Blind galaxy survey images Deconvolution with Shape Constraint

Jean-Luc Starck

http://jstarck.cosmostat.org

Collaborators:

Fadi Nammour Morgan Schmitz Fred Nole

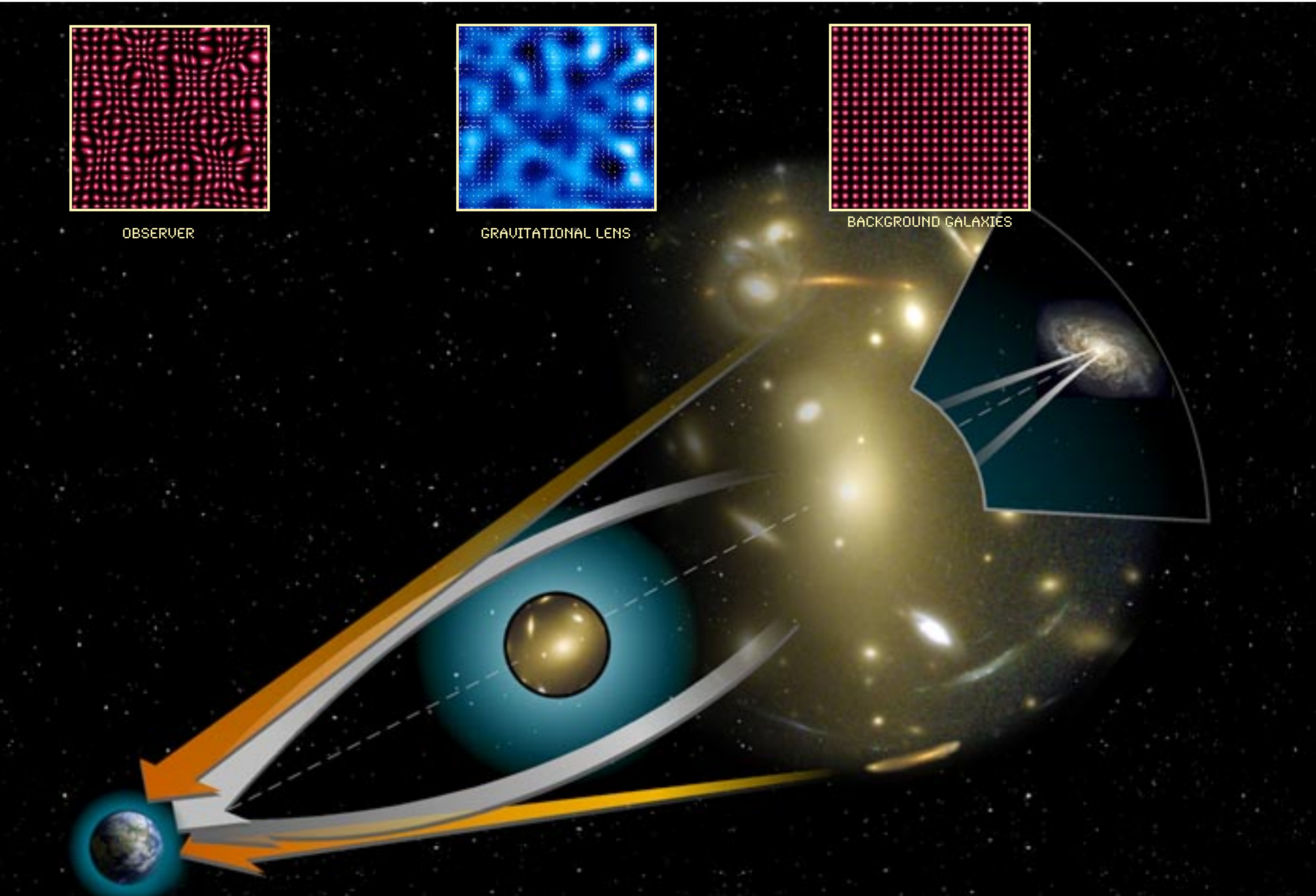
COSMOSTAT

- **Part I: Introduction to the Blind Deconvolution Problem**

- Part II: Point Spread Fonction (PSF) Field Recovery

- Part III: Shape Measurements & Deconvolution

Weak Lensing



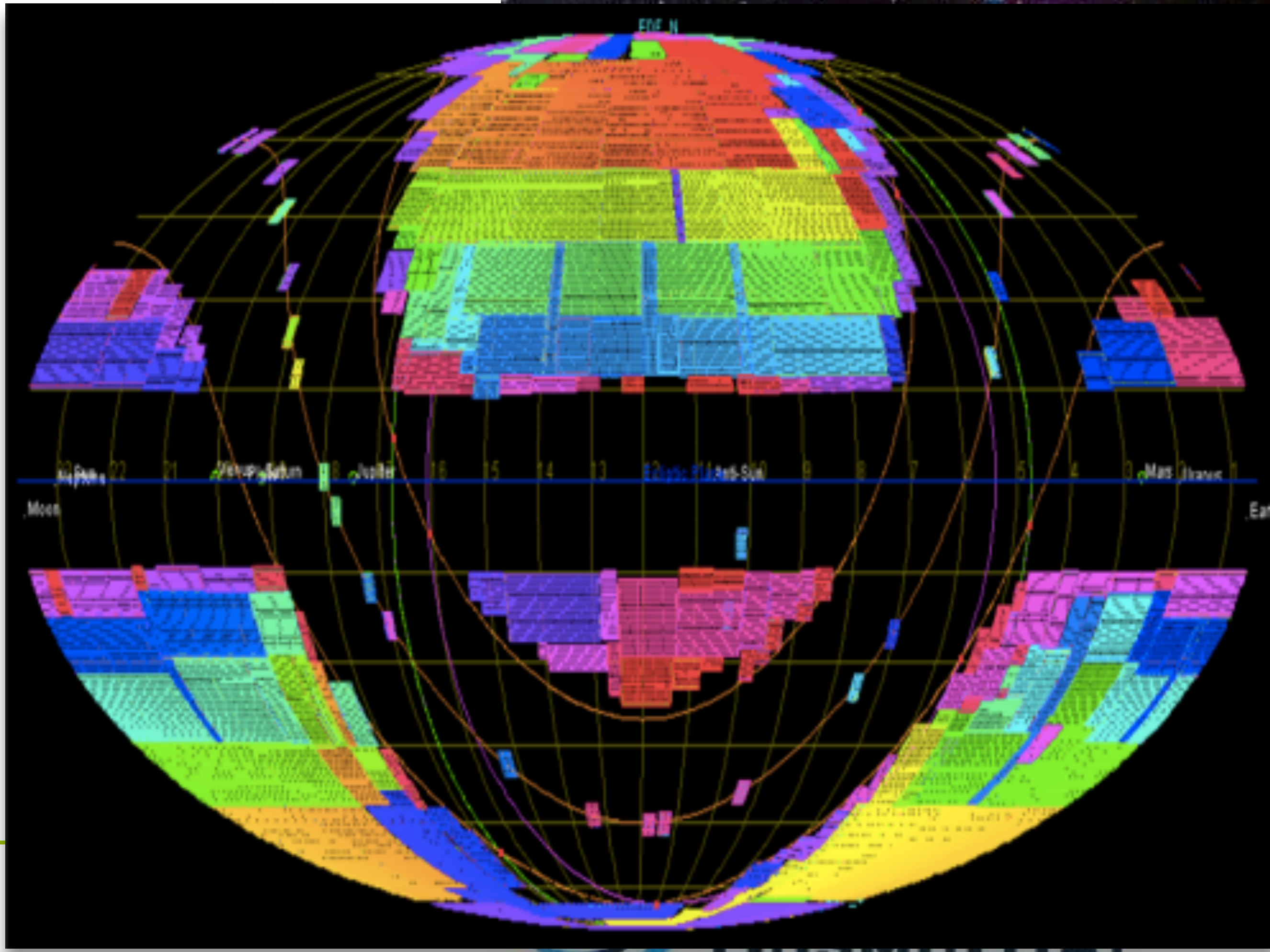
Euclid ESA Space Mission

Euclid Mission
- Medium-class mission in ESA's Cosmic Vision program
- 6 year survey, launch in Q4 2022

→probe the properties and nature of *dark energy, dark matter, gravity* and distinguish their effects decisively

2022

Optimized for weak lensing (and galaxy clustering)
— **15,000 deg2** survey area
— over **1.5 billion galaxies**
— redshifts out to **z = 2**



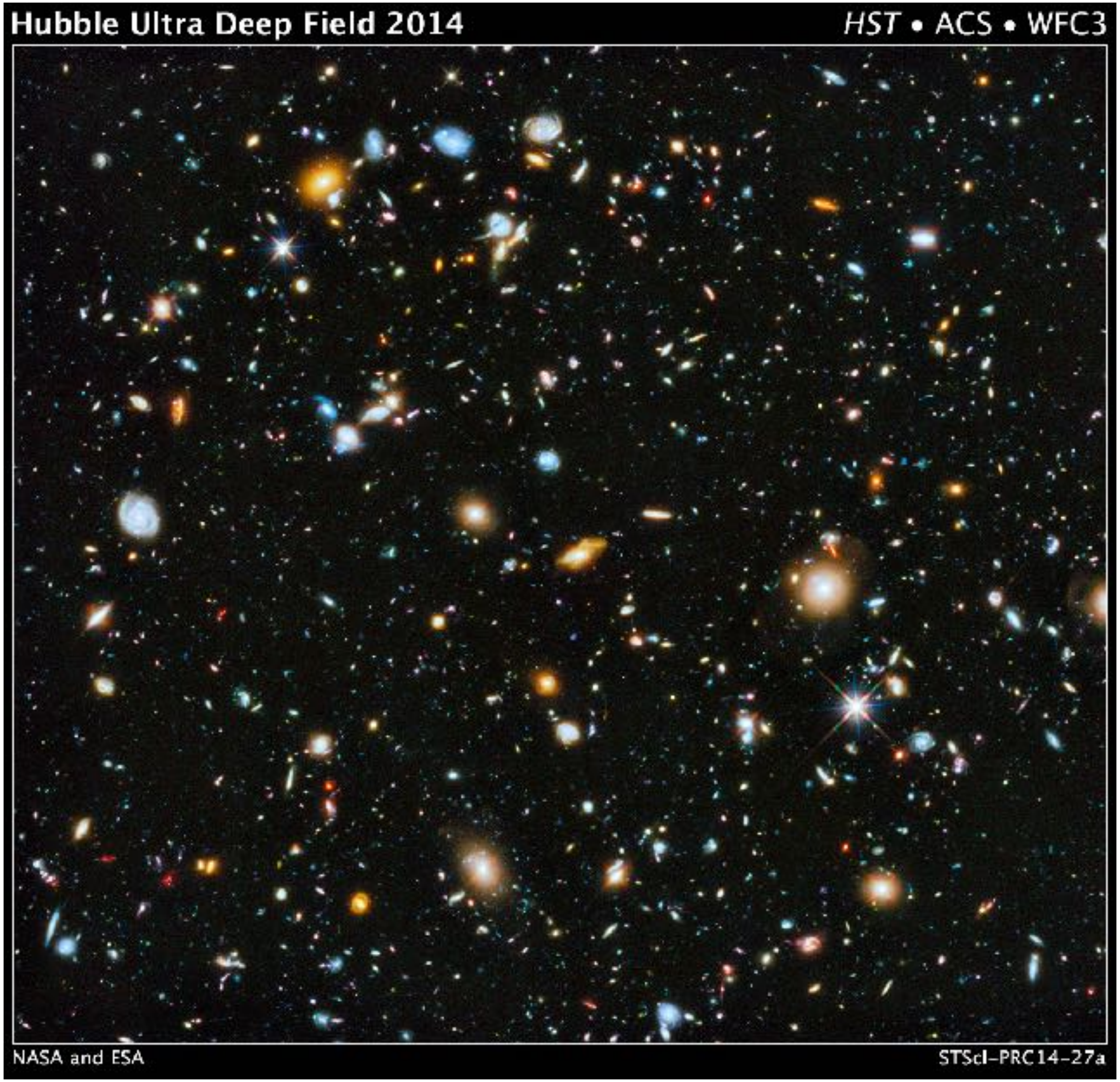
Gains in space:

Stable data: homogeneous data set over the whole sky

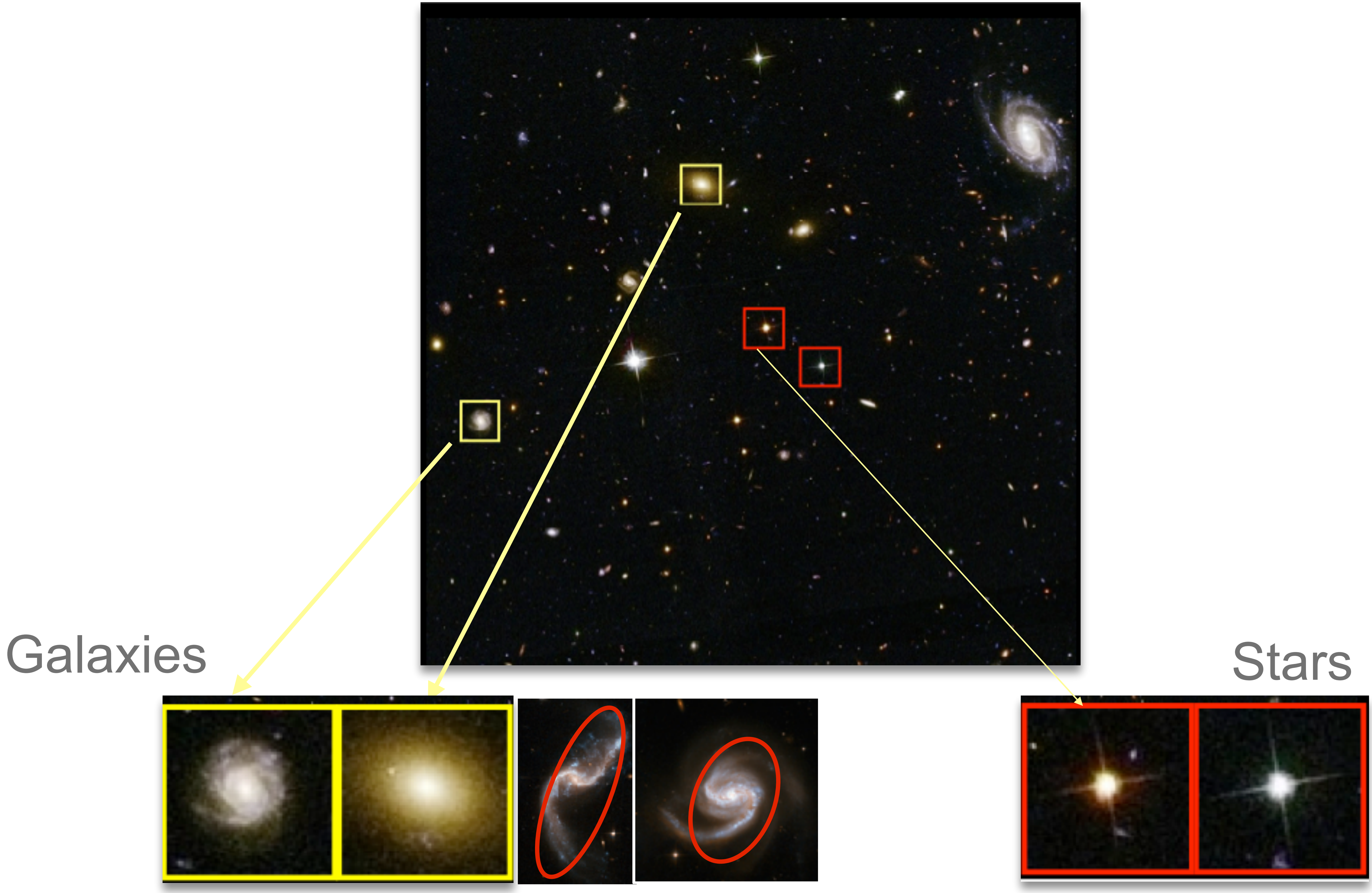
→**Systematics** are small, understood and controlled

→Homogeneity : Selection function perfectly controlled

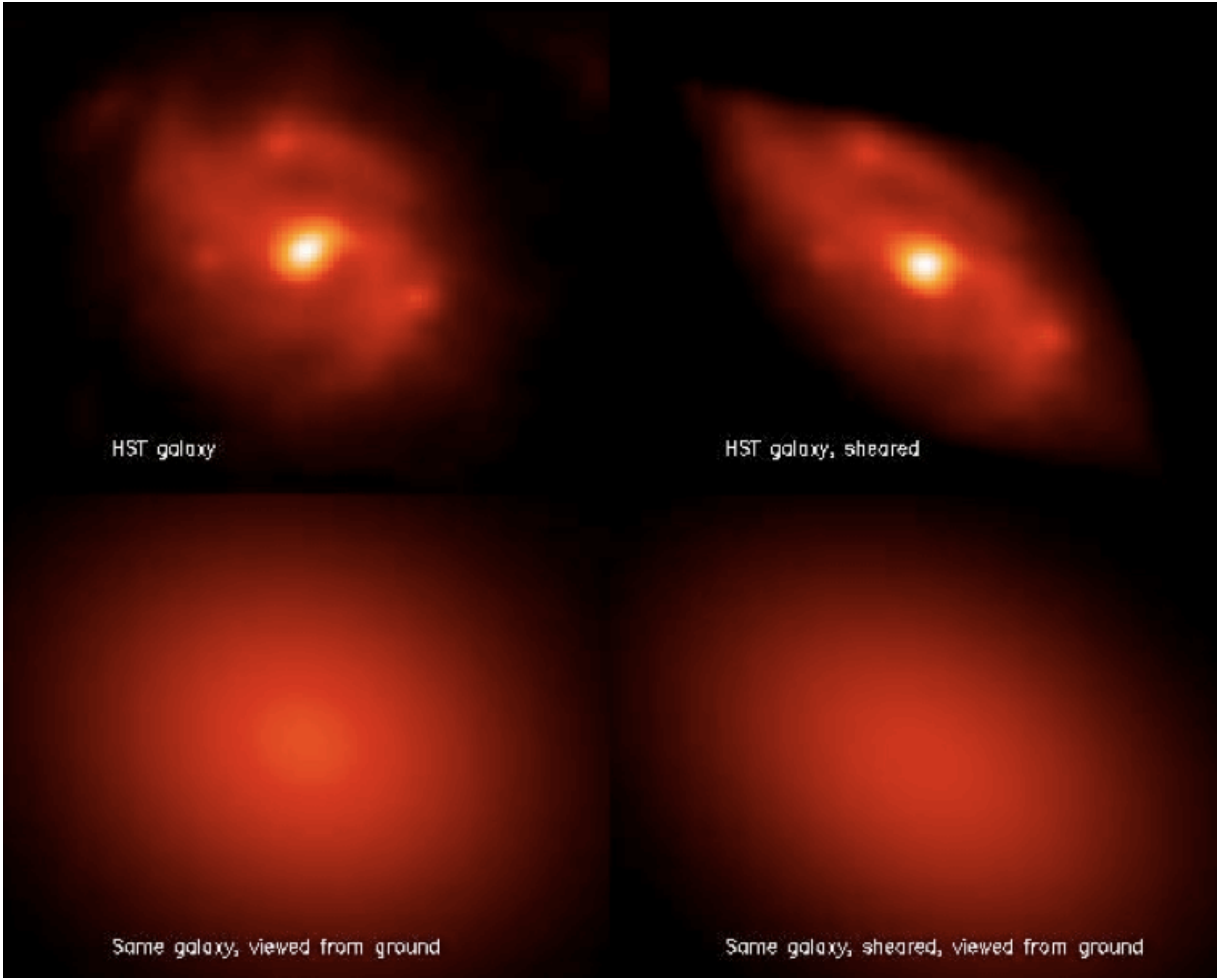
Galaxies


Hubble Ultra Deep Field 2014
HST • ACS • WFC3
NASA and ESA
STScI-PRC14-27a

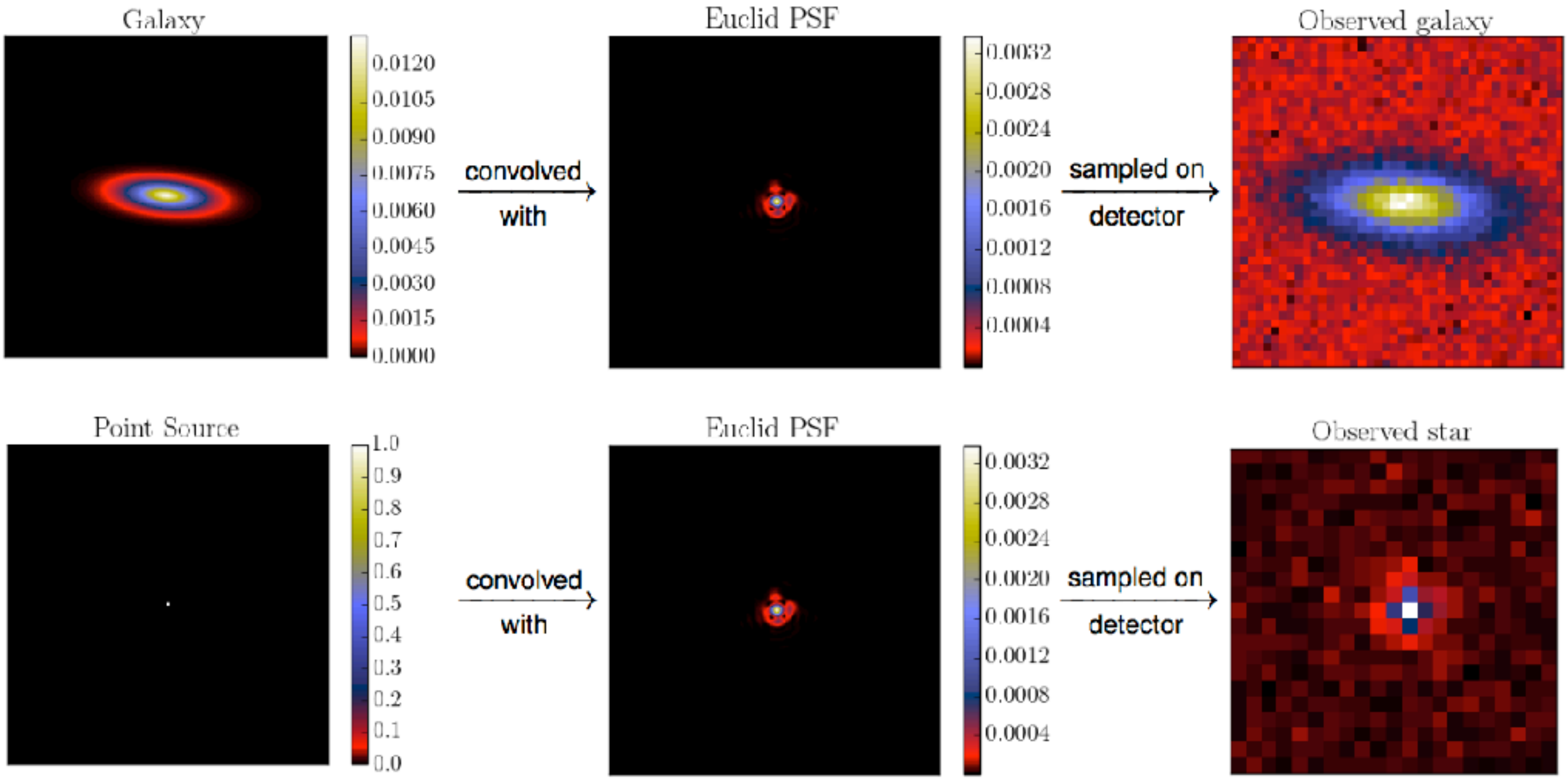
Detection + Classification stars/galaxies



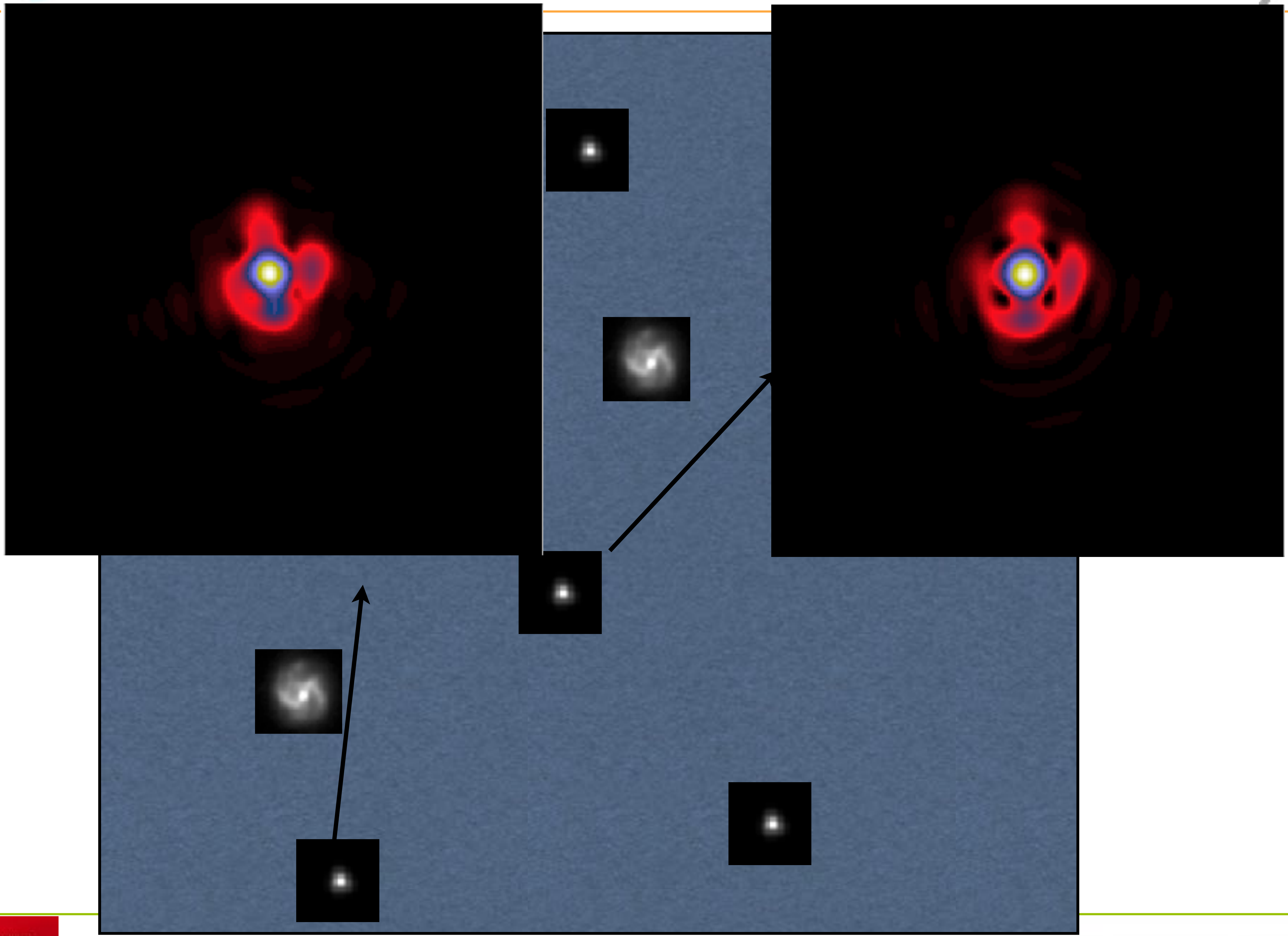
Motivation for spatial observations



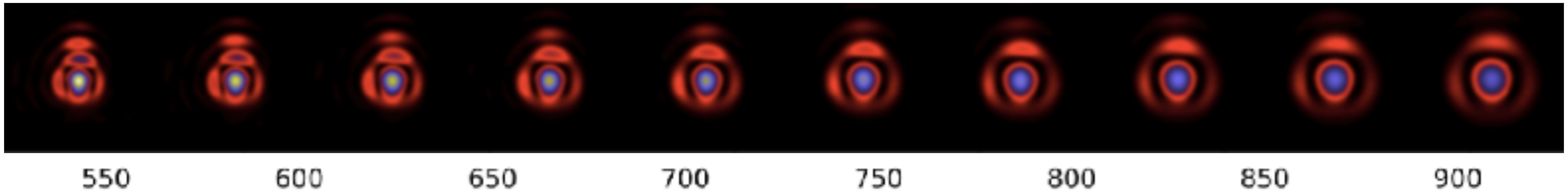
Convolution Operator + Sampling

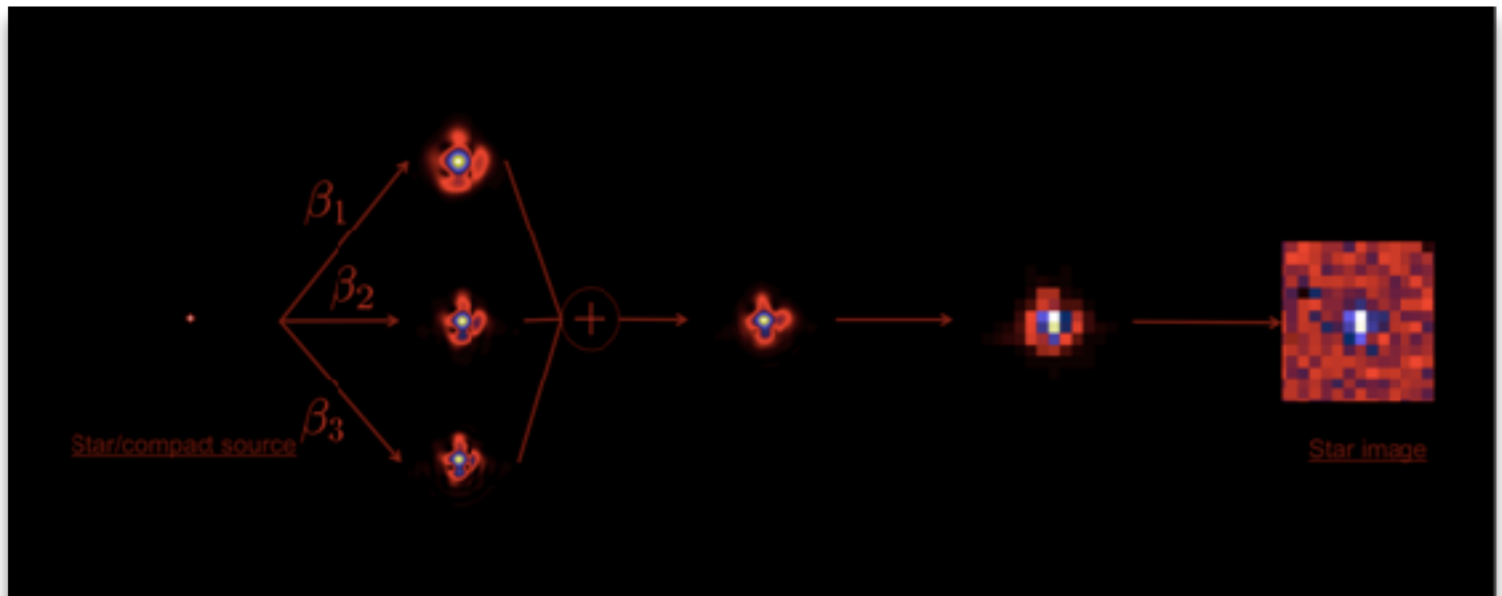


Space Variant PSF



Euclid PSF Wavelength Dependency



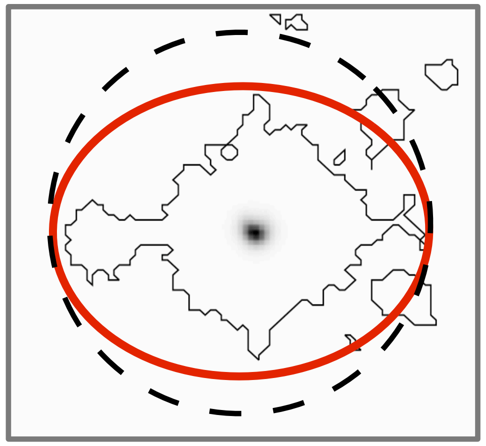


- PSF Modeling
  - ➡**Undersampling**
  - ➡**Space dependency**
  - ➡Wavelength dependency
  - ➡*Time dependency*

Shape Measurements

Galaxies are convolved by an **asymetric** PSF

+ Images are **undersampled**



Shape measurements must be deconvolved

Methods: Moments (KSB), Shapelets, Forward-Fitting, Bayesian estimation, etc

State of the art: PSFextractor [E. Bertin, 2011]

$$\tilde{y}_{k,ij} = \sum_{l} \sum_{m} h(l - N(i - i_k), m - N(j - j_k)) x_{lm}$$

where h is Lanczos interpolation kernel

$$h(x) = \begin{cases} 1 & \text{if } x = 0 \\ sinc(x) sinc(\frac{x}{4}) & \text{if } 0 < |x| < 4 \\ 0 & \text{else} \end{cases}$$

Regularization

$$J(X) = \sum_{k=1}^{p} \sum_{i=1}^{L} \sum_{j=1}^{L} \frac{(y_{k,ij} - f_k \tilde{y}_{k,ij})^2}{\sigma_k^2} + \frac{\left\| X - X^{(0)} \right\|_{l_2}^2}{\sigma}$$

State of the art: PSF Modeling

1. Forward Modelling approach (FM) leaded by Lance Miller
   - Model the exit pupil using pre-defined Zemax modes.
   - Fit to all stars.

➡ PRO: No other **existing** method can achieve the very strong requirement on the PSF.

➡ CON: But hard to validate since i) the simulations are done with the same model, and ii) the model may change (vibrations at launch).

2. Need a data driven approach
   - Validate the FM solution on real data.
   - All surveys ((KIDS, CFHTLenS, DES) have used the data driven approach.
   - HST: TinyTim HST modelling software (Krist 1995) for the Hubble Space Telescope not as good as a data driven approach (Jee et al, PASP, 2007; Hoffmann and Anderson, Instrument Science Report ACS 2017-8, 2017).
   - What does not exist can be invented.
   - Combination of both approaches could lead to the optimal solution.

Blind Deconvolution

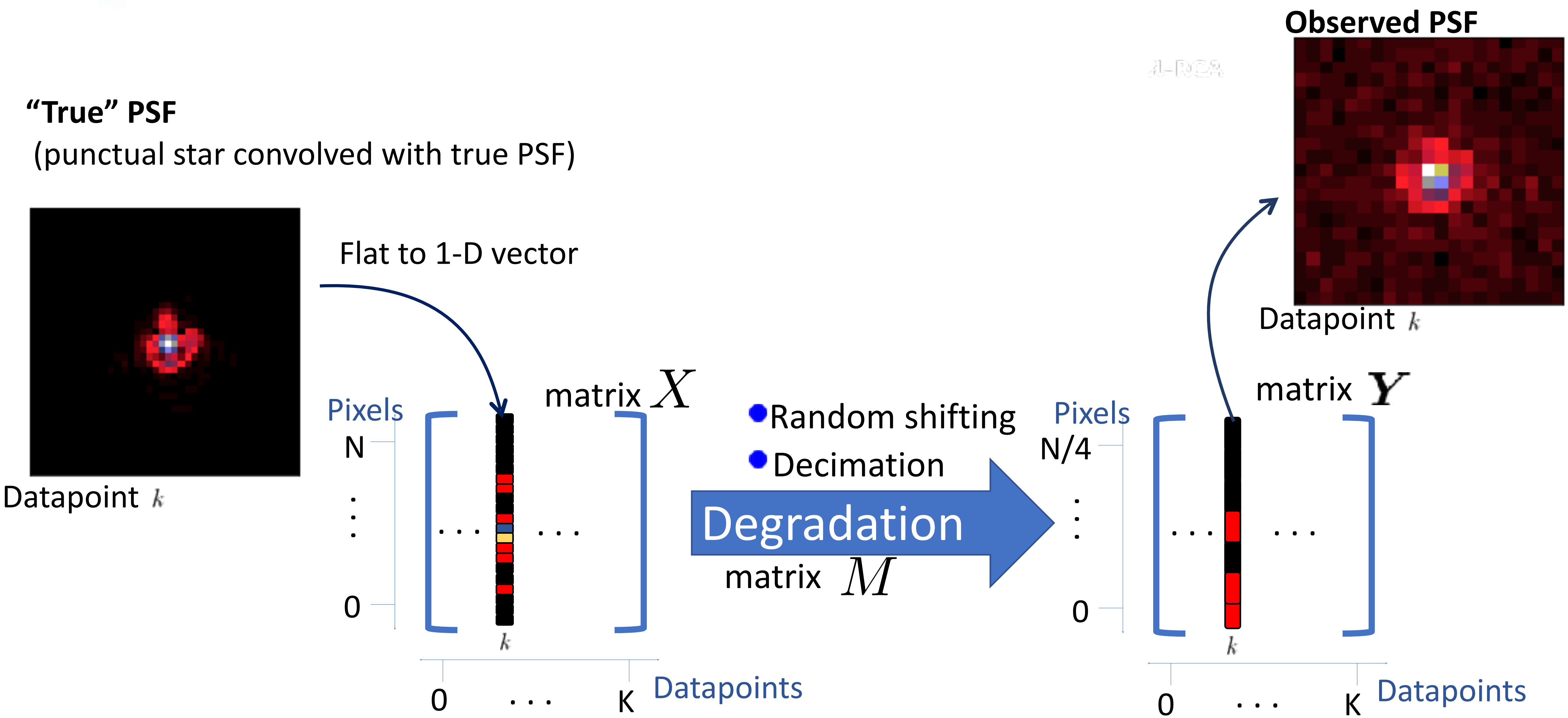
– Introduction to the Blind Deconvolution Problem

– **Euclid PSF Field Measurement**

- Monochromatic PSF: Matrix Factorization + Laplacian Graph + Sparsity
  - F. M. Mboula, J.-L. Starck, S. Ronayette, K. Okumura, and J. Amiaux, Super-resolution method using sparse regularization for point-spread function recovery. A&A, 575, id.A86, 2015.
  - F. Ngole, J.-L Starck, et al, "Constraint matrix factorization for space variant PSFs field restoration", Inverse Problems, 2016.
  - M.A. Schmitz, J.-L. Starck and F.M. Ngolè, "Euclid Point Spread Function field recovery through interpolation on a Graph Laplacian", submitted, 2018.
- Polychromatic PSF and Optimal Transport
  - F.M. Ngolè Mboula, and J.-L. Starck, "PSFs field learning based on Optimal Transport distances", SIAM Journal on Imaging Sciences, 10, 3, pp. 979-1004, 2017. DOI: 10.1137/16M1093677.
  - M. A. Schmitz et a; "Wasserstein Dictionary Learning:Optimal Transport-based unsupervised representation learning", SIAM Journal on Imaging Sciences, 11, 2018.

– Shape Measurements & Deconvolution

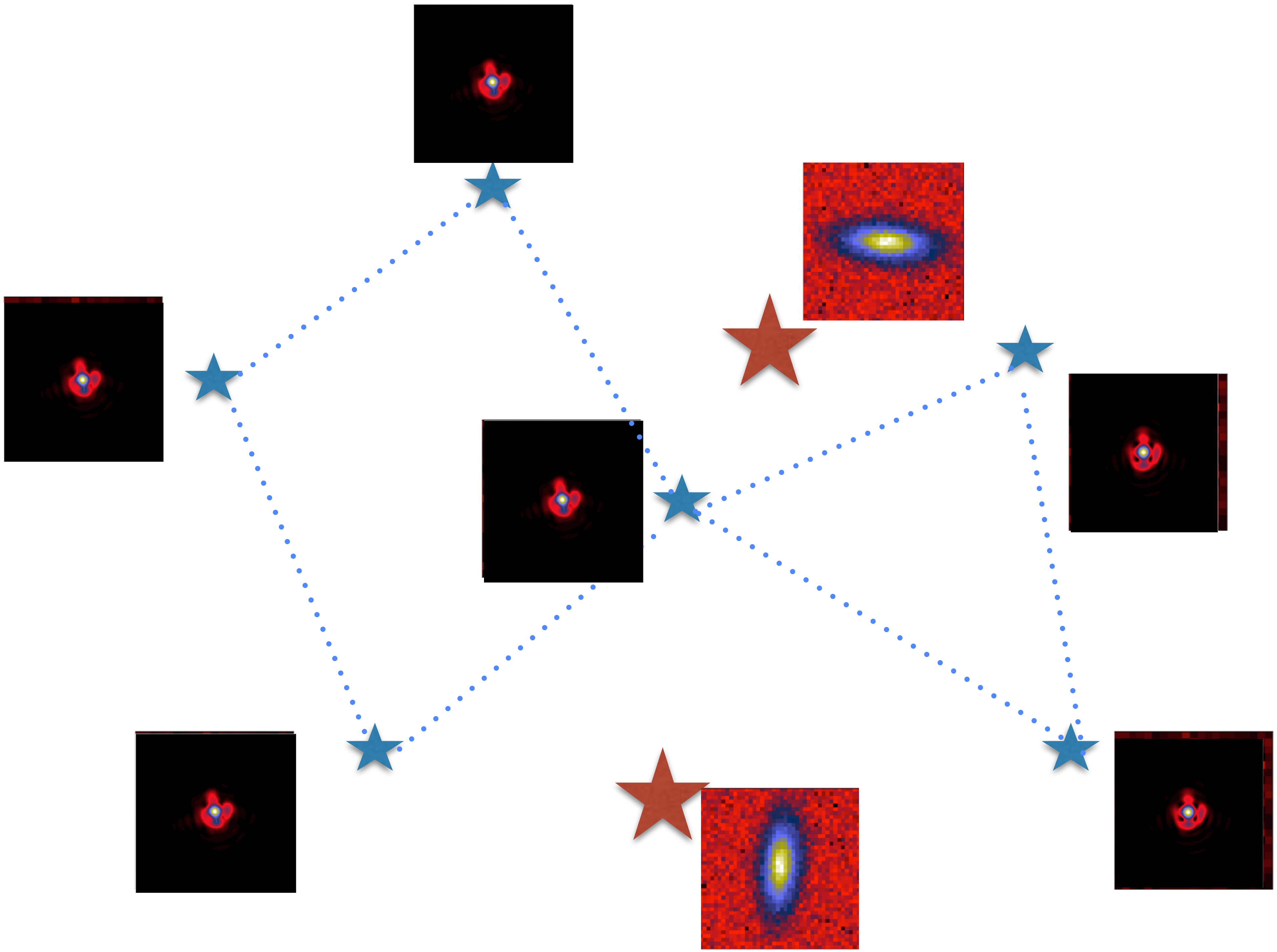
The PSF Field Recovery Problem



$$\min_{X} \| Y - MX \|^2$$

- $X \in \mathbb{R}_+^{N \times K}$: true PSFs at different positions of the FOV.
- $\boldsymbol{M} = [\boldsymbol{M}_1, \cdots, \boldsymbol{M}_K]$, $\boldsymbol{M}_k \in \mathbb{R}_+^{N' \times N}$: decimation and shifting.
- $\boldsymbol{Y} \in \mathbb{R}_+^{N' \times K}$: observed PSFs.

The PSF Super-resolution Challenge



Inverse Problems & Regularization

$$Y = HX + N$$

$$\min_{X} ||Y - HX||^2 \quad + \quad \mathcal{C}(X)$$

Physical Knowledge on X (ex: Gaussian Random Field, etc).
==> Gaussian smoothing, Wiener reconstruction, etc
==> Log normal distribution prior

X properties are understood through a representative data set.
==> Machine Learning

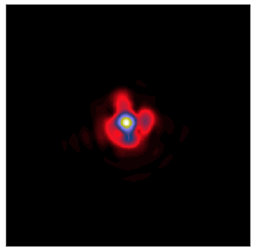
Knowledge on the histogram of X in pixel space or in another one.
==> Positivity constraint, sparsity constraint, etc.

Constraints

Joint estimations of super-resolved PSFs at stars positions

➡ Low rank constraint: Constraint the PSFs to be a linear combination of the ***eigenvectors*** PSFs:

Eigen PSF



$$\mathbf{PSF}^{(k)} = x_k = \sum_{i=1}^{r} a_{i,k} s_i$$

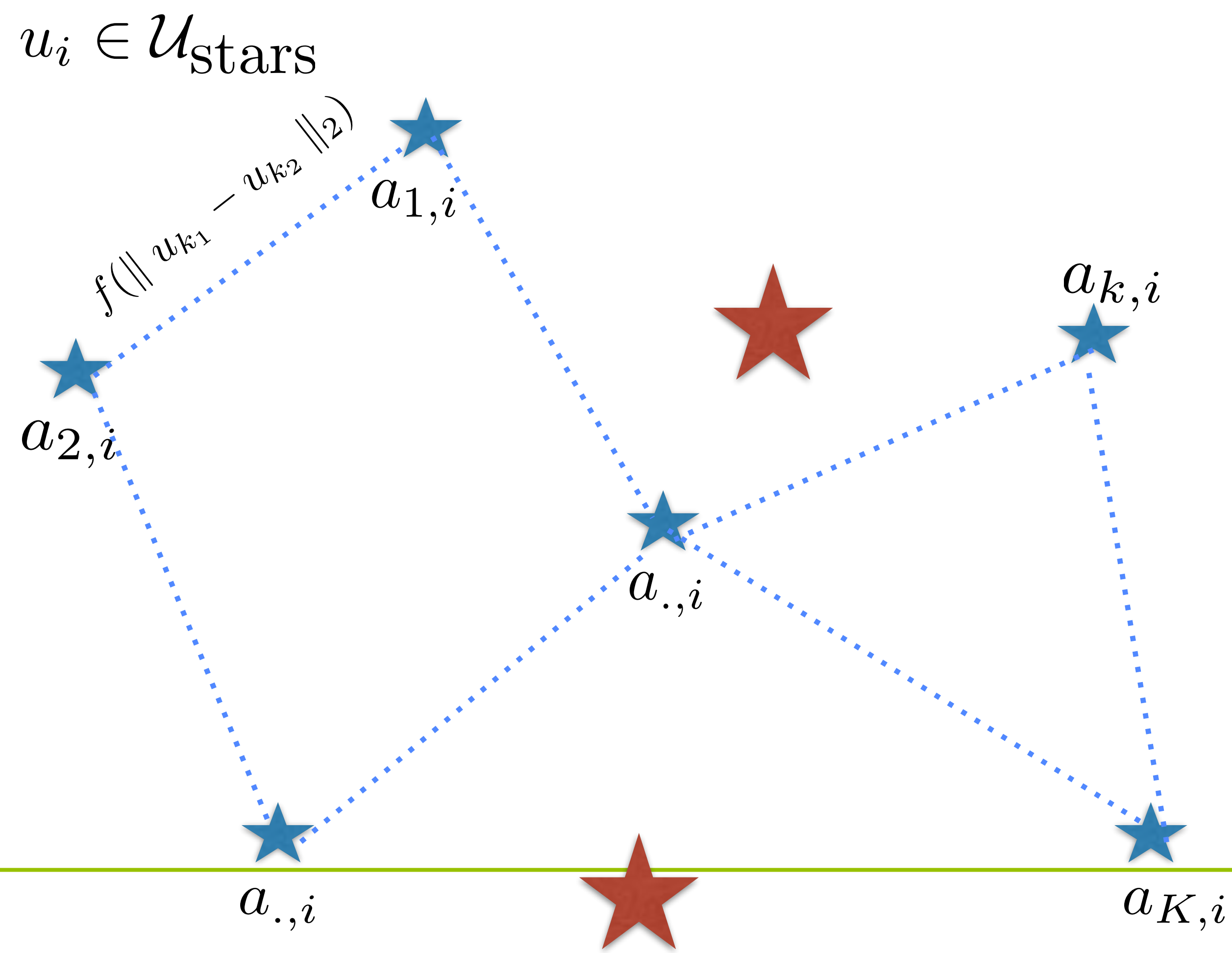
$s_i = \text{ith } \mathit{vector} \text{ (2D image)}$

$a_{i,k}$ = coefficient corresponding to contribution of the i-th vector to the k-th PSF.

➡ Positivity constraint (**$x_k > 0$**)

➡ Smoothness constraint on each s_i $\quad \sum_r \| \Phi s_i \|_1$

➡ Proximity constraint on the $a_{i,k}$ coefficients: the closer are the stars, the more the coefficients of linear combination are similar.



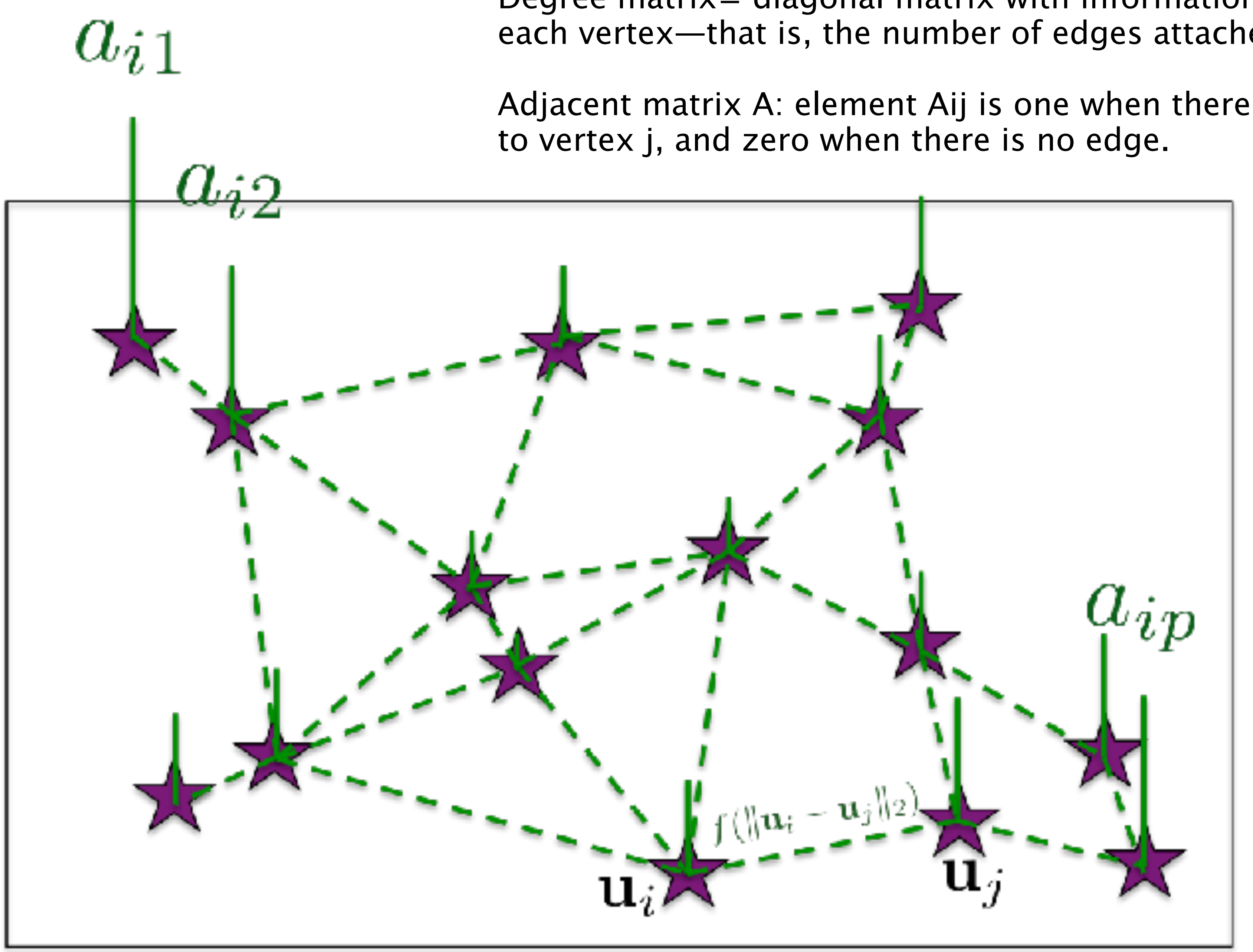
PSF Laplacian Graph

Laplacian Graph **P = D - J**,   where D is the degree matrix and J is the adjacent matrix of the graph

$$P^t P = V \Lambda V^t$$

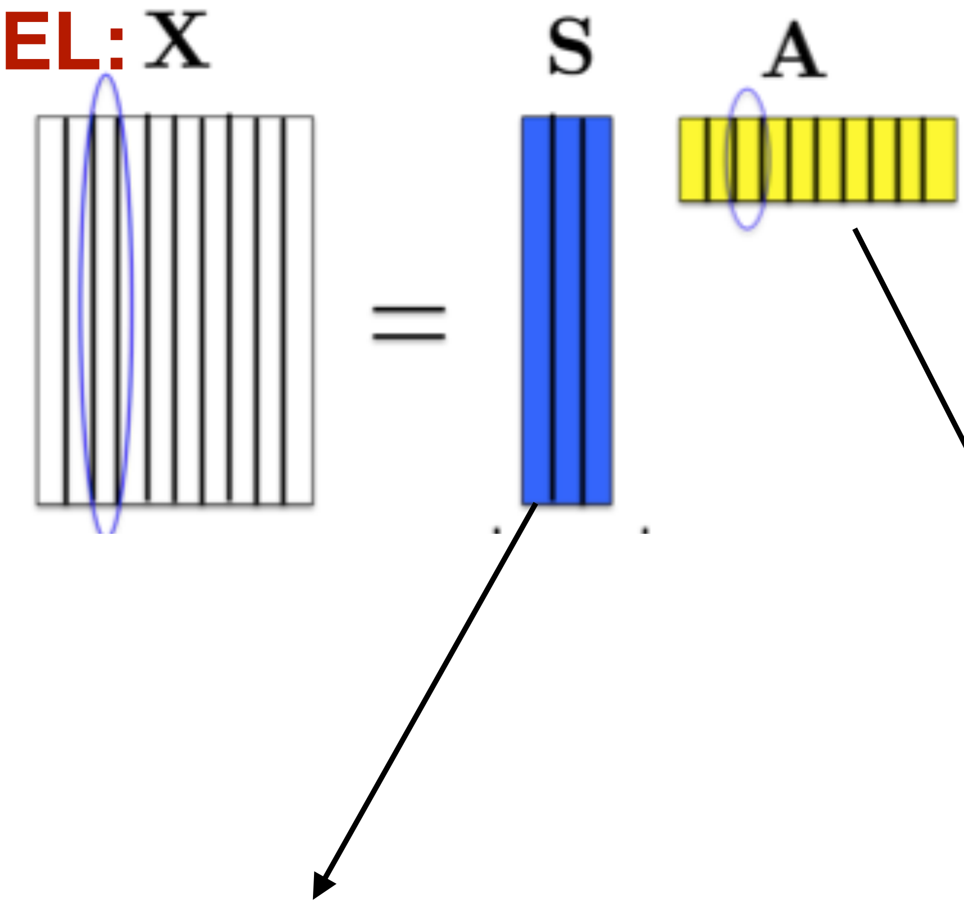
Degree matrix= diagonal matrix with information about the degree of each vertex—that is, the number of edges attached to each vertex.

Adjacent matrix A: element Aij is one when there is an edge from vertex i to vertex j, and zero when there is no edge.



Matrix Factorization

MODEL: $\mathbf{X}$



$$\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_p]$$

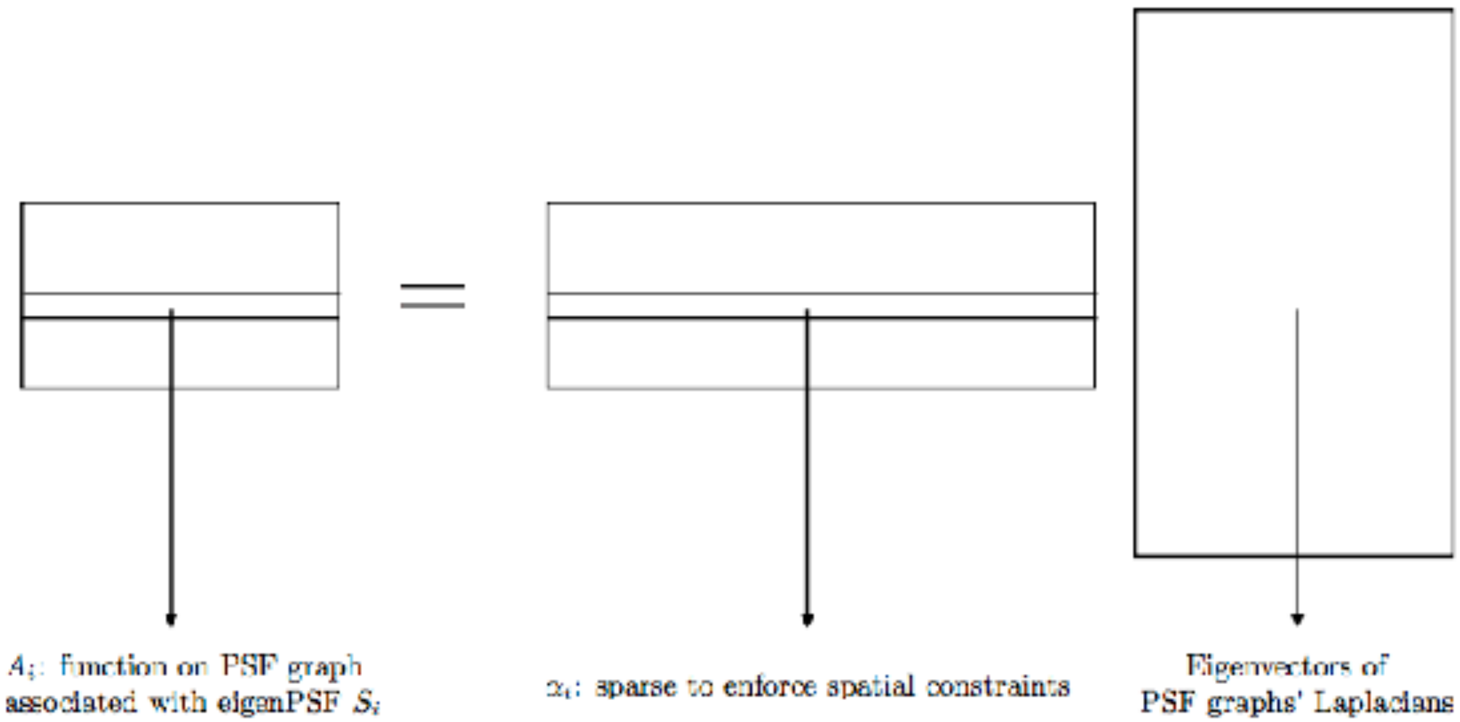
$$\mathbf{PSF}^{(k)} = x_k = \sum_{i=1}^{r} a_{i,k} s_i$$

s_i are "eigen PSF"

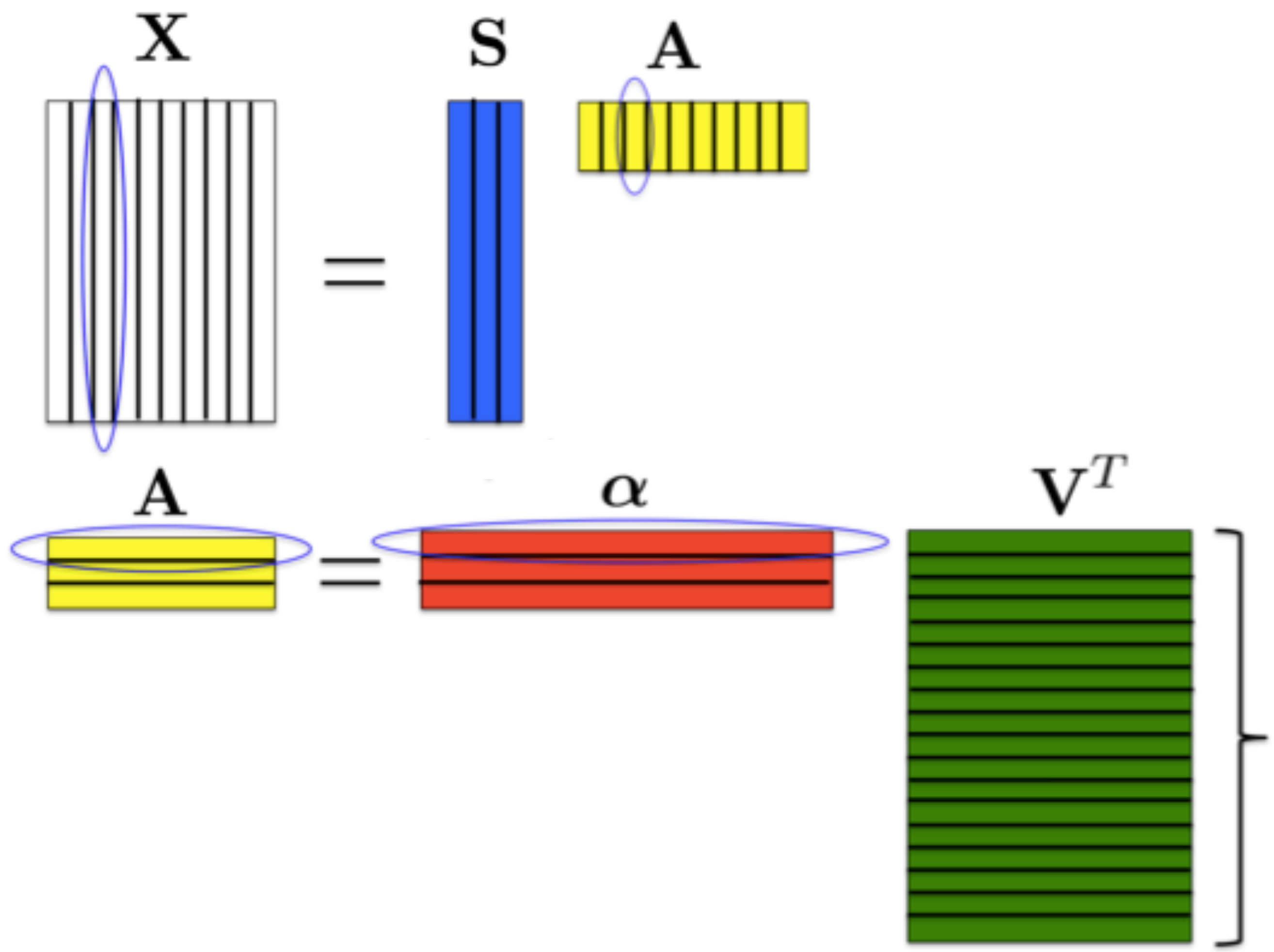
$$A = \alpha V^\top$$

Each eigen PSF is sparse in a wavelet dictionary

$$\sum_r \| \Phi s_i \|_1$$



Matrix Factorization



$$\min_{S,\alpha} \frac{1}{2}\|Y - M(S\alpha V^\top)\|_2^2 + \sum_{i=1}^{r} \|w_i \odot \Phi s_i\|_1 + \iota_+(S\alpha V^\top) + \iota_\Omega(\alpha)$$

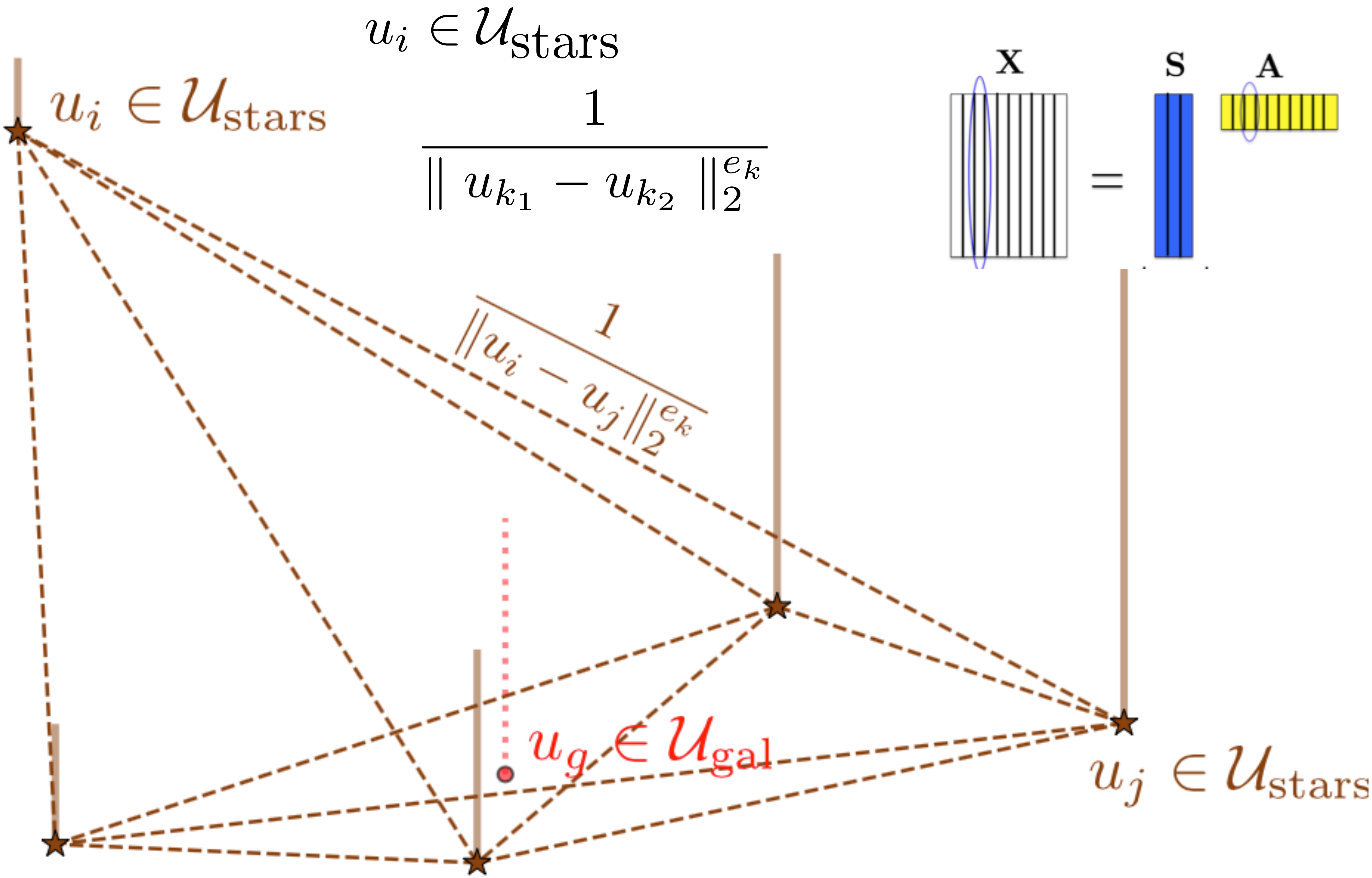
Data fidelity term

Sparsity on the eigen PSF: the PSF should have a sparse representation in an appropriate basis

Positivity Constraint

Smoothness of the PSF field constraint : the smaller the difference between two PSFs' positions u_i, u_j, the smaller the difference between their estimated representations

The PSF Interpolation Challenge

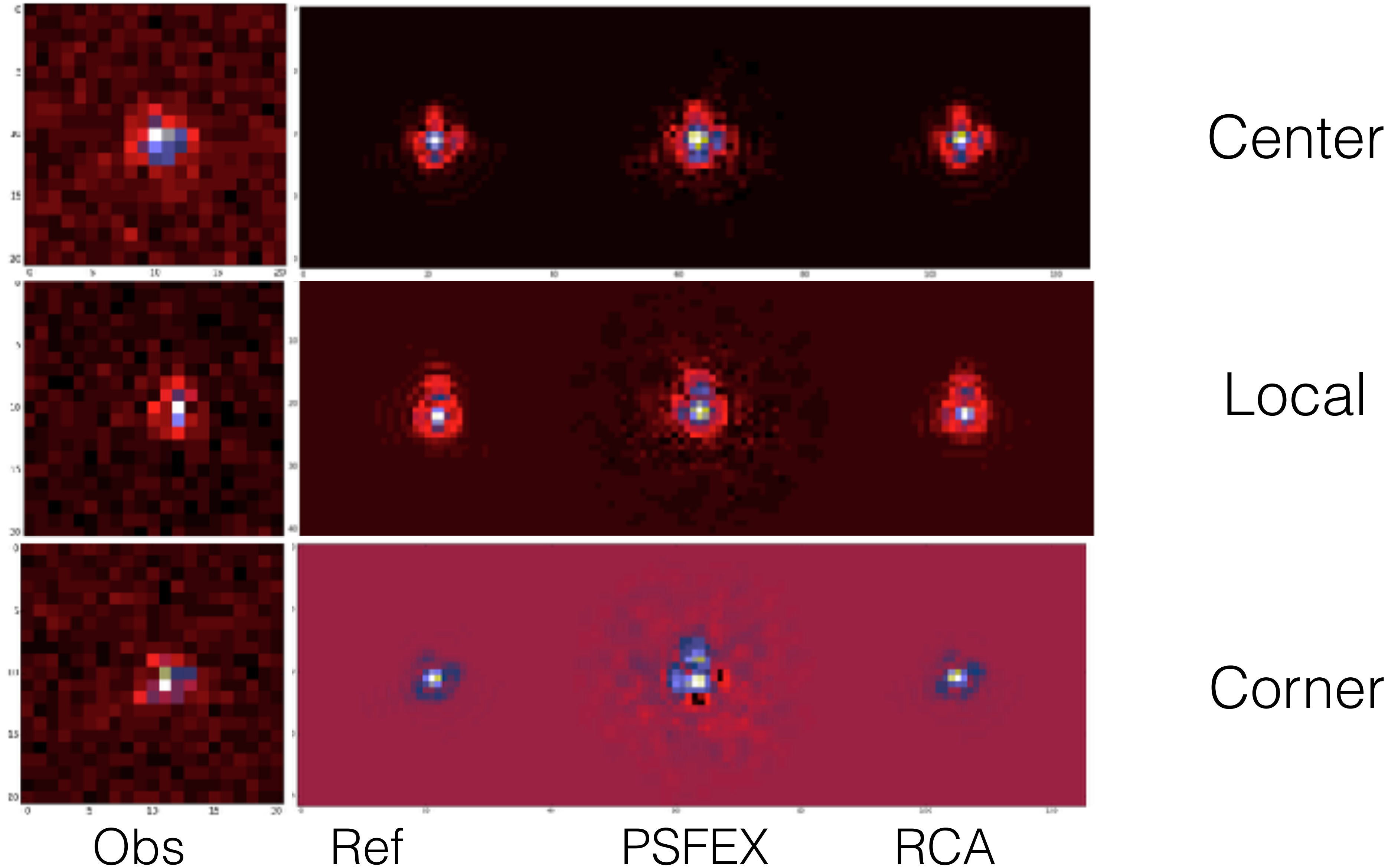


Numerical Experiments

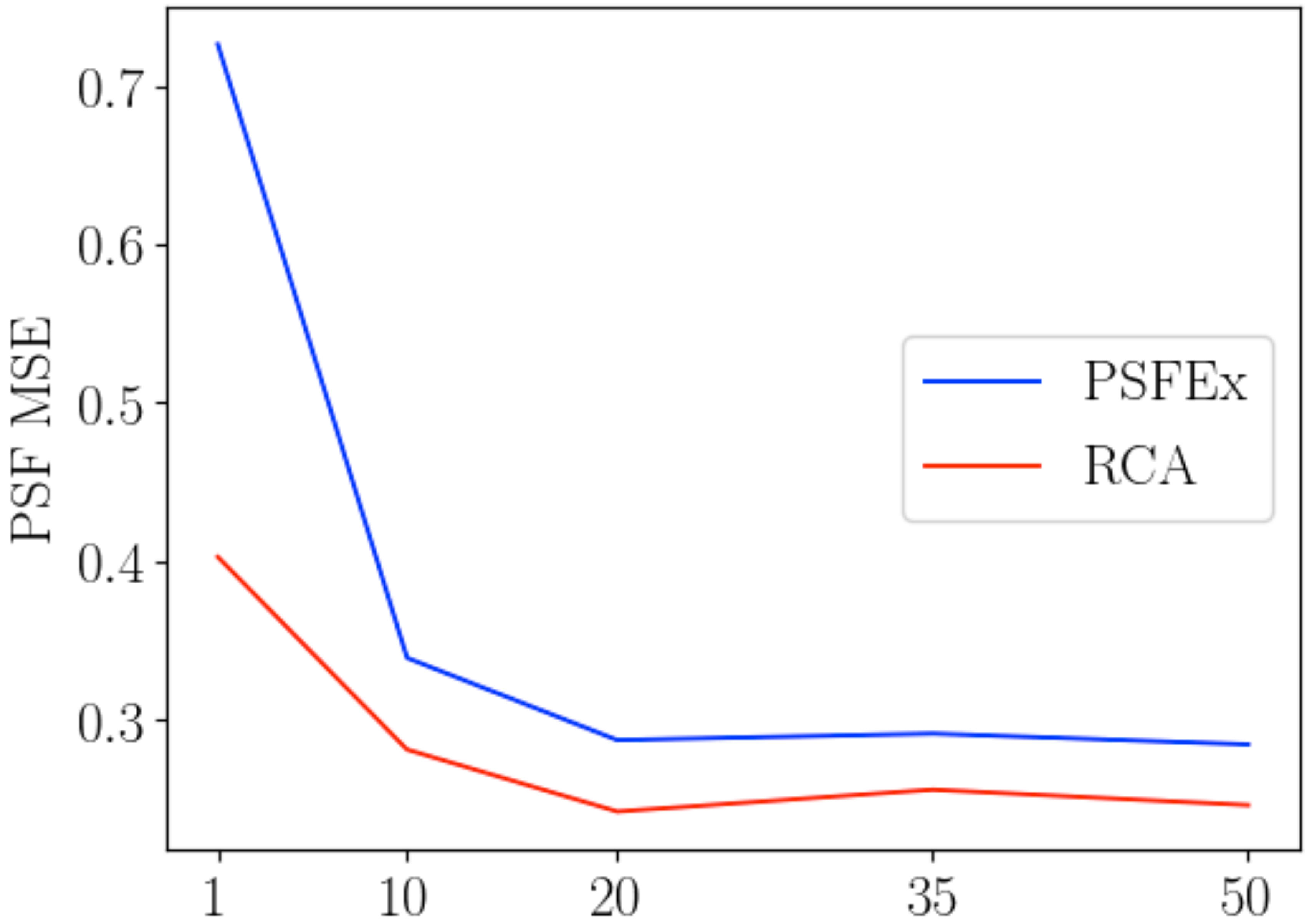
Data: 500 Euclid-like PSFs (Zemax), field observed with different SNRs

Theese PSFs account for mirrors polishing imperfections, manufacturing and alignments errors and thermal stability of the telescope.

With undersampling (upsampling factor of 2)



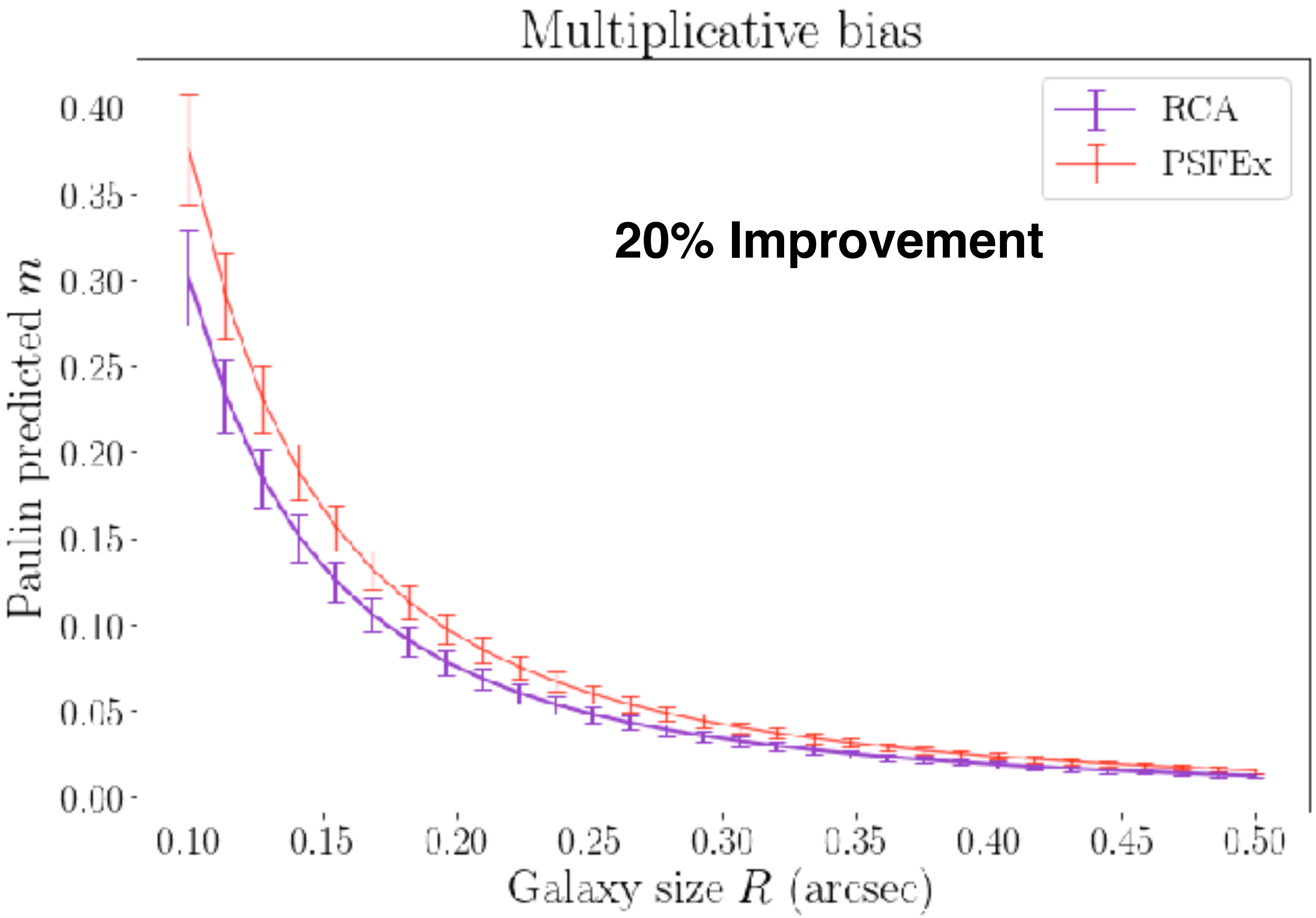
Numerical Experiments



Stars SNR

Multiplicative Bias

- M.A. Schmitz, J.-L. Starck and F.M. Ngolè, "Euclid Point Spread Function field recovery through interpolation on a Graph Laplacian", submitted, 2018.



PSF Wavelength Dependency

Optimal Transport

The Monge-Kantorovich problem

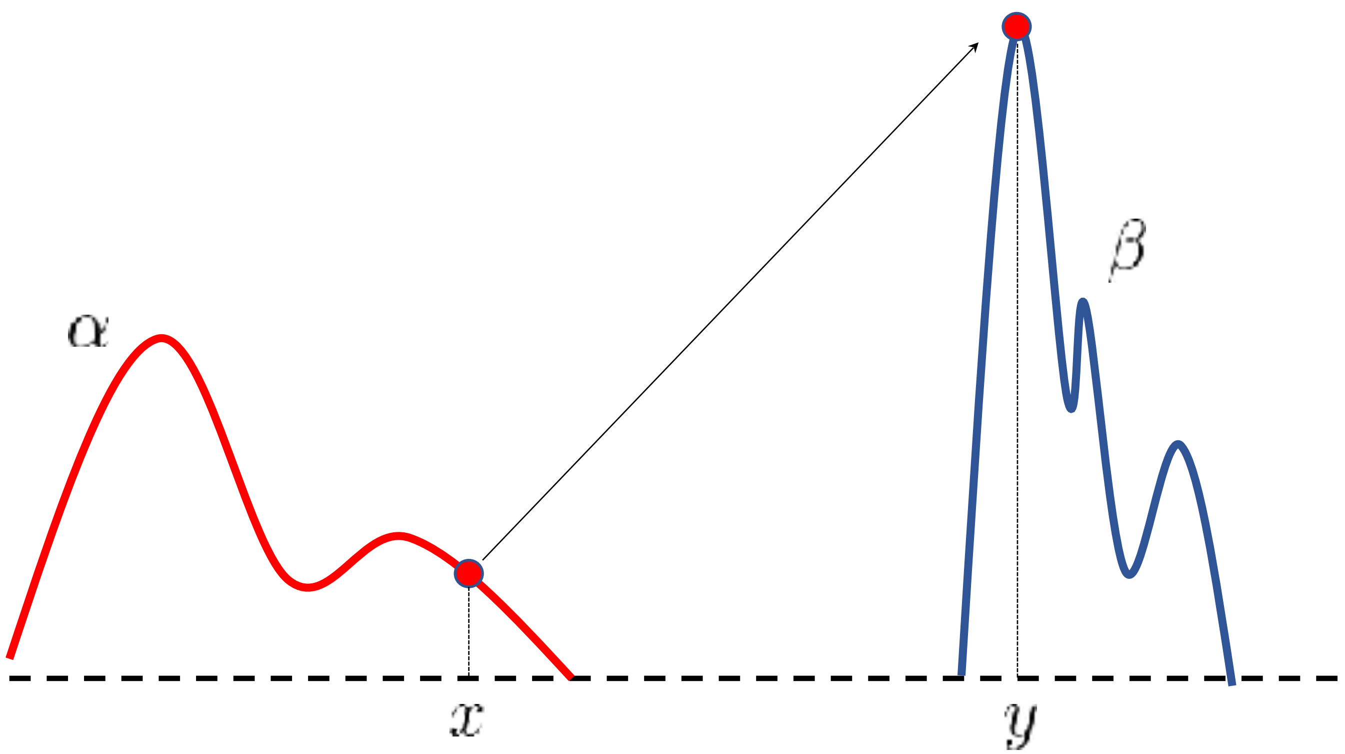
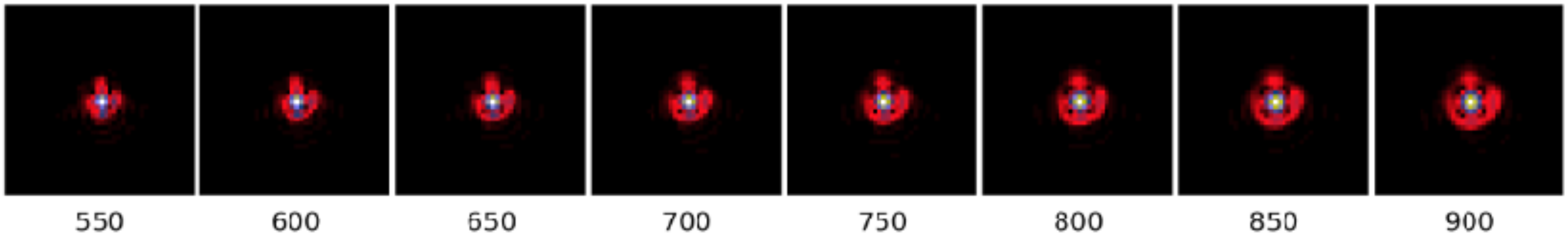


Fig: Graphical representation of mass transportation problem: find the optimal way of transporting a heap of sand α into the shape of a heap of sand β, knowing the cost of moving grains of sand to and from any position.

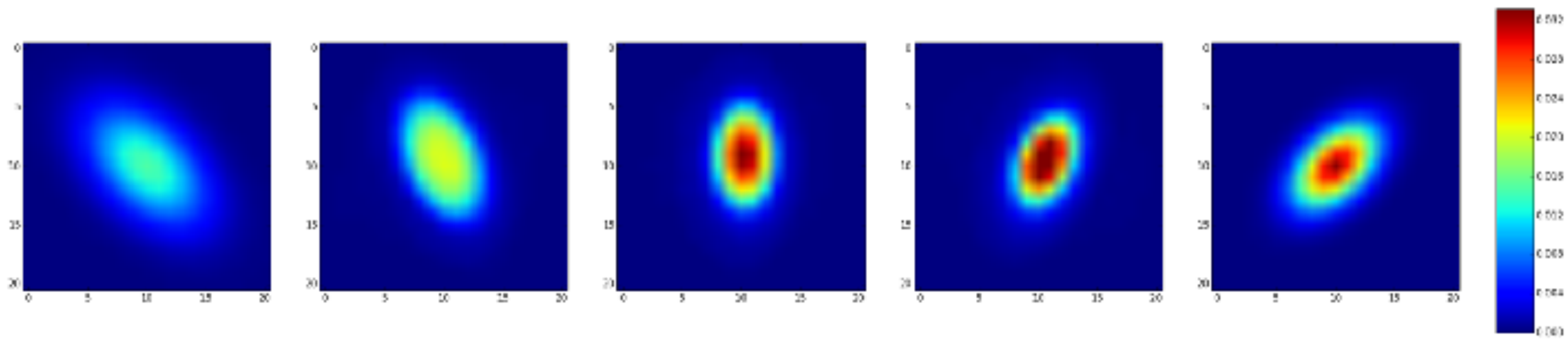
G. Monge, "Mémoire sur la théorie des déblais et des remblais", 1781

Optimal Transport & PSF

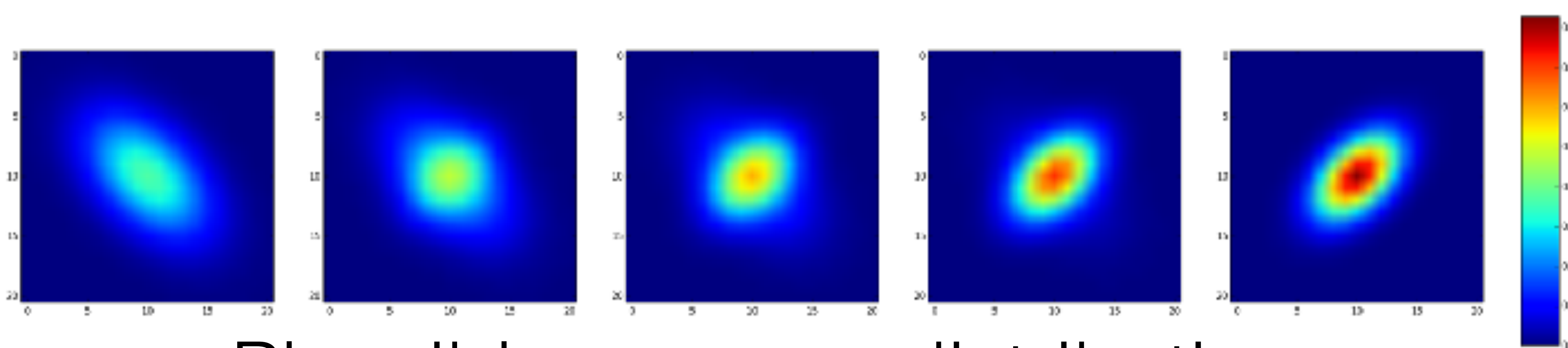


Barycenter

OT



Linear interpolation



- Plausible average distribution in presence of shifts and changing of shape

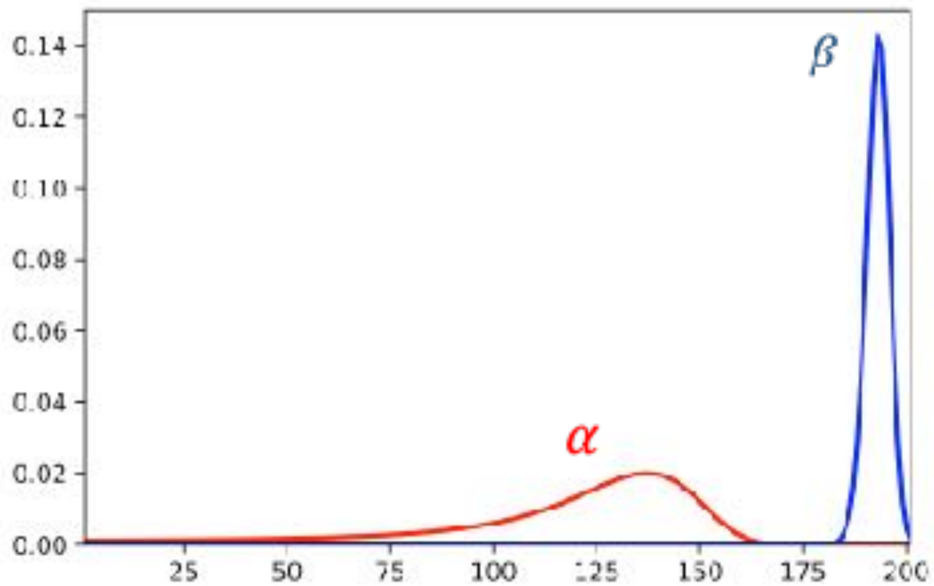
Optimal Transport

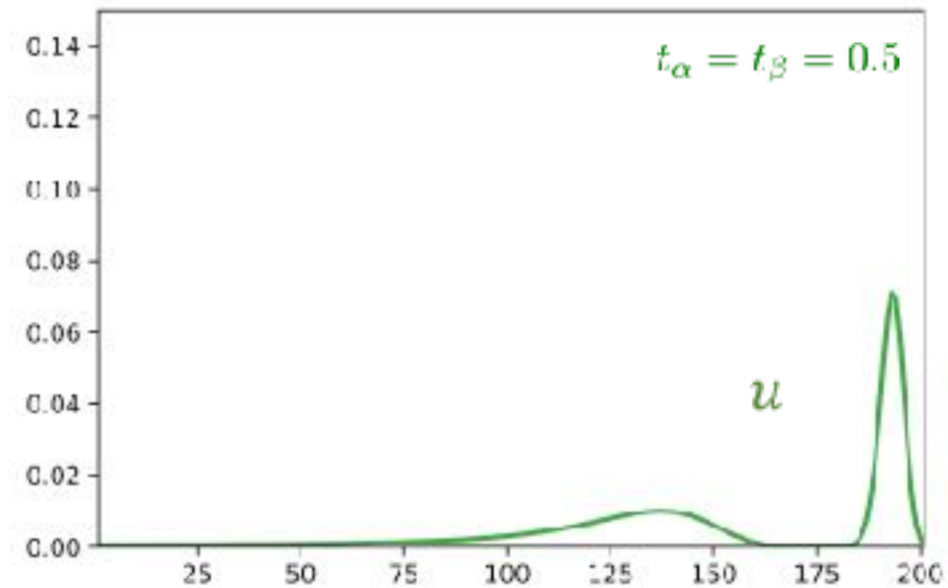
$\mathcal{W}_2^2(\boldsymbol{d}_i, \boldsymbol{u})$ If instead we replace the Euclidean distance by the Wasserstein distance..

Euclidean barycenter

$$\mathcal{P}((\boldsymbol{d}_i)_i, (t_i)_i) = \underset{\boldsymbol{u}}{\operatorname{argmin}} \sum_i t_i \underbrace{\|\boldsymbol{d}_i - \boldsymbol{u}\|_2^2} = \sum_i t_i \boldsymbol{d}_i$$

$\boldsymbol{d}_i \in \Sigma_N\,,\;\; t \in \Sigma_S$

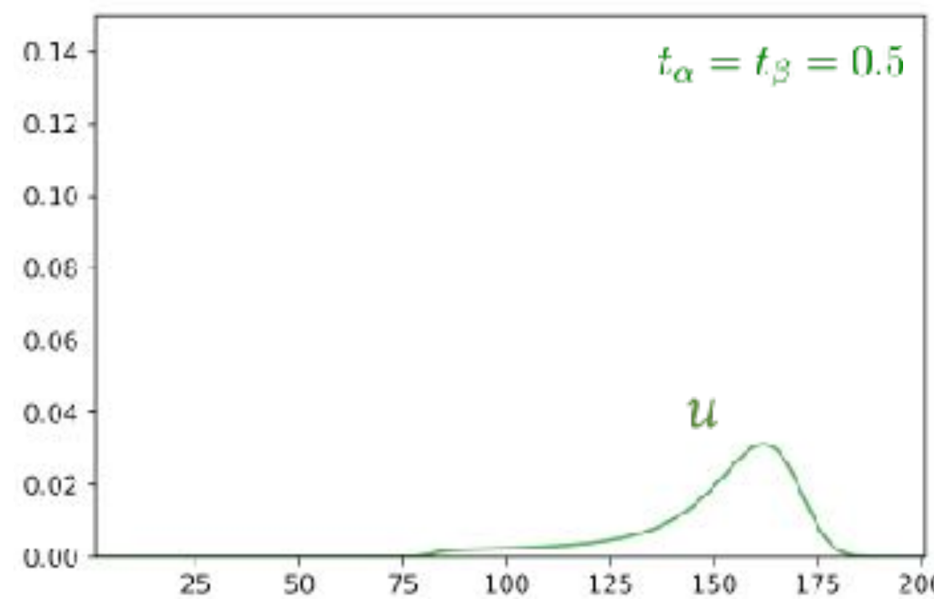




Wasserstein barycenter

$$\mathcal{P}((\boldsymbol{d}_i)_i, (t_i)_i) = \underset{\boldsymbol{u}}{\operatorname{argmin}} \sum_i t_i \mathcal{W}_2^2(\boldsymbol{d}_i, \boldsymbol{u})$$

- $\boldsymbol{d}_i \in \Sigma_N\,,\;\; t \in \Sigma_S$

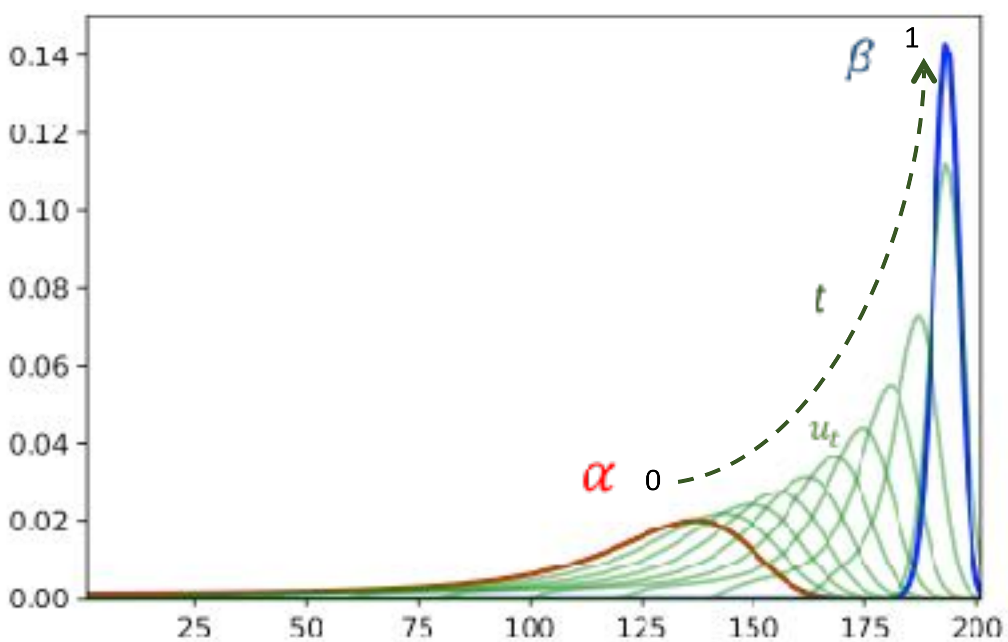


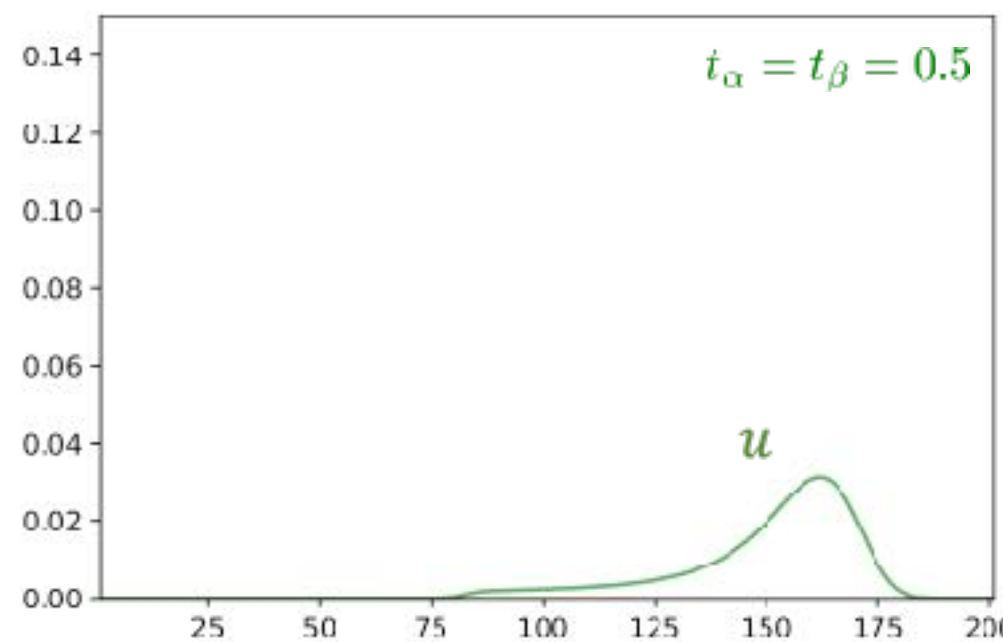
Optimal Transport

Wasserstein barycenter

$$\mathcal{P}((d_i)_i,(t_i)_i) = \underset{u}{\operatorname{argmin}} \sum_i t_i W_2^2(d_i, u)$$

- $d_i \in \Sigma_N, \quad t \in \Sigma_S$





ATTRACTIVE TOOL, BUT COSTLY TO COMPUTE: $\mathcal{O}(N^3)$

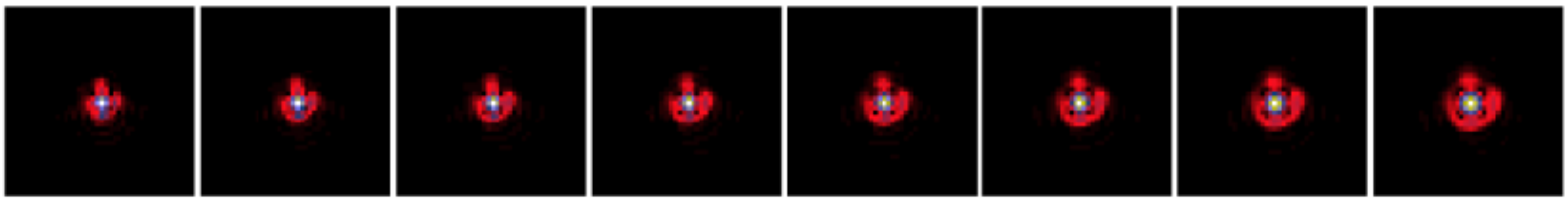
==> fixed using a regularisation (Sinkhorn iterations)

- M. A. Schmitz et a; "Wasserstein Dictionary Learning:Optimal Transport-based unsupervised representation learning", SIAM Journal on Imaging Sciences, 11, 2018.

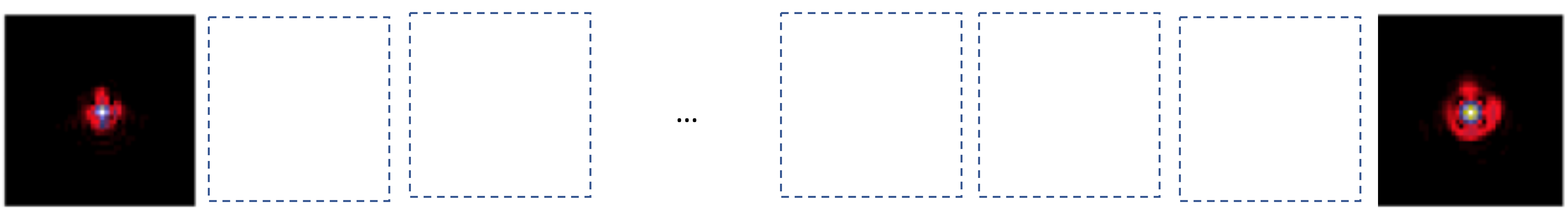
Optimal Transport

Back to the PSF estimation problem:

$$Y = M \sum_{v}^{m} \beta_v H_v + \text{noise}$$



How to break the degeneracy?



Hypothesis: monochromatic PSFs of a intermediary wavelength are Wasserstein barycenters of the extremes.

0

1.0

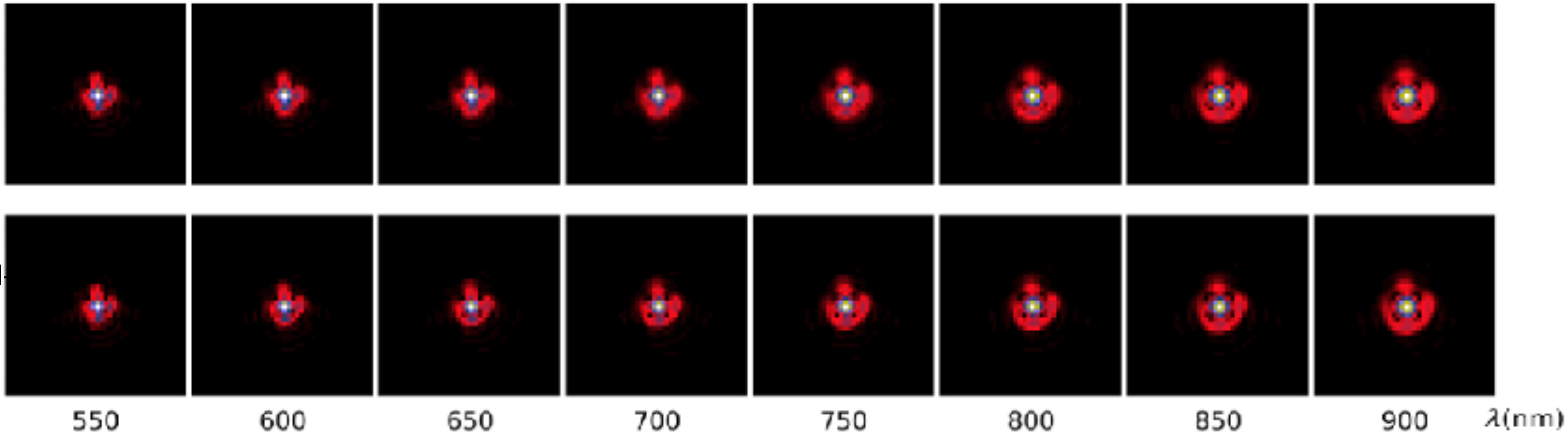
Optimal Transport

Hypothesis: monochromatic PSFs of a intermediary wavelength are Wasserstein barycenters of the extremes.

Experiment results:

OT Modeling

Simulated Euclid-like PSFs (ground truth)



- linear projection of wavelengths into [0,1].
- $\gamma=0.3$

λ-RCA

Very Preliminary Results

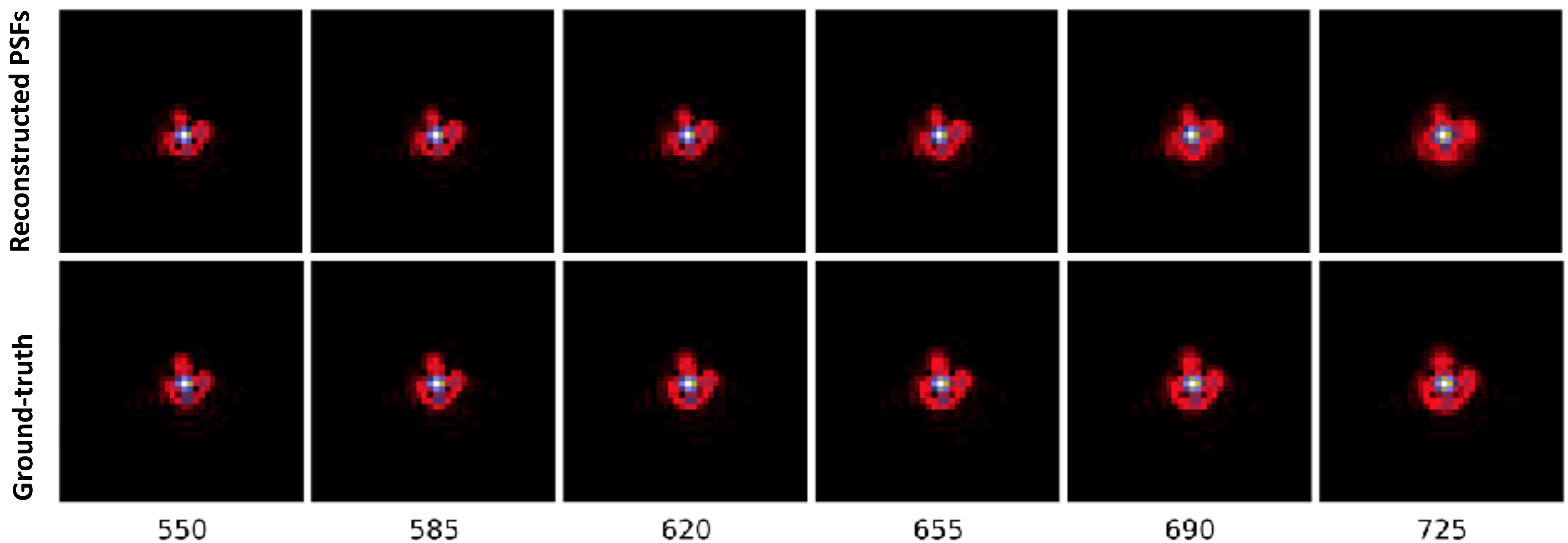
$$\min_{\boldsymbol{A},\boldsymbol{D}} \frac{1}{2}\sum_{k}^{K}\|\boldsymbol{Y}_k - \boldsymbol{M}\sum_{i}^{r} a_{i,k}\underbrace{\sum_v \beta_v \mathcal{P}(\boldsymbol{D}_i, \boldsymbol{t}_v)}_{\boldsymbol{s}_i}\|_2^2 + \|\boldsymbol{\Phi}\boldsymbol{D}\|_1 \quad \text{s.t.} \quad \boldsymbol{D}\in\Sigma_N$$

Simple constraint on dictionary: probability distribution.

- $\boldsymbol{D} = (\boldsymbol{d}_0^i, \boldsymbol{d}_1^i)_{1\leq i\leq S}, \quad \boldsymbol{d}_0^i \in \mathbb{R}_+^N$

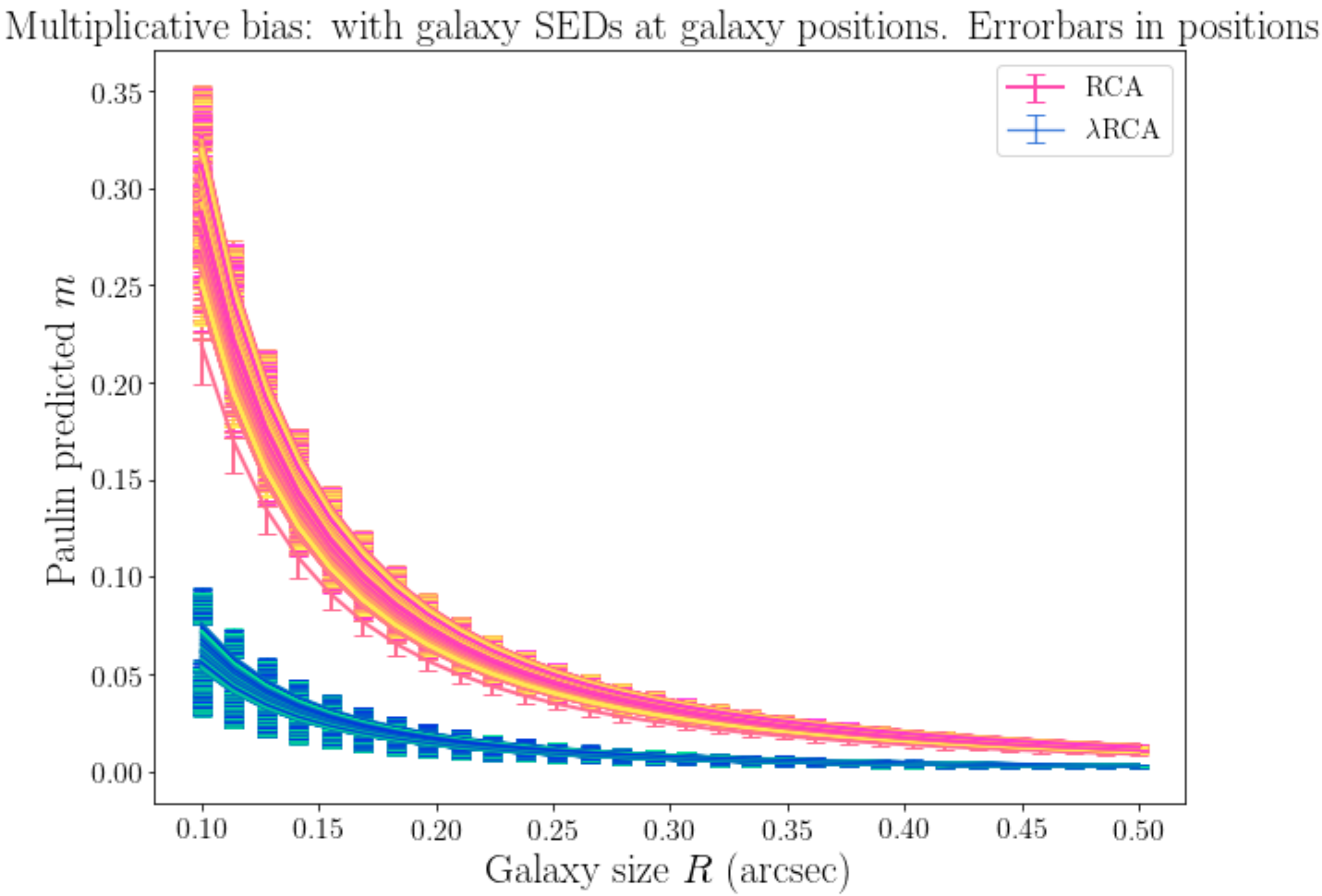
Eigen-PSF extreme left

Eigen-PSF extreme right



- M. A. Schmitz et al, in preparation, 2018

Numerical Experiments



- Part I: Weak Lensing and Euclid
- Part II: Point Spread Fonction (PSF) Field Recovery
- **Part III: Shape Measurements & Deconvolution**

Astronomical Image Deconvolution

Sandard deconvolution framework:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Noise
True Image
PSF Convolution
Observed Image

Sandard deconvolution framework:

$$\underset{\mathbf{X}}{\operatorname{argmin}} \quad \frac{1}{2}\|\mathbf{Y} - \mathbf{H}\mathbf{X}\|_2^2 + \|\Phi^t \mathbf{X}\|_p \quad \text{s.t.} \quad \mathbf{X} \geq 0$$

H is huge !!!

Object Oriented Deconvolution

For each galaxy, we use the PSF related to its center pixel:

$$\mathbf{Y} = \mathcal{H}(\mathbf{X}) + \mathbf{N}$$

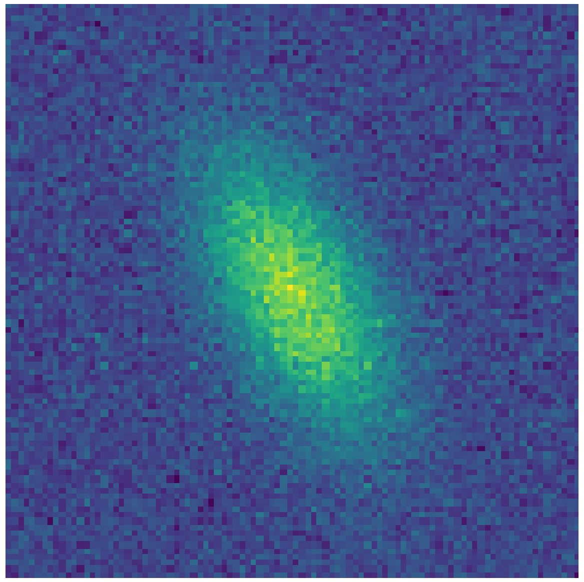
$[n^0, n^1, \ldots, n^n]$

$[H^0x^0, H^1x^1, \ldots, H^nx^n]$

$[y^0, y^1, \ldots, y^n]$

$$\underset{\mathbf{X}}{\operatorname{argmin}} \quad \frac{1}{2}\|\mathbf{Y} - \mathcal{H}(\mathbf{X})\|_2^2 + \lambda\|\Phi^t\mathbf{X}\|_p \quad \text{s.t.} \quad \mathbf{X} \geq 0$$

Inverse Problems: Sparse Recovery



$$\min_{\alpha} \|Y - H\Phi\alpha\|^2 + \lambda \|\alpha\|_p^p$$

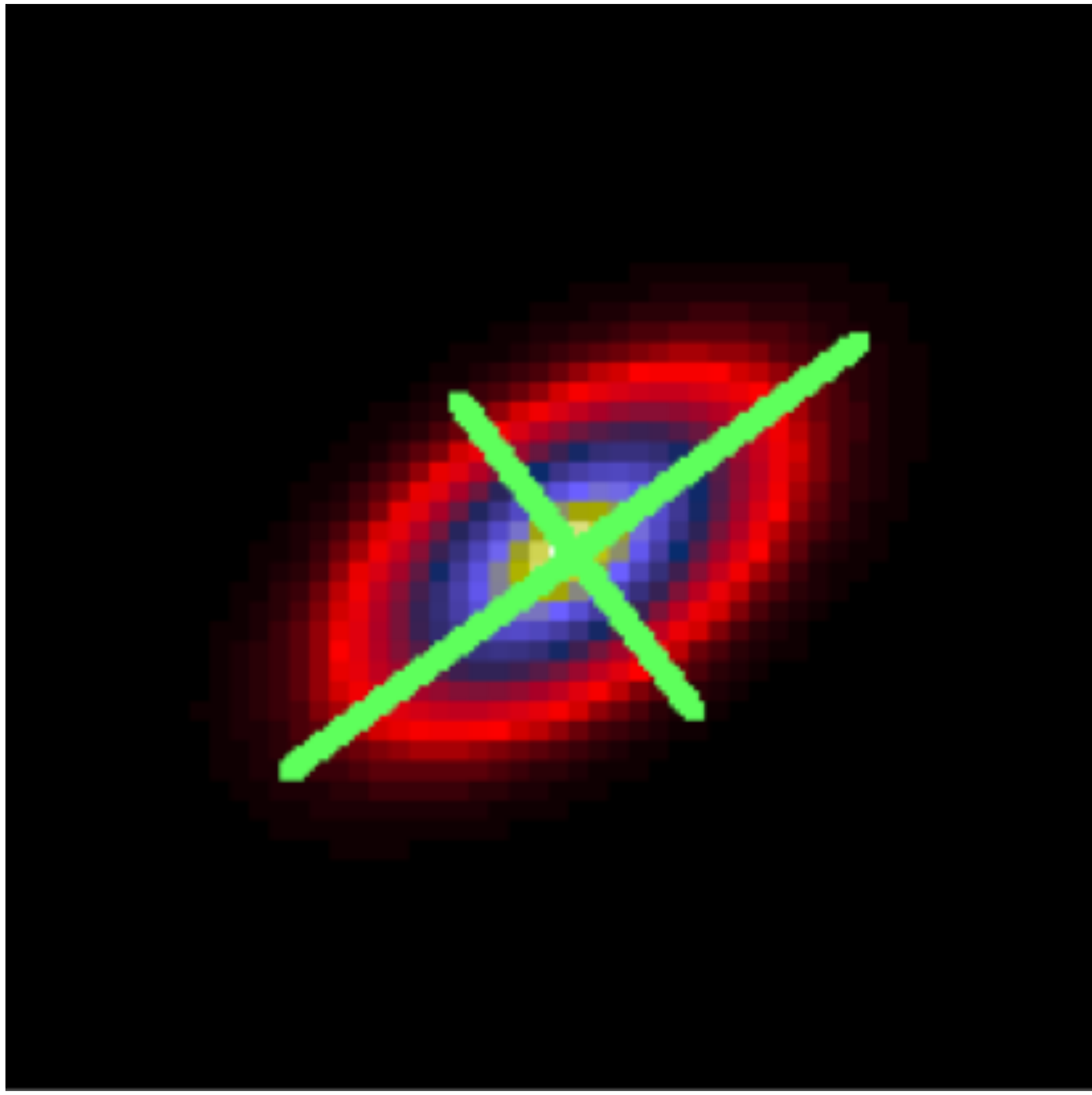
➡ **Wavelet Denoising introduces a bias.**

➡ **Fitting a galaxy model directly on significant wavelet coefficients introduces a bias.**

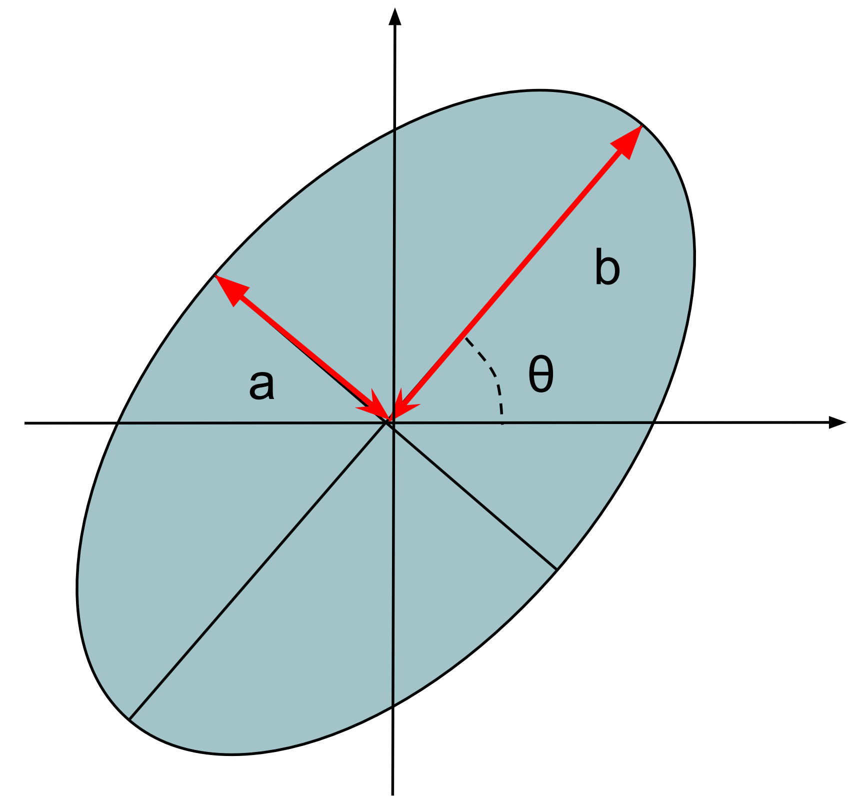
➡ **Need to find a way to preserve the ellipticity during the restoration process**

Measuring Galaxies shapes

The complex ellipticity of a galaxy image uses quadrupole moments



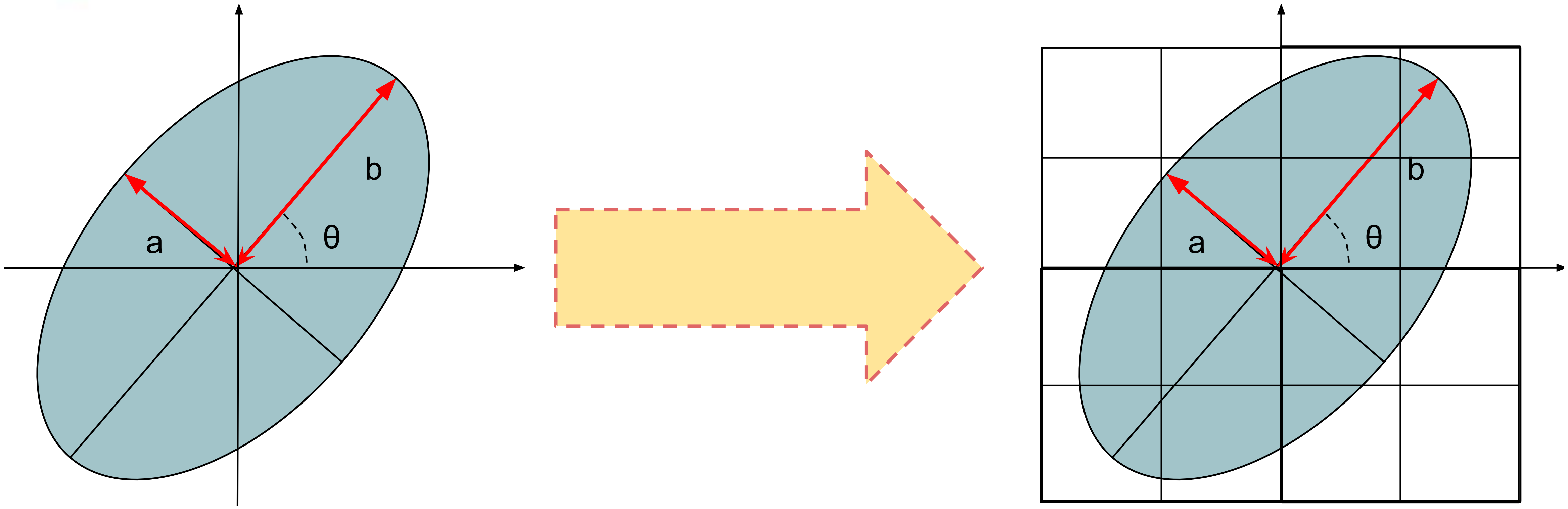
X : **Galaxy**



$$e(X) = \frac{1 - \left(\frac{b}{a}\right)^2}{1 + \left(\frac{b}{a}\right)^2} \exp(2i\theta) = e_1(X) + ie_2(X)$$

$e = \gamma + e_g$ and $< e > \simeq \gamma$

From Ellipticity to Moments



$$X \in \mathcal{M}_n(\mathbb{R})$$

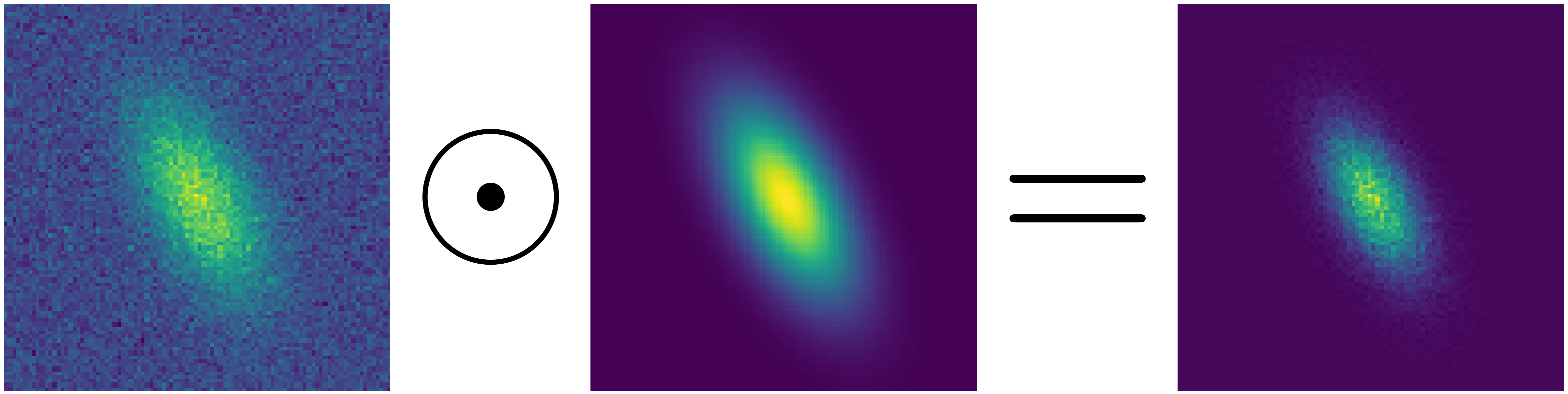
$$\mu_{s,t}(X) = \sum_{i,j} X[i,j](i - i_c)^s (j - j_c)^t$$

$$\gamma_1(X) = \frac{\mu_{2,0}(X) - \mu_{0,2}(X)}{\mu_{2,0}(X) + \mu_{0,2}(X)} \qquad \gamma_2(X) = \frac{2\mu_{1,1}(X)}{\mu_{2,0}(X) + \mu_{0,2}(X)}$$

$\gamma = \gamma_1 + i\gamma_2 = \dfrac{\mu_{2,0} - \mu_{0,2} + 2\mu_{1,1}}{\mu_{2,0} + \mu_{0,2}}$ is an unbiased estimator of “ellipticity” (Schneidez & Seitz 1994)

Moments Windowing

Does not work in presense of noise! Need to apply a window function.



Observed galaxy Y

Matched Gaussian window W

Windowed galaxy $W \odot Y$

- "HSM" (or adaptive moments, Hirata & Seljak 2003; Mandelbaum et al 2004): match the gaussian window to the object
- Handling the PSF: Kaiser, Squires & Broadhurst 1995 (**KSB**).

High Accuracy Requirements

$$\gamma = \gamma_1 + i\gamma_2 = |\gamma| e^{i2\Phi}$$

$$\tilde{\gamma} = (1 + \underbrace{m}_{\text{Multiplicative bias}})\gamma + \underbrace{c}_{\text{Additive bias}}$$

$$m = m_1 + i m_2 = |m| e^{i2\Phi_m}$$

$$c = c_1 + i c_2 = |c| e^{i2\Phi_c}$$

Requirements for the ESA Euclid Space Telescope

$$\sigma_m < 2 \times 10^{-3}$$

$$\sigma_c < 5 \times 10^{-4}$$

Criteria for a good method

- ➡ Fast Calculation.
- ➡ Level of bias (multiplicative and additive bias).
- ➡ Capacity to measure the bias (number of simulations, depth, resolution, etc).
- ➡ **Robustness to calibration errors (PSF measurement errors, detectors effects background subtraction, etc).**

$$\sigma(m) = \frac{m(X_0)}{\sqrt{N_{X,Cal}}} \left[\sum_X \left(\frac{\partial \ln m}{\partial \ln X} \right)^2 \right]^{1/2}$$

X = image properties

Moments using Inner Products

$X \in \mathcal{M}_n(\mathbb{R})$

$$e_1(X) = \frac{\mu_{2,0}(X) - \mu_{0,2}(X)}{\mu_{2,0}(X) + \mu_{0,2}(X)} \quad , \quad e_2(X) = \frac{2\mu_{1,1}}{\mu_{2,0}(X) + \mu_{0,2}(X)}$$

$$\mu_{s,t}(X) = \sum_{i,j} X[i,j](i - i_c)^s (j - j_c)^t$$

$$e_1(X) = \frac{\langle X, U_5\rangle\langle X, U_3\rangle - \langle X, U_1\rangle^2 + \langle X, U_2\rangle^2}{\langle X, U_5\rangle\langle X, U_3\rangle - \langle X, U_1\rangle^2 - \langle X, U_2\rangle^2}$$

$$e_2(X) = \frac{2(\langle X, U_6\rangle\langle X, U_3\rangle - \langle X, U_1\rangle\langle X, U_2\rangle)}{\langle X, U_5\rangle\langle X, U_3\rangle - \langle X, U_1\rangle^2 - \langle X, U_2\rangle^2}$$

Matrices of indices combination

$$U_1 = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 2 & 2 & \dots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ n & n & \dots & n \end{pmatrix} \quad U_2 = \begin{pmatrix} 1 & 2 & \dots & n \\ 1 & 2 & \dots & n \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & \dots & n \end{pmatrix} \quad U_3 = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{pmatrix}$$

$$U_4 = \begin{pmatrix} 1^2+1^2 & 1^2+2^2 & \dots & 1^2+n^2 \\ 2^2+1^2 & 2^2+2^2 & \dots & 2^2+n^2 \\ \vdots & \vdots & \ddots & \vdots \\ n^2+1^2 & n^2+2^2 & \dots & n^2+n^2 \end{pmatrix}$$

$$U_5 = \begin{pmatrix} 1^2-1^2 & 1^2-2^2 & \dots & 1^2-n^2 \\ 2^2-1^2 & 2^2-2^2 & \dots & 2^2-n^2 \\ \vdots & \vdots & \ddots & \vdots \\ n^2-1^2 & n^2-2^2 & \dots & n^2-n^2 \end{pmatrix} \quad U_6 = \begin{pmatrix} 1 & 2 & \dots & n \\ 2 & 4 & \dots & 2n \\ \vdots & \vdots & \ddots & \vdots \\ n & 2n & \dots & n^2 \end{pmatrix}$$

Ellipticities as inner products

$$e_1(X) = \frac{\langle X, U_5\rangle\langle X, U_3\rangle - \langle X, U_1\rangle^2 + \langle X, U_2\rangle^2}{\langle X, U_5\rangle\langle X, U_3\rangle - \langle X, U_1\rangle^2 - \langle X, U_2\rangle^2} \quad , \quad e_2(X) = \frac{2(\langle X, U_6\rangle\langle X, U_3\rangle - \langle X, U_1\rangle\langle X, U_2\rangle)}{\langle X, U_5\rangle\langle X, U_3\rangle - \langle X, U_1\rangle^2 - \langle X, U_2\rangle^2}$$

Idea : Conserving the inner products is equivalent to conserving the ellipticity parameters.

Advantage : These inner products are linear functions of the image we want to restore so it is mathematically easier to work with.

Question : How to formulate a constraint using these inner products? What would be the additional contribution of this constraint regarding the data fidelity term?

The Moment Shape Constraint

The Shape Contraint :

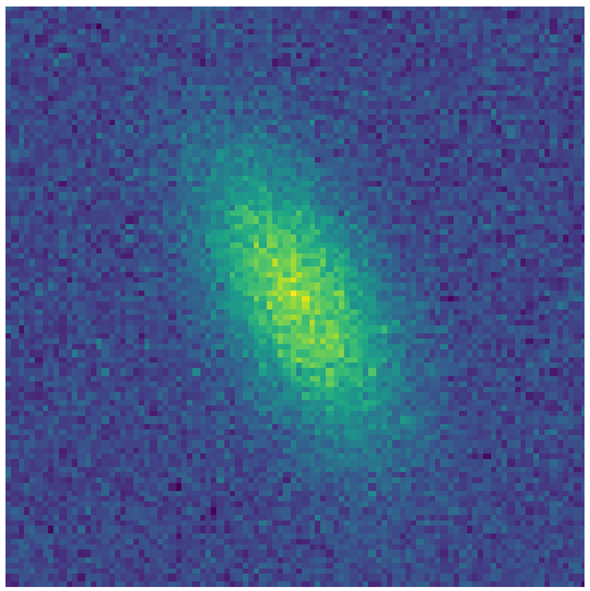
$$M(X) = \sum_{i=1}^{6} \mu_i \left\langle X * H - Y, U_i \right\rangle^2$$

The constraint looks just like a data fidelity term expressed in the moments space.

Idea :

- Remove noise as much as possible inside the constraint in a inverse pb.

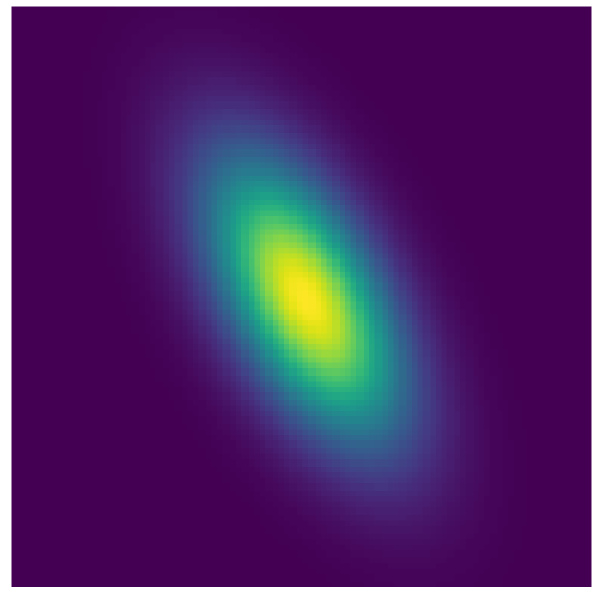
Constraint with a Gaussian Window



Observed galaxy

Y

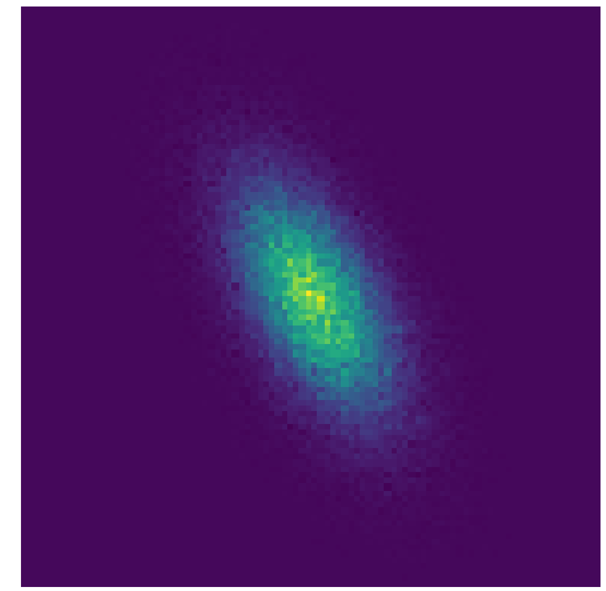
$\odot$



Matched Gaussian window

W

$=$



Windowed galaxy

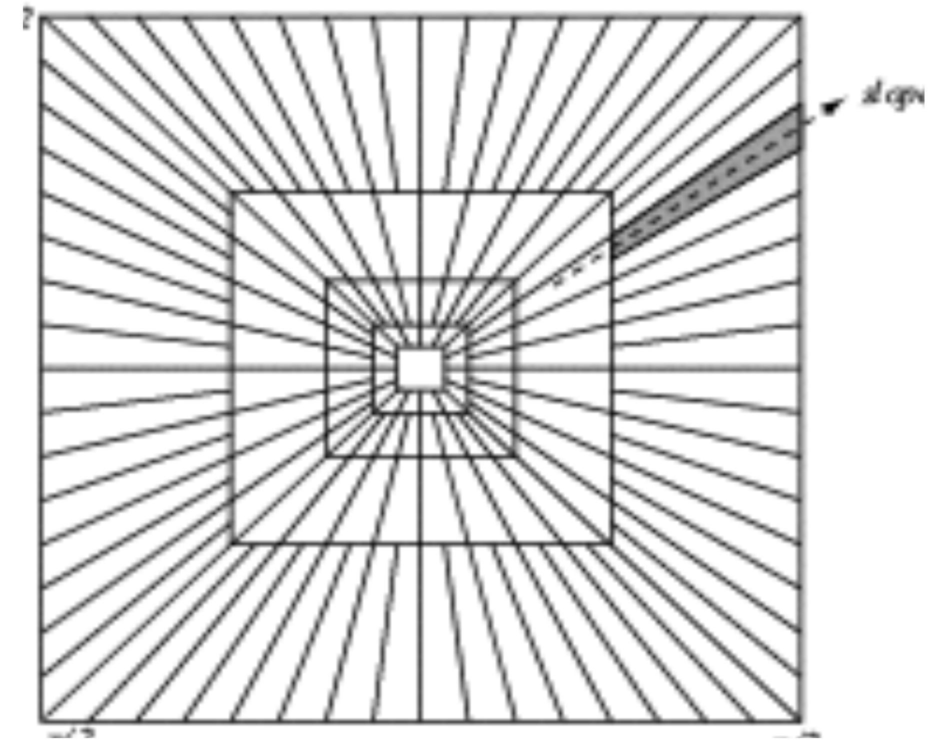
$W \odot Y$

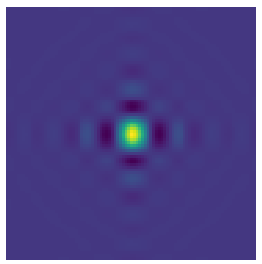
<u>**Formulation with a Gaussian window :**</u>

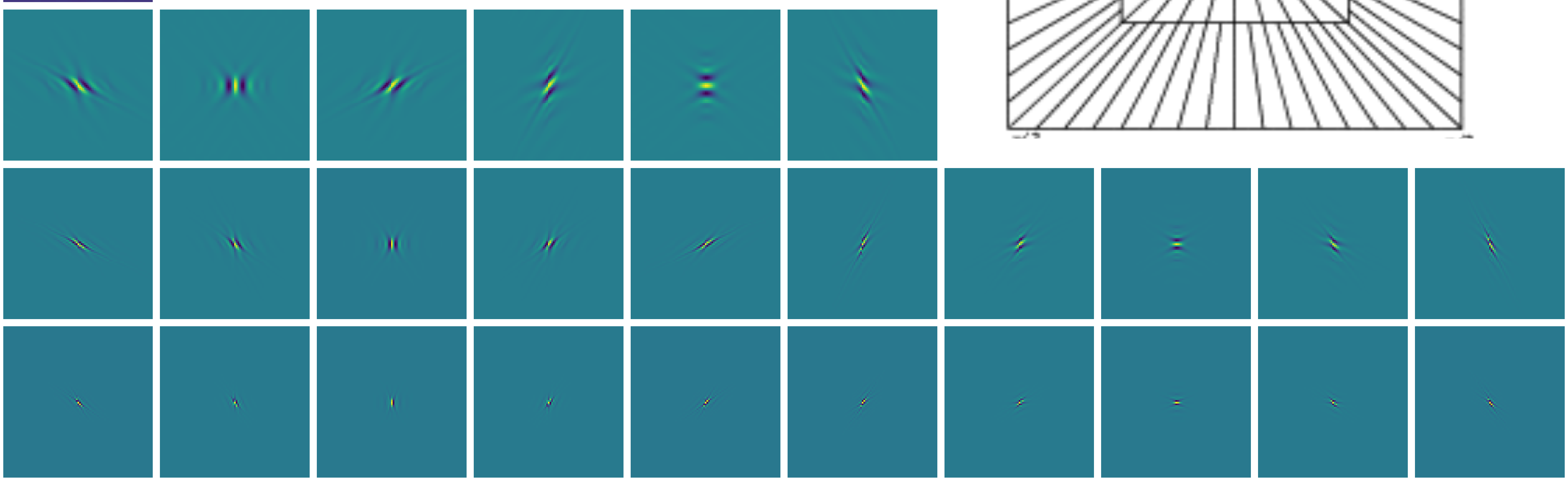
$$M(X) = \sum_{i=1}^{6} \mu_i \left\langle W \odot (X * H - Y), U_i \right\rangle^2$$

Multi-Window Constraints

Rather than trying to fit a Gaussian window to the data, why not trying all windows in all directions and all scales ?

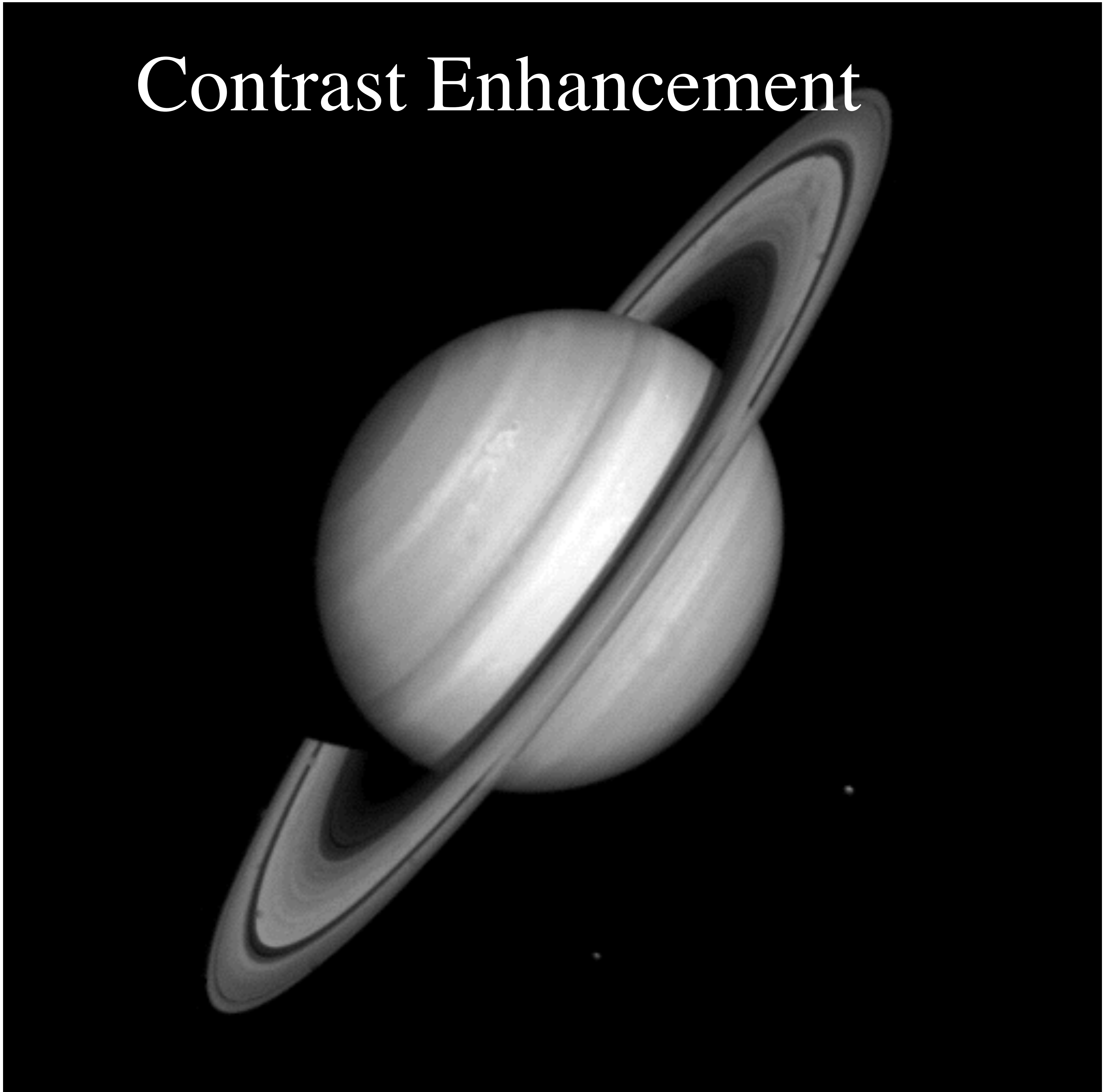


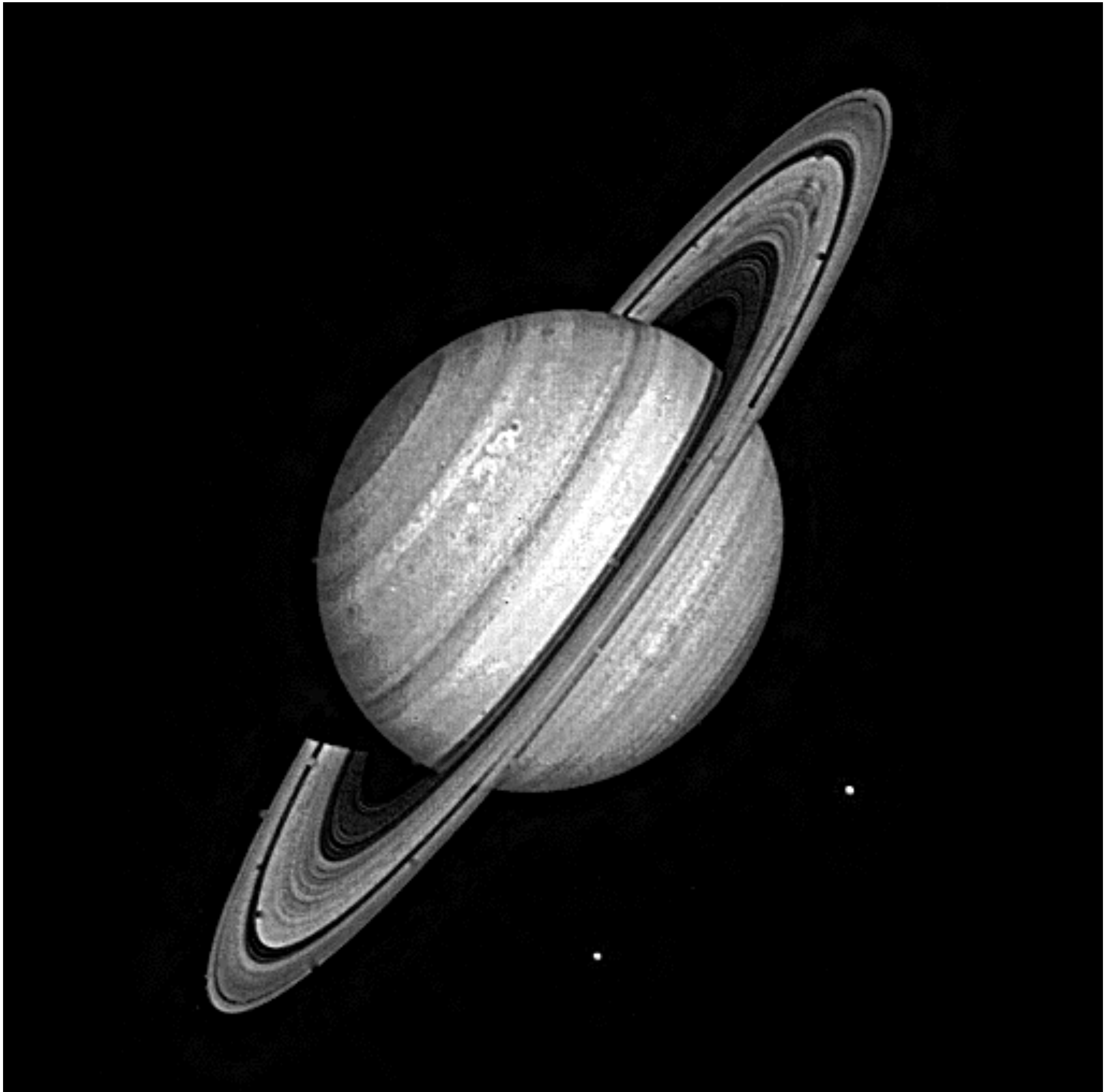




This can be done easily using a Curvelet like Decomposition.

Contrast Enhancement



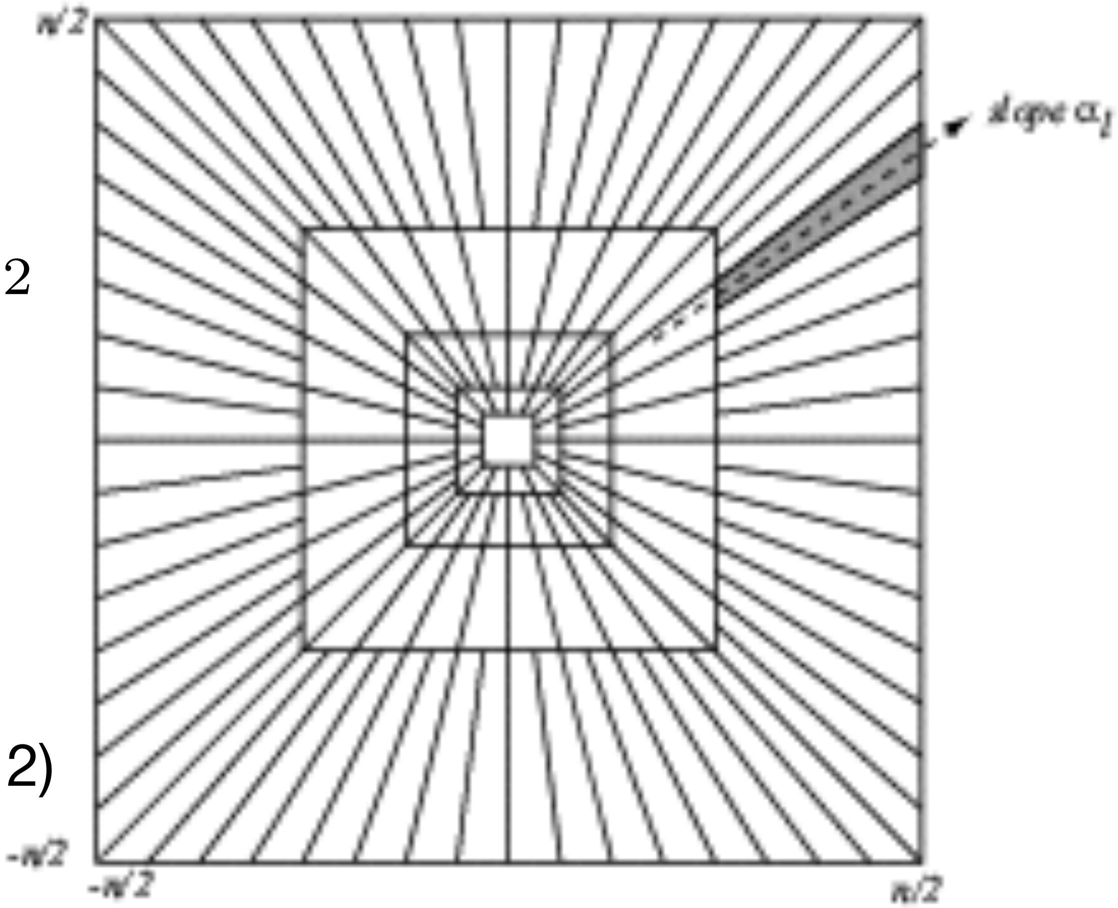


Measuring the Multi-Scale Moments

Formulation with shearlets :

$$M(X) = \frac{1}{6}\sum_{i=1}^{6}\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}\left\langle \Psi_j(X * H - Y), U_i \right\rangle^2$$

- Number of bands
- Ψ_j : Curvelet/Shearlet transform (Kutyniok & Labate, 2012)



(a)

Aleternative form :

$$M(X) = \frac{1}{6}\sum_{i=1}^{6}\frac{1}{N}\sum_{j=1}^{N}\mu_{ij}\left\langle X * H - Y, \Psi_j^*(U_i) \right\rangle^2$$

- Ψ_j^* : adjoint of the shearlet transform operator

Advantage : $\Psi_j^*(U_i)$ is a constant.

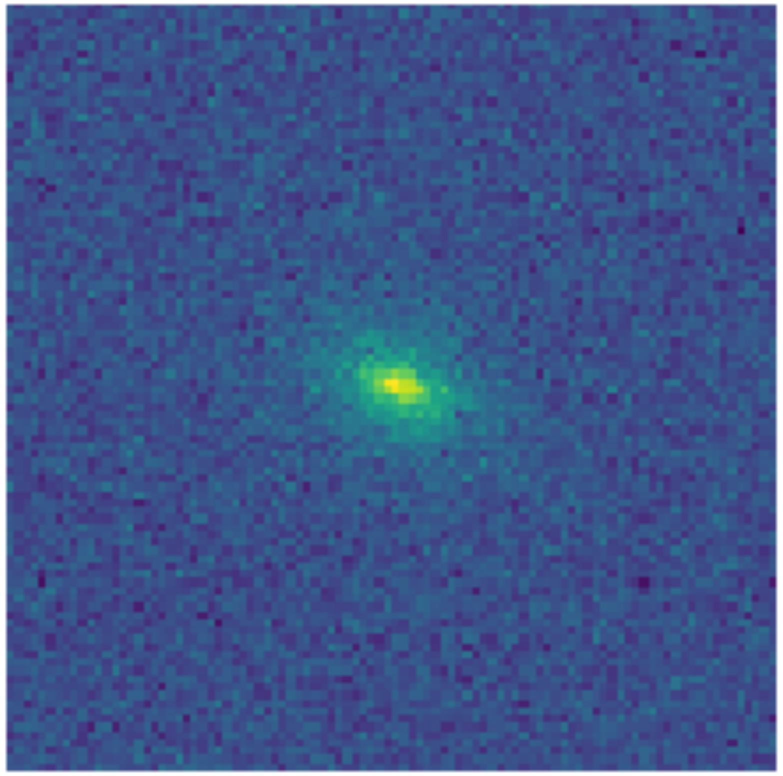
Functional to Minimise

$$L(X) = \frac{1}{2\sigma^2}\|X * H - Y\|_{\mathrm{F}}^2 + \frac{\gamma}{2\sigma^2}M(X) + \|\lambda_{\kappa-\sigma_{\mathrm{MAD}}} \odot \Phi X\|_0$$

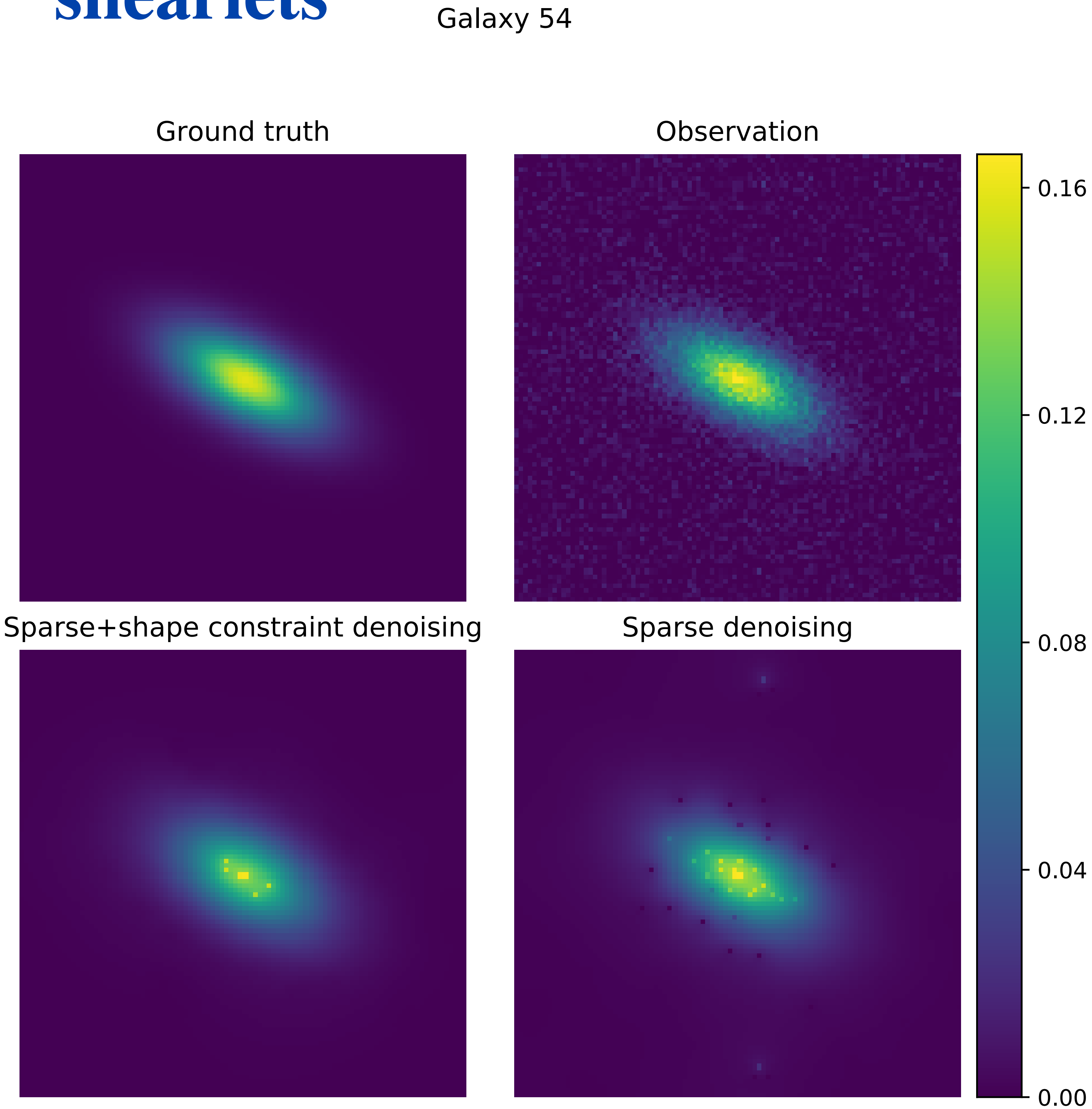
- $Y \in \mathcal{M}_n(\mathbb{R})$: galaxy observation
- $H \in \mathcal{M}_n(\mathbb{R})$: Point Spread Function
- σ : noise standard deviation in Y
- γ : trade-off parameter between the data fidelity term and the moments constraint
- $\lambda_{\kappa-\sigma_{\mathrm{MAD}}}$: weighting tensor
- Φ : starlet transform operator
- $M(.)$: moments constraint

Denoising Experiment: Gaussian window v/s shearlets

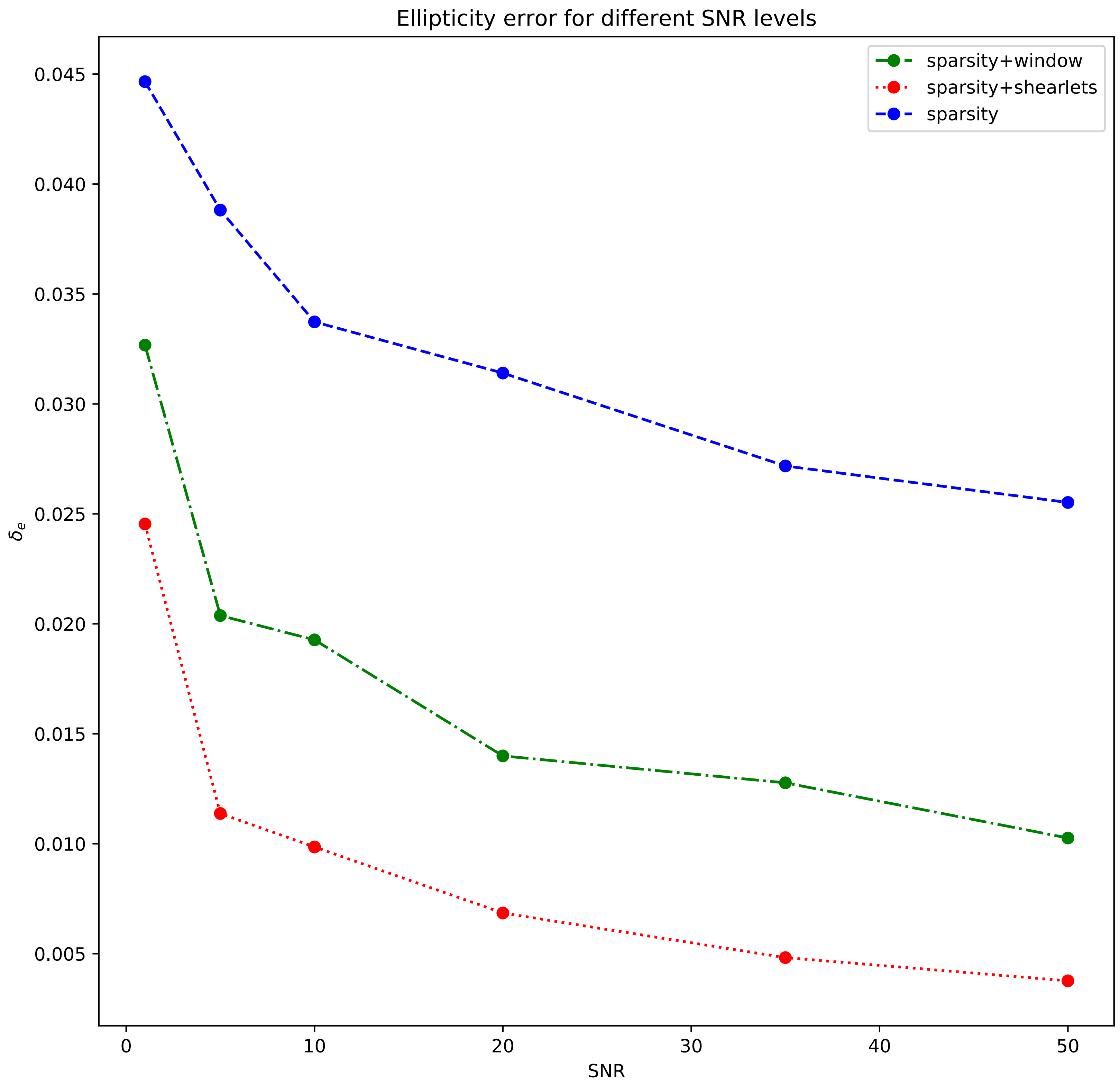
300 GALSIM galaxies



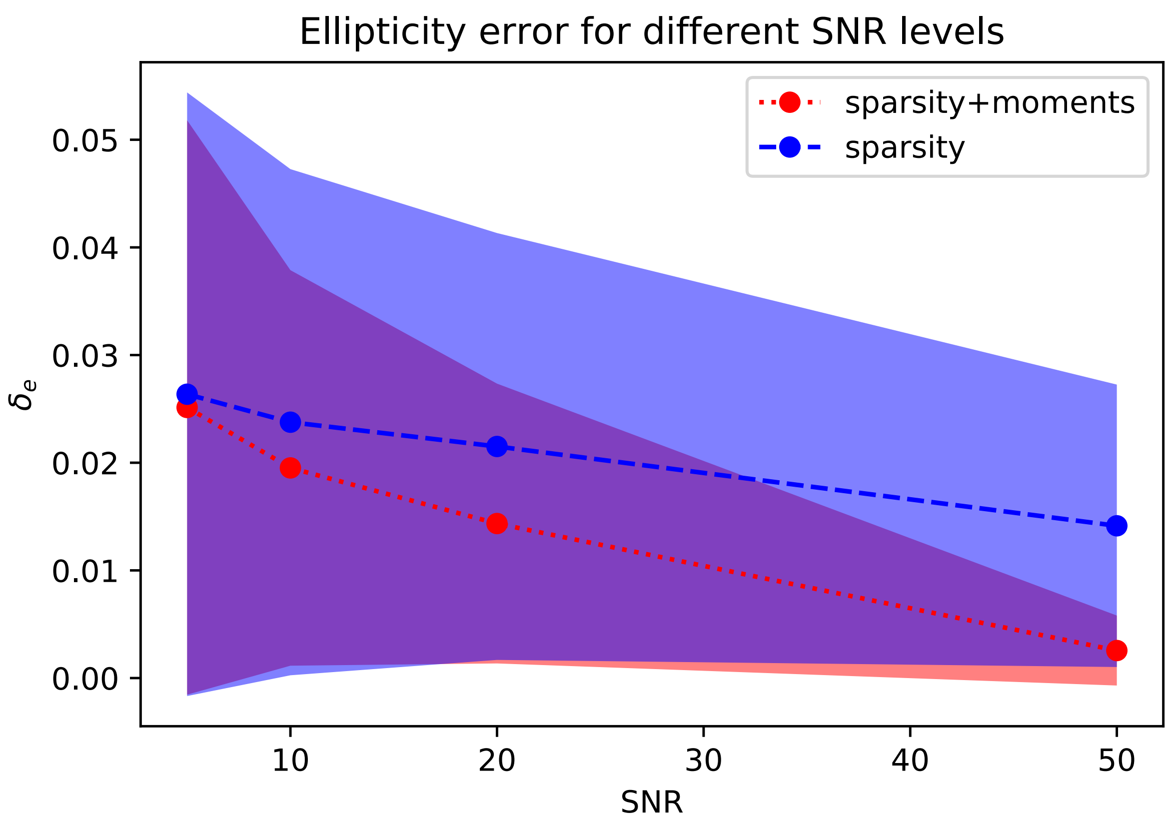
Observed image (Y=X*PSF+N)

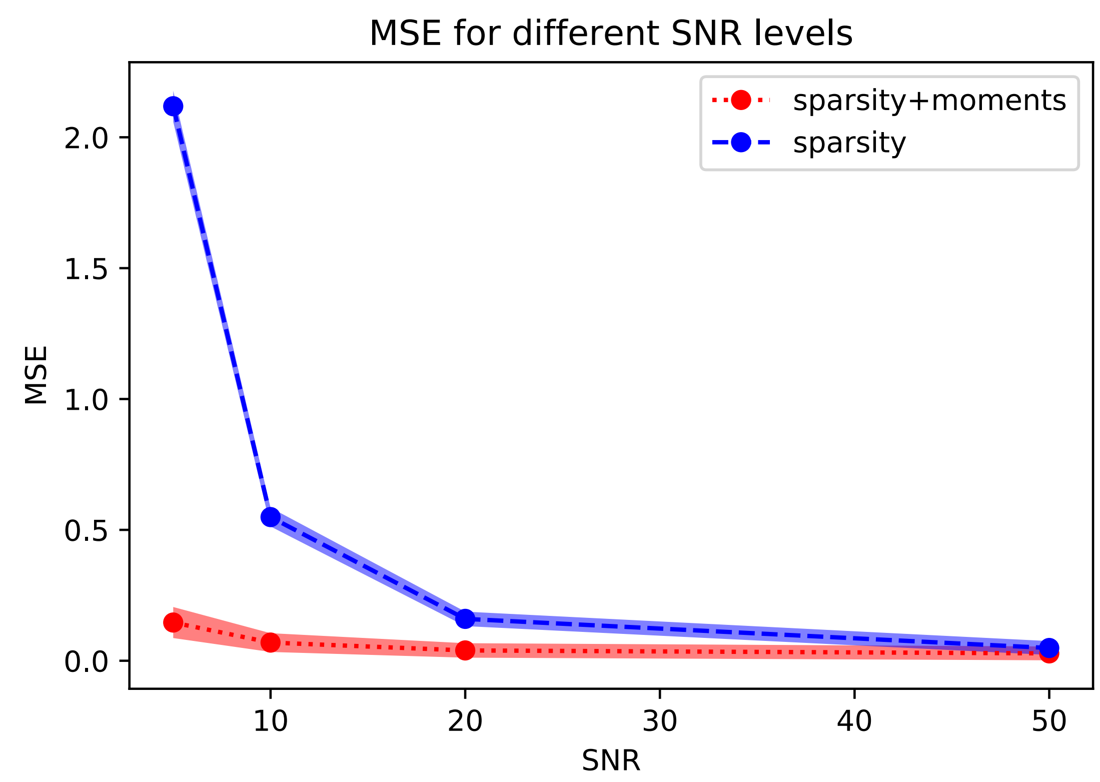


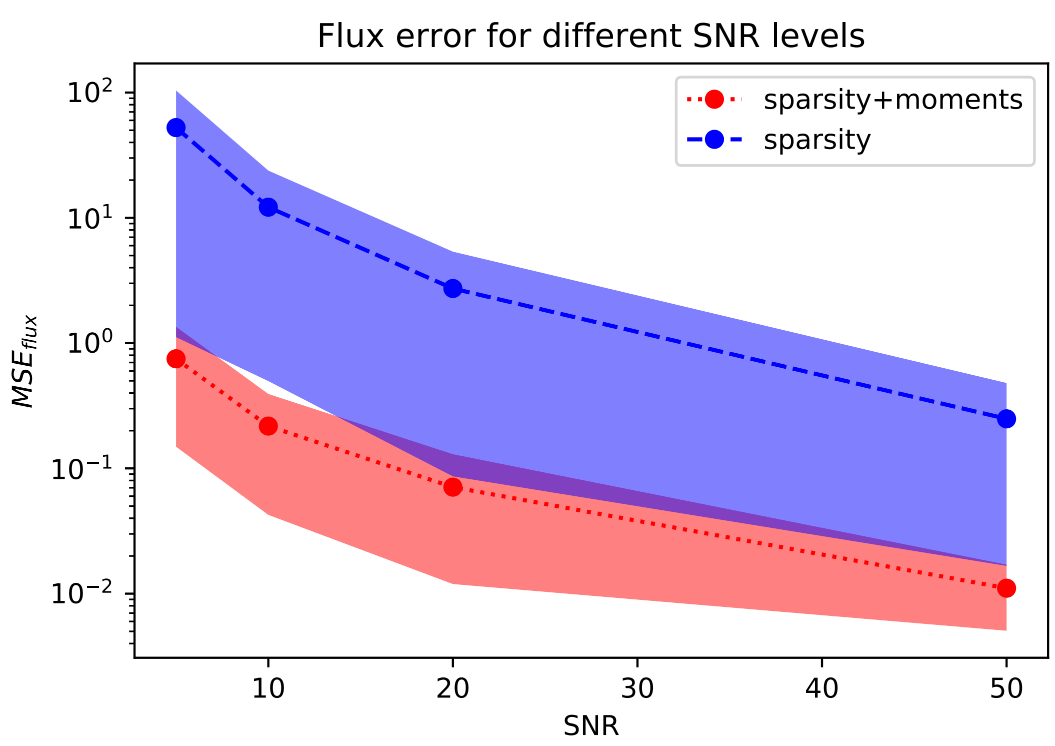
Gaussian window v/s shearlets

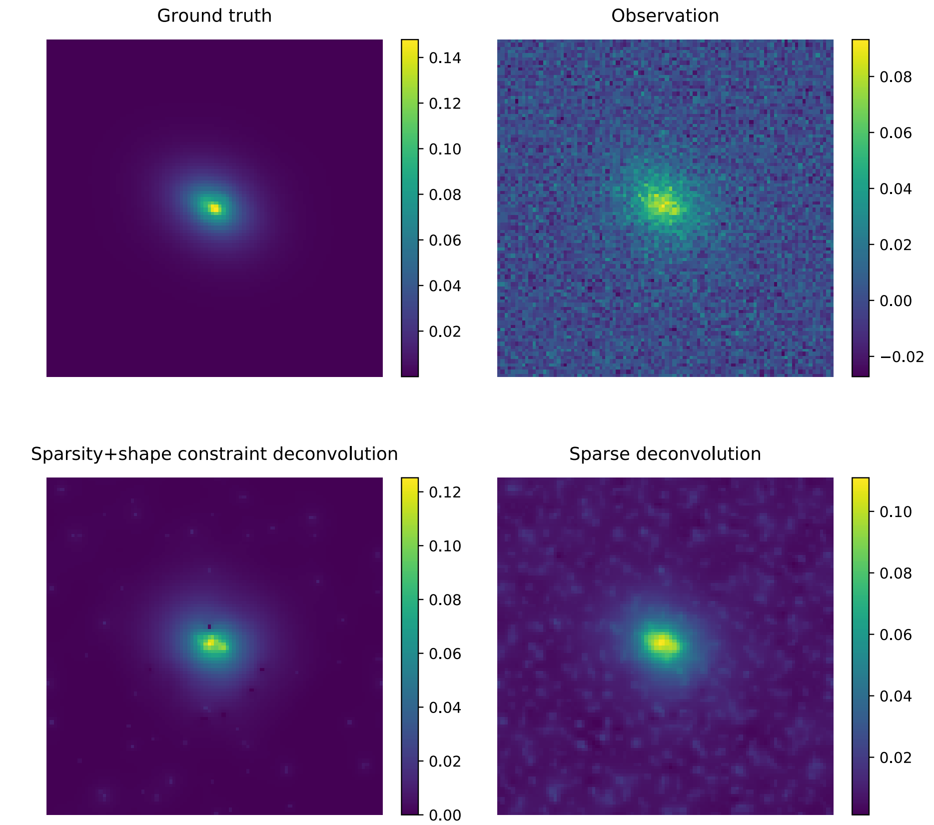


Experiment: Deconvolution



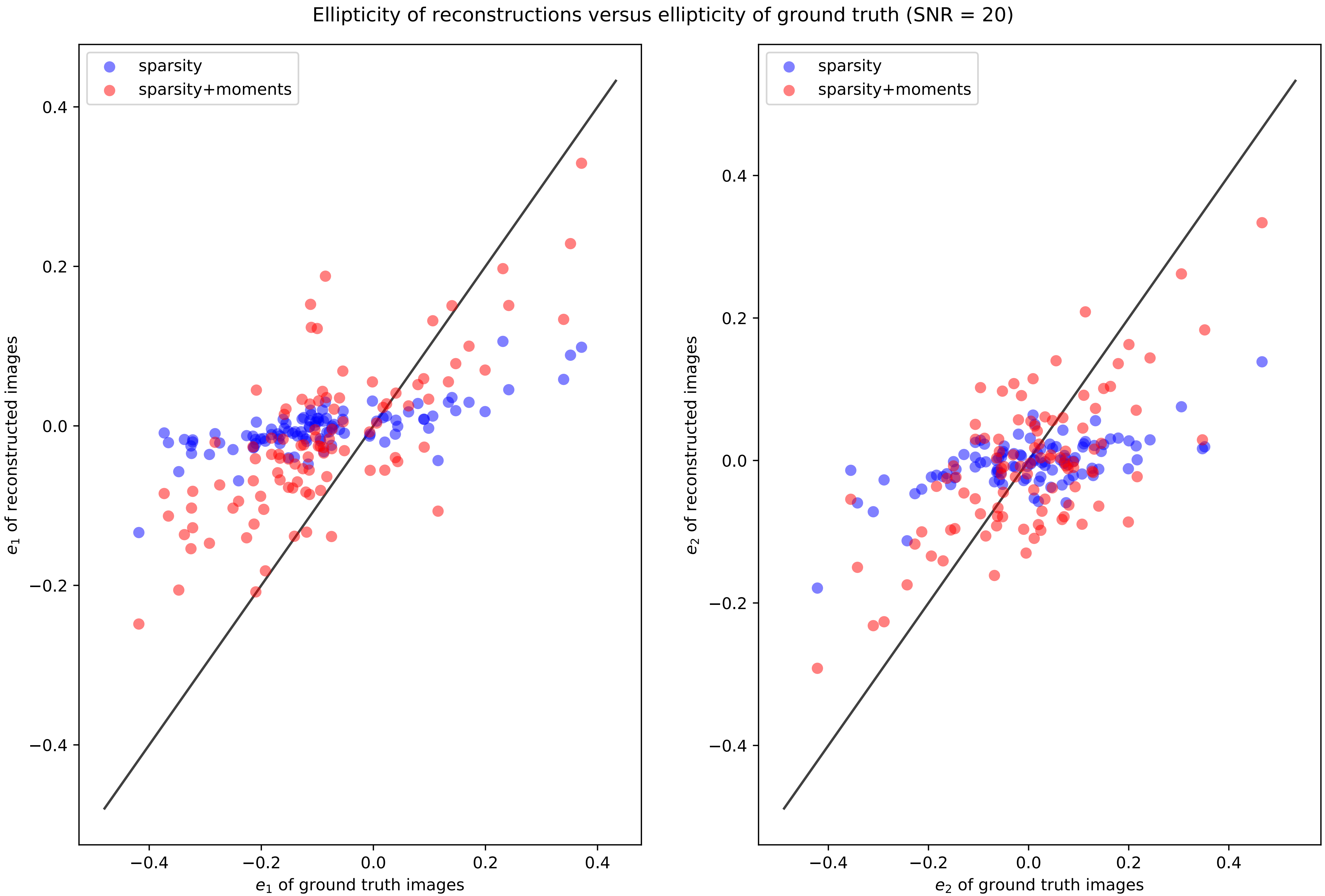




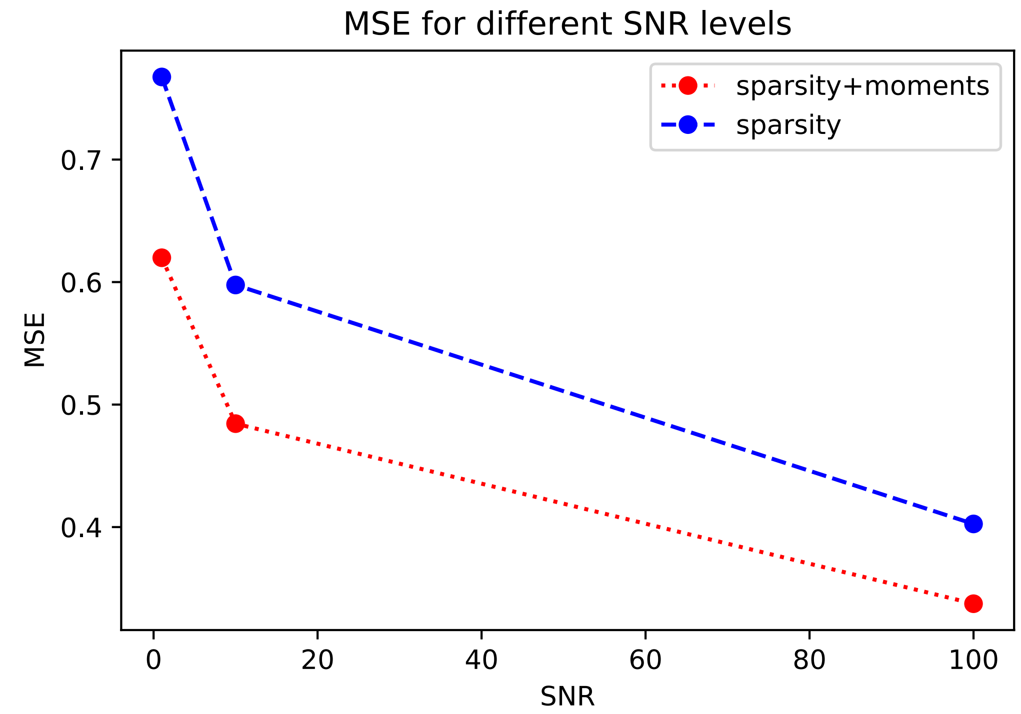


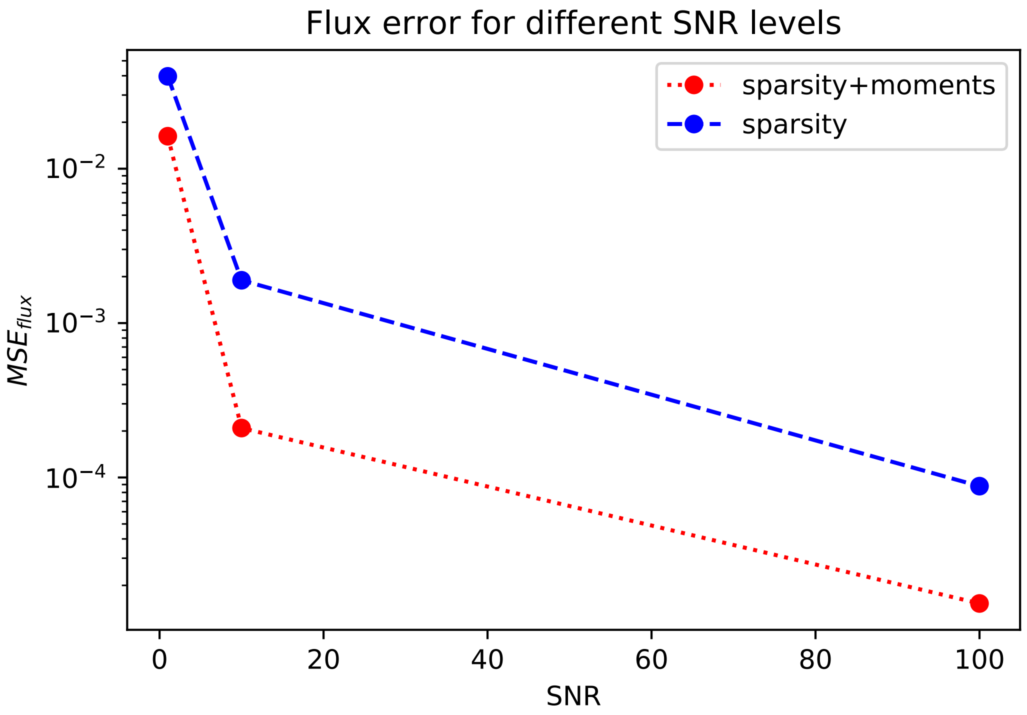
100 GALSIM galaxies + Optical GALSIM PSF

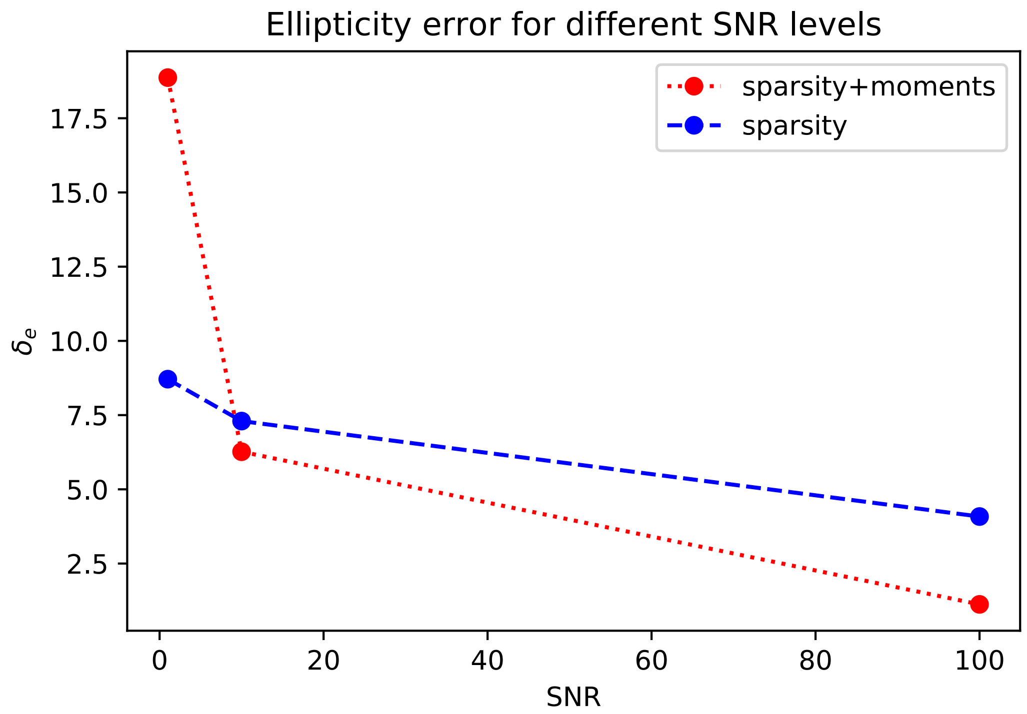
Experiment: Deconvolution



Deconvolution : GALSIM galaxies + MeerKat PSF







1024 GALSIM galaxies + Radio PSF (MeerKat, CASA)

Conclusions

- ✓ **Weak Lensing and Shape Measurements**
  - ➡ Serious mathematical challenges to well control systematics.
  - ➡ Need the best methods.

- ✓ **Euclid PSF: RCA** is new method which uses all available PSFs to derive the PSF field.
  - ➡ Use Optimal Transport, Graph Laplacian and Sparsity.

- ✓ **Moments Constraint:**
  - ➡ A new approach to preserve the ellipticity information during the restoration process.
  - ➡ Shape constraints using shearlets outperform the standard Gaussian windowing.

- ✓ **Perspectives:**
  - ➡ **Radio:** Is this shape constraint reconstruction method competitive with galaxy model fitting in Fourier space ?
  - ➡ **Optical:** Compare the constraint denoising method to the Gaussian windowing.