Mathematical Analysis 2

Call 2 – 16/02/2024

Part 1 – 10:00-11:30

Do **not** turn this sheet over until instructed to do so.

Name: [ ],
Mat: [ ] Seat: **A**

This test is in two parts, each part consists of 3 questions, for each part you have 1.5 hours to attempt to solve the problems.

Respond to the questions with **fully and clearly argued solutions**. If there is something that you are uncertain about, do the best you can and demonstrate your knowledge of the relevant concepts.

- Students are permitted to bring only the following items to their desk in the test room: pens / pencils, drinking bottle. Calculators and any other electronic items are not permitted.
- Paper for rough calculations will be provided in the test room. After the test the paper used during the test remains in the test room.
- Under penalty of exclusion, during the test it is forbidden to communicate, using any means, with anyone except the test invigilators. If you want to communicate, raise your hand and an invigilator will come to you.
- It is forbidden to have anything that can be used for communication (e.g., telephone, smartwatch, computer, headphones, earbuds, messenger pigeon) on your person or close to you during the test.
- You may choose to leave the test early but only after receiving confirmation from an invigilator.

Question 1. For each of the following statements: (1) identify if the statement is true; (2) write the negation of the statement (without using the word "not" or similar).

- There exists a positive number x such that for every positive number y, $y = x^2$.
- There exists a positive number x such that for every positive number y, we have $y^2 = x$.
- There exist positive numbers x, y, such that $y = x^2$.
- For every positive number y, there exists a positive number x such that $y^2 = x$.
- There does not exist a positive number y such that for every positive number x, we have $y^2 = x$.

Question 2. Consider the surface $x^2 + y^2 - 4z^2 = 4$.

- Verify that the point $(2, 2, 1)$ is contained in the surface.
- Find the tangent plane to this surface at this point. *Hint: write this surface as a level set* $\{(x, y, z) : f(x, y, z) = c\}$, *calculate* ∇f *at the specified point and use the connection between gradient and tangent plane.*
- Consider the intersection of this surface with the plane $z = 2$. Find a parametrization of this curve.

Question 3. Find and classify all the stationary points of the scalar field,

$$f(x, y) = (3x + x^3)(y^2 + y).$$

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Part 2 – 11:45-13:15

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Question 4.

- Consider the vector field

$$\mathbf{F}(x,y) = \begin{pmatrix} -y(x^2+y^2)^{-1} \\ x(x^2+y^2)^{-1} \end{pmatrix}$$

  defined on $S = \mathbb{R}^2 \setminus (0,0)$. Let $\boldsymbol{\alpha}(t)$ denote the path which traverses clockwise the circle of radius $r > 0$ centred at the origin. Evaluate the line integral $\int \mathbf{F} \cdot d\boldsymbol{\alpha}$.

- Evaluate the line integral of $\mathbf{F}$ along the straight line segment from $(1,0)$ to $(0,1)$.

- Evaluate $\int \nabla g \cdot d\boldsymbol{\alpha}$ where $g(x,y) = ye^{x^2-1} + 4xy$ and the path is $\boldsymbol{\alpha}(t) = (1 - t, 2t^2 - 2t)$ for $0 \leq t \leq 2$.

Question 5. Using a double integral determine the volume of the solid that is contained within the cylinder $x^2 + y^2 = 16$, lies below $z = 2x^2 + 2y^2$ and above the xy-plane.

Question 6. Consider the surface S defined to be the half of the sphere of radius 4 with $z \geq 0$ and let $\mathbf{n}$ denote the normal with positive z-component. Consider also the vector field

$$\mathbf{F}(x,y,z) = \begin{pmatrix} y \\ -x \\ yx^3 \end{pmatrix}.$$

Use Stokes' Theorem to evaluate the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$.