



1.1 Modulo 1. Francesca Carocci (Tor Vergata)

Titolo: Moduli spaces of semi-stable vector bundles on curves

Modalità di esame: Oral exam: the student can expand (choosing) on a portion of the lecture where details were missing.

Periodo di erogazione: Da metà
Gennaio a metà/fine Marzo (1
lezione a settimana da 2 ore,
Martedì 11:00–13:00)



Abstract

This course aims to introduce the concept of moduli spaces as well as some of the classical concepts/techniques/results often used for the construction and the local and global study of such moduli spaces, focusing in particular on the example of moduli of semi-stable vector bundles on algebraic curves. Wherever, for time reason, some details of the more technical proofs will be omitted, the class will be complemented by a series of exercises, with the aim of fixing the main ideas via computations of easy examples/counterexamples.



Programma Dettagliato

- **Part I (4–6 hours):** Coherent/Locally free sheaves and their cohomology on smooth projective varieties; Serre duality and Riemann-Roch for locally free sheaves on curves; cohomology and base change.

- **Part II (4–6 hours):**

Moduli spaces and Quot schemes: Introduction to moduli Functors; sheaves in flat families and Hilbert polynomial; Quot scheme, representability and projectivity.

- **Part III (6 hours):**

$M(r, d)$ construction: Semi-stability, Jordan-Hölder and Harder-Narasimhan filtration; Boundedness of families of semi-stable bundles; GIT and conclusion of the construction.

- **Part IV (4 hours):**

Geometry of $M(r, d)$: Tangent to $M(r, d)$; Smoothness criterion at stable points; irreducibility.

Prerequisiti

The students should have some familiarity with algebraic varieties and coherent sheaves on them, as well as have seen singular cohomology for a smooth manifold. Ideally, they would also have seen Chern classes of vector bundles and the computation of cohomology for line bundles on projective space and on curves. However, these will be quickly reviewed in the first part of the course.

Bibliografia

- **Corso:** J. Le Potier, *Lectures on Vector Bundles*.
- **Prerequisiti:** R. Hartshorne, *Algebraic Geometry*, Chapter I/II (except 9), working over an algebraically closed field.