WASP | WALLENBERG AUTONOMOUS SYSTEMS AND SOFTWARE PROGRAM

digital futures

KTH VETENSKAP OCH KONST

Homological algebra and persistence

TDA group at KTH in Stockholm

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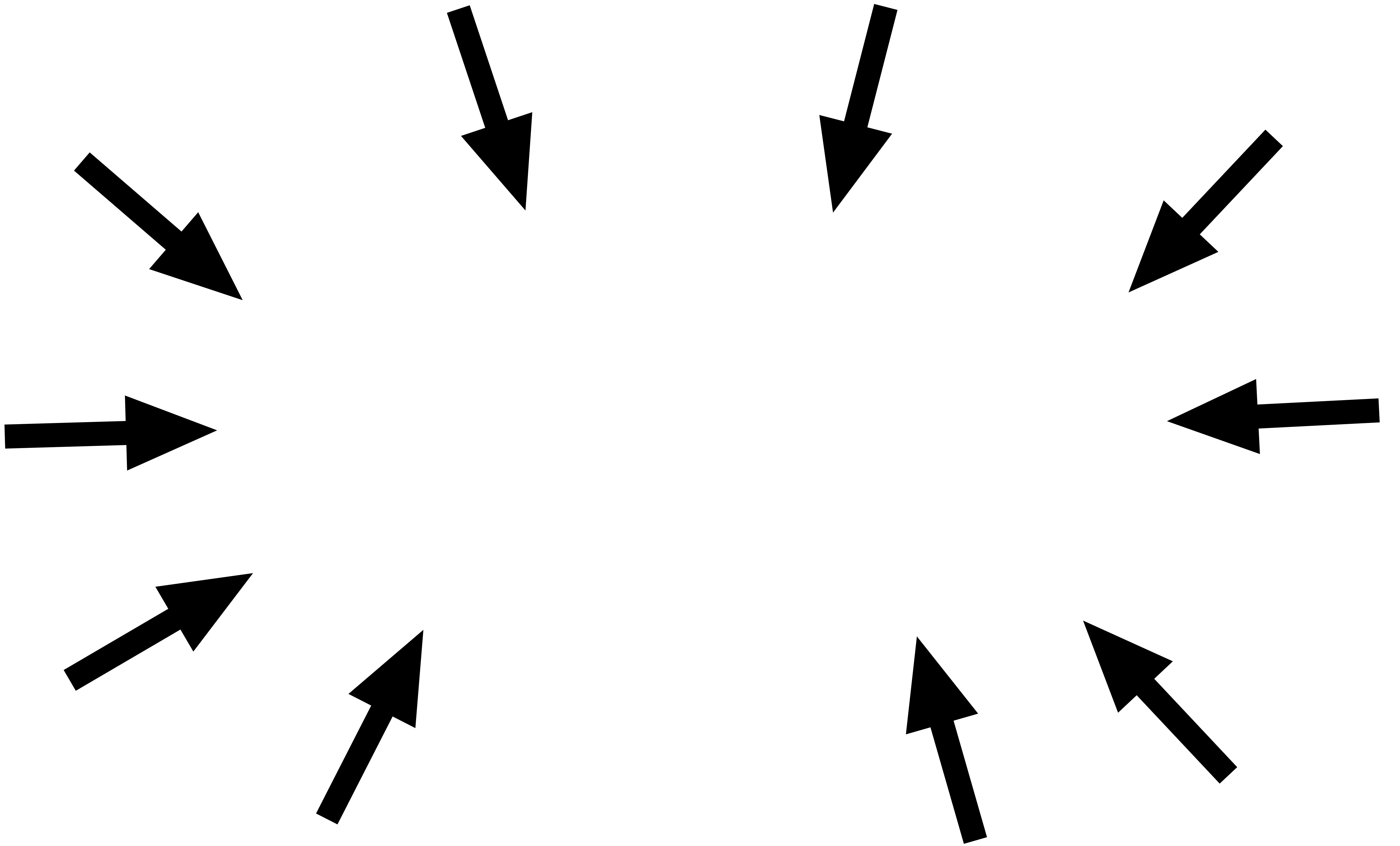
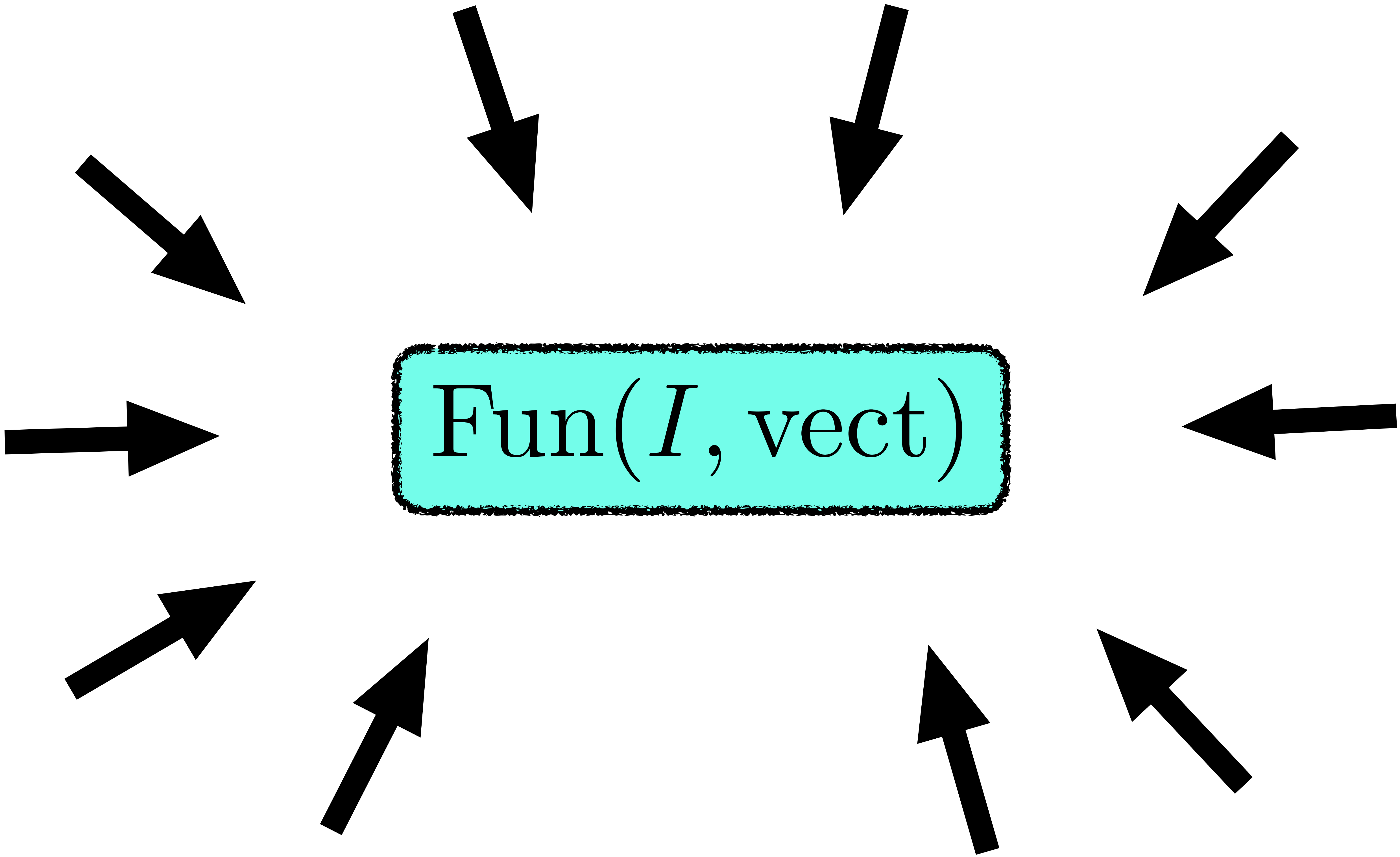
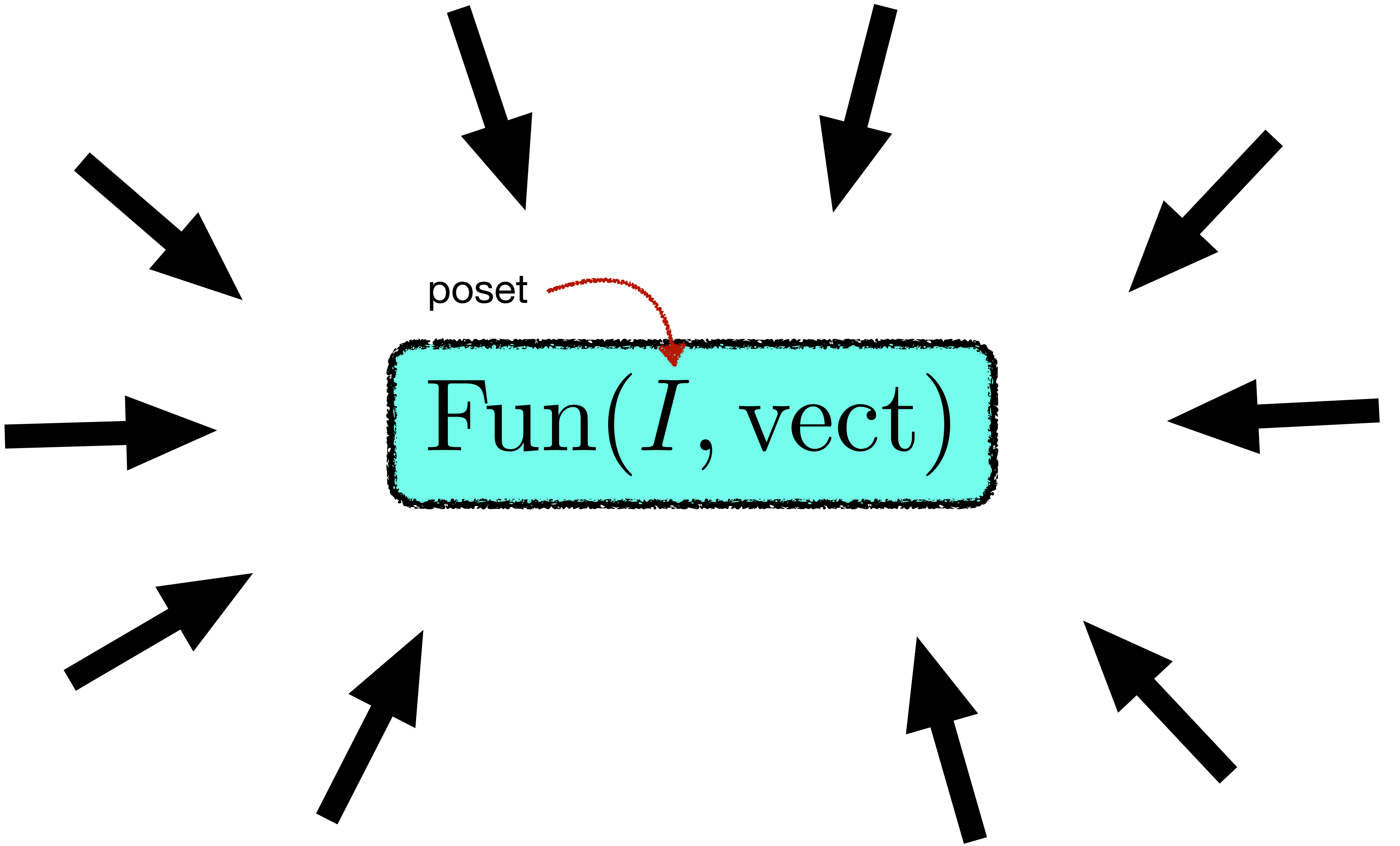
Barbara Mahler
Isaac Ren
Henri Riihimäki
Martina Scolamiero
Francesca Tombari

Topology of Data in Rome, 2022




Fun(I, vect)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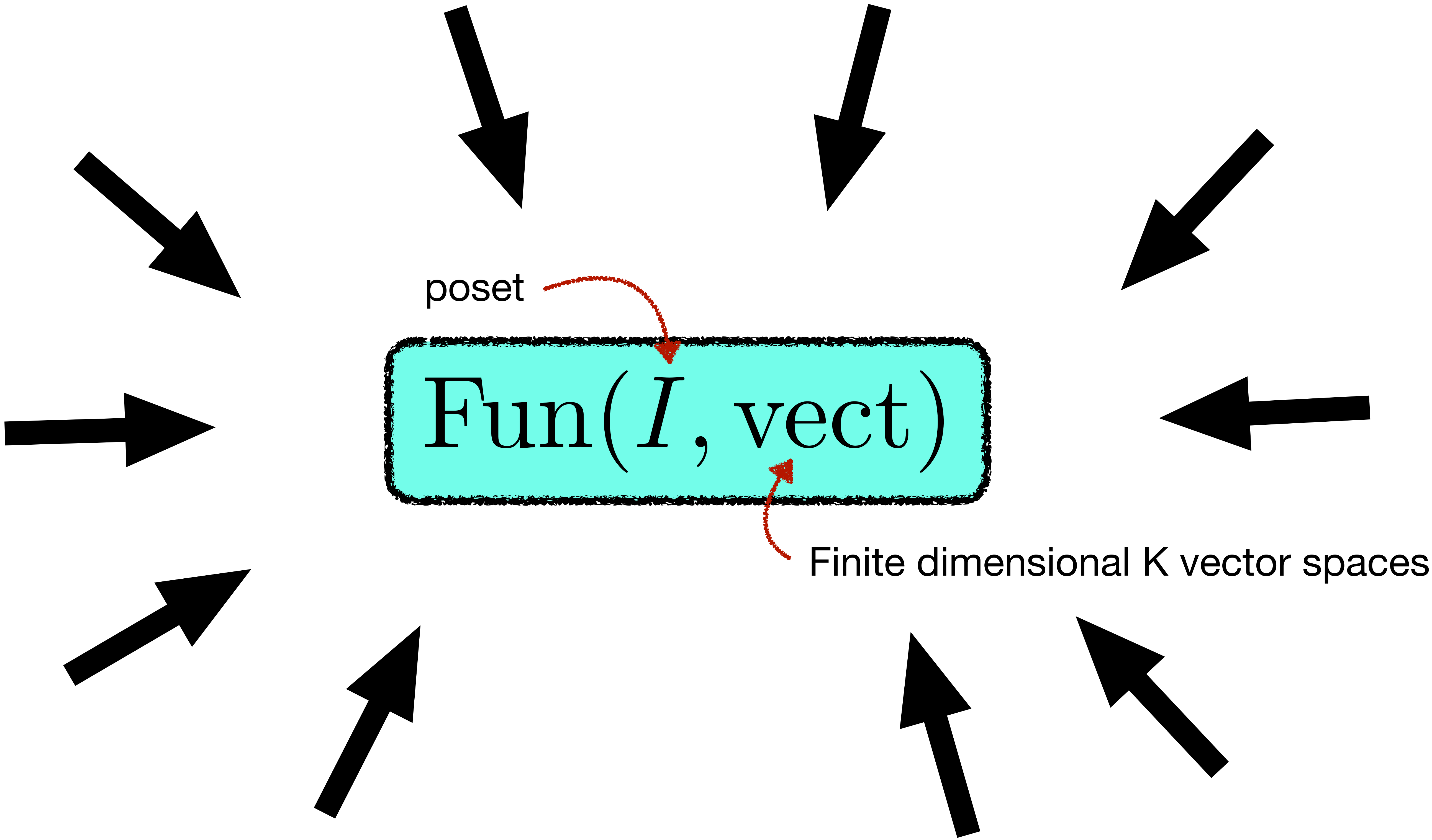
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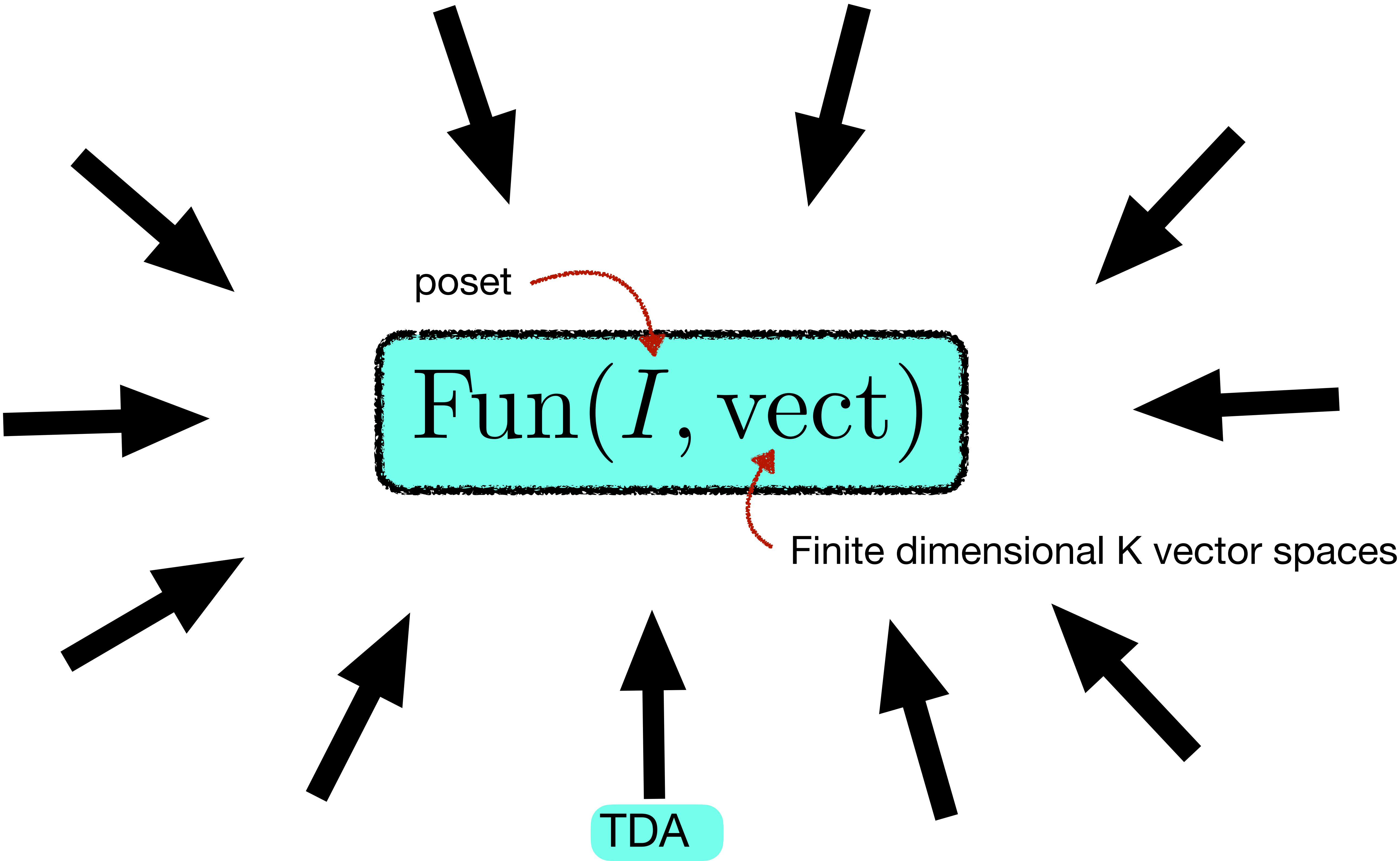
Finite dimensional K vector spaces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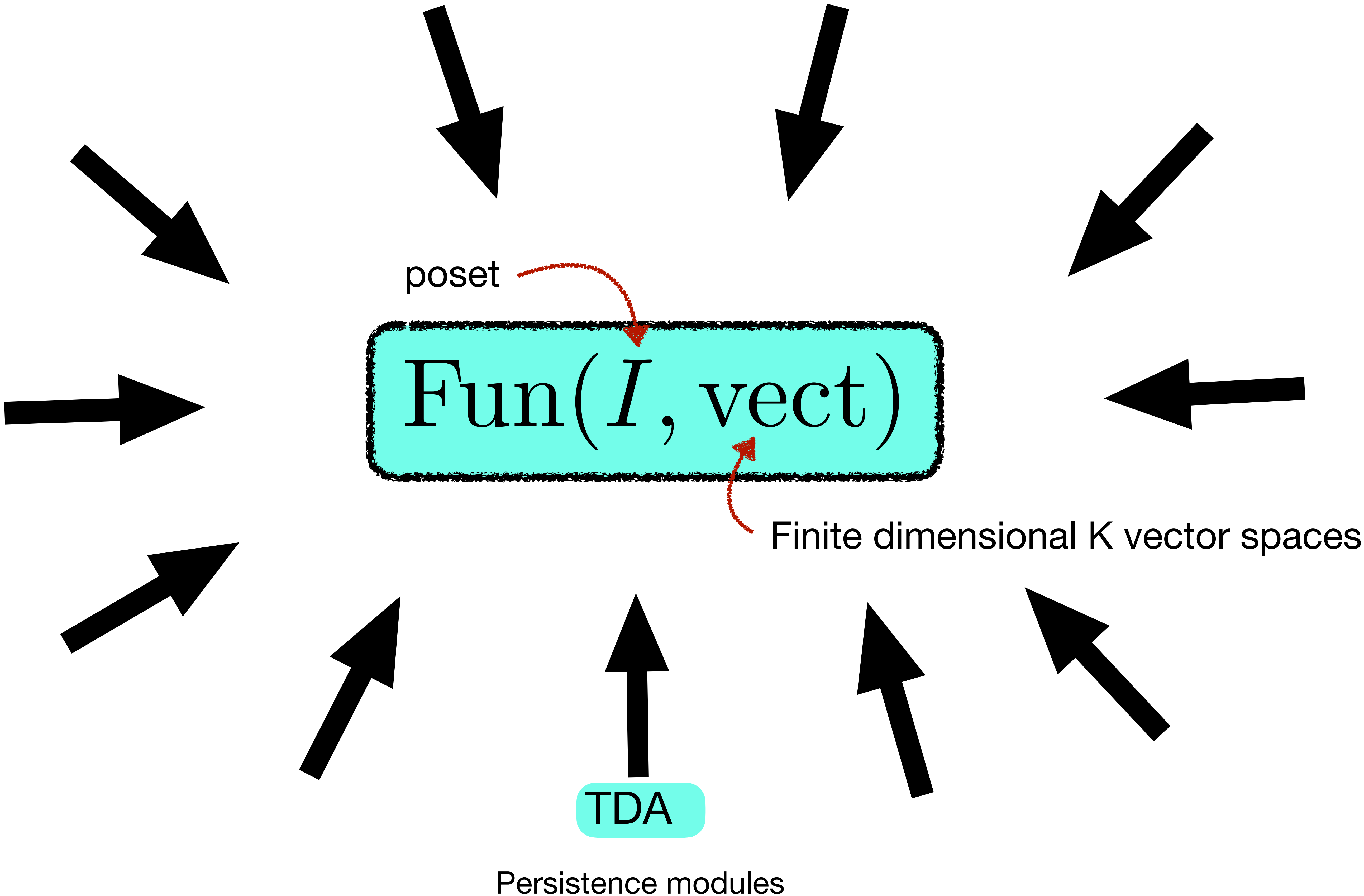
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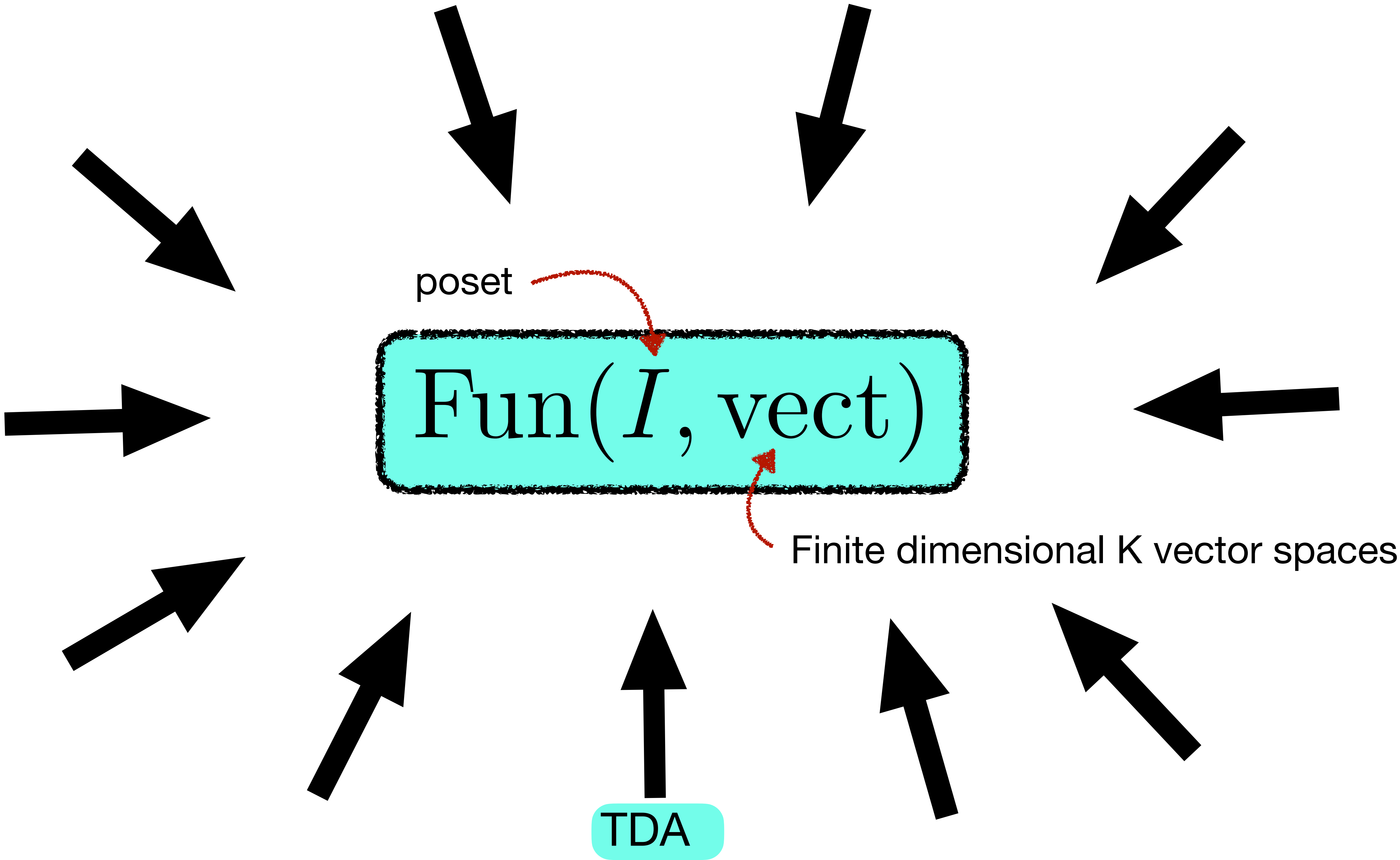
Persistence modules



Persistence modules

$$I: \qquad [n]^k := [0 < \cdots < n]^k, \quad \mathbb{N}^k, \quad [0,\infty)^k, \quad \mathbb{R}^k$$

Homological properties of persistence modules

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Finite direct sums of elements in *P* are called *P*-free

P is called **independent** if for every P-free C, there is a unique function $\beta C\colon \mathcal{P} \to \mathbb{N}$ such that:

$$C \simeq \bigoplus_{A \in \mathcal{P}} A^{\beta C(A)}$$

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If every $\mathcal{P}$ projective is $\mathcal{P}$ free, then $\mathcal{P}$ is called **acyclic.**

Covers

Choose an independent and acyclic collection $\mathcal{P}$

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A $\mathcal{P}$ cover of M is a $\mathcal{P}$ exact sequence

$$C_0 \to M \to 0$$

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A minimal $\mathcal{P}$ cover of M leads to an invariant:

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***M* is *P* free if and only if its minimal *P* cover is an isomorphism.**

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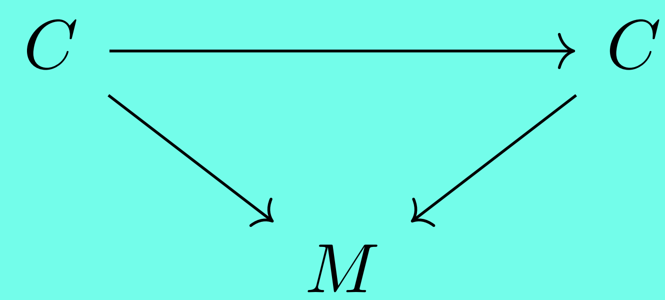

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A minimal $\mathcal{P}$ resolution of M leads to a sequence of invariants:

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How to calculate the obtained invariants?

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finite

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$$\mathcal{S} := \left\{ \quad K(v,-)\colon I \to \mathrm{vect} \mid K(v,w) = \begin{cases} K & \text{if } v \le w \\ 0 & \text{if } v \not\le w \end{cases} \text{ for } v \text{ and } w \text{ in } I \quad \right\}$$

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requires constructing differentials and their kernels

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$$\mathcal{K}_v M := \cdots \to \bigoplus_{\substack{S \subset \mathcal{U}(v) \\ S \text{ is bounded below} \\ |S|=2}} M(\wedge S) \to \bigoplus_{\substack{S \subset \mathcal{U}(v) \\ S \text{ is bounded below} \\ |S|=1}} M(\wedge S) \to M(v)$$

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$$\beta^d M(v) = \dim\ H_d(\mathcal{K}_v M)$$

Grading on P

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Pair of adjoin functors

$$M \longmapsto \mathrm{Nat}_I(\mathcal{P}(-), M)$$

$$\mathrm{Fun}(I, \mathrm{vect}) \underset{\mathcal{L}}{\overset{\mathcal{R}}{\rightleftarrows}} \mathrm{Fun}(J, \mathrm{vect})$$

$$\mathcal{P}(a) \longleftarrow\!\shortmid K(a, -)$$

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Every *a* in *J* leads to a natural transformation:

$$\mu_a\colon K(a,-) \to \mathcal{R}\mathcal{L}K(a,-) = \mathcal{R}\mathcal{P}(a) = \mathrm{Nat}_I(\mathcal{P}(-), \mathcal{P}(a))$$

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The functor is called **thin** if μ_a is surjective for every a in J

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Every *a* in *J* leads to a natural transformation:

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The functor is called **thin** if μ_a is surjective for every a in J

μ_a is thin if for every $a \leq b$ in *J*, every natural transformation $\mathcal{P}(b) \to \mathcal{P}(b)$ is of the form: $\lambda\mathcal{P}(a \leq b)$ for some $\lambda \in K$

Grading on P

poset

Assume: Instead of a collection P, we have a functor $\mathcal{P}\colon J^{op} \to \mathrm{Fun}(I, \mathrm{vect})$ called grading

Pair of adjoin functors

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In particular for every $a \leq b$ in *J*, $\dim \mathrm{Nat}_I(\mathcal{P}(b), \mathcal{P}(a)) \leq 1$

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For "many" a in *J*:

$$\beta^d_{\mathcal{P}} M(\mathcal{P}(a)) = \dim H_d(\mathcal{K}_a(\mathcal{R}M))$$

$I = \mathbb{N}^2$

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Lower hooks

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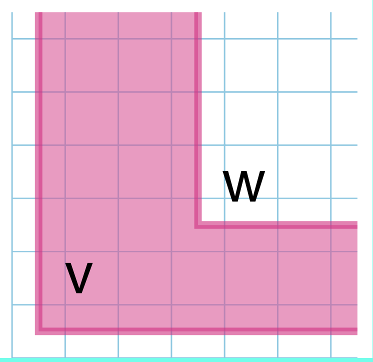
$\mathcal{P} = \Big\{$ 

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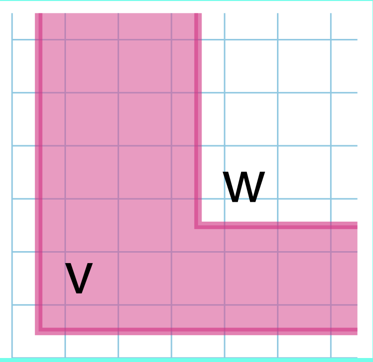
$\mathcal{P} = \Bigg\{$  $\mid v < w \text{ in } I \cup \{\infty\} \Bigg\}$

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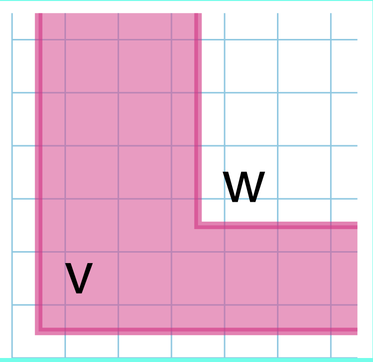

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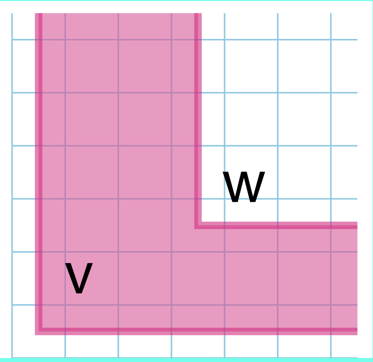
$\mathcal{P} = \left\{ \right.$  $\left. \mid v < w \text{ in } I \cup \{\infty\} \right\}$

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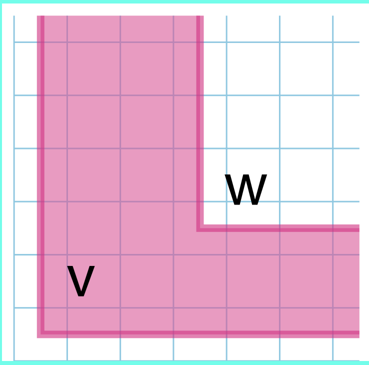
$\mathcal{P} = \Big\{$  $\mid v < w \text{ in } I \cup \{\infty\} \Big\}$

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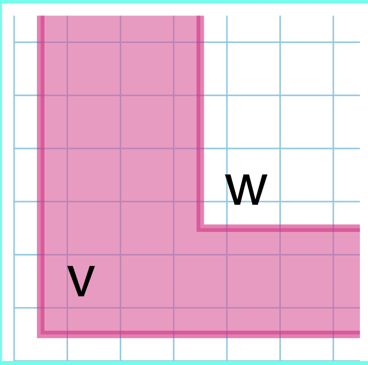
$\mathcal{P} = \Bigg\{$ 

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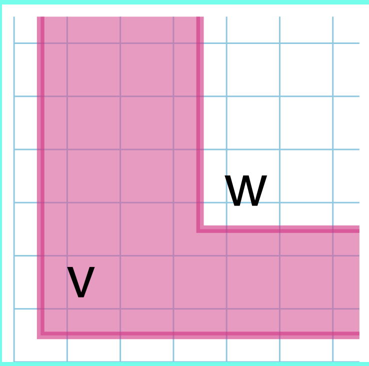

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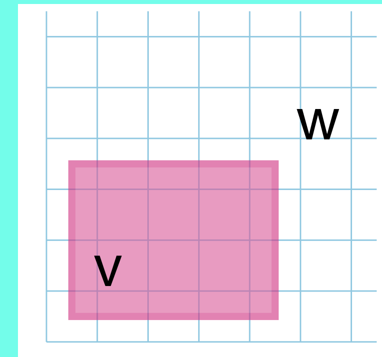

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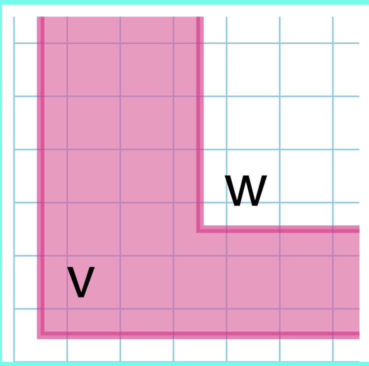
$\mathcal{P} = \Big\{$ 

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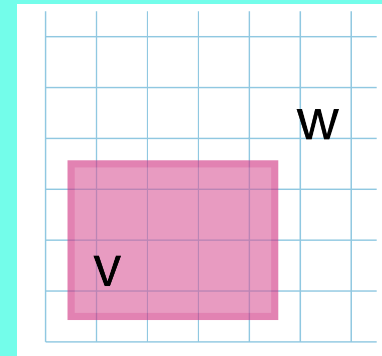
$\mathcal{P} = \Big\{$ 

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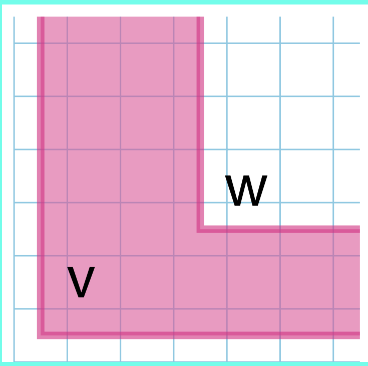

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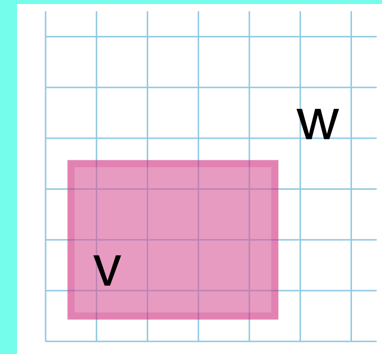

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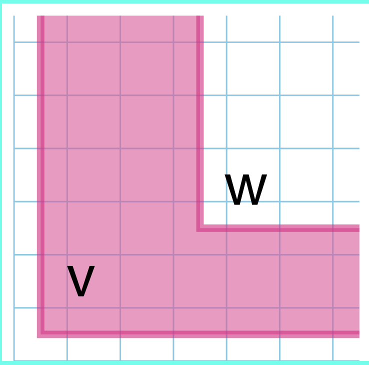

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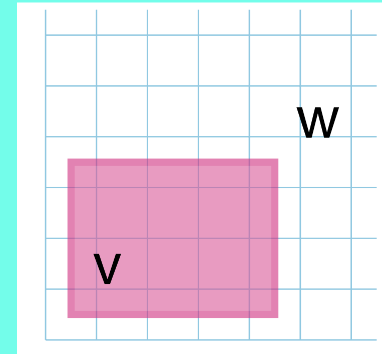

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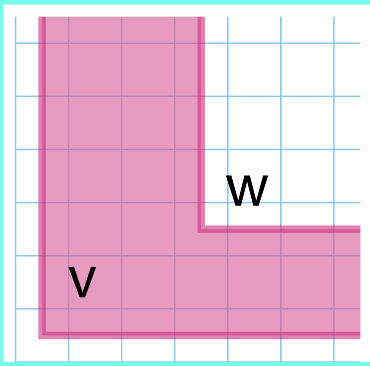
$\mathcal{P} = \Big\{$ 

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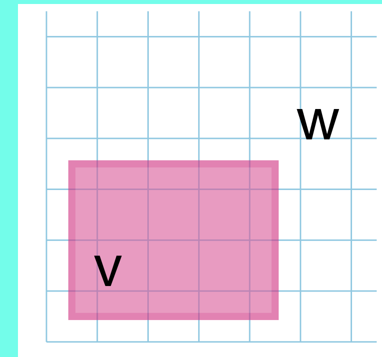
$\mathcal{P} = \Big\{$ 

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$$\beta^d_{\mathcal{P}} M(\mathcal{P}(v < w)) = \dim H_d(\mathcal{K}_{v<w}(\mathcal{R}M))$$

Fast and effective way of checking if
M is a direct sum of non zero area rectangles
and describing its components

Conclusion

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Thank you