Workshop

Mathematical Models and Methods in Earth and Space Sciences

Department of Mathematics
University of Rome Tor Vergata
Aula Magna Gismondi

Rome, March 19–22, 2019

Book of Abstracts

Gravitational wave and lensing inference from the CMB polarization

Ethan Anderes (University of California at Davis, USA)

In the last decade cosmologists have spent a considerable amount of effort mapping the radially-projected large-scale mass distribution in the universe by measuring the distortion it imprints on the CMB. Indeed, all the major surveys of the CMB produce estimated maps of the projected gravitational potential generated by mass density fluctuations over the sky. These maps contain a wealth of cosmological information and, as such, are an important data product of CMB experiments. However, the most profound impact from CMB lensing studies may not come from measuring the lensing effect, per se, but rather from our ability to remove it, a process called "delensing". This is due to the fact that lensing, along with emission of millimeter wavelength radiation from the interstellar medium in our own galaxy, are the two dominant sources of foreground contaminants for primordial gravitational wave signals in the CMB polarization. As such delensing, i.e. the process of removing the lensing contaminants, and our ability to either model or remove galactic foreground emission effectively sets the noise floor on upcoming gravitational wave science.

In this talk we will present a complete Bayesian solution for simultaneous inference of lensing, delensing and gravitational wave signals in the CMB polarization as characterized by the tensor-to-scalar ratio r parameter. Our solution relies crucially on a physically motivated re-parameterization of the CMB polarization which is designed specifically, along with the design of the Gibbs Markov chain itself, to result in an efficient Gibbs sampler—in terms of mixing time and the computational cost of each step—of the Bayesian posterior. This re-parameterization also takes advantage of a newly developed lensing algorithm, which we term LenseFlow, that lenses a map by solving a system of ordinary differential equations. This description has conceptual advantages, such as allowing us to give a simple non-perturbative proof that the lensing determinant is equal to unity in the weak-lensing regime. The algorithm itself maintains this property even on pixelized maps, which is crucial for our purposes and unique to LenseFlow as compared to other lensing algorithms we have tested. It also has other useful properties such as that it can be trivially inverted (i.e. delensing) for the same computational cost as the forward operation, and can be used for fast and exact likelihood gradients with respect to the lensing potential. Incidentally, the ODEs for calculating these derivatives are exactly analogous to the backpropagation techniques used in deep neural networks but are derived in this case completely from ODE theory.

Dynamics of the global meridional ice flow of Europa's icy shell

Yosh Ashkenazy (Ben-Gurion University of the Negev, Israel)

Europa is one of the most probable places in the solar system to find extra-terrestrial life, motivating the study of its deep (~ 100 km) ocean and thick icy shell. The chaotic terrain patterns on Europa's surface have been associated with vertical convective motions within the ice. Horizontal gradients of ice thickness are expected due to the large equator-to-pole gradient of surface temperature and can drive a global horizontal ice flow, yet such a flow and its observable implications have not been studied. We present a global ice flow model for Europa composed of warm, soft ice flowing beneath a cold brittle rigid ice crust. The model is coupled to an underlying (diffusive) ocean and includes the effect of tidal heating and convection within the ice. We show that Europa's ice can flow meridionally due to pressure gradients associated with equator-to-pole ice thickness differences, which can be up to a few km and can be reduced both by ice flow and due to ocean heat transport. The ice thickness and meridional flow direction depend on whether the ice convects or not; multiple (convecting and non-convecting) equilibria are found. Measurements of the ice thickness and surface

temperature from future Europa missions can be used with our model to deduce whether Europa's icy shell convects and to constrain the effectiveness of ocean heat transport.

Ocean eddies, submesoscale turbulence and vertical transport

Annalisa Bracco (Georgia Institute of Technology, USA)

In 1984 Aref introduced the term chaotic advection to describe mixing in a flow controlled by coherent vortices. In such a flow the distribution of a passive tracer is stirred into small-scale structures by the stretching and folding property of the advective flows. As a result, any smooth initial distribution evolves into a complex pattern of filaments that, when sufficiently stretched, can be smoothed out by diffusion. Vortices, or eddies, are ubiquitous in the ocean, and also in many other geophysical flows characterized by a small aspect ratio (vertical scale / horizontal scale). As a result, a large number of studies have applied ideas from chaotic advection to oceanic problems often assuming that oceanographic flows are nearly two-dimensional or quasi-geostrophic in the horizontal plane. Under such approximation the vertical velocity field w is weak and its derivative is negligible to the leading order in the continuity equation. Relatively less attention has been paid to the investigation of dispersion in oceanic—or more broadly, geophysical—flows that are fully three-dimensional while maintaining a small aspect ratio, and for which its derivative cannot be neglected. This is the case for the so-called submesoscale circulations, for which the Rossby and Richardson numbers are $O(1)$. Submesoscale circulations develop preferentially at the vertical boundaries of geophysical flows in presence of density gradients; in the ocean they concentrate at the surface and bottom boundary layers. Here I will discuss the contribution to vertical transport of submesoscale dynamics at the edge and within the core of vortices, using available observations and simulations obtained using a state-of-the-art regional ocean model. This challenging problem is key to our understanding of biogeochemical interactions, primary productivity and export production in the ocean, and may be of relevance to other geophysical flows, on our planet and elsewhere.

COMPASS: Control for orbit manoeuvring enhancing natural perturbations

Camilla Colombo (Politecnico di Milano, Italy)

Space benefits mankind through the services it provides to Earth. Future space activities progress thanks to space transfer and are safeguarded by space situation awareness. Natural orbit perturbations are responsible for the trajectory divergence from the nominal two-body problem, increasing the requirements for orbit control; whereas, in space situation awareness, they influence the orbit evolution of space debris that could cause hazard to operational spacecraft and near-Earth objects that may intersect the Earth.

However, the COMPASS project proposes to leverage the dynamics of natural orbit perturbations to significantly reduce current extreme high mission cost and create new opportunities for space exploration and exploitation. It bridges over the disciplines of orbital dynamics, dynamical systems theory, optimisation and space mission design by developing novel techniques for orbit manoeuvring by "surfing" through orbit perturbations. The use of semi-analytical techniques and tools of dynamical systems theory will lay the foundation for a new understanding of the dynamics of orbit perturbations.

In this work we consider the exploitation of luni-solar perturbations for the disposal of high-altitude orbits about the Earth. The dynamics of highly altitude orbits is mainly influenced by the effects of the third body perturbation due to the gravitational attraction of the Moon and the Sun, coupled

with the Earth's oblateness. The luni-solar attraction induces long-term and secular variation in the eccentricity, inclination, argument of the node and argument of perigee. The representation in orbital elements with respect to the perturbing body plane can be described by a two degree of freedom Hamiltonian system. An analytical model is proposed for designing the disposal manoeuvre to be given to a spacecraft in these orbits to achieve natural re-entry by exploiting the long-term effects of these natural perturbations, enhanced by an impulsive manoeuvre. The optimal initial conditions during the natural evolution of the argument of perigee and the orbit eccentricity is selected so that, through an impulsive manoeuvre, the new orbit conditions will lead to a natural increase of the orbit eccentricity until re-entry is reached.

The proposed idea of optimal navigation through orbit perturbations can address various major engineering challenges in space situation awareness, for application to space debris evolution and mitigation, re-entry prediction, deflection of potentially hazardous asteroids and planetary protection, and in space transfers, for perturbation-enhanced trajectory design in the planetary and interplanetary space.

Locally Refined Splines for compact representation and analyses of geospatial big data

Tor Dokken (SINTEF, Norway)

The volume of geospatial data acquired by satellites, from the air, on the ground, on the sea or in the sea using image, lidar or sonar technology is continuously growing. Surveys are often conducted in locations that have existing data from before, providing updated datasets. The data acquired can either confirm already existing information, or it can represent changes important to detect and understand. In the FP7 IP `www.iqmulus.eu` (2012–2016) technology for the representation of huge datasets of the ocean floor was developed using a novel technology called locally refined splines. The work is continued in the national Norwegian project `www.sintef.no/en/projects/analyst/` (2017–2021). The smooth component of the data is represented using spline surfaces where local degrees of freedom are added where required by the data behaviour. The result is a spline surface representing the smooth component and a point set that is not part of the smooth behaviour. Data compression experienced are typically two orders of magnitude. The representation is well suited for GPU accelerated visualization. The spline surface representation also makes it easy to check if new data sets confirm the already existing information, or if deviations are found that must be checked and analysed in more detail, with a possible update of the spline model. So far, the approach has been tried out on lidar and sonar data. However, we see a great potential for testing the approach on sets of multi-spectral image satellite data to represent the data as a compact locally refined spline in a consistent coordinate system.

On the restricted three-body problem with crossing singularities

Giovanni Gronchi (Università di Pisa, Italy)

In this talk we deal with the long term behaviour of the solutions of the restricted three-body problem, say Sun-Earth-asteroid, when the osculating Keplerian trajectory of the asteroid can cross the trajectory of the Earth.

Crossings of trajectories correspond to singularities of the averaged Hamilton equations, where the average is made over the two fast Delaunay variables of the asteroid and the Earth. The solutions of the averaged equations with crossing singularities have been studied starting from the 70's by Lidov and Ziglin, who dealt with a particular case, and more recently in a series of papers by myself and different collaborators. The results of these investigations can be applied to some relevant problems in Astronomy, such as the computation of proper elements (i.e. quasi-integrals of motion) for near-Earth asteroids and the detection of parent bodies of meteor streams. Another interesting application is a qualitative measure of the hazard of a near-Earth asteroid, allowing a quick estimate of possible Earth-crossing times valid from a statistical point of view. First I'll review what is known about this problem. Then I'll present some new preliminary results, obtained in collaboration with M. Fenucci, concerning the relation between the solutions of the averaged equations and the corresponding components of the solutions of the full equations.

Invariant manifolds for a Solar sail

Àngel Jorba (Universitat de Barcelona, Spain)

Solar Sailing is a form of spacecraft propulsion, where large membrane mirrors reflect the solar radiation pressure to push the spacecraft. Although this acceleration is smaller than the one achieved by a traditional engine, it is continuous and unlimited. This makes some long term missions more accessible, and opens a wide new range of possible applications that cannot be achieved by a traditional spacecraft.

In this presentation we will focus on the dynamics of a Solar sail in a couple of situations. The first is the motion near collinear equilibrium points of the Earth-Sun system, and the second example is the dynamics of a Solar sail close to an asteroid. The main tools used are the computation of stable, unstable and centre manifolds.

Generating sparse representations by adaptive multiscale approximations

Angela Kunoth (Universität zu Köln, Germany)

Facing the challenge to extract and represent information inherent in masses of data or in complex systems, adaptive approaches based on multiscale representations play an important role in current scientific research in many areas of science. The leading paradigm is to spend a minimal amount of degrees of freedom and work while extracting and representing the maximal amount of information. This approach allows one to principally work with computer platforms of any size since it is ultimately steered by the availability of a certain budget.

I want to discuss two classes of problems for which adaptive multilevel schemes can be developed along this line. The first class concerns explicitly given information and discusses the problem of fitting nonuniformly distributed data to approximate multivariate functions. The second class deals with approximating optimization problems for operator equations, specifically, control problems constrained by elliptic and parabolic partial differential equations which may, in addition, have stochastic coefficients. Here the information—the state and control of a system—is contained implicitly. Both applications have in common that the solution method is based on minimizing a quadratic functional and that the concept of adaptivity in a coarse-to-fine fashion plays a central role. For developing solution schemes of optimal complexity, I also address fast iterative solvers.

Mathematical modelling of ice with numerical experiments in alpine, polar and extraterrestrial environment

Daniela Mansutti (IAC-CNR, Roma, Italy)

In this talk I shall present different mathematical models aimed to describe evolving thermo-mechanics of ice in different topo-morphological and climatic conditions. Up-to-date computational glaciology address to the intensive use of the large amount of data, gathered in (alpine or polar) on-field campaigns, and to the "brute force" adaptation of the mathematical modelling of glacier evolution based on Glen's law via phenomenological multi-parametrical functional factors and/or addenda. Although, reasonable to fully satisfactory numerical results have been being obtained with this approach adopted by the most popular open-source computational glaciology codes, with the aim to improve the comprehension of the physical mechanisms and processes, I shall discuss extensions of such models by explicit inclusion of natural phase transition occurrence (inherent and/or at a boundary interface) and by expansion of Glen's constitutive equation in order to take into account the effects of the presence of sand and rock fragments in glacier interstices. Several problems will be discussed: the description of the thermo-mechanical evolution of the icy crust of Europa, Juppiter's satellite; the check of the compatibility of the existence of a subglacial lake at Svalbard archipelago; the reproduction of the borehole measurements at the Murtel–Corvatsch glacier, Grisons Alps, Switzerland. Thus extraterrestrial, polar and alpine environments, respectively, will be considered. Whether the added value of the numerical results obtained in each case impacts either on fundamental fluid dynamics or applied glaciology will be, eventually, clarified by further numerical testing on real problems. Certainly, by facing the mentioned problems, meaningful, very complex, partial differential systems have come to my attention as a mathematician, and their coefficient calibration, solution characterization and treatment, still represent an excellent opportunity of future involvement for mathematical numerical deep skills and expertise.

Parameter determination for Energy Balance Models with Memory

Patrick Martinez (Université Toulouse III – Paul Sabatier, France)

We are interested in a problem arising in climate dynamics, coming more specifically from the classical Energy Balance Models introduced independently by Budyko and Sellers. These models describe the evolution of temperature as the effect of the balance between the amount of energy received from the Sun and radiated from the Earth, and were developed in order to understand the past and future climate and its sensitivity to some relevant parameters on large time scales (centuries). After averaging the surface temperature over longitude, they take the form of a one-dimensional nonlinear parabolic equation with degenerate diffusion. In order to take into account the long response times that cryosphere exhibits (for instance, the expansion or retreat of huge continental ice sheets occurs with response times of thousands of years), it is important to let the coalbedo function depend on a memory term (see Bhattacharya–Ghil–Vulis [1]).

The mathematical analysis of such quasilinear Energy Balance Models with Memory has been the subject of many deep works for a long time, see in particular Díaz [3]. Here we consider a class of Budyko's and Sellers' type models, and we study the following inverse problem: is it possible to recover the insolation function (which is a part of the incoming solar flux) from measurements of the solution? We provide existence and regularity results for these models, and we obtain uniqueness and stability estimates that are useful for the determination of the insolation function in Sellers' type model with memory. This completes in particular results of Roques et al. [4].

References

[1] K. Bhattacharya, M. Ghil, and I.L. Vulis. Internal variability of an energy-balance model with delayed albedo effects. *J. Atmos. Sci.*, **39** (1982), 1747–1773.

[2] P. Cannarsa, M. Malfitana, and P. Martinez. Parameter determination for Energy Balance Models with Memory. *Springer INdAM Series*, in press (hal-01834739).

[3] J.I. Díaz. On the mathematical treatment of energy balance climate models. In: J.I. Díaz (eds) *The Mathematics of Models for Climatology and Environment. NATO ASI Series (Series I: Global Environmental Change)*, **48** (1997), 217-251, Springer.

[4] L. Roques, M.D. Checkroun, M. Cristofol, S. Soubeyrand, and M. Ghil. Determination and estimation of parameters in Energy Balance Models with Memory. *Proc. Royal Soc. A*, **470** (2014), 20140349.

Wavelet localisation of isotropic random fields on the sphere and cosmological implications: searching for primordial gravitational waves

Jason McEwen (University College, London, UK)

Cosmological observations of the relic radiation of the Big Bang, the cosmic microwave background (CMB), are made on the celestial sphere and so inherently live on the twosphere (hereafter sphere). Wavelet techniques on the sphere have become a common tool for cosmologists to analyse observational data of the CMB to improve our understanding of the physical processes describing the early Universe, which remain poorly understood. Scale discretised wavelets yield a directional framework on the sphere where a signal can be probed not only in scale and position but also in orientation. Furthermore, a signal can be synthesised from its wavelets coefficients exactly, in theory and practice. I will review directional scale-discretised wavelets on the sphere and discuss their quasi-exponential localisation properties and asymptotic uncorrelation properties for isotropic Gaussian random fields on the sphere. I will then discuss the use of spin scale-discretised wavelets on the sphere to recover E- and B-mode signals for CMB polarisation, in order to search for the extremely weak signature of primordial gravitational waves that is anticipated but has so far escaped direct detection.

A percolation approach for the detection of bottlenecks in air traffic networks

Salvatore Miccichè (Università di Palermo, Italy)

Our main aim is detecting critical links in air traffic networks by using a percolation approach. While the percolation approach has already been considered in other model complex systems, to our knowledge it is the first time that an empirical study using this methodology is performed at the level of the network that originates from the navigation points that appear in the aircraft flight plans. In our network, the navigation points are the nodes and we set a link between two nodes whenever there is a flight between them. In general, percolation is a random process exhibiting a phase transition. In the simplest setting percolation is investigated in simple geometrical systems such as regular lattices covering a 2D surface. Even in the simplest setting there are different variants of the percolation problem. Specifically, one speaks about bond percolation when a link between two neighbouring sites is present with probability q and its absence is observed with probability $(1-q)$. In this variant all sites are present in the system and links between any pair of them may or may not be present. In the other variant of node percolation the links of the lattice are always present between two occupied sites, but each site is occupied with probability q and empty with probability $(1-q)$. In our context, percolation refers to probabilistic, network-wide emergent behaviour, between sites or sub-systems (clusters). Specifically, percolation refers to the phenomenon by which, according

to the value of q, which play the role of an order parameter, the network shows a transition from a phase where all nodes are connected in a unique giant connected component to a phase where nodes belong to clusters of elements not connected between each others, i.e. the network gets fragmented. As such while in the first case it is always possible to reach any point in the network starting from any other point, as soon as q increase the network gets fragmented into smaller pieces, which prevents a potential user to reach any point in the network starting from any other point. The investigations we have so far performed, although preliminary, yet clearly shows that percolation occurs in the navigation point network generated starting from the aircraft trajectories flying over the ECAC airspace in a cartain day and this approach is therefore able to detect links, characterized by certain q values, which carry a potential risk of disruption in the network. We will also discuss the implications of our results at the level of air traffic sectors. These are the elementary blocks in which the airspace is partitioned for the purpose of a better management of the aircraft trajectories when performing conflict detection and resolution. One key parameter characterizing sectors is capacity, i.e. the maximal number of aircraft present in the sectors in a certain time-interval. We will specifically discuss the role of capacity in relation to the value qc of the order parameter q that characterizes the critical links.

Extracting cosmological information from CMB experiments and cross-correlation with other probes

Gianluca Polenta (Agenzia Spaziale Italiana, Italy)

The Cosmic Microwave Background is a fundamental source of information for cosmology. In the standard inflationary scenario, the CMB can be described to first order as a Gaussian random field on the sphere, and as such all cosmological information is encoded in its angular power spectrum. While a simple estimator can be defined for an ideal full-sky, noise-free CMB map, this is not the case for real applications where partial sky coverage, instrumental effects, and contamination from other astrophysical sources have to be considered. Hence, a number of estimators have been developed following different trade-off between optimality, unbiasedness, and computational resources, also depending on the characteristics of the dataset to be analysed. Being a tensor on the sphere, CMB polarisation is adding further complexity to this problem. Moreover, power spectrum estimation is an important, yet intermediate step during CMB data processing, since a likelihood function is actually needed to estimate the cosmological parameters within a Bayesian framework. In this talk I will review the different classes of estimators used in the CMB data analysis, focusing in particular to the solutions adopted for the ESA Planck satellite, which represents the current reference in the CMB field. Finally, next generation of space-borne and ground based experiments are expected to produce a huge amount of data, such as large galaxy surveys from e.g. Euclid and LSST, and maps of 21cm radiation from SKA. I will highlight how cross-angular power spectrum techniques can be used to correlate different cosmological probes and constrain cosmology.

Climate models for deep time and deep space

Antonello Provenzale (IGG-CNR, Pisa, Italy)

Current state-of-the-art global climate models include many interacting components—atmospheric physics and chemistry, ocean dynamics, soil and vegetation processes, and abridged representations of biogeochemical cycles—providing a global view of Earth's climate with a spatial resolution that can be as high as a few tens of kilometers. Owing to computational constraints, however, such models cannot be run for very long times and, perhaps most importantly, contain several parame-

terizations of unresolved processes that are tuned to the current climate of our planet. As such, they can be adapted to slightly different climates (such as those expected in the coming decades) but can have problems when exploring very different conditions such as those encountered in ancient times (e.g., the very warm period of the Paleocene - Eocene Thermal Maximum about 55 million years ago) or the climate of other planets, for example to assess the possible existence of climatic conditions supporting the presence of life. Correspondingly, these huge climate models may have troubles identifying possible "tipping points" and other occurrences of nonlinear behaviour that can drastically and suddenly alter the climatic state. To explore the climate of deep time and deep space, used as a source of information on the full spectrum of climate possibilities, more basic research is needed to unravel the dynamics occurring in such different conditions. In this framework, a first step is to use simplified climate models such as modern versions of the ancient "Energy Balance Models" or of atmospheric column models, or even more simplified models addressing the interaction of geosphere and biosphere. In this talk I'll review some of these approaches, focusing on some of these simplified models and their application to distant climates.

Isogeometric analysis for plasma physics applications

Eric Sonnendrücker (IPP, Garching, Germany)

Isogeometric analysis and more precisely Finite Element Exterior Calculus based on B-splines has proven to be a very promising tools for different models from plasma physics in particular for simulation magnetic fusion devices like tokamaks. After describing the mathematical and numerical framework, we shall present some recent results for mesh generation, simulations based on MHD and kinetic models and specific problems related to the singularities arising at the magnetic axis and on the X-point in Tokamak geometry as well as fast multigrid solvers and their preconditioning.

Blind galaxy survey images deconvolution with shape constraint

Jean-Luc Starck (CEA, Paris-Saclay, France)

Removing the aberrations introduced by the Point Spread Function (PSF) is a fundamental aspect of astronomical image processing. The presence of noise in observed images makes deconvolution a nontrivial task that necessitates the use of regularisation. This task is particularly difficult when the PSF varies spatially as is the case for big surveys such as LSST or Euclid surveys. It becomes a fantastic challenge when the PSF field is unknown. The first step is therefore to estimate accurately the PSF field. In practice, isolated stars provide a measurement of the PSF at a given location in the telescope field of view. Thus we propose an algorithm to recover the PSF field, using the measurements available at few these locations. This amounts to solving an inverse problem that we regularize using mathematical concepts such as optimal transport, graph theory and sparsity. We also show how a shape constraint can be added in the inverse problem which improves significantly the shape measurements on the reconstructed galaxies.

Optimal control in aerospace

Emmanuel Trélat (Sorbonne Université – Paris 6, France)

I will report on nonlinear optimal control theory and show how it can be used to address problems in aerospace, such as orbit transfer. The knowledge resulting from the Pontryagin maximum principle

is in general insufficient for solving adequately the problem, in particular due to the difficulty of initializing the shooting method. I will show how the shooting method can be successfully combined with numerical homotopies, which consist of deforming continuously a problem towards a simpler one. In view of designing low-cost interplanetary space missions, optimal control can also be combined with dynamical system theory, using the nice dynamical properties around Lagrange points that are of great interest for mission design.

On the design of space trajectories under severe uncertainty: the case of the exploration of the solar system with small satellites

Massimiliano Vasile (University of Strathclyde, Glasgow, UK)

The dynamics of spacecraft is subject to different types of uncertainty that affect the overall reliability and robustness of a space mission. The use of safety and system margins, extensive qualification campaigns and accurate but costly operations mitigate the effect and propagation of this uncertainty. However, the emergence of small, inexpensive and fast-responsive spacecraft for space exploration has induced mission designers to reconsider the traditional approach to uncertainty treatment and to develop more rigorous methods for uncertainty quantification and optimisation under uncertainty.

This presentation discusses the different sources of uncertainty that can affect the realisation of small exploration missions and introduces some computational techniques to account for epistemic uncertainty in dynamics and systems design.