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# Introduction to PDE

Advanced topics in Analysis

This is a 20 hours (10 lessons) course starting March 2024.

PROGRAMMA:
The aim of the course is to provide an introduction to the basic notions about LaplacePoisson, Heat and Wave equations. There will be three lessons of two hours each a week. Lecture notes of the course will be available. The Lectures will be delivered in presence, possibly in mixed (online) form if needed.
Topics covered
◦ Laplace and Poisson equations. Harmonic functions. Fundamental solutions. ◦ Mean value formulas. Maximum principles, uniqueness. Mollifiers, convolutions and smoothing.
◦ Regularity and local estimates for harmonic functions. The Liouville Theorem, classification of solutions of the Poisson equation in RN , N ≥ 2.
◦ The Harnack inequality for harmonic functions. The Green function. The Green function on a ball. The Poisson Kernel.
◦ Variational (Energy) methods. The Dirichlet principle.
◦ The Heat equation. The fundamental solution. The Cauchy problem for the homogeneous and non homogeneous equation. Mean value formula and the heat ball.
◦ Maximum principle for the heat equation. Uniqueness. Regularity of solutions of the heat equa- tion.
◦ Transport equations. The Wave equation. D'Alambert formula (N=1), Euler-PoissonDarboux equation, Kirchoff's formula (N=3). Descent method, Poisson's formula (N=2). ◦ Nonhomogeneous wave equations, retarded potentials. Energy methods, finite speed propagation.

Bibliography

• D. Bartolucci, Lecture notes of the course.
• L.C. Evans, Partial Differential Equations. Second Edition. American Mathematical Society 2010.