Mathematical Analysis 2

Call 1 – 31/01/2024

Part 1 – 10:00-11:30

Do not turn this sheet over until instructed to do so.

Name: [ ],
Mat: [ ] Seat: **A**

This test is in two parts, each part consists of 3 questions, for each part you have 1.5 hours to attempt to solve the problems.

Respond to the questions with **fully and clearly argued solutions**. If there is something that you are uncertain about, do the best you can and demonstrate your knowledge of the relevant concepts.

- Students are permitted to bring only the following items to their desk in the test room: pens / pencils, drinking bottle. Calculators and any other electronic items are not permitted.
- Paper for rough calculations will be provided in the test room. After the test the paper used during the test remains in the test room.
- Under penalty of exclusion, during the test it is forbidden to communicate, using any means, with anyone except the test invigilators. If you want to communicate, raise your hand and an invigilator will come to you.
- It is forbidden to have anything that can be used for communication (e.g., telephone, smartwatch, computer, headphones, earbuds, messenger pigeon) on your person or close to you during the test.
- You may choose to leave the test early but only after receiving confirmation from an invigilator.

Question 1. For each of the following statements, identify if the statement is true and write the negation of the statement.

- For every positive number x, and every positive number y, we have $y^2 = x$.
- There exists a positive number x such that for every positive number y, we have $y^2 = x$.
- There exists a positive number x, and there exists a positive number y, such that $y^2 = x$.
- For every positive number y, there exists a positive number x such that $y^2 = x$.
- There exists a positive number y such that for every positive number x, we have $y^2 = x$.

Question 2. Consider the functions,

$$\boldsymbol{\alpha} : [0, \infty) \to \mathbb{R}^2; \quad t \mapsto (\sin t, \sqrt{t}),$$
$$f : \mathbb{R}^2 \to \mathbb{R}; \quad (x, y) \mapsto xy^2 - 2y.$$

Let $g = f \circ \boldsymbol{\alpha}$. Calculate $g'(t)$ both by using the chain rule and by first calculating $g(t)$ and then differentiating and confirm that the answer is the same using either method.

Question 3. Find and classify all the stationary points of the scalar field,

$$f(x, y) = (3x + 4x^3)(y^2 + 2y).$$

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Part 2 – 11:45-13:15

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Question 4. Consider the vector fields,

$$\mathbf{F}(x,y) = \begin{pmatrix} 2x^2y \\ x^3 \end{pmatrix}, \quad \mathbf{G}(x,y) = \begin{pmatrix} y^2 \\ x^2 \end{pmatrix}, \quad \mathbf{H}(x,y) = \begin{pmatrix} 2xy^3 + e^y \\ 3x^2y^2 + xe^y \end{pmatrix}.$$

Identify which is conservative on $\mathbf{R}^2$ and show that the others are not conservative. For the conservative one, find φ such that the vector field is equal to $\nabla\varphi$.

Question 5. Consider the vector field

$$\mathbf{F}(x,y) = \begin{pmatrix} 3y \\ x^2 - y \end{pmatrix}.$$

Let α denote the path composed of the upper half of the circle centred at the origin of radius 1 with counter clockwise rotation and the portion of $\{y = x^2 - 1\}$ from $x = -1$ to $x = 1$. Evaluate the line integral, $\int \mathbf{F} \cdot d\alpha$.

Question 6. Consider the surface $S = \{y = 3x^2 + 3z^2, y \leq 6\}$ and let $\mathbf{n}$ denote the normal with positive $y$component. Consider also the vector field

$$\mathbf{F}(x,y) = \begin{pmatrix} -x \\ 2y \\ -z \end{pmatrix}.$$

Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n} dS$.