INTRODUCTION TO PDE

PhD Course 24/25

Prof. D. Bartolucci

Course description

This is a 20 hours (10 lessons) course whose aim is to provide an introduction to the basic notions about Laplace-Poisson, Heat and Wave equations. There will be three lessons of two hours each a week. The exact schedule will be posted few days before the first lesson, which will be held the first week of March 2025. Lecture notes of the course will be available. The Lectures will be delivered in presence, possibly in mixed (online) form if needed.

Topics covered ([B] D. Bartolucci, Lecture notes of the course).

- Laplace and Poisson equations. Harmonic functions. Fundamental solutions. ([B] Lecture 1)
- Mean value formulas. Maximum principles, uniqueness. Mollifiers, convolutions and smoothing. ([B] Lecture 2)
- Regularity and local estimates for harmonic functions. The Liouville Theorem, classification of solutions of the Poisson equation in $\mathbb{R}^N$, $N \geq 2$. ([B] Lecture 3)
- The Harnack inequality for harmonic functions. The Green function. The Green function on a ball. The Poisson Kernel. ([B] Lecture 4)
- Variational (Energy) methods. The Dirichlet principle. ([B] Lecture 4)
- The Heat equation. The fundamental solution. The Cauchy problem for the homogeneous and non homogeneous equation. Mean value formula and the heat ball.([B] Lecture 4-5)
- Maximum principle for the heat equation. Uniqueness. Regularity of solutions of the heat equation. ([B] Lecture 5-6)
- Transport equations. The Wave equation. D'Alambert formula (N=1), Euler-Poisson-Darboux equation, Kirchoff's formula (N=3). Descent method, Poisson's formula (N=2). ([B] Lecture 7)
- Nonhomogeneous wave equations, retarded potentials. Energy methods, finite speed propagation. ([B] Lecture 7)
- Crash introduction to: weak derivatives, Sobolev spaces; weak solutions of second order elliptic equations; The Theorem of Lax-Milgram; The Fredholm alternative for second order elliptic equations. ([B] Lecture 8)
- Crash introduction to: eigenvalues of selfadjoint elliptic operators; Elliptic regularity theory. ([B] Lecture 9)
- Crash introduction to: maximum principle for second order elliptic operators; maximum principle and the first eigenvalue ([B] Lecture 10, Lecture 10-MP).