Mathematical Analysis 2

Call 5 – 31/08/2023

Do **not** turn this sheet over until instructed to do so.

Name: **De Nym Sue**,
Mat: **'0000000'** Seat: **0**

This exam consists of 5 questions, each are worth 6 points (no negative scoring), you have 3 hours to attempt the exam. Solutions available online when the exam finishes.

Fill the blanks with the correct **integer**, $-\infty$ or ∞.

- Students are permitted to bring only the following items to their desk in the exam room:
  - Two A4 sheets of paper with course notes (writing permitted on both sides, or equivalent area);
  - Pens and pencils;
  - An identity document;
  - Drinking bottle.
- Paper for rough calculations will be provided in the exam room. After the exam the paper used during the exam remains in the exam room.
- During the exam it is forbidden to communicate, using any means, with anyone except the exam invigilators. If you want to communicate, raise your hand and an invigilator will come to you.
- It is forbidden to have anything that can be used for communication (e.g., telephone, smartwatch, computer, headphones, earbuds, messenger pigeon) on your person or close to you during the exam.
- Under penalty of exclusion, during written tests the use of electronic devices and applications, is not allowed. Calculators are not permitted.
- You may choose to leave the exam early but only after receiving confirmation from an invigilator.

I affirm that I will not give or receive any unauthorized help on this exam. If I am offered unauthorized help I will notify an invigilator.

Name: De Nym Sue, Signature:

Question 1. Consider the differential equation $y' = x^2 - y^2$ with initial condition $y(0) = 1$. We observe that $y'' = \boxed{\mathbf{2}}\, x + \boxed{\mathbf{-2}}\, yy'$ and the differential equation has the following power series solution. (2)

$$y(x) = \boxed{\mathbf{1}} + \boxed{\mathbf{-1}}\, x + \frac{2}{2!}x^2 + \frac{\boxed{\mathbf{-4}}}{3!}x^3 + \frac{\boxed{\mathbf{20}}}{4!}x^4 + \cdots$$ (4)

Question 2. Use Lagrange's multipliers to find the maximum and minimum values of $f(x, y, z) = y^2 - 10z - 19$ subject to the constraint $x^2 + y^2 + z^2 = 36$. The absolute minimum is -79 and the absolute maxima is $\boxed{\mathbf{42}}$. (6)

Question 3. Let C be the curve $\{(x, y) : x^2 + y^2 = 16, y \geq 0\}$ in $\mathbb{R}^2$ And let

$$\mathbf{f}(x, y) = \begin{pmatrix} 2 \\ xy - 2x \end{pmatrix}$$

be a vector field on $\mathbb{R}^2$. Let $\boldsymbol{\alpha}(t)$ be a parametrization of C which starts at $(4, 0)$ and ends at $(-4, 0)$. We calculate that,

$$\int \mathbf{f} \cdot d\boldsymbol{\alpha} = \frac{\boxed{\mathbf{32}}}{3} + \boxed{\mathbf{-16}}\,\pi.$$ (3+3)

Question 4. Let S denote the region determined by $y \leq 4 - x^2, x \geq 0, y \geq 0$. Sketch this region and calculate that the area of S is $A = \iint_S dxdy = \frac{16}{3}$. The two moments are

$$M_y = \iint_S y\, dxdy = \frac{128}{\boxed{\mathbf{15}}} \quad \text{and} \quad M_x = \iint_S x\, dxdy = \boxed{\mathbf{4}}.$$ (2+2)

Using this we conclude that the centre of mass of S is $(\boxed{\mathbf{3}}/4, 8/5)$. *Hint: centre of mass (x_0, y_0) satisfies $x_0 A = M_x$ and $y_0 A = M_y$.* (2)

Question 5. Let S denote the parametric surface $z = x^2 + y^2, 0 \leq z \leq 1$. If the density is $\mu(x, y, z) = z$, the mass of the surface is equal to

$$\left(\frac{\boxed{\mathbf{5}}}{12}\sqrt{5} + \frac{1}{\boxed{\mathbf{60}}}\right)\pi.$$ (3 + 3)

Hint: Evaluate the integral $\iint_S \mu(x, y, z)\, dS$.