment.)

Timeline of the course:

- 04 November

01: Definition of the zeta function and basic properties 02: Poisson summation, functional equation for the zeta function

- 11 November

03: Prerequisites on Euler's Gamma functions, integral functions of order 1 04: Infinite products (Weierstrass' theorem)

- 18 November

05: Zero free region 06: Number of zeros in boxes

- 25 November

Explicit formula

- 02 December

09: Proof of the Prime Number Theorem, statement and consequence of the Riemann Hypothesis.

10: Selberg--Delange method, Large sieve for quadratic characters

- 09 December

11 The field of p -adic numbers, Hilbert symbols, Hilbert reciprocity 12 Hasse's local--global principle for quadratic hypersurfaces

- 16 December

Friedlander--Iwaniec's 2010 paper "Ternary quadratic forms with rational zeros" on Serre's problem.

- 05 and 13 January

Theory of elliptic curves

- 20 January

Heath-Brown's 1993 paper "The size of Selmer groups for the congruent number problem" that is connected to averages of ranks of elliptic curves