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LOCALLY REFINED SPLINES FOR COMPACT REPRESENTATION AND ANALYSES OF GEOSPATIAL BIG DATA

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Background

- SINTEF has addressed research on and industrial uses of polynomial splines for four decades in cooperation with the University of Oslo
- In the start the focus was Computer Aided Design. Now applications are within Big Data representation and Analysis, Artificial Intelligence, Additive Manufacturing (Representation, design and simulation)
- 2012- 2016 fp7 IP IQmulus targeting big geospatial data (Lidar, sonar,…).
  - SINTEF focus on compact representation and analysis of sea bottom data
- 2017-2021 IKTPluss ANALYST (Norwegian AI and big data project)
  - Hydrographic office of the Norwegian Mapping Authorities as partner.

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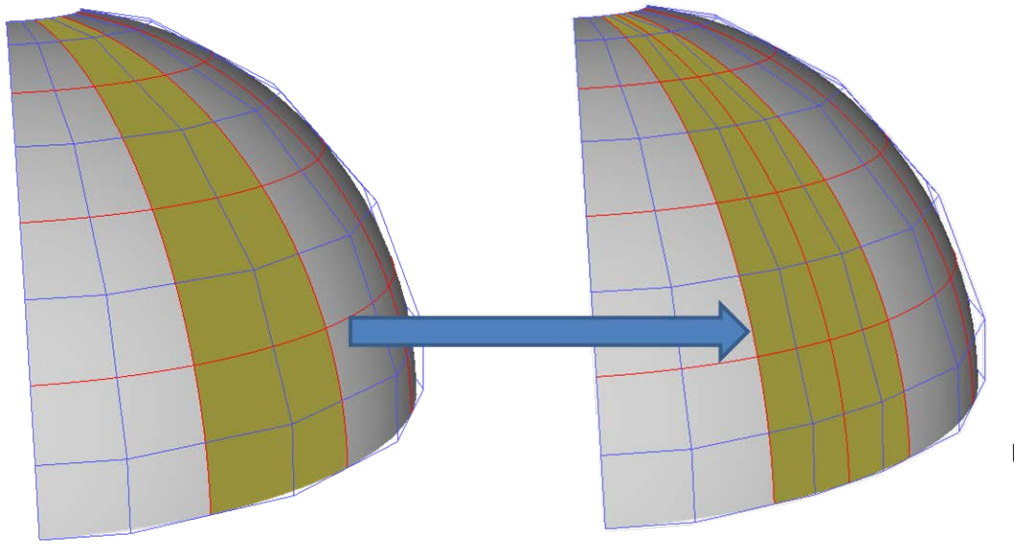
Polynomial splines

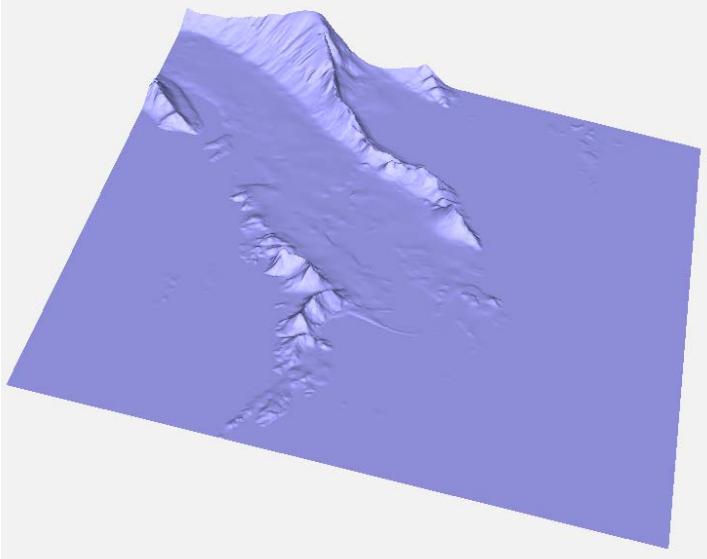
- A freeform curve (e.g., contour curve in a map) can be represented as a sequence of polynomial segments (degree 1, 2, 3 or higher) where adjacent segments meet with a specified continuity
- A sculptured surface can be represented as a patchwork of bi-variate polynomial patches where adjacent patches meet a specified continuity
- For smooth curves and surfaces piecewise polynomial representation (splines) are much more compact than linear pieces (chains of straight lines or triangulations)
- B-splines are regarded as well suited for the representation of polynomial splines

Tensor product B-splines surfaces

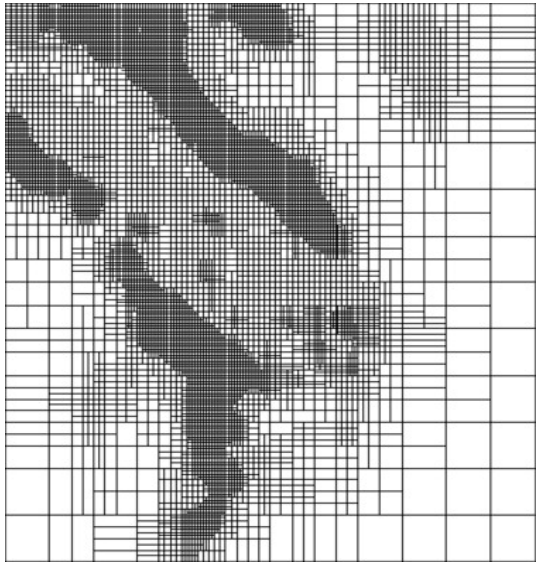
- Traditionally B-splines surfaces have been represented using spline spaces that are a tensor product of two univariate spline spaces
- The resulting parameter domain will have a regular grid structure
- Refinement (adding new degrees of freedom) will have effect across the domain



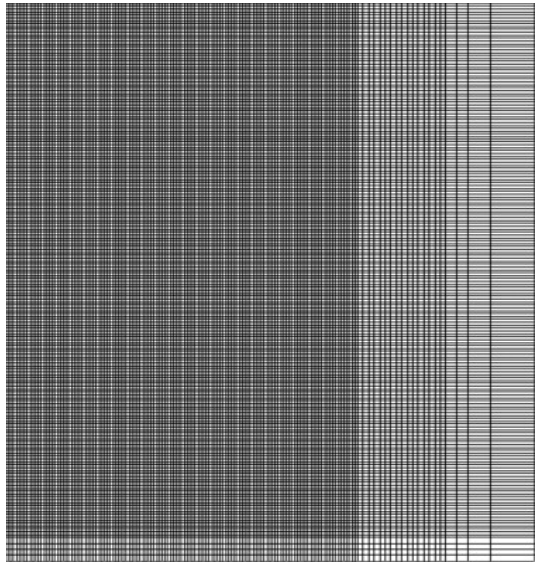
Locally refined spline surfaces – add degrees of freedom where needed



Surface



Polynomial patches, locally refined spline surface



Polynomial patches, tensor-product spline surface version

LR B-spline refinement

Please click the link below to view the video of
K.A. Johannessen, SINTEF Digital
https://youtu.be/vFyXs-72qYY

- At the start of the video we have a bi-variate tensor product B-spline space of bi-degree (2,2)
- Each yellow button represent the vertex (coefficient) of a B-spline. (At the start a regular grid)
- Successive knotline segments are inserted, splitting B-splines and adding new vertices.
- Note: The right continuity over T-joints is coded into the B-splines.

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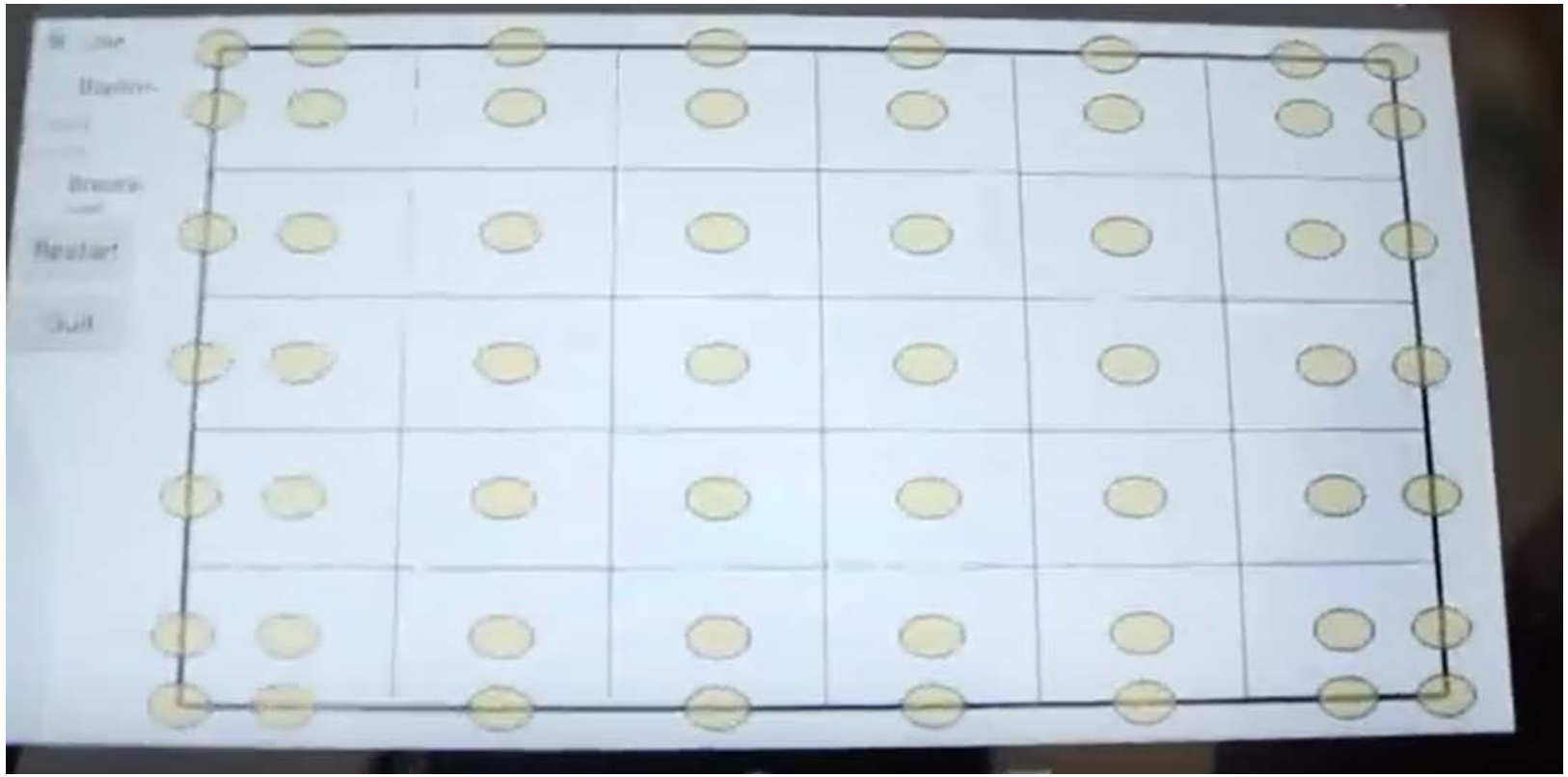
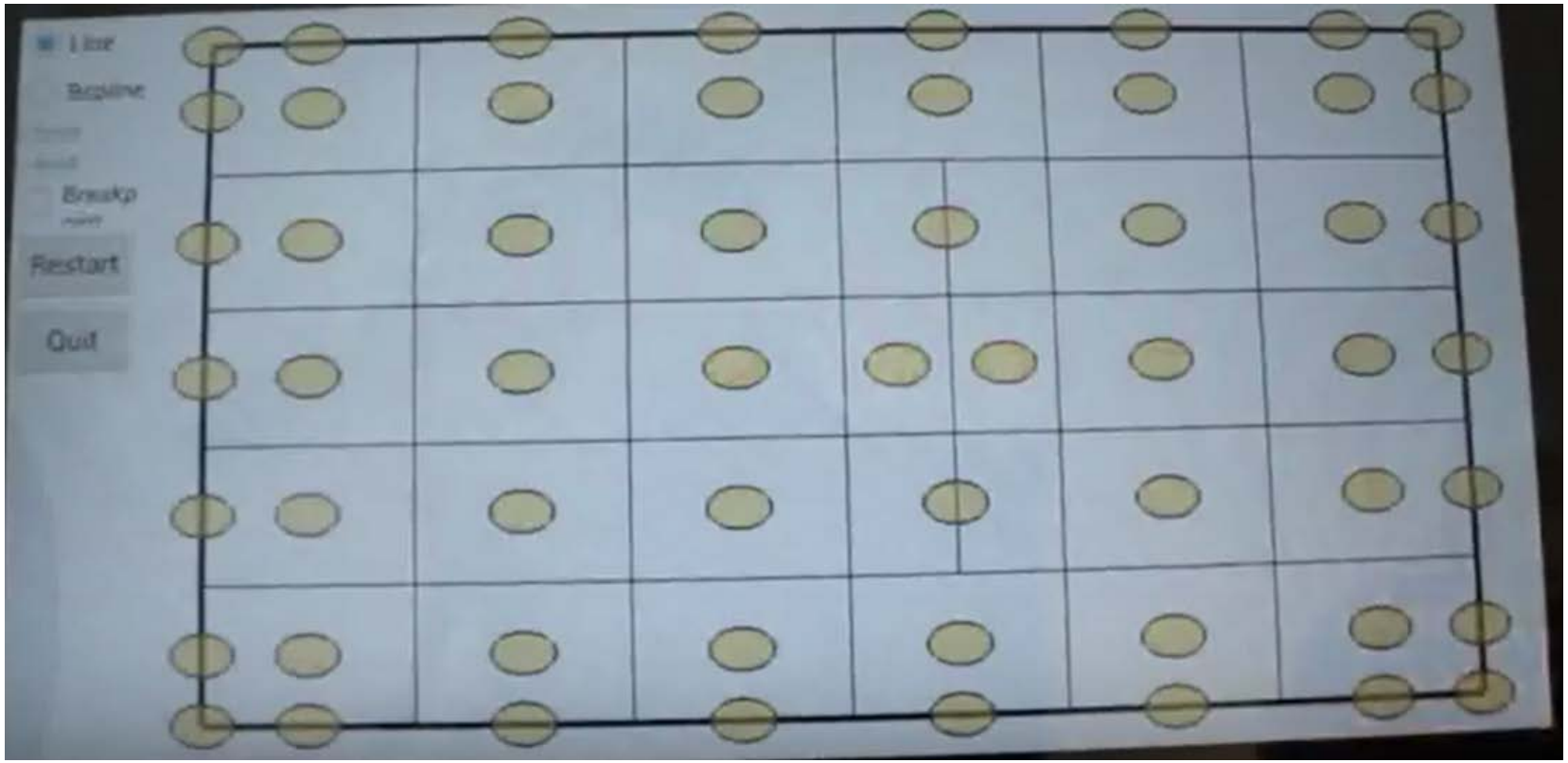
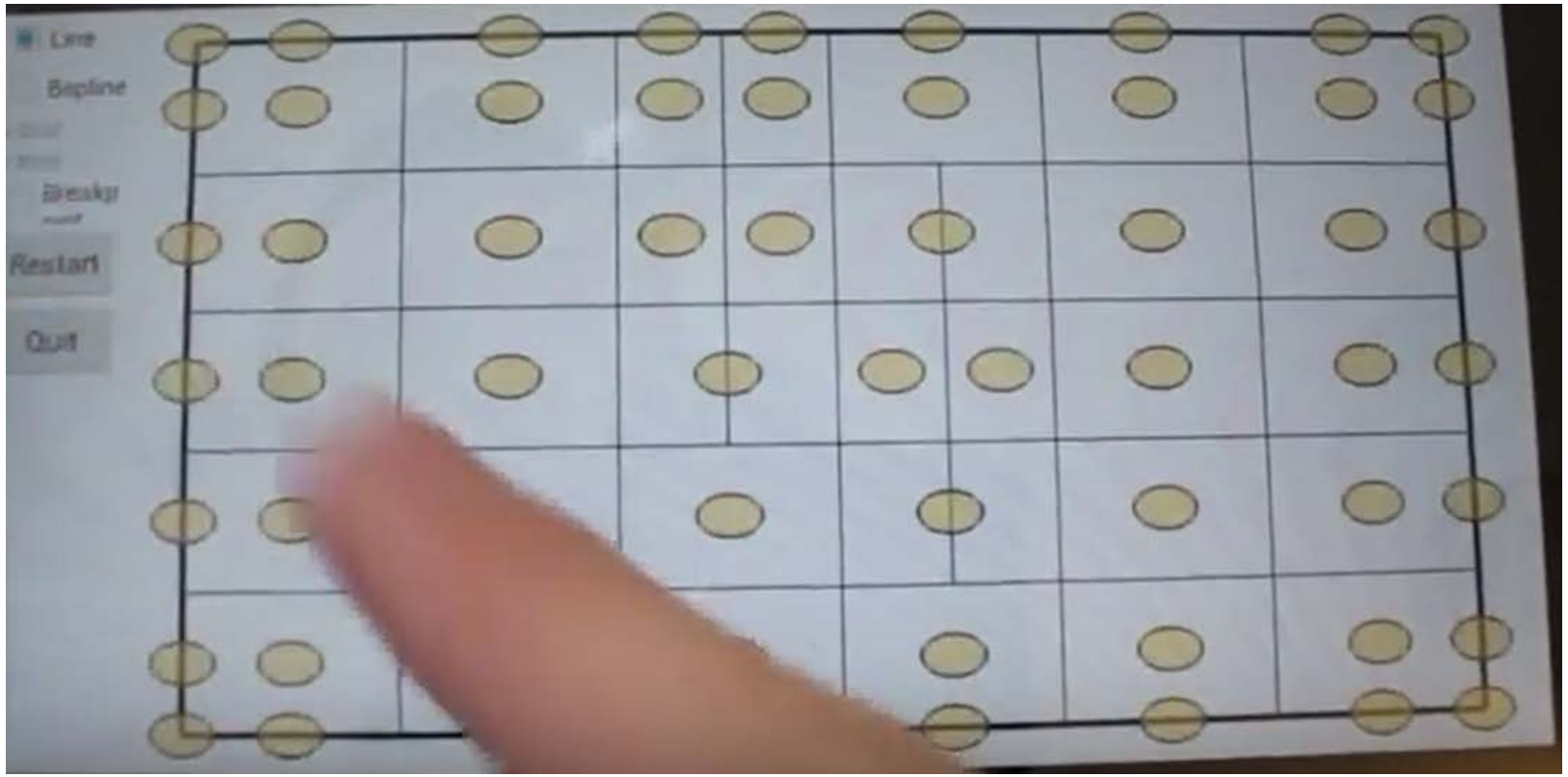
Starting from tensor product spline space of bi-degree (2,2) – open knots



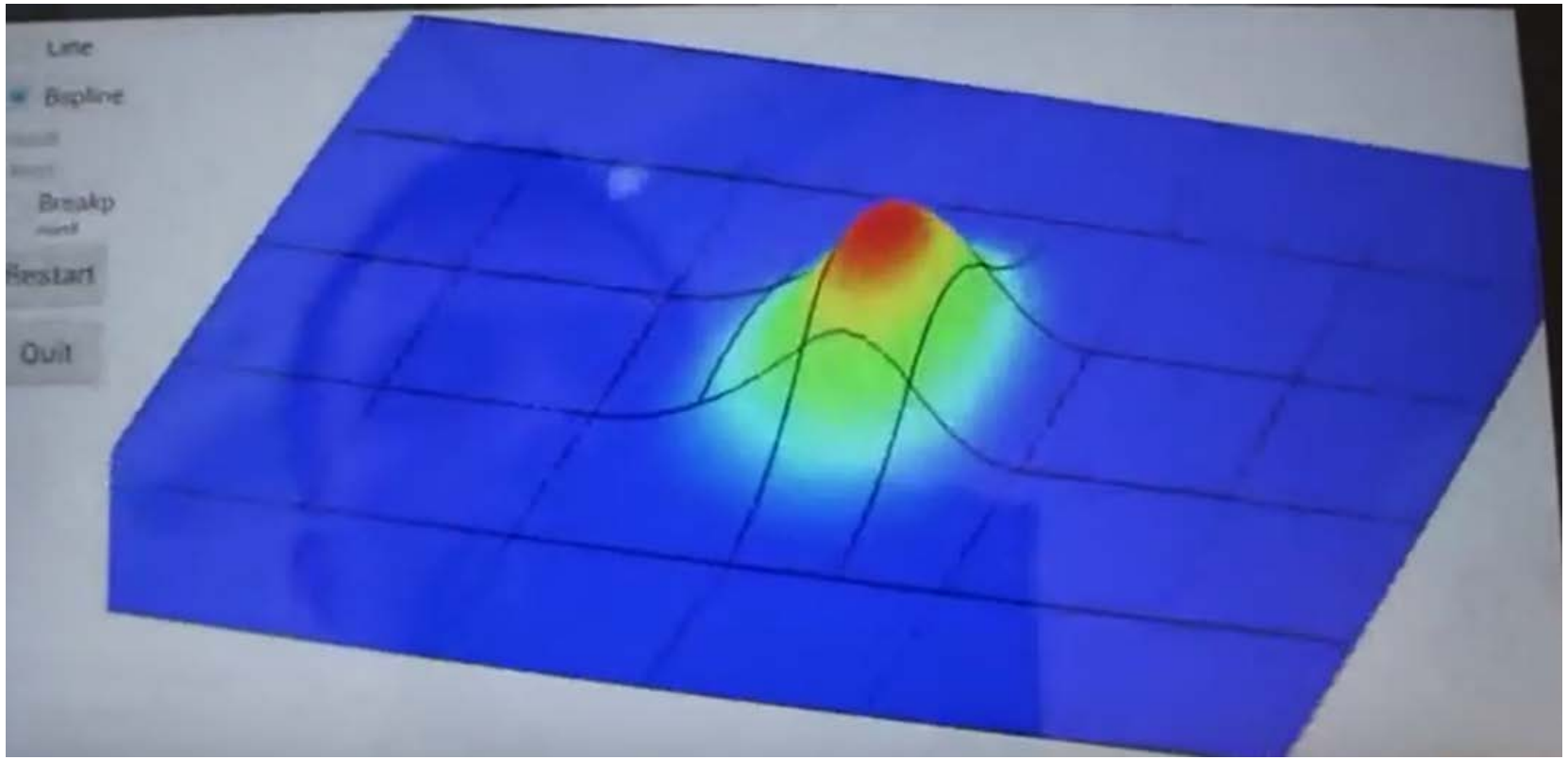
Split one B-spline – increase by one degree of freedom



Split from boundary - insert three degrees of freedom



Example of B-spline



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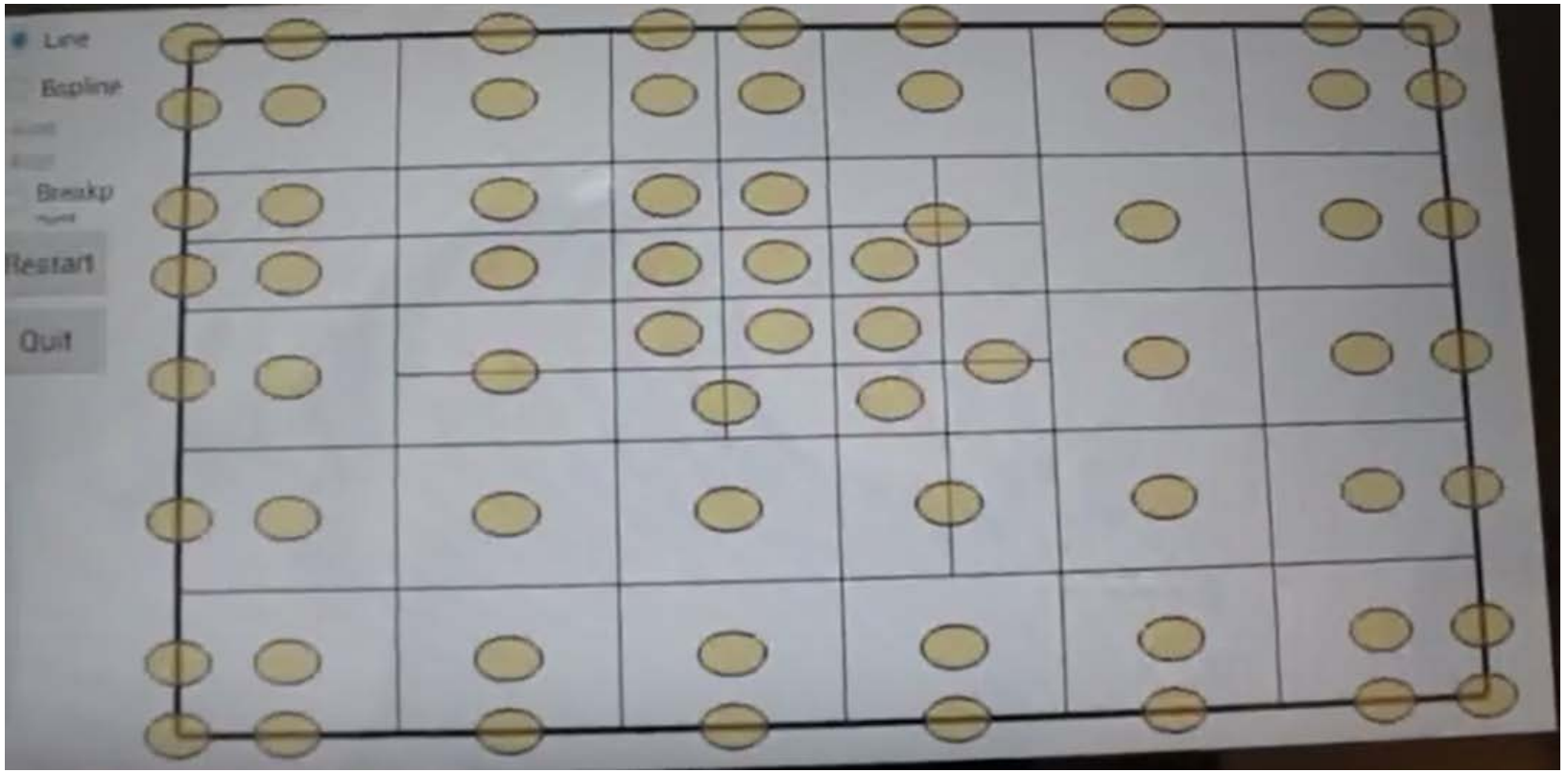
Additional refinements



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Iterative algorithm for LR B-spline surface approximation

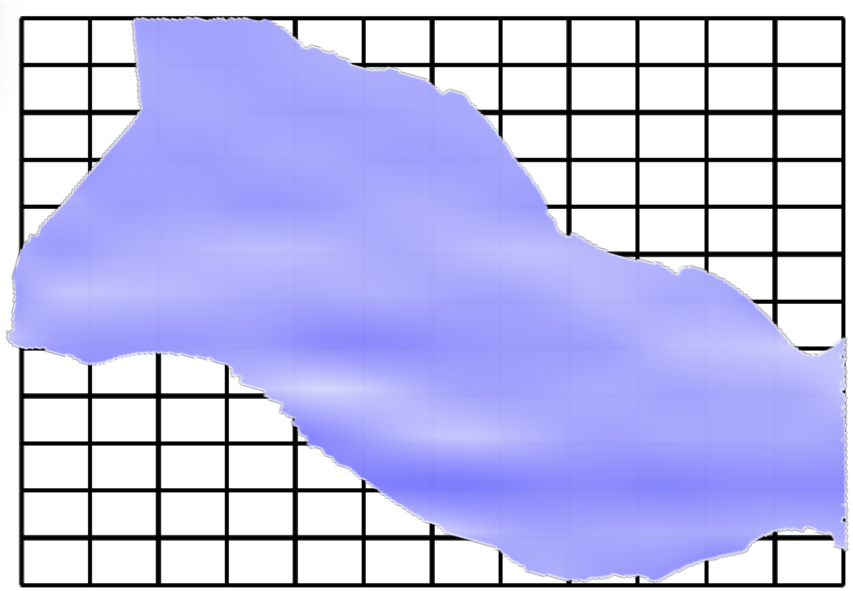
- **Input:** point cloud, threshold, maximum number of iterations
- **Algorithm:**
  - Make a lean approximation on a regular grid
  - Compute distances between points and surface approximation
  - While (max err > threshold AND number of iterations < max number)
    - Refine the surface in regions where the error is above threshold
    - Compute approximation given the current degrees of freedom
- **Output:** LR B-spline surface, accuracy information

The examples are based on point clouds acquired by single- and multi-beam sonar

- The point clouds consequently have a striped structure
- Multiple point clouds have been registered into the same coordinate system
- We have experienced that making smooth surface approximations of geospatial data highlight features, outliers and registration problems.

Examples from IQmulus fp7 IP British Channel

- Work performed together with HR Wallingford
- Data courtesy HR Wallingford, SeaZone
  - 14.6 Million points (280 Mbyte)
  - Tolerance 0.5 m
  - 6 levels of refinement
  - Starting from a tensor product B-spline space that is trimmed to only address the areas that are covered by points.
- Bi-degree (2,2) LR B-spline approximation

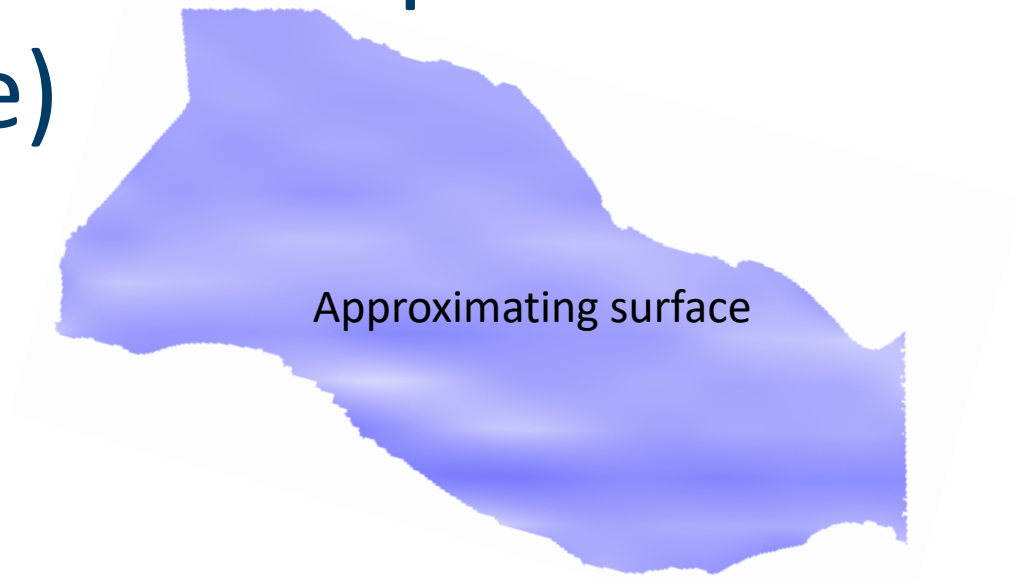


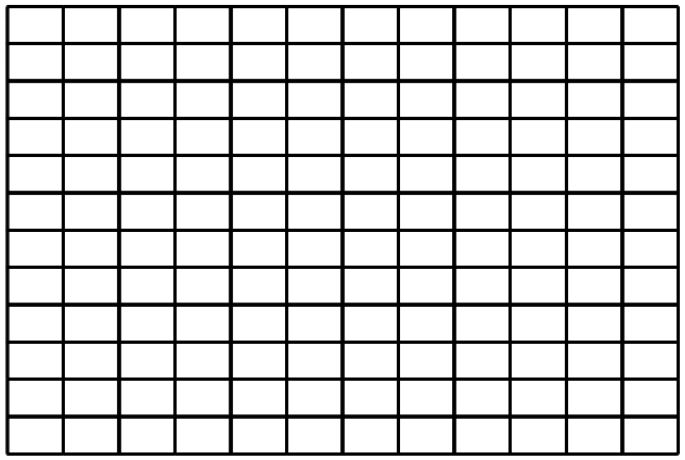
Initial approximation of 14.6 mill points (280 Mbyte)

Number of points	14.6 mill
No. of coefs.	196
Surface file size	26 KB
Max. dist	12.8 m.
Average dist	1.42 m.
# points, dist > 0.5 m	9.9 mill

Data courtesy HR Wallingford, SeaZone

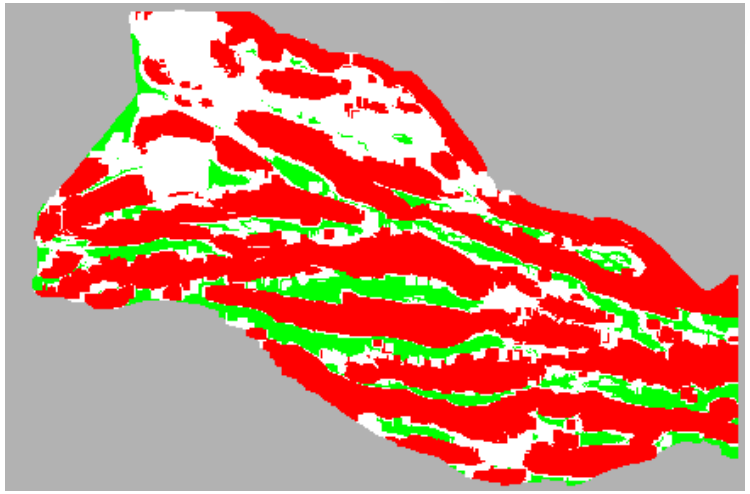
Elevation interval: ~50 m.





Polynomial patches in the parameter domain of the surface (bi-quadratic)

Green points at least 0.5m below surface
White points within 0.5 m of surface
Red points at least 0.5 m above surface

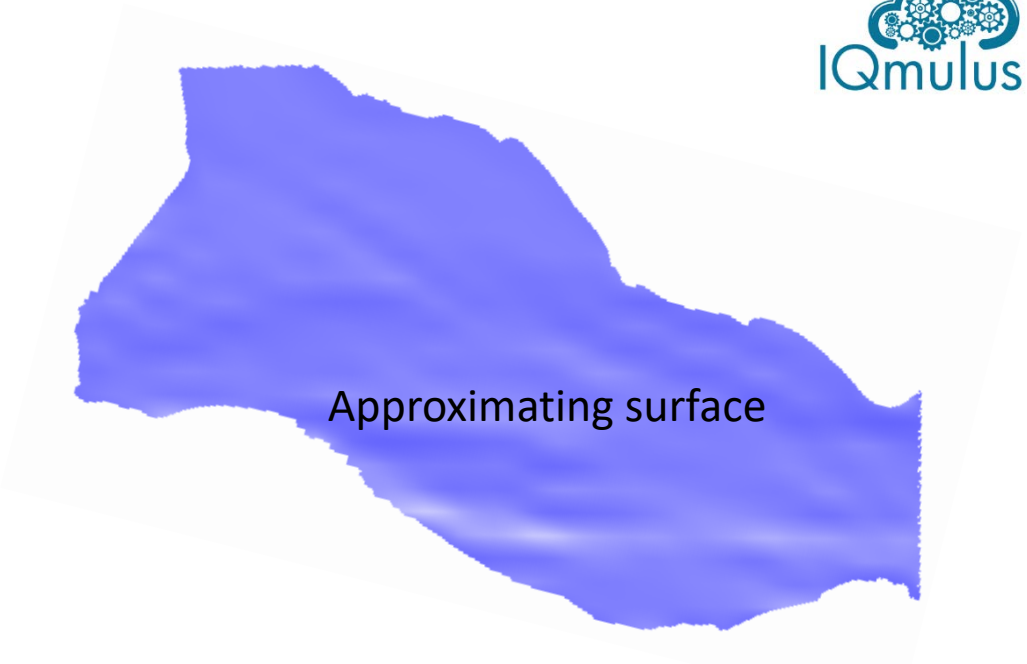


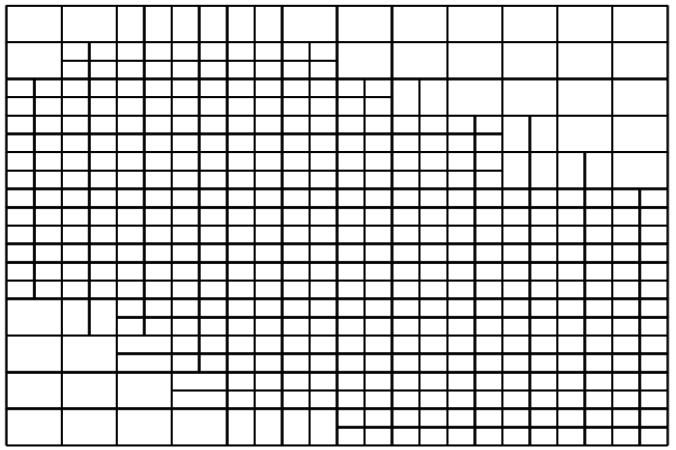
First iteration

Number of points	14.6 mill
No. of coefs.	507
Surface file size	46 KB
Max. dist	10.5 m.
Average dist	0.83 m.
# points, dist > 0.5 m	7.3 mill

Data courtesy HR Wallingford, SeaZone

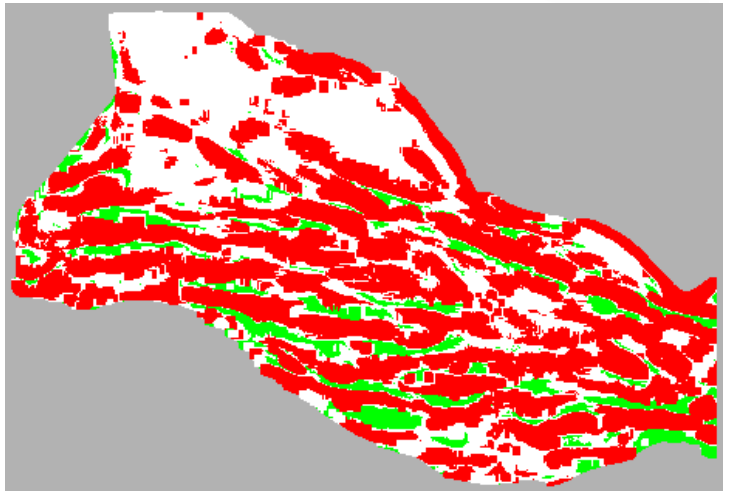
Elevation interval: ~50 m.





Polynomial patches in the parameter domain of the surface

Green points at least 0.5m below surface
White points within 0.5 m of surface
Red points at least 0.5 m above surface

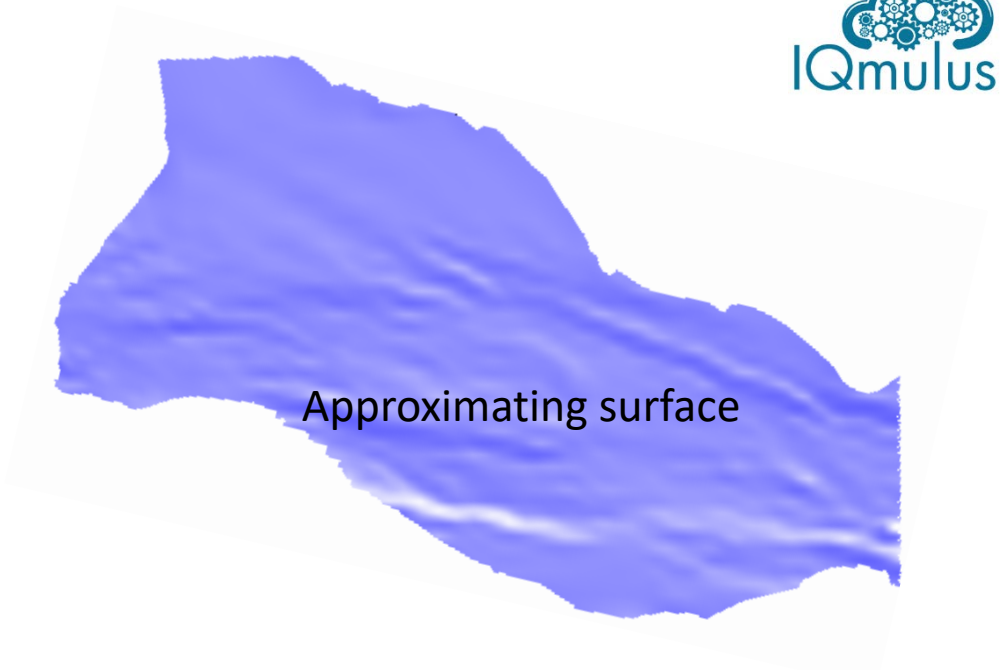


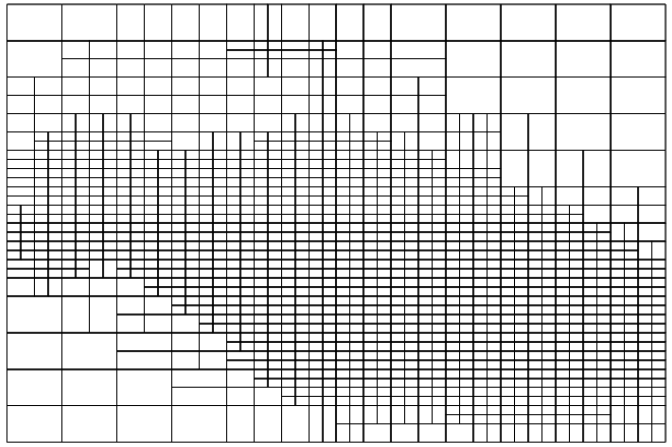
Second iteration

Number of points	14.6 mill
No. of coefs.	1336
Surface file size	99 KB
Max. dist	8.13 m.
Average dist	0.41 m.
# points, dist > 0.5 m	3.9 mill

Data courtesy HR Wallingford, SeaZone

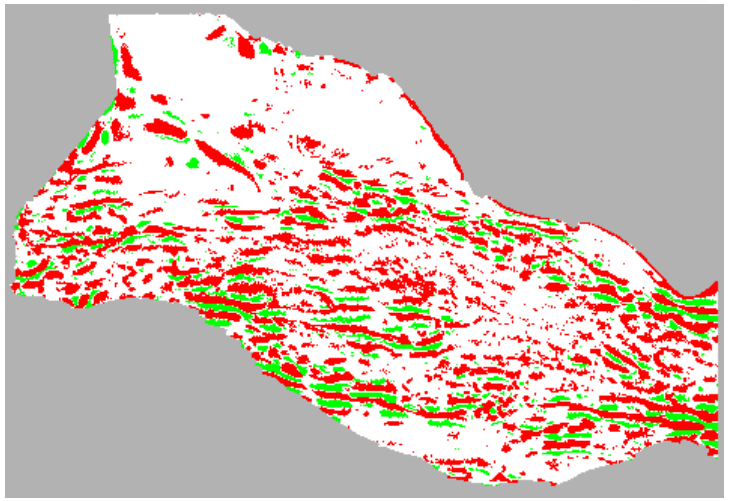
Elevation interval: ~50 m.





Polynomial patches in the parameter domain of the surface

Green points at least 0.5m below surface
White points within 0.5 m of surface
Red points at least 0.5 m above surface

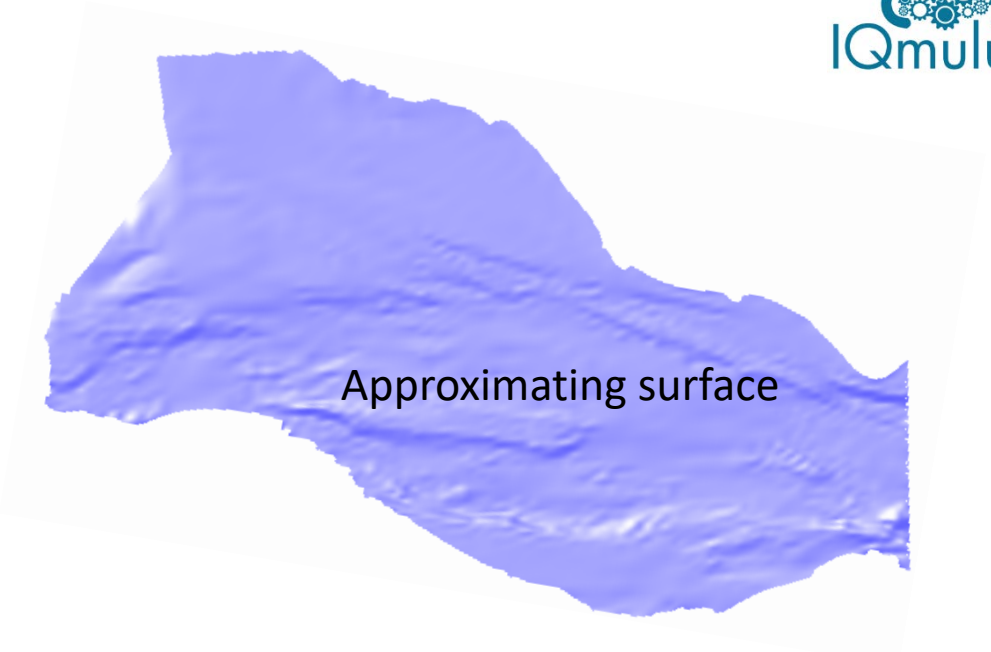


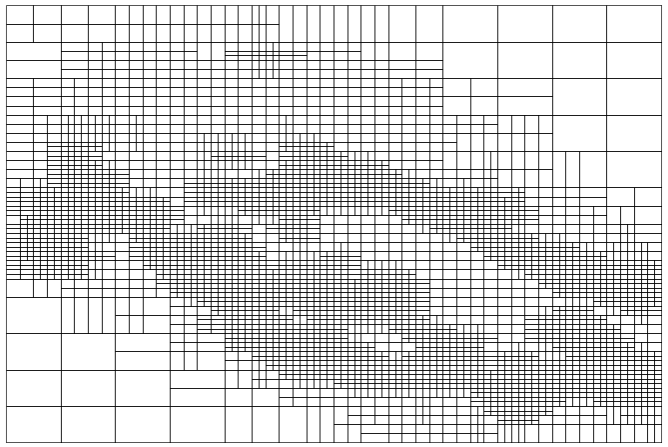
Third iteration

Number of points	14.6 mill
No. of coefs.	3563
Surface file size	241 KB
Max. dist	6.1 m.
Average dist	0.22 m.
# points, dist > 0.5 m	1.4 mill

Data courtesy HR Wallingford, SeaZone

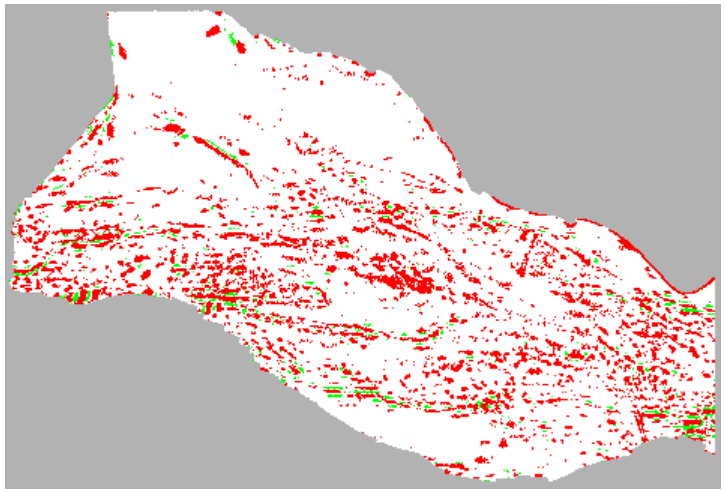
Elevation interval: ~50 m.





Polynomial patches in the parameter domain of the surface

Green points at least 0.5m below surface
White points within 0.5 m of surface
Red points at least 0.5 m above surface

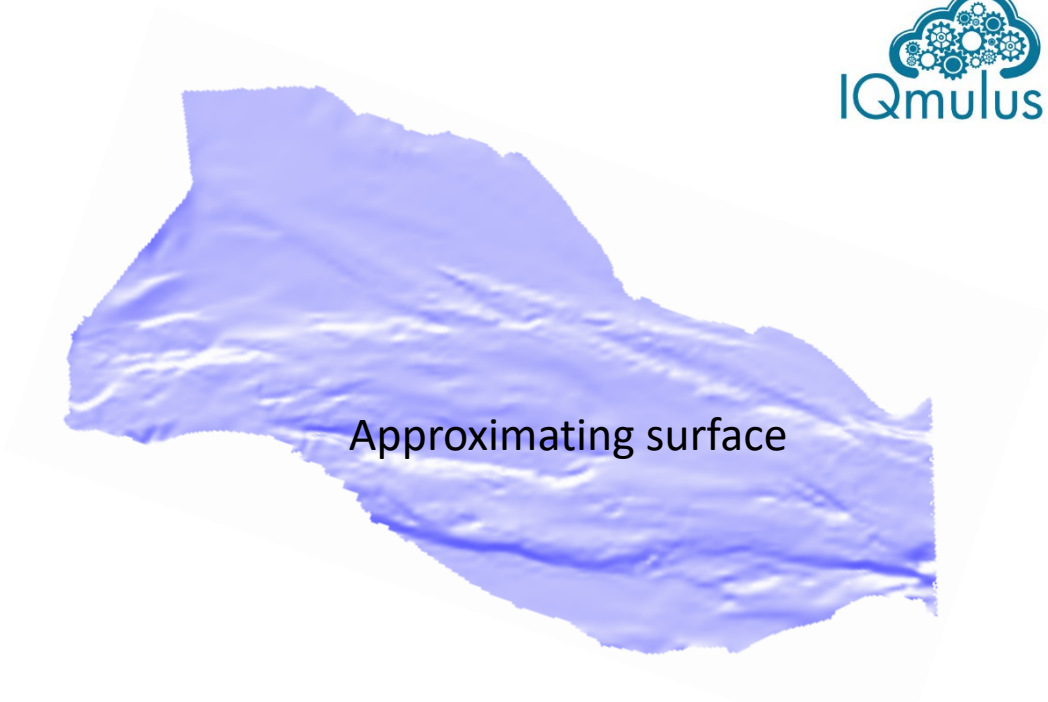


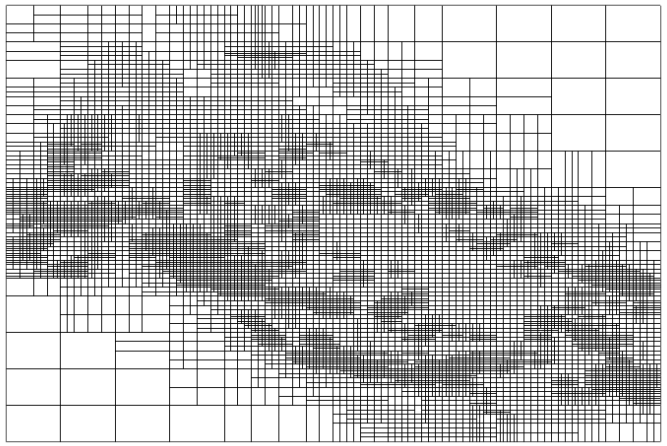
Fourth iteration

Number of points	14.6 mill
No. of coefs.	9273
Surface file size	630 KB
Max. dist	6.0 m.
Average dist	0.17 m.
# points, dist > 0.5 m	0.68 mill

Data courtesy HR Wallingford, SeaZone

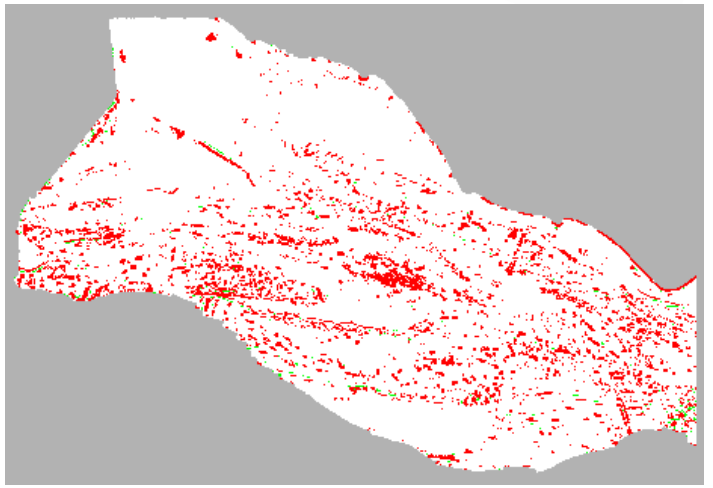
Elevation interval: ~50 m.





Polynomial patches in the parameter domain of the surface

Green points at least 0.5m below surface
White points within 0.5 m of surface
Red points at least 0.5 m above surface

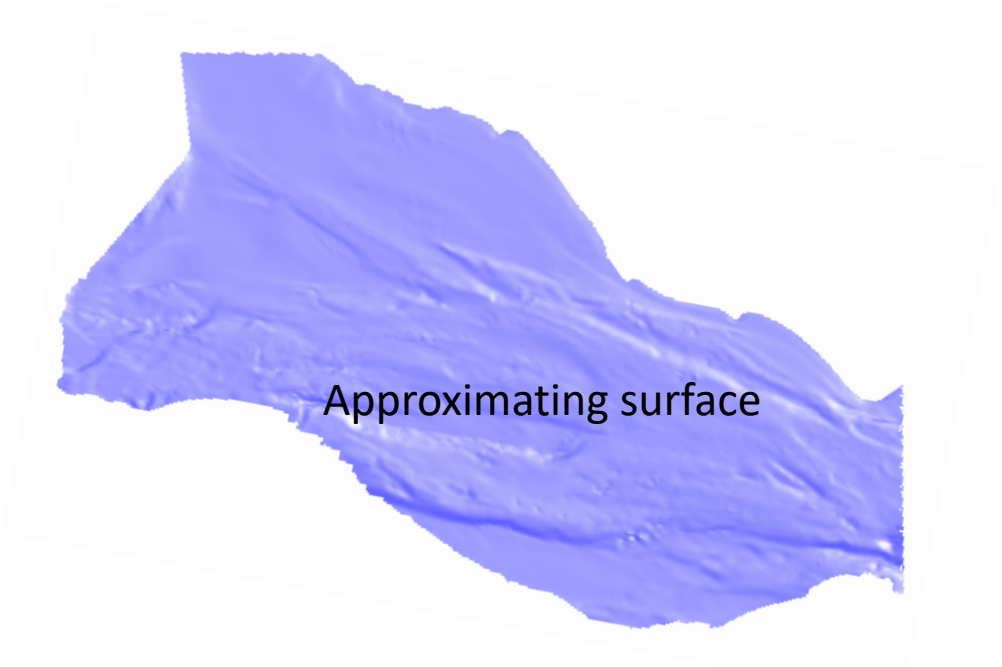


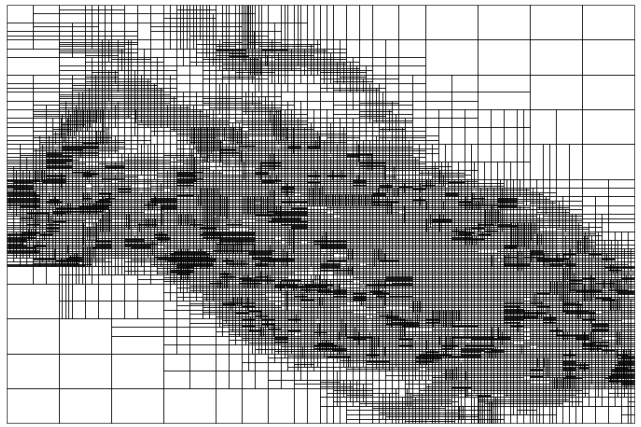
Fifth iteration

Number of points	14.6 mill
No. of coefs.	23002
Surface file size	1.6 MB
Max. dist	5.3 m.
Average dist	0.12 m.
# points, dist > 0.5 m	244 850

Data courtesy HR Wallingford, SeaZone

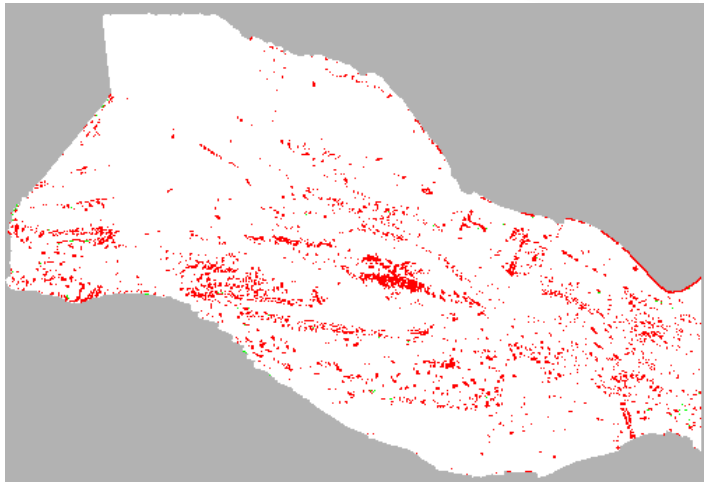
Elevation interval: ~50 m.





Polynomial patches in the parameter domain of the surface

Green points at least 0.5m below surface
White points within 0.5 m of surface
Red points at least 0.5 m above surface

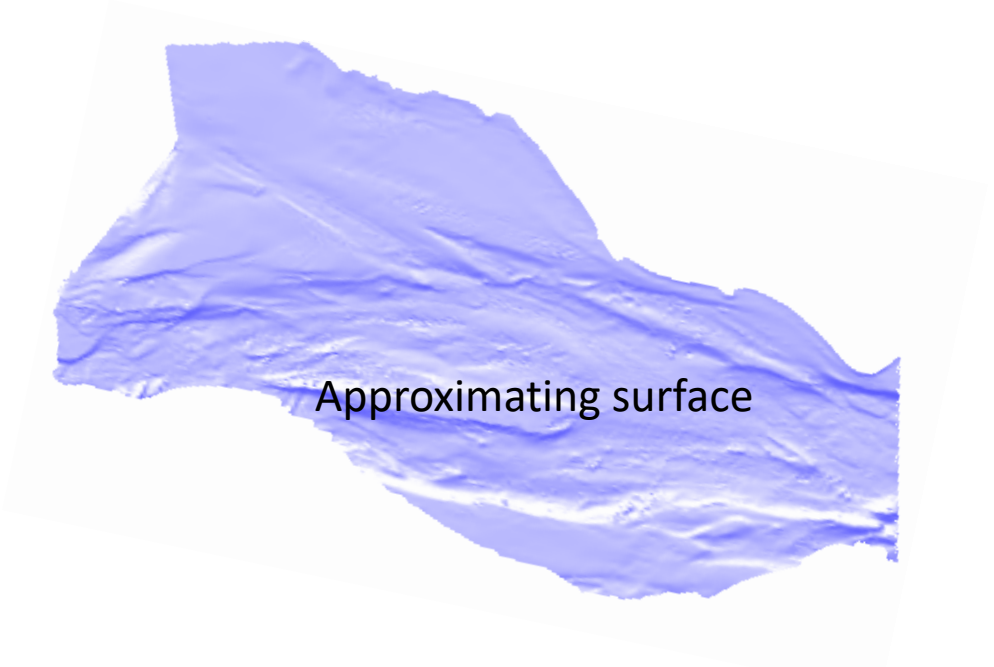


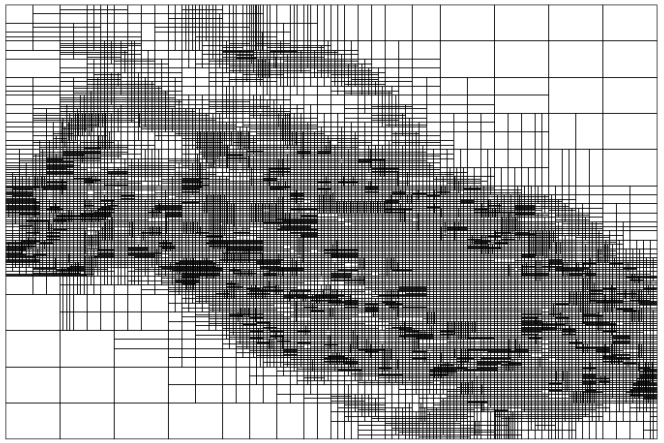
Sixth iteration

Number of points	14.6 mill
No. of coefs.	52595
Surface file size	3.7 MB
Max. dist	5.4 m.
Average dist	0.09 m.
# points, dist > 0.5 m	75 832

Data courtesy HR Wallingford, SeaZone

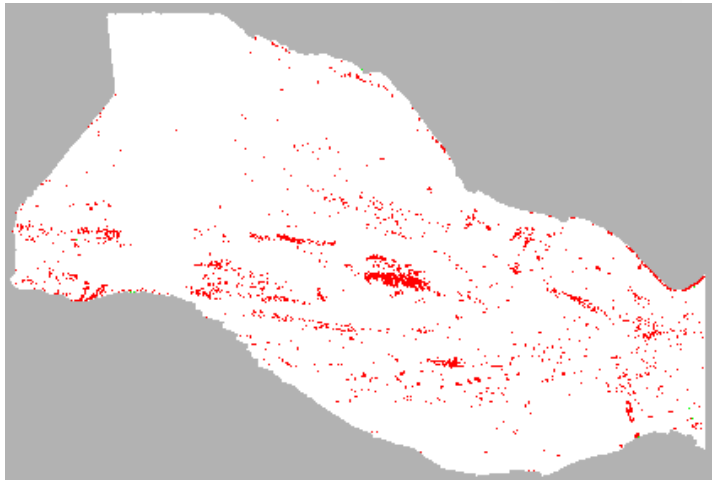
Elevation interval: ~50 m.





Polynomial patches in the parameter domain of the surface

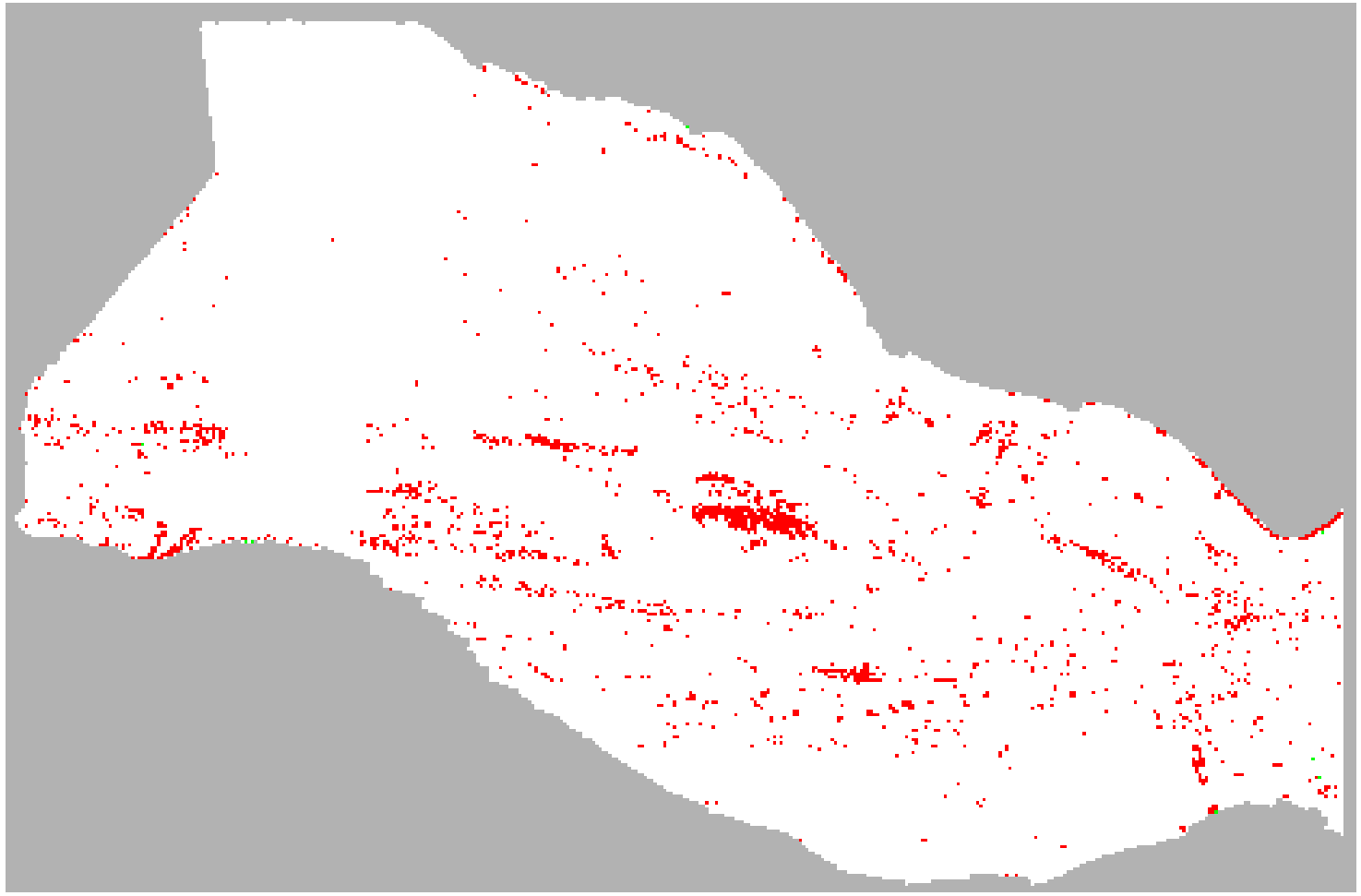
Green points at least 0.5m below surface
White points within 0.5 m of surface
Red points at least 0.5 m above surface



Running through the iterations

Data courtesy HR Wallingford, SeaZone

- Green points at least 0.5m below surface
- White points within 0.5 m of surface
- Red points at least 0.5 m above surface



196 coefficients
(9.9 mill outside tol.)

507 coefficients
(7.3 mill outside tol.)

1 336 coefficients
(3.9 mill outside tol.)

3 563 coefficients
(1.4 mill outside tol.)

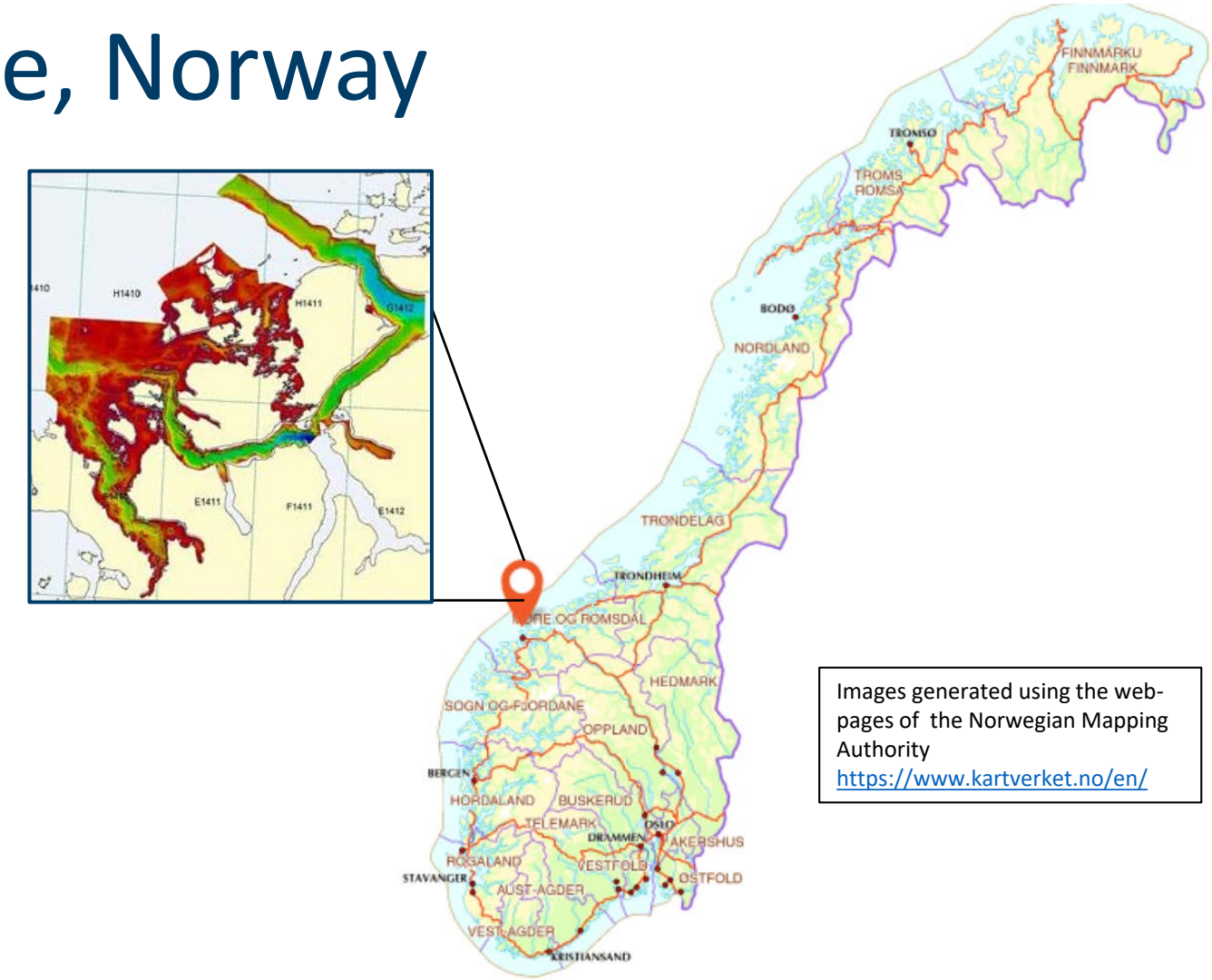
9 273 coefficients
(0.68 mill outside tol.)

23 002 coefficients
(244 850 outside tol.)

52 595 coefficients
(75 832 outside tol.)

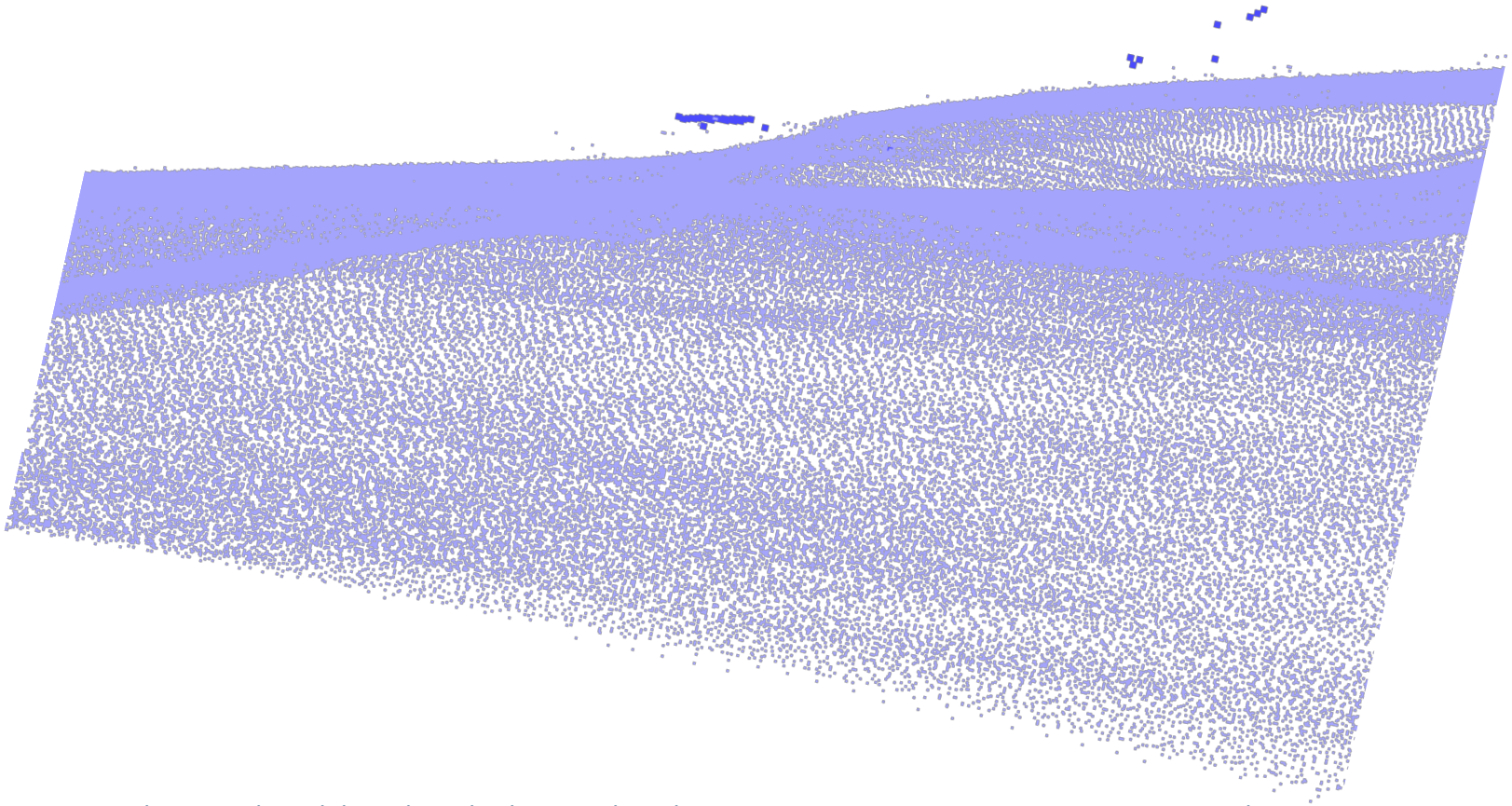
Small example from a large data set from Søre Sunnmøre, Norway

- The Norwegian Hydrographic Service is the authoritative source for nautical publications in Norwegian marine areas, at total of 2.3 million square km.
- Data acquisition rates for bathymetry alone is in the range of multiple petabytes (Pb) per year.



Images generated using the web-pages of the Norwegian Mapping Authority
https://www.kartverket.no/en/

Details from pointset with outliers.
Idea: Approximate smooth part of point set



Example: Extraction of the smooth component

We use a small data set to be able to show visual details

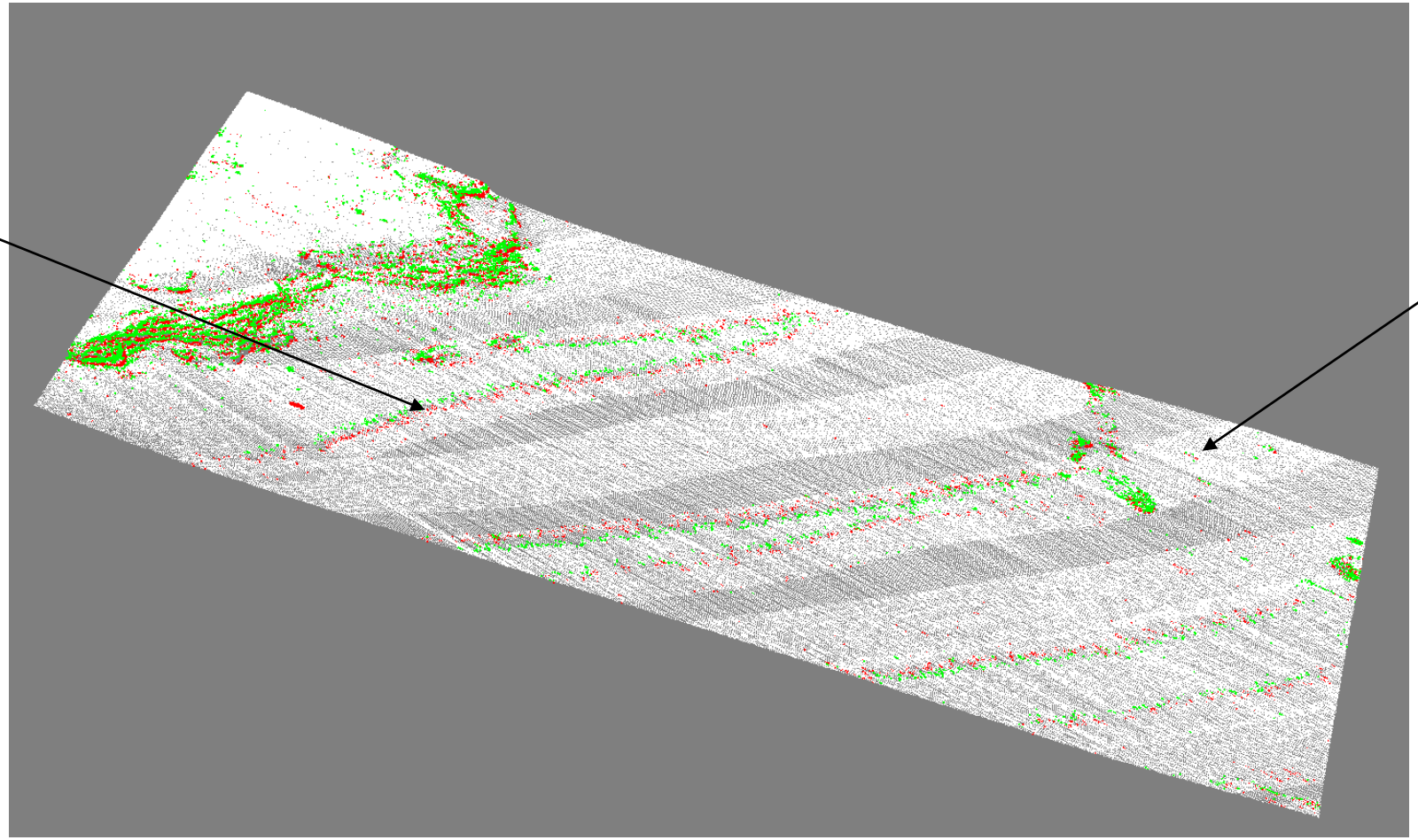
- Data set 641 141 points , file size 26 Mbyte
- LR B-spline approximation on the complete data set: Surface 7.3 Mbyte

Comparing with approximation where non-smooth data points are removed

- Compute rough LR B-spline approximation on the complete data set
- Identify and remove points outside of tolerance, 623 722 points remain
- Calculate the smooth component by using LR B-spline approximation on resulting data set of 25Mbyte giving a LR B-spline surface of 1.7 Mbyte

Total data set. In the reduced (smooth) dataset red and green points are removed, white are kept

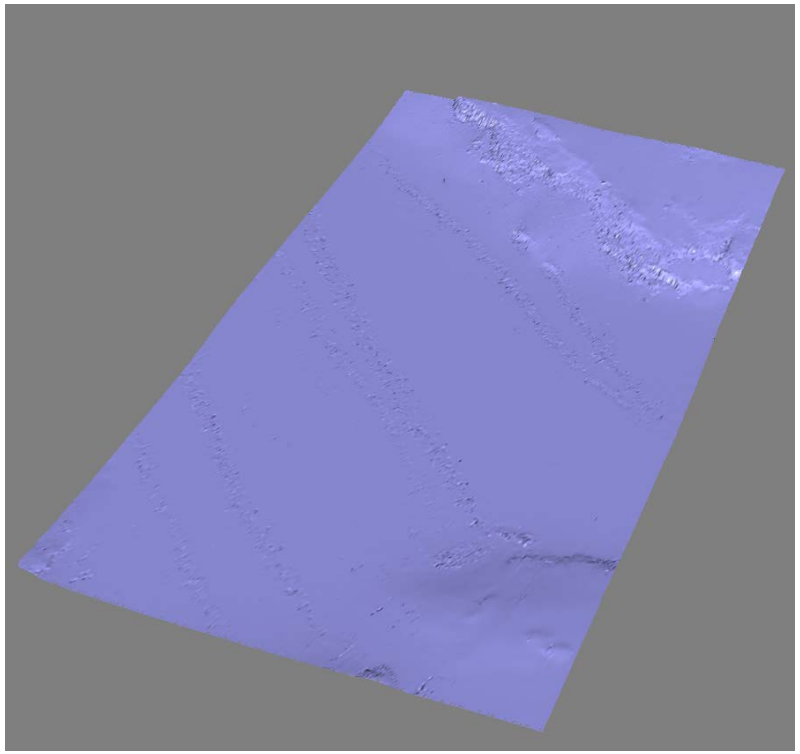
Red/green structure related to variable angular accuracy of sonar?



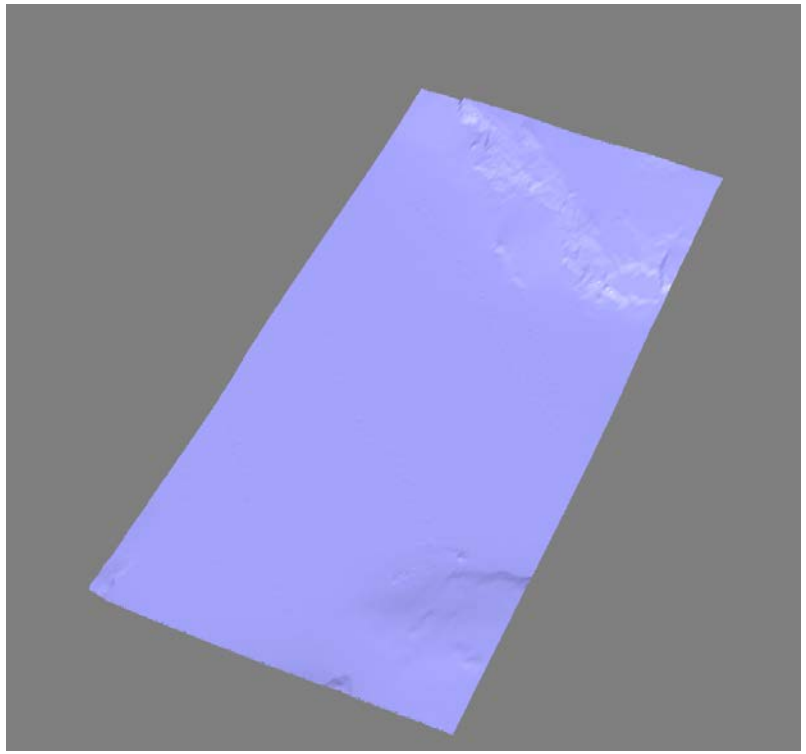
White means higher density of points

Whole point set vs smooth subset: Surface

Approximation of whole point set (26 Mbyte)
Resulting surface 7.3 Mbyte



Approximation of smooth component (25 Mbyte)
Resulting surface 1.7 Mbyte



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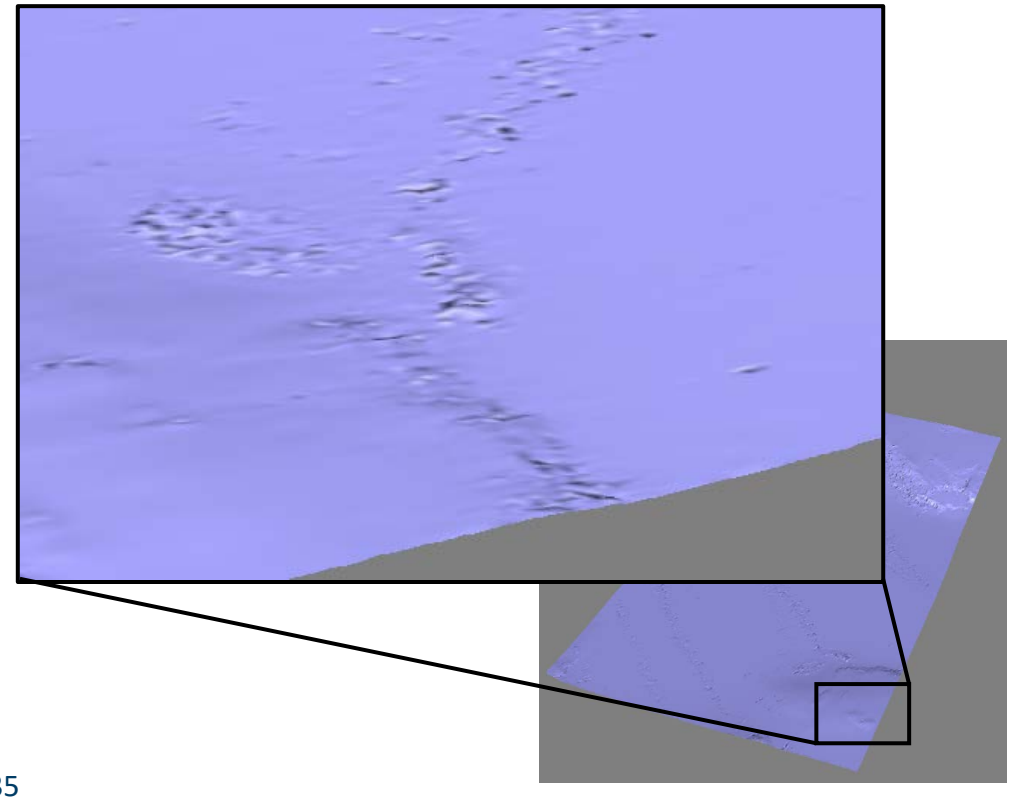
Comparison of surfaces

Approximation of whole data set	
# points approximated	641 141
# coefficients	106 789
(# coefficients)/# (points approximated)	16,66%
# of elements	122 063
Max distance (Note: Large outlier)	23.7m
Average distance	0.08m
Average distance points outside tolerance	0.91m
# Original points outside tolerance	9 350
Surface size	7.3 Mbyte

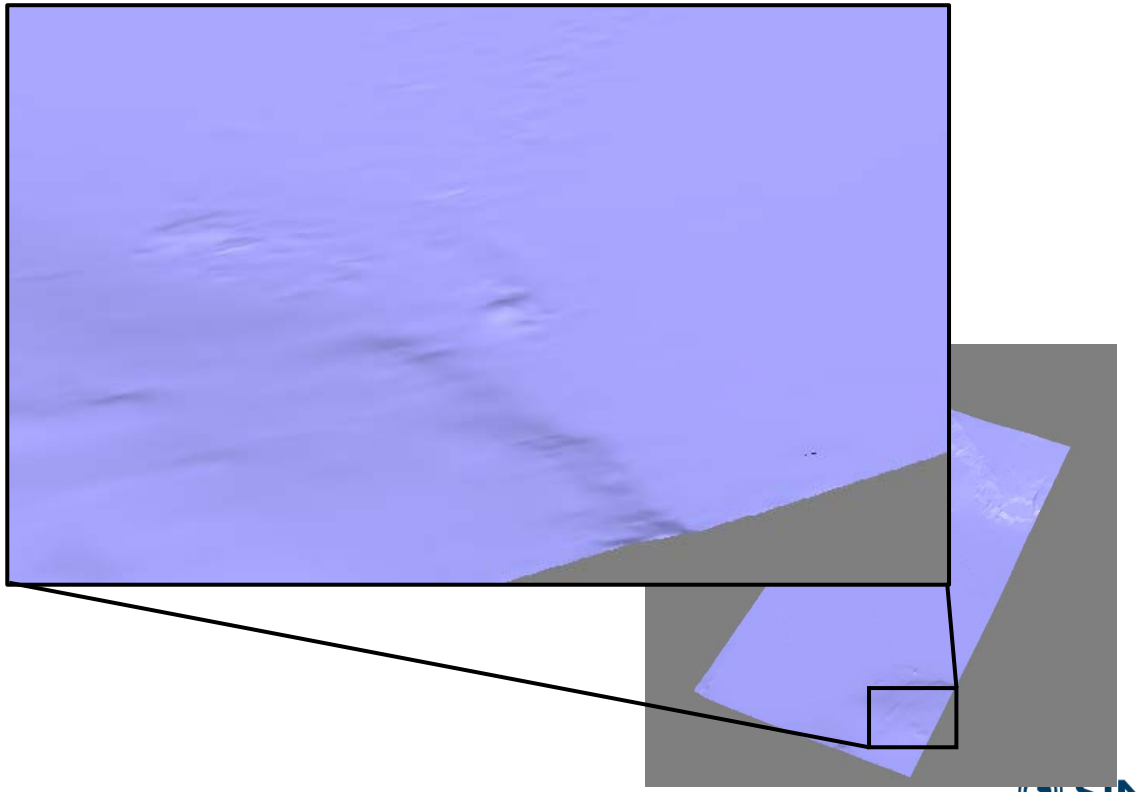
Approximation of smooth subset	
# points approximated	623 722
# coefficients	24 837
(# coefficients)/# (points approximated)	3.98%
# of elements	28 473
Max distance (Note: No large outlier)	0.80m
Average distance	0.068m
Average distance points outside tolerance	0.55m
# Original points outside tolerance	18 002
Surface size	1.7 Mbyte

Whole point set vs smooth subset: Surface

Approximation of whole point set (26 Mbyte)
Resulting surface 7.3 Mbyte

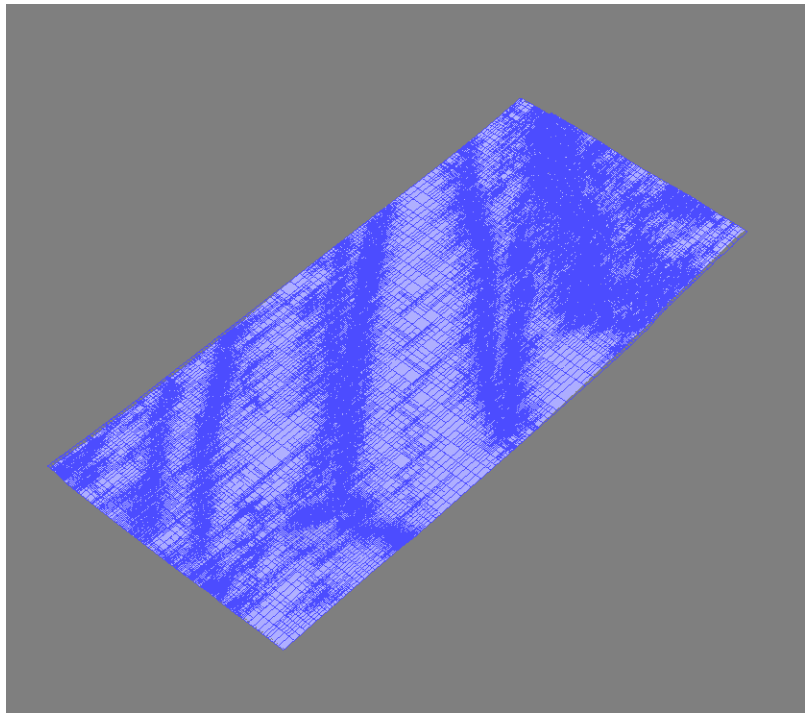


Approximation of smooth component (25 Mbyte)
Resulting surface 1.7 Mbyte

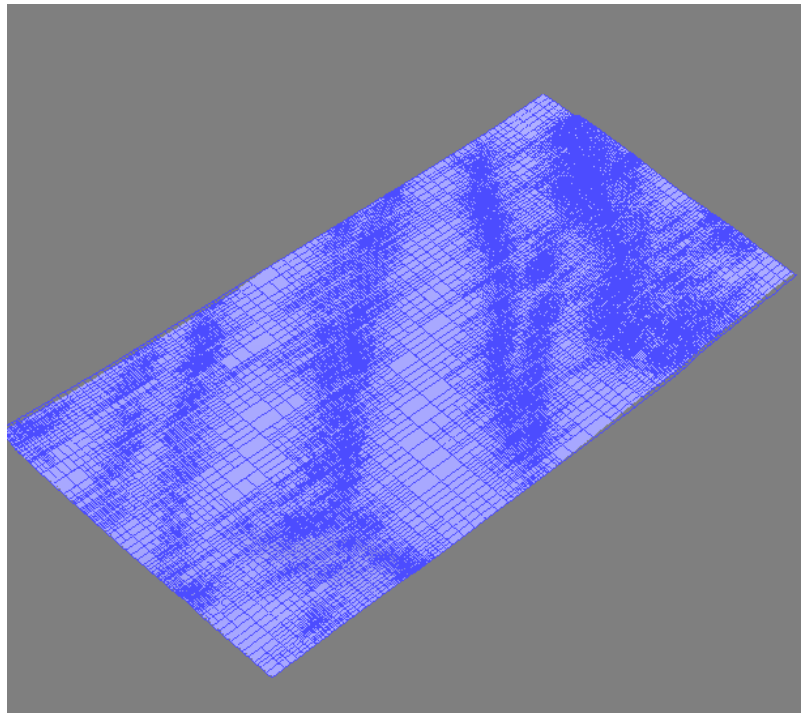


Whole vs smooth subset: Polynomial pieces

Approximation of whole point set (26 Mbyte)
Resulting surface 7.3 Mbyte (122 063 elements)



Approximation of smooth component (25 Mbyte)
Resulting surface 1.7 Mbyte (28 473 elements)



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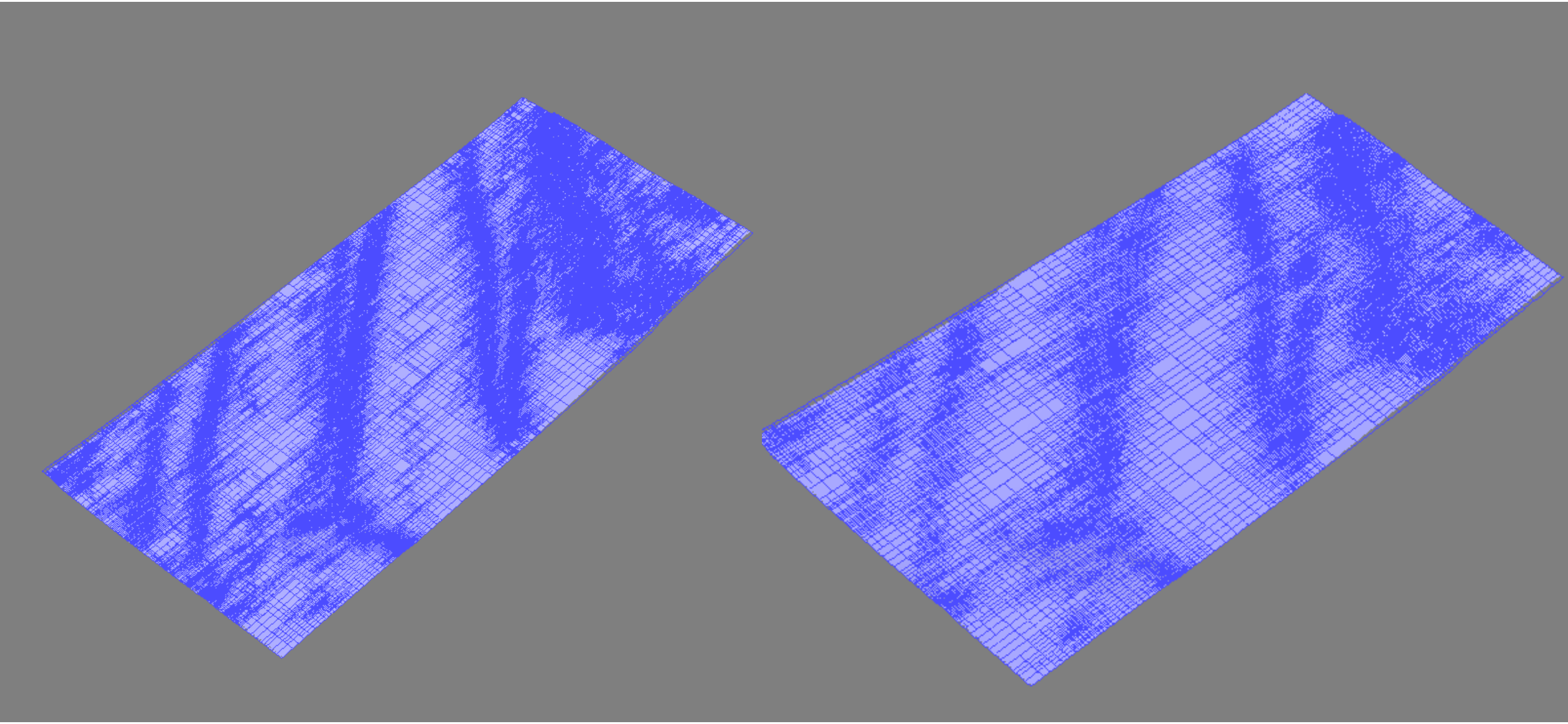
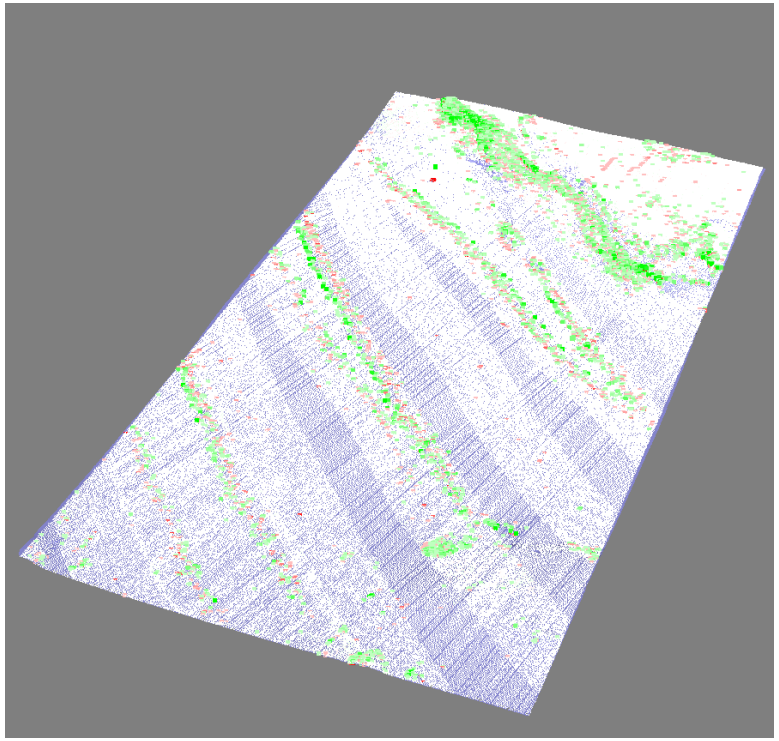
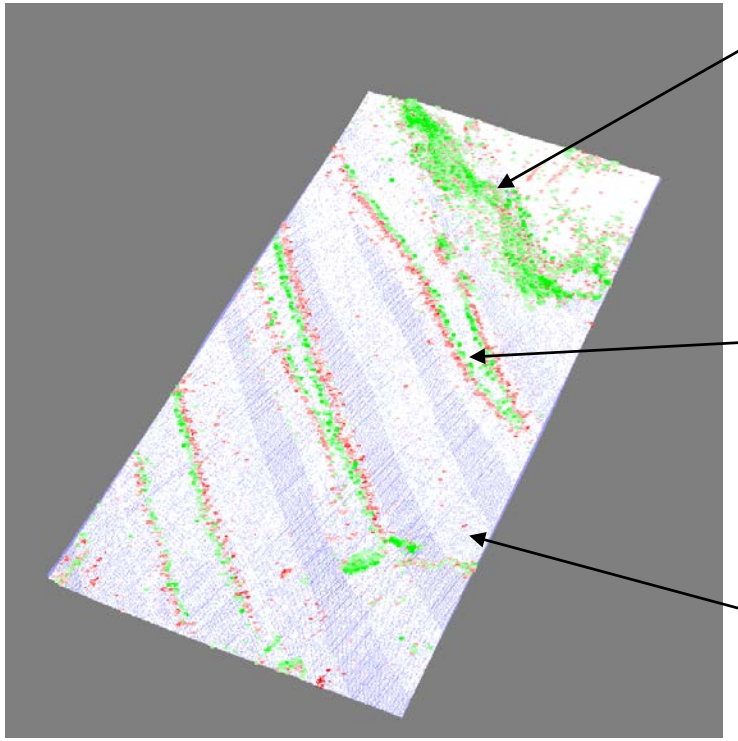


Points outside tolerance
Comparison with complete data set

Approximation of whole point set (26 Mbyte)
Resulting surface 7.3 Mbyte



Approximation of smooth component (25 Mbyte)
Resulting surface 1.7 Mbyte



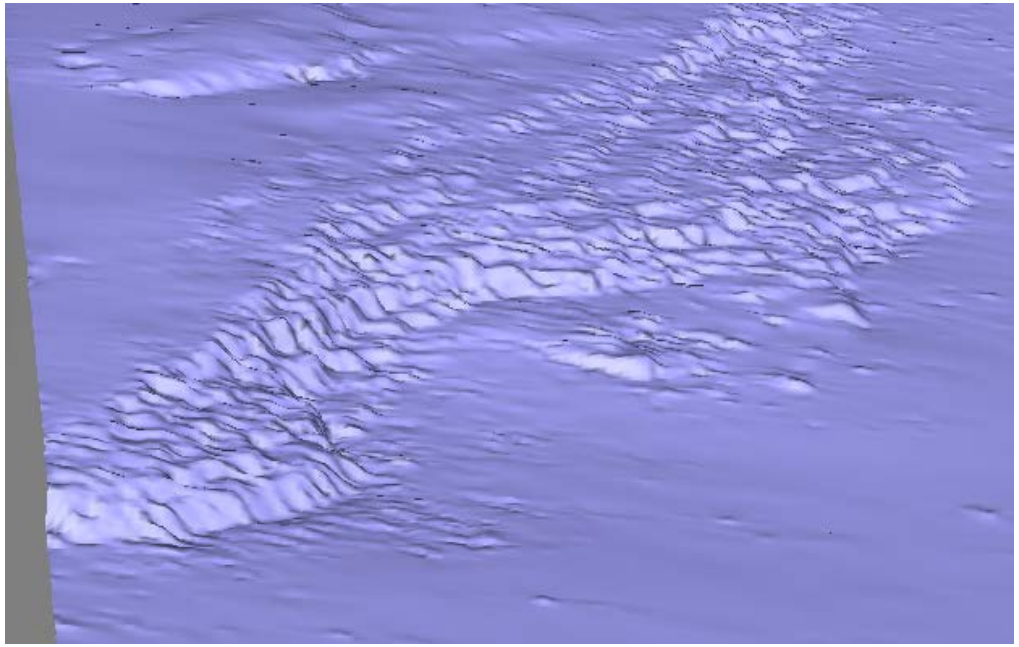
Rocky surface not soft. Better to use parametric surface

Red/green structure related to variable angular accuracy of sonar?

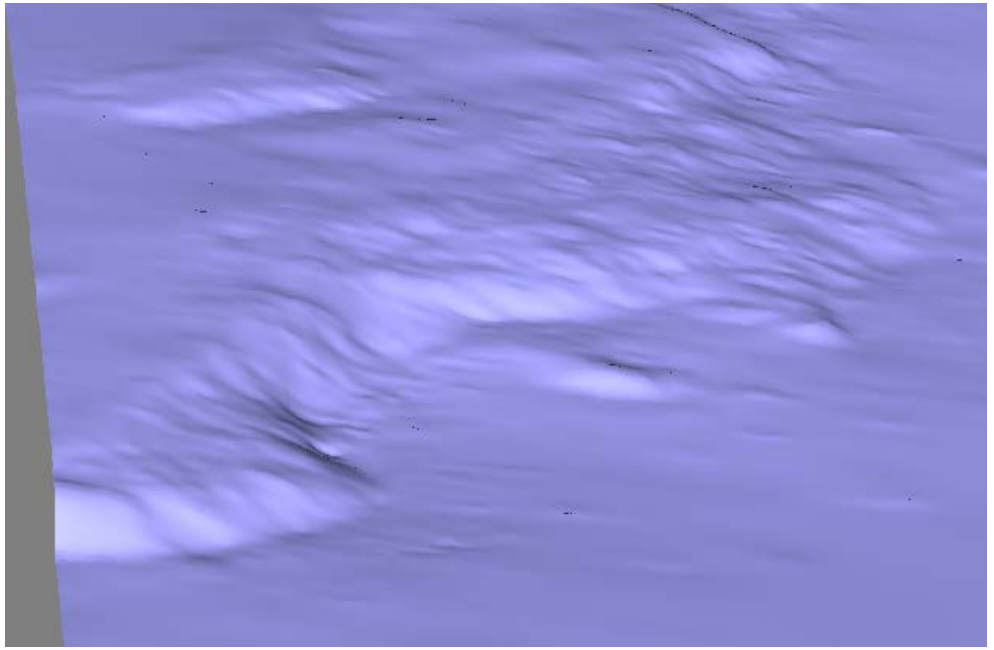
White means higher density of points

Whole vs smooth subset: Rocky portion

Approximation of whole point set (26 Mbyte)
Resulting surface 7.3 Mbyte



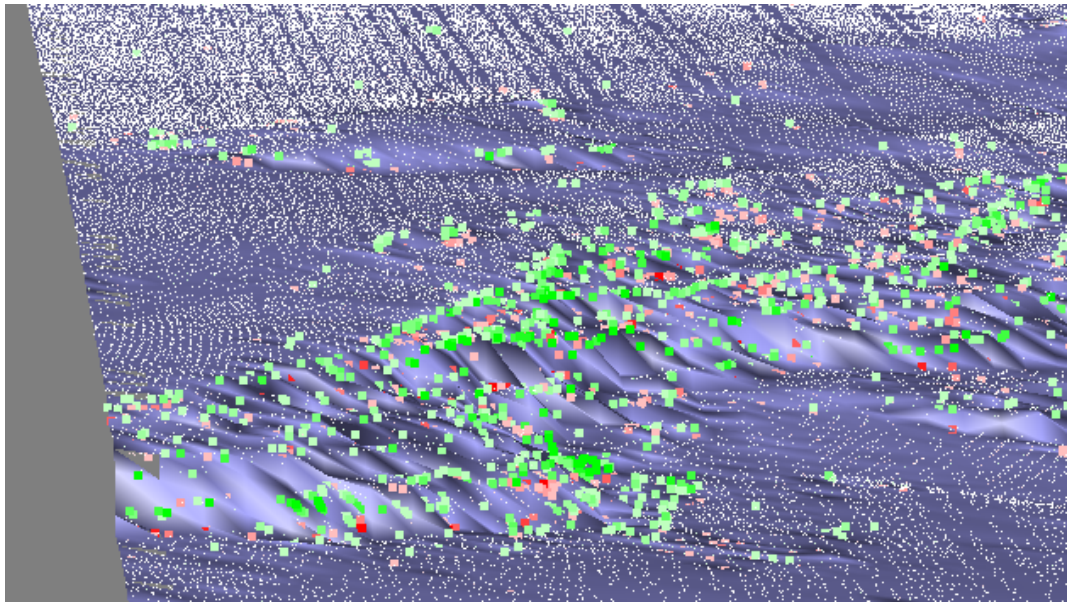
Approximation of smooth component (25 Mbyte)
Resulting surface 1.7 Mbyte



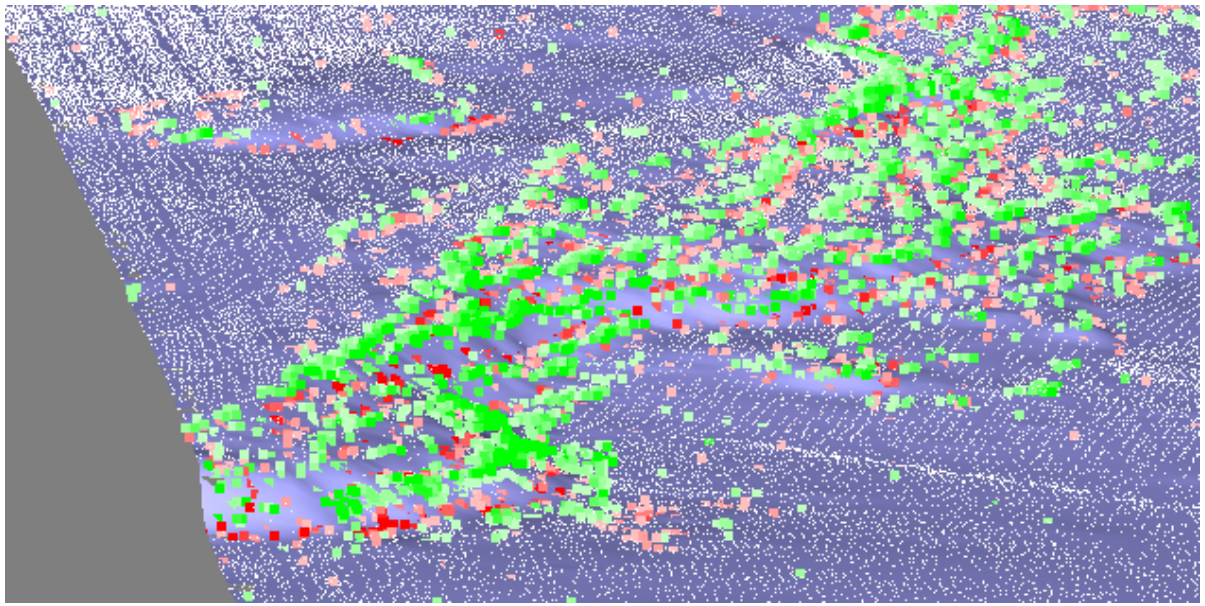
The many extra degrees of freedom to the left might introduce oscillations giving artificial details.

Whole vs smooth subset: Points outside tol.

Approximation of whole point set (26 Mbyte)
Resulting surface 7.3 Mbyte



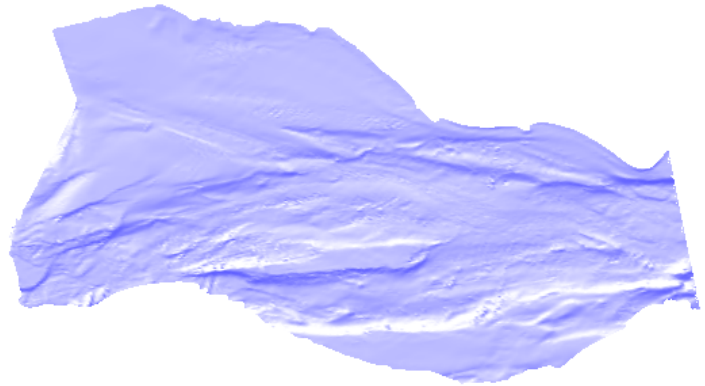
Approximation of smooth component (25 Mbyte)
Resulting surface 1.7 Mbyte



Alternative handling of points outside tolerance either local triangulation or local parametric surface.

Surface size versus point cloud size

- Original data set contains approx. 58 million points
- We perform successive thinning of the point cloud and approximate with fixed parameters:
- 0.5 m threshold, 6 iteration levels
- Results are very stable showing that the resulting LR B-spline grid is more dependent on the features of the terrain than the number of points in a scan.



Surface size ≈ 3.7 MB

No. points	File size	No. coefs.	Max. error	Average error	Average outside	Prop. OOT points
58 578 420	1.1 GB	53 454	5.55	0.092	0.66	0.56%
29 289 210	559 MB	52 709	5.39	0.092	0.66	0.55 %
14 644 406	280 MB	52 595	5.39	0.093	0.65	0.52 %
7 322 302	140 MB	52 611	5.33	0.093	0.65	0.47 %
3 661 151	70 MB	53 628	5.25	0.093	0.65	0.41%
1 830 575	35 MB	51 124	3.24	0.094	0.65	0.40 %

Relevance of LR B-spline approximations for grid structured data

- Each wavelength in a **multispectral image** can be approximated individually or simultaneously with LR B-splines
- In scattered data there are holes in the data coverage, grid structured data has less holes
- Multi imagery can be put in a common coordinate system and a reference LR B-spline model calculated combining information from the individual images
- Later check for changes can be performed by (fast) evaluation of the LR B-spline model

Local spline approximation technologies

- T-splines (2003). Refinement in the B-spline vertex mesh, works well for bi-cubic surfaces.
- Truncated Hierarchical B-splines (THB) (2012). Refinement based on uniform spline spaces defined over dyadic sequence of grids. Works for all degrees and dimensions. Enforces many extra degrees of freedom.
- Locally Refined B-splines (LRB) (2013). Direct refinement of the spline space by insertion of "mesh rectangles". Works for all degrees and dimensions. Allows lean refinements.

LR B-spline the most flexible approach and contains the spline spaces of T-splines and THB.

Conclusion

- Approximation of big data by locally refined spline spaces allows compact representation of the smooth component of the data
- The difference between the smooth component and the original dataset highlights the none smooth components. The can represent:
  - Outliers and noise in the data
  - Problems in registration of multiple data sets into the same coordinate system
  - Actual none smooth features in the data set

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