

**Programma:**

This course introduces the language, concepts, and methods for nonlinear PDEs in unbounded domains. We will review classical approaches mainly based on the direct method of the Calculus of Variations, to study some classes of nonlinear elliptic PDEs on  $\mathbb{R}^N$ . We will mainly consider two 'toy problems':

the Choquard equation, and the nonlinear Schrödinger-Poisson-Slater equation. Although these equations share some features, such as non-linearity and nonlocality, they require a separate analysis. Emphasis will be given to these equations' variational formulation, functional setting, and relevant compactness properties.

Known results and suggestions for new research projects in this area will be discussed.

The first part of the course will be devoted to some basic concepts of the variational approach to nonlinear analysis, based on extending Calculus to infinite dimensional Banach spaces. Some elementary tools in critical point theory will be introduced, such as the Lagrange multipliers rule and the Mountain Pass Theorem. These can help identify nontrivial solutions to PDEs characterized by 'energy levels'. Some exercises will be discussed as a preparation for the second part of the course. Students who already know these tools can skip this part and attend directly the second part of the course.

NB: orari (e aule) possono cambiare in base alle esigenze degli studenti interessati