Mathematical Analysis 2

Midterm 1 – 30/10/2023 – 14:00-15:30

<u>Do **not** turn this sheet over until instructed to do so.</u>

Name: –,   Seat: –

This test consists of 5 questions, you have 1.5 hours to attempt to solve them.

Respond to the questions with **fully and clearly argued solutions**. If there is something that you are uncertain about, do the best you can and demonstrate your knowledge of the relevant concepts.

Include your seat number (–) on each sheet of paper on which you write your solutions.

- Students are permitted to bring only the following items to their desk in the test room: pens / pencils, drinking bottle. Calculators and any other electronic items are not permitted.
- Paper for rough calculations will be provided in the test room. After the test the paper used during the test remains in the test room.
- Under penalty of exclusion, during the test it is forbidden to communicate, using any means, with anyone except the test invigilators. If you want to communicate, raise your hand and an invigilator will come to you.
- It is forbidden to have anything that can be used for communication (e.g., telephone, smartwatch, computer, headphones, earbuds, messenger pigeon) on your person or close to you during the test.
- You may choose to leave the test early but only after receiving confirmation from an invigilator.
- In case of any inconsistencies with the written exam, an interview may be requested to confirm the grade.

I affirm that I will not give or receive any unauthorized help on this test. If I am offered unauthorized help I will notify an invigilator.

Name: –,   Signature:

Question 1. Consider, for $x, y, z, u, v \in \mathbb{R}$, the functions defined as

$$f(u,v) = (u \sin v, u \cos v, uv^2) \quad \text{and} \quad g(x,y,z) = x^2 + xy^2 + 2e^{yz}.$$

1. Identify the domain and codomain of f and g (i.e., $f : \mathbb{R}^? \to \mathbb{R}^?$, etc.).
2. Determine if $f \circ g$ and $g \circ f$ are well defined. For each that is well defined, write the function explicitly and determine the domain and codomain.
3. Compute the partial derivatives $\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}$.

Question 2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x,y) = x^2 + 3 + e^{x-2y}$.

1. Calculate ∇f, the gradient of f.
2. Determine the directional derivative, $D_v f(4,2)$, in the direction of $v = \left(\frac{3}{5}, \frac{4}{5}\right)$.

Question 3. Compute the Jacobian matrix of the transformation

$$h : (u,v,w) \mapsto (u \cos v, u \sin v, w).$$

Compute also the determinant of this matrix.

Question 4. Consider the functions,

$$\begin{aligned} \boldsymbol{\alpha} &: [0,\infty) \to \mathbb{R}^2; \quad t \mapsto (\cos t, \sqrt{t}), \\ f &: \mathbb{R}^2 \to \mathbb{R}; \quad (x,y) \mapsto xy^2 + y. \end{aligned}$$

Let $g = f \circ \boldsymbol{\alpha}$. Calculate $g'(t)$ both by using the chain rule and by first calculating $g(t)$ and then differentiating and confirm that the answer is the same using either method.

Question 5. Find the negation of the following statements:

1. Every prime number is odd.
2. All cats hate dogs.
3. For each $y \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that $x^2 = y$.

The negation of "If P, then Q" is "P and not Q". Describe a non mathematical example to illustrate this. (Funny examples don't score higher but are always appreciated!)