

Mathematical Analysis 2

Midterm 2 – 20/11/2023 – 14:00-15:30

Do **not** turn this sheet over until instructed to do so.

Name: –, Seat: –

This test consists of 5 questions, you have 1.5 hours to attempt to solve them.

Respond to the questions with **fully and clearly argued solutions**. If there is something that you are uncertain about, do the best you can and demonstrate your knowledge of the relevant concepts.

Include your seat number (–) on each sheet of paper on which you write your solutions.

- Students are permitted to bring only the following items to their desk in the test room: pens / pencils, drinking bottle. Calculators and any other electronic items are not permitted.
- Paper for rough calculations will be provided in the test room. After the test the paper used during the test remains in the test room.
- Under penalty of exclusion, during the test it is forbidden to communicate, using any means, with anyone except the test invigilators. If you want to communicate, raise your hand and an invigilator will come to you.
- It is forbidden to have anything that can be used for communication (e.g., telephone, smartwatch, computer, headphones, earbuds, messenger pigeon) on your person or close to you during the test.
- You may choose to leave the test early but only after receiving confirmation from an invigilator.
- In case of any inconsistencies with the written exam, an interview may be requested to confirm the grade.

I affirm that I will not give or receive any unauthorized help on this test. If I am offered unauthorized help I will notify an invigilator.

Name: –, Signature:

Question 1. Apply the Lagrange multiplier method to find the maximum and minimum values of

$$f(x, y) = 8x^2 - 2y$$

subject to the constraint

$$x^2 + y^2 = 1.$$

Hint: the maximum is $\frac{65}{8}$, the minimum is an integer.

Question 2. Find and classify all the stationary points of the scalar field,

$$f(x, y) = 7x - 8y + 2xy - x^2 + y^3.$$

Hint: there are two stationary points, the determinant of the Hessian at one is equal to -8 and at the other is equal to 8 .

Question 3. Sketch the two vector fields:

$$\mathbf{F}(x, y) = \begin{pmatrix} 1 \\ x \end{pmatrix}, \quad \mathbf{G}(x, y) = \begin{pmatrix} x \\ y \end{pmatrix}.$$

Question 4. Consider the vector fields,

$$\mathbf{F}(x, y) = \begin{pmatrix} 2x^2y \\ x^3 \end{pmatrix}, \quad \mathbf{G}(x, y) = \begin{pmatrix} y^2 \\ x^2 \end{pmatrix}, \quad \mathbf{H}(x, y) = \begin{pmatrix} 2xy^3 + e^y \\ 3x^2y^2 + xe^y \end{pmatrix}.$$

Identify which is conservative on \mathbf{R}^2 and show that the others are not conservative. For the conservative one, find φ such that the vector field is equal to $\nabla\varphi$.

Question 5. Consider the vector field

$$\mathbf{F}(x, y) = \begin{pmatrix} 6x - 2y \\ x^2 \end{pmatrix}.$$

Let α denote the path defined to be the line segment from $(6, -3)$ to $(0, 0)$ followed by the line segment from $(0, 0)$ to $(6, 3)$. Evaluate the line integral, $\int \mathbf{F} \cdot d\alpha$.

Hint: the final answer is a two digit square number.