Optimal control in aerospace

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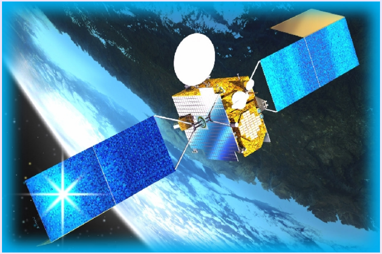
The orbit transfer problem with low thrust

Controlled Kepler equation

$$\ddot{q} = -q\frac{\mu}{|q|^3} + \frac{F}{m}$$

$q \in \mathbb{R}^3$: position, F: thrust, m mass:

$$\dot{m} = -\beta|F|$$



Maximal thrust constraint

$|F| = (u_1^2 + u_2^2 + u_3^2)^{1/2} \leqslant F_{\max} \simeq 0.1N$

Orbit transfer

from an initial orbit to a given final orbit

Controllability properties studied in

- B. Bonnard, J.-B. Caillau, E. Trélat, *Geometric optimal control of elliptic Keplerian orbits*, Discrete Contin. Dyn. Syst. Ser. B **5**, 4 (2005), 929–956.
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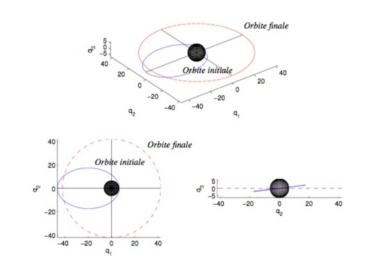

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Modelling in terms of an optimal control problem

State: $x(t) = \begin{pmatrix} q(t) \\ \dot{q}(t) \end{pmatrix}$

Control: $u(t) = F(t)$

Optimal control problem

$$\dot{x}(t) = f(x(t), u(t)), \qquad x(t) \in M, \qquad u(t) \in \Omega$$

$$x(0) = x_0, \quad x(T) = x_1$$

$$\min C(T, u), \quad \text{where} \;\; C(T, u) = \int_0^T f^0(x(t), u(t))\, dt$$

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Definition

End-point mapping

$$\begin{array}{rcl} E_{x_0,T} : L^\infty([0,T],\Omega) & \longrightarrow & M \\ u & \longmapsto & x(T;x_0,u) \end{array}$$

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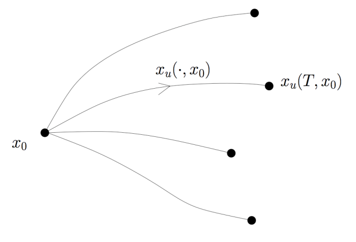

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$\longrightarrow$ Optimization problem

$$\min_{E_{x_0,T}(u)=x_1} C(T, u)$$

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Definition

A control u (or the trajectory $x_u(\cdot)$) is *singular* if $dE_{x_0,T}(u)$ is not surjective.

Lagrange multipliers (or KKT in general)

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Optimization problem

$$\min_{E_{x_0,T}(u)=x_1} C(T,u)$$

Lagrange multipliers (if $\Omega = \mathbb{R}^m$)

$$\exists(\psi,\psi^0) \in (T^*_{x(T)}M \times \mathbb{R}) \setminus \{(0,0)\} \quad | \quad \psi.dE_{x_0,T}(u) = -\psi^0 dC_T(u)$$

In terms of the Lagrangian $L_T(u,\psi,\psi^0) = \psi.E_{x_0,T}(u) + \psi^0 C_T(u)$:

$$\frac{\partial L_T}{\partial u}(u,\psi,\psi^0) = 0$$

- Normal multiplier: $\psi^0 \neq 0 \quad (\rightarrow \psi^0 = -1)$.
- Abnormal multiplier: $\psi^0 = 0 \quad (\Leftrightarrow u$ singular, if $\Omega = \mathbb{R}^m)$.

Pontryagin Maximum Principle

Optimal control problem

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Pontryagin Maximum Principle

Every minimizing trajectory $x(\cdot)$ is the projection of an *extremal* $(x(\cdot), p(\cdot), p^0, u(\cdot))$ solution of

$$\dot{x} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial x}, \qquad H(x, p, p^0, u) = \max_{v \in \Omega} H(x, p, p^0, v)$$

where $H(x, p, p^0, u) = \langle p, f(x,u) \rangle + p^0 f^0(x,u)$.

An extremal is said *normal* whenever $p^0 \neq 0$, and *abnormal* whenever $p^0 = 0$.

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$$(p(T), p^0) = (\psi, \psi^0) \quad \text{up to (multiplicative) scaling.}$$

An extremal is said *normal* whenever $p^0 \neq 0$, and *abnormal* whenever $p^0 = 0$.

Singular trajectories coincide with projections of abnormal extremals s.t. $\frac{\partial H}{\partial u} = 0$.

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$$u(t) = u(x(t),p(t))$$

$\Big($locally, e.g. under the strict Legendre assumption:

$\dfrac{\partial^2 H}{\partial u^2}(x,p,u)$ negative definite$\Big)$

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Shooting method:

Extremals $z = (x, p)$ are solutions of

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$$\dot{p} = -\frac{\partial H}{\partial x}(x,p), \quad p(0) = p_0$$

where the optimal control maximizes the Hamiltonian.

Exponential mapping

$$\exp_{x_0}(t, p_0) = x(t, x_0, p_0)$$

(extremal flow)

$\longrightarrow$ Shooting method: determine p_0 s.t. $\boxed{\exp_{x_0}(t, p_0) = x_1}$

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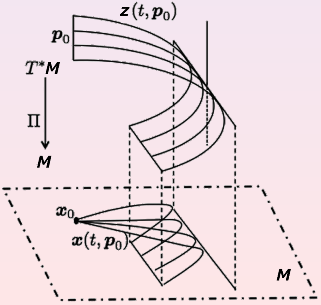


(fold singularity)

Remark

- PMP = first-order necessary condition for optimality.
- Necessary / sufficient (local) second-order conditions: **conjugate points**.

$\rightarrow$ test if $\exp_{x_0}(t, \cdot)$ is an immersion at p_0.

There exist other numerical approaches to solve optimal control problems:

- direct methods: discretize the whole problem
  $\Rightarrow$ finite-dimensional nonlinear optimization problem with constraints
- Hamilton-Jacobi methods.

The shooting method is called an indirect method.

In aerospace applications, shooting methods are privileged in general because of their numerical accuracy.

BUT: difficult to make converge... (Newton method)

To improve performance and facilitate applicability, PMP may be combined with:

(1) continuation or homotopy methods

(2) geometric control

(3) dynamical systems theory

E. Trélat, *Optimal control and applications to aerospace: some results and challenges*, JOTA 2012.

Minimal time orbit transfer

Maximum Principle $\Rightarrow$ the extremals (x,p) are solutions of

$$\dot{x} = \frac{\partial H}{\partial p},\ x(0) = x_0,\ x(T) = x_1, \quad \dot{p} = -\frac{\partial H}{\partial x},\ p(0) = p_0,$$

with an optimal control saturating the constraint: $\|u(t)\| = F_{max}$.

$\longrightarrow$ *Shooting method: determine p_0 s.t. $x(T; x_0, p_0) = x_1$*

combined with a homotopy on $F_{max} \mapsto p_0(F_{max})$

Heuristic on t_f:

$$t_f(F_{max}) \cdot F_{max} \simeq \text{cste}.$$

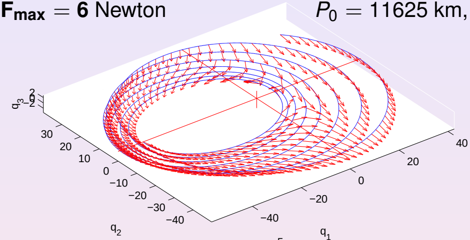
(the optimal trajectories are "straight lines", Bonnard-Caillau 2009)

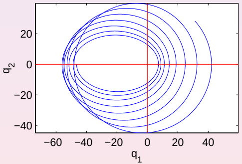
F_{max}	t_f	Exécution	F_{max}	t_f	Exécution
60	14.800	1	1.4	606.13	33
24	34.716	5	1	853.31	44
12	70.249	3	0.7	1214.5	64
9	93.272	7	0.5	1699.4	234
6	141.22	6	0.3	2870.2	223
3	285.77	22	0.2	4265.7	226
2	425.61	22			

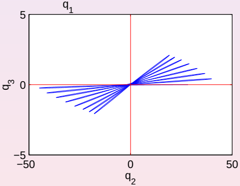
(Caillau, Gergaud, Haberkorn, Martinon, Noailles, ...)

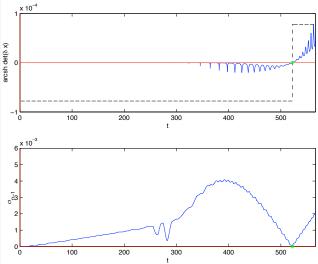
Minimal time orbit transfer

$\mathbf{F_{max}} = \mathbf{6}$ **Newton** $\quad P_0 = 11625$ km, $|e_0| = 0.75$, $i_0 = 7^o$, $P_f = 42165$ km



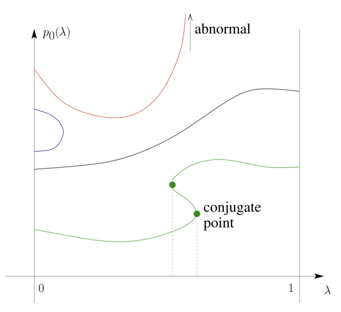






Minimal time: 141.6 hours ($\simeq$ 6 days). First conjugate time: 522.07 hours.

Continuation method



Main tool used: continuation (homotopy) method
$\rightarrow$ continuity of the optimal solution with respect to a parameter λ

Theoretical framework (sensitivity analysis):

$$F(p_0(\lambda), \lambda) \; = \; \exp_{x_0,\lambda}(T, p_0(\lambda)) - x_1 \; = \; 0$$

Local feasibility is ensured:
in the absence of conjugate points

$\downarrow$

Numerical test of Jacobi fields
(Bonnard Caillau Trélat, COCV 2007)

Global feasibility is ensured:
in the absence of abnormal minimizers

$\downarrow$

True for *generic* systems having more than 3 controls (Chitour Jean Trélat, JDG 2006)

Continuation method

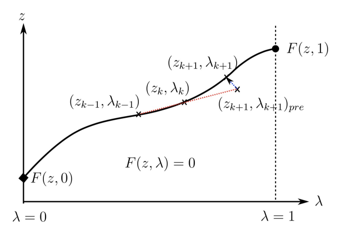
Work with ArianeGroup (Max Cerf):

Minimal consumption transfer for Ariane launchers

$\rightarrow$ automatic and instantaneous software (used since 2012).

Examples of continuations (on the dynamics, on the cost):

- Parameters, like $F_{\max}$ (maximal thrust), I_{sp}, gravity, ...
- Curvature of the Earth.
- A third, a fourth body.
- State constraints (hybrid systems), obstacles, activation constraints.
- State and control time-delays
  (continuity of extremals: Bonalli Hérissé Trélat SICON 2019)
- L^1, L^2 cost, ...



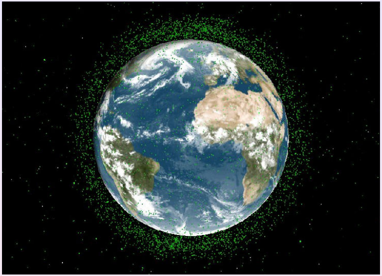
(piecewise-linear continuation
by prediction-correction)

Debris cleaning

A challenge (urgent!!)

Collecting space debris:

- 22000 debris of more than 10 cm (cataloged)
- 500000 debris between 1 and 10 cm (not cataloged)
- millions of smaller debris



In low orbit

→ difficult mathematical problems combining optimal control, continuous / discrete / combinatorial optimization

Max Cerf (JOTA 2013, JOTA 2015, RAIRO 2017)

Ongoing studies: ArianeGroup, CNES, ESA, NASA

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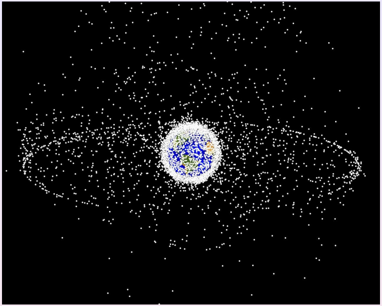

Around the geostationary orbit

$\rightarrow$ difficult mathematical problems combining optimal control, continuous / discrete / combinatorial optimization

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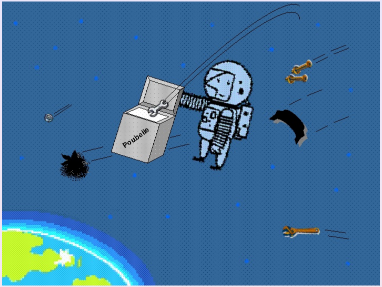

The space garbage collectors

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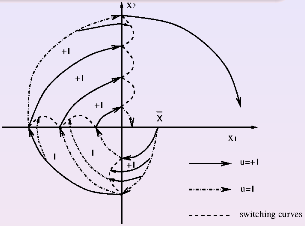
Geometric control

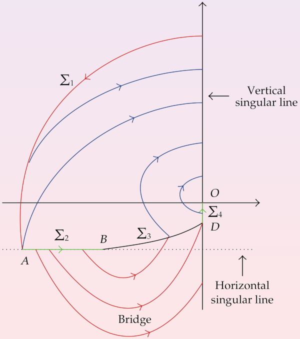
Describe the (local or global) structure of optimal trajectories: optimal synthesis.

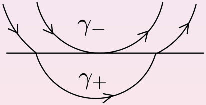


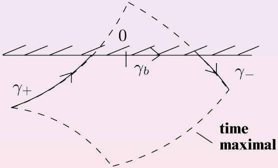
Example: for single-input control-affine systems

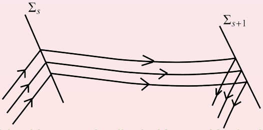
$$\dot{x}(t) = f_0(x(t)) + u(t)f_1(x(t)) \qquad |u(t)| \leqslant 1$$

describe the structure of optimal controls: number of switchings, order of switchings, singular arcs, boundary arcs.









Agrachev Bonnard Boscain Brockett Bullo Caillau Chyba Gauthier Hermes Jurdjevic Krener Kupka Lewis Lobry Miele Piccoli Poggiolini Sachkov Sarychev Schättler Sussmann Sigalotti Stefani Trélat Zelikin...

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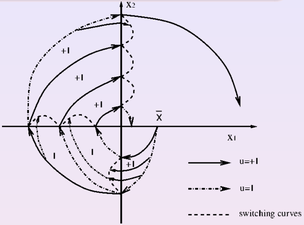

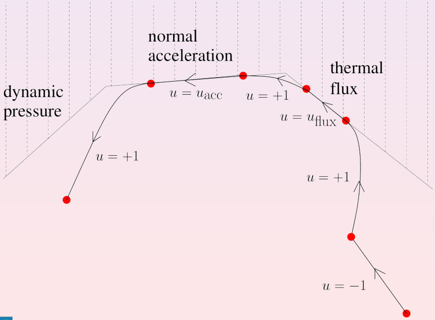

Objective: “reduction” of the shooting problem

Example of application: atmospheric re-entry

(Bonnard Trélat 2005)

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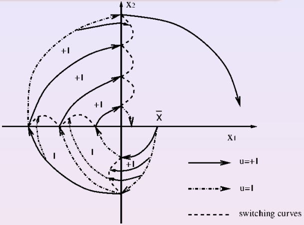
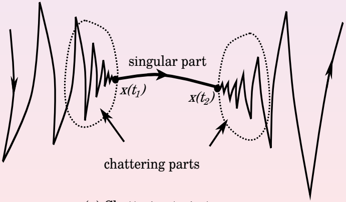
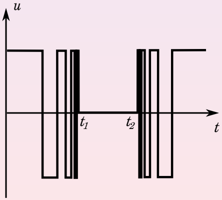


Possible problem with optimal chattering (Zelikin Borisov 1994):



(a) Chattering trajectory



(b) Chattering control

occuring for:

- missile guidance or interception (Bonalli Hérissé Trélat 2018)
- rocket attitude and trajectory guidance (coupling attitude and orbit dynamics) (Zhu Trélat Cerf 2016)

$\Rightarrow$ sub-optimal strategies, “averaging” the chattering part or penalizing by a BV term in the cost (Caponigro Ghezzi Piccoli Trélat, TAC 2017)

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Dynamical systems theory

Circular restricted three-body problem: dynamics of a body with negligible mass in the gravitational field of two massive bodies (primaries) having circular orbits.

Newton equations of motion (rotating frame)

$$\ddot{x} - 2\dot{y} = \frac{\partial \Phi}{\partial x}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Phi}{\partial y}$$

$$\ddot{z} = \frac{\partial \Phi}{\partial z}$$

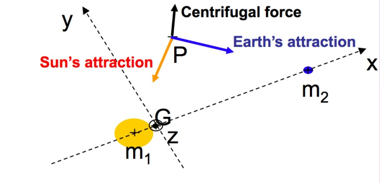
with

$$\Phi(x, y, z) = \frac{x^2 + y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2}$$

and

$$r_1 = \sqrt{(x+\mu)^2 + y^2 + z^2}$$

$$r_2 = \sqrt{(x-1+\mu)^2 + y^2 + z^2}$$



Bernelli-Zazzera, Bonnard, Celletti, Chenciner, Farquhar, Gómez, Jorba, Koon, Laskar, Llibre, Lo, Marsden, Masdemont, Mingotti, Ross, Szebehely, Simó, Topputo, Trélat, ...

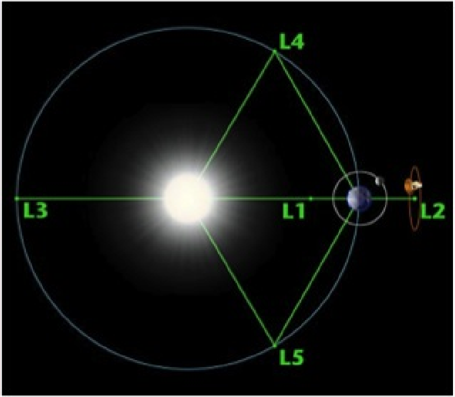
Lagrange points

Jacobi integral $J = 2\Phi - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$ $\rightarrow$ 5-dimensional energy manifold

Five equilibrium points:

- 3 collinear equilibrium points: L_1, L_2, L_3 (unstable); (Euler)
- 2 equilateral equilibrium points: L_4, L_5 (stable). (Lagrange)

(see Szebehely 1967)



Extension of a Lyapunov theorem (Moser) $\Rightarrow$ same behavior than the linearized system around Lagrange points.

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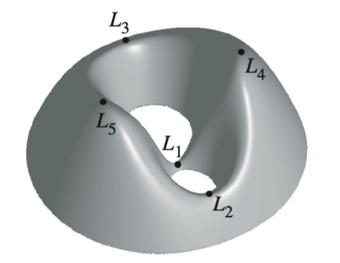
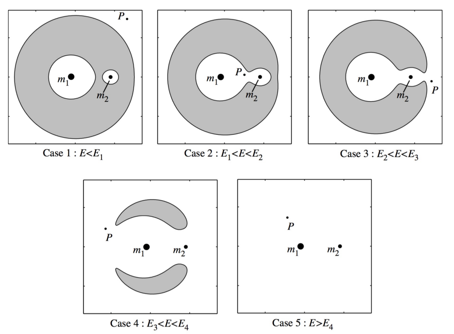


Hill region

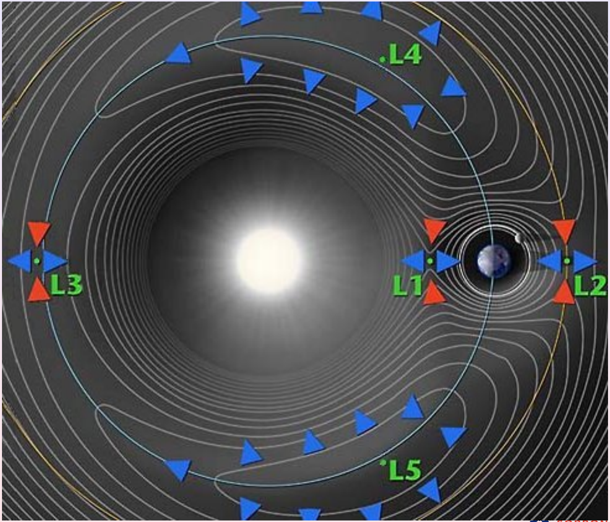


Extension of a Lyapunov theorem (Moser) $\Rightarrow$ same behavior than the linearized system around Lagrange points.

Lagrange points in the Earth-Sun system

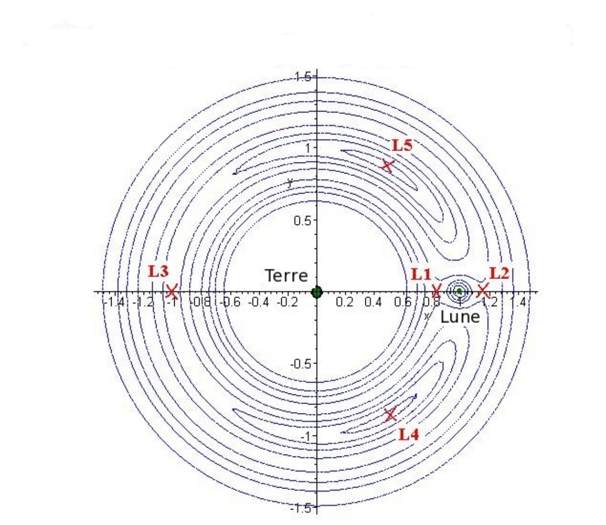
From Moser's theorem:

- L_1, L_2, L_3: unstable.
- L_4, L_5: stable.



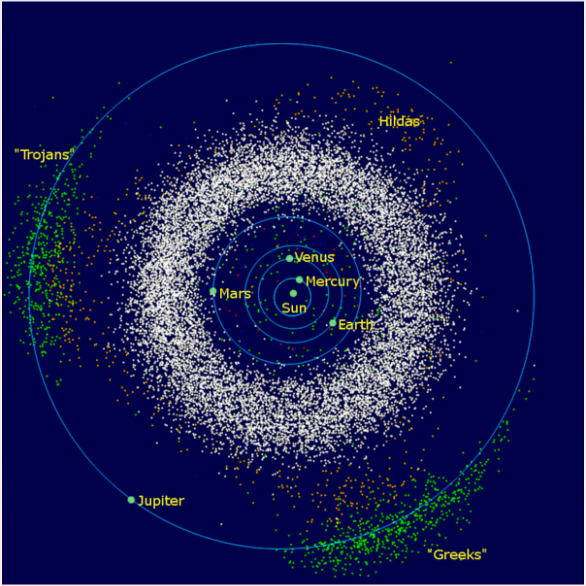
Lagrange points in the Earth-Moon system

- L_1, L_2, L_3: unstable.
- L_4, L_5: stable.



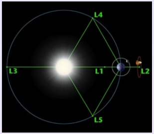
Examples of objects near Lagrange points

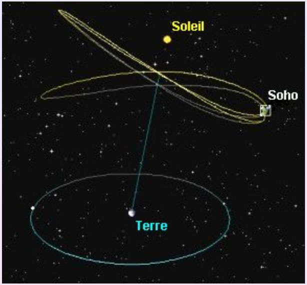
Points L4 and L5 (stable) in the Sun-Jupiter system:
Trojan asteroids



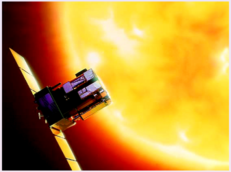
Examples of objects near Lagrange points

Sun-Earth system:

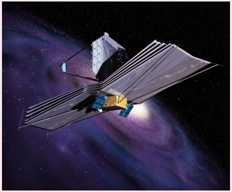




Point L1: SOHO



Point L2: JWST

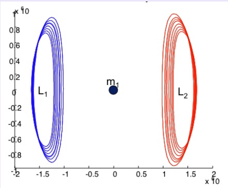


Point L3: planet X...

Periodic orbits

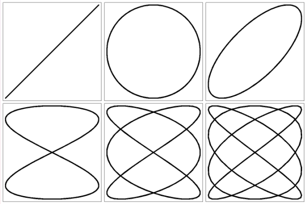
From a Lyapunov-Poincaré theorem, there exist:

- a 2-parameter family of periodic orbits around L_1, L_2, L_3
- a 3-parameter family of periodic orbits around L_4, L_5



Among them:

- planar orbits called Lyapunov orbits;
- 3D orbits diffeomorphic to circles called halo orbits;
- other 3D orbits with more complicated shape called Lissajous orbits.



(Richardson 1980, Gomez Masdemont Simo 1998)

Eight-Lissajous orbits

Analytical approximation by Lindstedt-Poincaré method:

Collinear Lagrange points are of type saddle×center×center, with eigenvalues $(\pm\lambda, \pm i\omega_p, \pm i\omega_v)$. Bounded solutions of the linearized system are written as

$$\begin{aligned} x(t) &= A_x \cos(\omega_p t + \phi) \\ y(t) &= \kappa A_x \sin(\omega_p t + \phi) \\ z(t) &= A_z \sin(\omega_v t + \psi) \end{aligned}$$

Nonlinearities change the eigenfrequencies of the solutions:

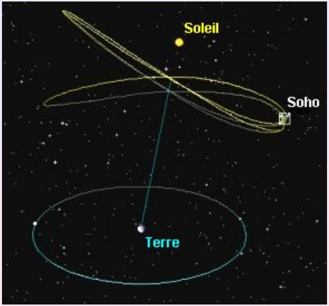
- halo orbits are obtained by imposing $\omega_p = \omega_v$ (Richardson, 1980)
- quasi-periodic orbits are obtained whenever $\omega_p/\omega_v \in \mathbb{R} \setminus \mathbb{Q}$
- Lissajous orbits are obtained whenever $\omega_p/\omega_v \in \mathbb{Q} \setminus \{1\}$

To get eight-shaped orbits, we impose $\omega_p = 2\omega_v$.

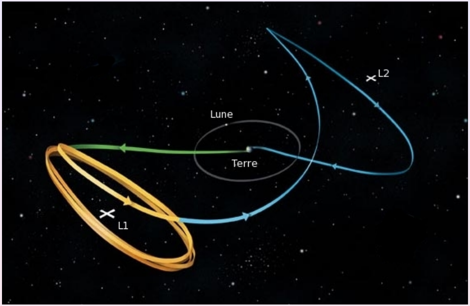
Third-order approximation obtained: used as initial guess in a shooting method, combined with a continuation method (homotopy parameter: z-excursion, or energy) $\Rightarrow$ compute families of periodic orbits.

(see also Gómez)

Examples of the use of halo orbits:



Orbit of SOHO around L1



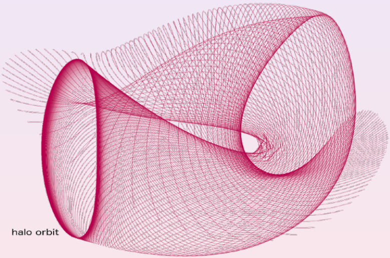
Orbit of the probe Genesis (2001–2004)

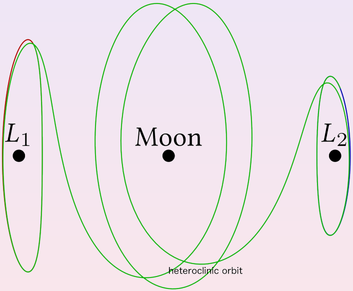
(requires control by stabilization)

Invariant manifolds

Invariant manifolds (stable and unstable) of periodic orbits:
4-dimensional tubes ($\mathcal{S}^3 \times \mathbb{R}$) inside the 5-dimensional energy manifold
(they play the role of separatrices)

$\rightarrow$ invariant “tubes”, kinds of “gravity currents” $\Rightarrow$ low-cost trajectories





video

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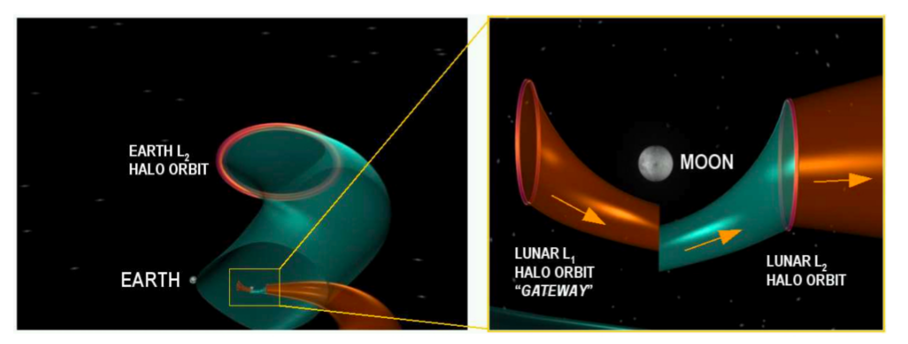

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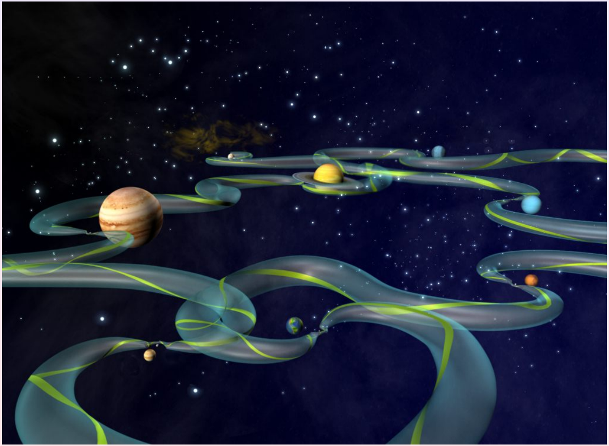

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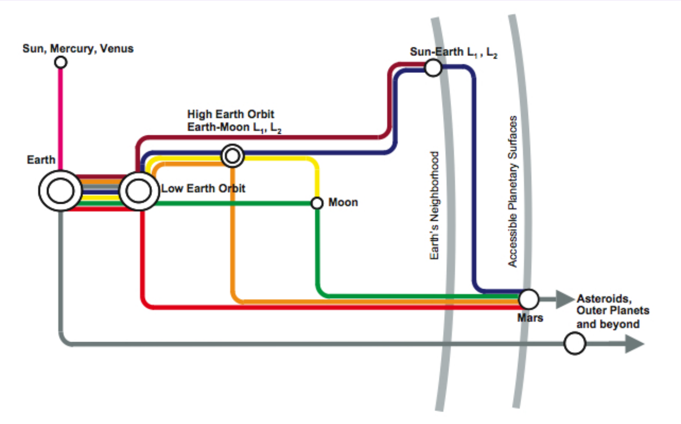


Cartography $\Rightarrow$ design of low-cost interplanetary missions

Meanwhile...

Back to the Moon

$\Rightarrow$ lunar station: intermediate point for interplanetary missions

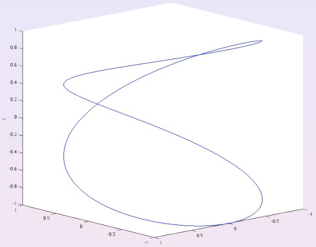
Challenge: design low-cost trajectories to the Moon and flying over all the surface of the Moon.

Mathematics used:
dynamical systems theory, differential geometry,
ergodic theory, control, scientific computing, optimization

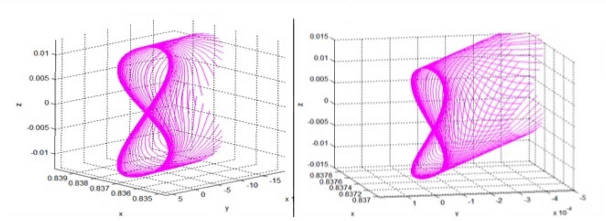
Eight Lissajous orbits

(PhDs of G. Archambeau 2008 and of Maxime Chupin 2016)

Periodic orbits around L_1 et L_2 (Earth-Moon system) having the shape of an eight:



$\Rightarrow$ they generate eight-shaped invariant manifolds:



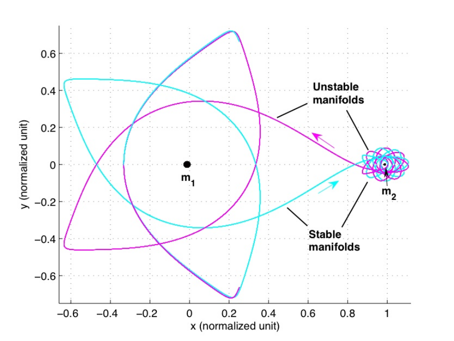
Invariant manifolds of Eight Lissajous orbits

(PhDs of G. Archambeau 2008 and of Maxime Chupin 2016)

We observe numerically two nice properties:

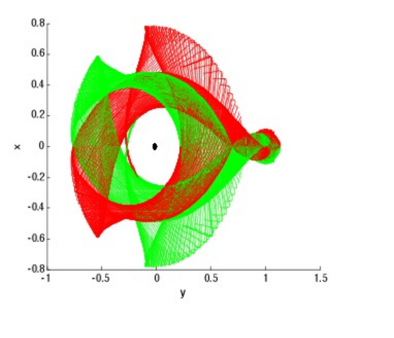
1) Stability in long time of invariant manifolds

Invariant manifolds of an Eight Lissajous orbit:



$\neq$

Invariant manifolds of a halo orbit:



→ global structure conserved

→ chaotic structure in long time

(numerical validation by computation of local Lyapunov exponents)

Details

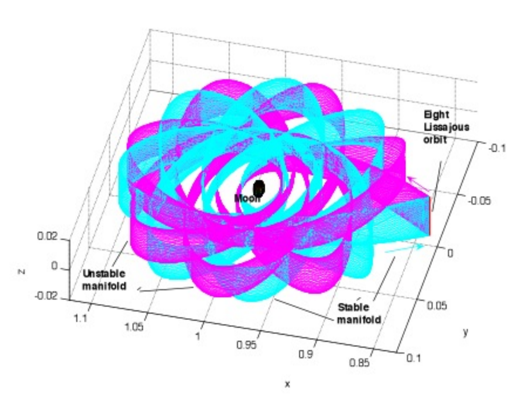
Invariant manifolds of Eight Lissajous orbits

(PhDs of G. Archambeau 2008 and of Maxime Chupin 2016)

We observe numerically two nice properties:

2) Flying over almost all the surface of the Moon

Invariant manifolds of an eight-shaped orbit around the Moon:



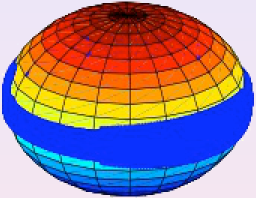
- oscillations around the Moon
- global stability in long time
- minimal distance to the Moon: 1500 km.

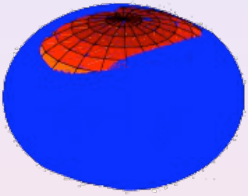
(Archambeau Augros Trélat 2011,
Chupin Haberkorn Trélat 2017)

Invariant manifolds of Eight Lissajous orbits

(PhDs of G. Archambeau 2008 and of Maxime Chupin 2016)

Moon surface overflown by invariant manifolds:





Possibility of “cargo missions”

- Missions using the properties of Eight Lissajous orbits.
- Fly over almost all the surface of the Moon with low cost.
- Compromise between lowt cost and long time.

Perspectives

- Using gravity currents:
    - Planning low-cost ”cargo” missions to the Moon
    - Interplanetary missions: compromise between low cost and long transfer time; gravitational effects (swing-by)
- collecting space debris (urgent! too late?)
- Optimal design:
    - optimal design of space vehicles
    - optimal placement problems (vehicle design, sensors)
- Inverse problems: reconstructing a thermic, acoustic, electromagnetic environment (coupling ODE’s / PDE’s)
- Robustness problems
- ...

$\Phi(\cdot, t)$: transition matrix along a reference trajectory $x(\cdot)$
$\Delta > 0$.

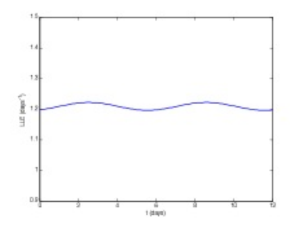
Local Lyapunov exponent

$$\lambda(t, \Delta) = \frac{1}{\Delta} \ln \left(\text{maximal eigenvalue of } \sqrt{\Phi(t+\Delta, t)\Phi^T(t+\Delta, t)} \right)$$

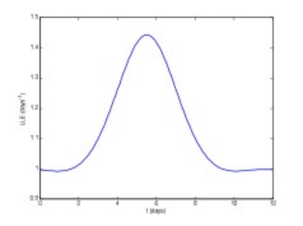
Simulations with $\Delta = 1$ day.

Return

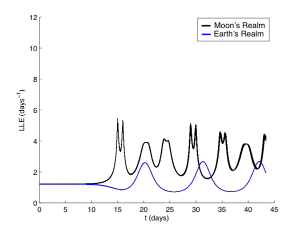
LLE of an eight-shaped Lissajous orbit:



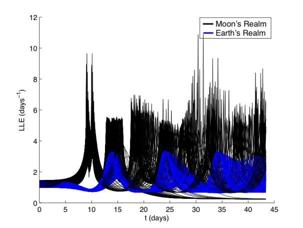
LLE of an halo orbit:



LLE of an invariant manifold of an eight-shaped Lissajous orbit:



LLE of an invariant manifold of an halo orbit:



Return