TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA
PhD Course 2026

Title: Locally Refined B-splines

Schedule: January 11th - January 29th
2/3-hour classes per week)

Prerequisites: Previous knowledge of the B-spline theory is useful but not mandatory.

Abstract

B-splines and Non-Uniform Rational B-Splines (NURBS) are fundamental tools in geometric modeling and engineering. Their importance has further increased with the advent of Isogeometric Analysis, where they are employed for the numerical solution of partial differential equations. However, the tensor-product structure of classical B-splines and NURBS makes refinement inherently global: improving the approximation in a localized region generally requires the insertion of degrees of freedom also in areas where no additional resolution is needed. To overcome this limitation, several spline technologies have been developed to relax the tensor-product structure while retaining the main properties of standard B-splines. Among them, Locally Refined (LR) B-splines provide a particularly flexible framework for local refinement and have proven effective in a variety of approximation applications, including surface modeling and data fitting.

This course presents the theoretical foundations of LR B-splines. After reviewing the definition of spline spaces over box partitions, we discuss the dimension of such spaces and its dependence on a homological term, which may lead to instability of the dimension formula. Conditions under which the dimension stabilizes naturally motivate the construction of LR meshes and LR B-splines.

We then introduce LR B-splines through knot insertion and examine their fundamental properties, including locality, non-negativity, and piecewise polynomial structure. Particular attention is devoted to their spanning properties, with emphasis on completeness, linear dependence, and local linear independence. Finally, we discuss characterization results and refinement strategies that guarantee the preservation of the key approximation and stability properties associated with tensor-product B-splines.

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Program

- Spline spaces (univariate, bivariate and multivariate cases)
- Locally Refined B-splines (univariate, bivariate and multivariate cases)
- Spanning properties and (local) linear independence
- Automatic refinement strategies

Bibliography

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3. Patrizi, F., & Dokken, T. (2020). Linear dependence of bivariate Minimal Support and Locally Refined B-splines over LR-meshes. Computer aided geometric design, 77, 101803.
4. Patrizi, F., Manni, C., Pelosi, F., & Speleers, H. (2020). Adaptive refinement with locally linearly independent LR B-splines: Theory and applications. Computer Methods in Applied Mechanics and Engineering, 369, 113230.
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Site : https://www.mn.uio.no/math/tjenester/kunnskap/kompendier/lyche-morken-spline-methods.pdf