PHD COURSE

Rational Curves and Surfaces for Geometric Modelling

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Prerequisites

- Projective spaces, projective coordinates and scales
- Duality: dual spaces, dual curves & surfaces
- Projective maps, collineations, correlations, perspectivities, polarities, dual map
- Linear rational transformations, cross ratio, harmonic ratio
- Pencils and bundles
- Quadrics, Intersections of quadrics with subspaces, tangent planes, singular points
- Tangent quadric, Generators of quadrics, Quadric pencils
- Conics, Intersection of conics
- Steiner construction
- Theorems by Pascal and Brianchon

Content

■ *Rational Curves in homogenous Bézier form: Week 1*

- Introduction and notation
- Properties of triangular and tensor-product Bézier nets for polynomials
- Polar forms and Bézier polygons (blossoming)
- Rational curves, base points, uniqueness, projective Bézier form
- Inner and outer weight points, shoulder points, derivatives
- Effect of weight changes, reparameterizations, complimentary segments
- Haase's algorithm (projective de Casteljau algorithm), dual Bézier form
- Primal and dual Bézier forms of conics
- Rational curves with rational offsets / rational arc length

■ *Rational splines : Week 2*

- Circle splines and their degree
- Curvature interpolation, curvature continuous splines
- Three- and four-point / tangent contacts
- C^k constructions, splines (NURBS)
- Quadric spline surfaces

■ *Rational surfaces in homogenous Bézier form: Week 3*

- Triangular Bézier form, weight points and lines, reparametrizations
- Complimentary segments, base curves, projective de Casteljau algorithm
- Rational de Casteljau algorithm, dual Bézier form, offset surfaces
- C^k constructions, free form spline surfaces, projective structures
- Ruled and developable surfaces: lines of regression, class and degree
- Primal and dual representation, joining developable surfaces smoothly
- Developable surfaces with cubic duals

■ *Rational parametrization of quadrics: Week 4*

- Stereographic projection and its generalization
- Triangular patches on quadrics
- Tensor-product patches on quadrics and Miquel's theorem
- Canal and pipe surfaces

■ *Dupin's cyclides: Week 5*

- Power of a point, orthogonal spheres, similarity centers
- Cyclides
- Classes of cyclides
- Smooth joints
- Sphere inversions, cyclides and tori