Some recent progress in geometric methods for instability of Hamiltonian systems.

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Report on joint work with M. Capinski, M. Gidea, T.M. Seara and work of other people.

The problem of instability

In the early 60's it was understood that for near-integrable systems, we see two phenomena:

- The effect of small perturbations averages out:
  - Large measure of initial conditions (KAM Theorem)
  - For a long time (Nekhoroshev theorem).
- There are situations where the perturbations do not average out and they grow (Arnol'd example)

The role in applications

- In some applications (e.g. plasma Physics, accelerators) accumulation of perturbations is the main **annoyance** since it breaks the confinement.
- In some applications (e.g. astrodynamics) accumulation of effects is **useful** since one can design maneuvers with very small thrusts. (0.05 N in 500Kg spacecraft).
- In some application (motion of asteroids, chemistry), one needs to understand when does it happen since it creates interesting effects: (rapid change of orbital elements; rearrangement of molecules).

How can we understand when the phenomenon of stability/instability takes place?

Some extra remarks

In applications, we are often interested in analyzing very concrete systems. (e.g. the forces have to be Newton/Coulomb).

In chemistry, all atoms of an element have the same mass and exert same forces. This leads to systems that are *degenerate* from the point of view of genericity theory.

The point of view of this lecture

The phenomenon is ubiquitous anyway.

We want to understand how it happens in concrete systems given to us.

We also want to study the design problem. (applying thrusts to spacecraft to enhance the phenomenon, change design of accelerators to suppress it).

- We will try to identify some geometric structures whose presence implies interesting (or useful) motions.
- We will try that the geometric structures are persistent under small perturbations.
  The perturbations do not need to be Hamiltonian; e.g we allow thrusters, albedo effects, solar pressure
  - Persistent in C^1 open sets.
  - Can be verified in concrete systems by a finite computation.
  - Give quantitative information (e.g. times of instability, Hausdorff dimension of orbits which diffuse, statistical description...)

Some alternative points of view

The problem of stability/instability is a very old problem.

There are many other philosophies and methodologies

- Numerical experiments
- Asymptotic expansions
- Construct the phenomenon by modifying the system, hence showing genericity properties.
- Study stochastic properties

Even with the same philosophy, there are many different techniques:

- 1960s Transition Whiskered Tori
- 1970's Topological methods: correctly aligned windows, Conley index.
- 1980's Local variational methods
- 1990 Global variational methods (several versions)
- Normally hyperbolic manifolds.
  - Separatrix map
  - Scattering map
  - Normally hyperbolic laminations.
  - Normally hyperbolic cylinders.
  - Kissing cylinders.
  - Blenders.
- Mixed methods (geometric/variational)
- Probabilistic methods

It is a very active field (a partial list).
Classical period

V. Arnold, A. Pustilnikov, L. Chierchia, G. Gallavotti, R. Douady, J. P. Marco, J. Cresson, R. Moeckel, E. Fontich, P. Martín.

V. Chirikov, G. Zavslavsky, J. Meiss, I. Percival, M. Kruskal, A. Tennyson. J. Herrera.

Around 2000

- E. Fontich, P. Martín, J. Cresson, J.P. Marco (lambda lemmas)
- U. Bessi, S. Bolotin, R. S. McKay, L. Chierchia, L. Biasco M. Berti, M. Bolle (local variational methods)
- J. Mather, J. Xia (Global variational methods; anouncements)
- J. Bourgain, V. Kaloshin (PDE's)
- A. Delshams, R. L., T. Seara (NHIM/scattering map)
- C.Q. Cheng (global variational/NHIM)
- D. Treshev (NHIM, separatrix map)
- P. Bernard (Lagrangian graphs)
- A. Delshams, R. L., T. Seara (NHIM/scattering map/secondary tori)
- M. Gidea, R. L (NHIM/ correctly aligned windows)
- M. Gidea,C. Robinson, J.P. Marco (Topological methods)
- R. Moeckel, R. L. V. Gelfreich, D. Turaev (Normally Hyperbolic Laminations)
- Nassiri-Pujals (symplectic blenders/NHIL)

- A. Delshams, G. Huguet/ A. Delshams, R. L., T. Seara
- R. L, V. Gelfreich, D. Turaev
- Dolgopyat, De Simoi
- P. Bernard, V. Kaloshin, K. Zhang
- C.Q. Cheng
- J.P. Marco
- V. Kaloshin, M. Levi, M. Saprykina
- V. Kaloshin, J. Fejoz, M. Guardia, P. Rold'an
- J. Xue
- A. Delshams, M. Gidea, P. Roldán
- Capinski, Zglyczinski,
- Arnold, Zarnitsky

In this lecture, I will just present one method based on:

https://arxiv.org/abs/1405.0866

- Based on a very simple geometric structure
- Results hold in C^1 open sets of systems (even non-hamiltonian).
- No non-generic assumptions such as positive definite, twist conditions.
- Can be verified by finite computations in concrete systems. Even in models for celestial mechanics.
- The method relies only on *"soft"* properties of Normally Hyperbolic Invariant Manifolds (NHIM) and their homoclinic orbits
  - Works in ∞ dimensional problems.
  - No need to use Aubry-Mather theory, local variational shadowing No convexity assumptions needed
  - No need to use averaging theory
  - No need to use KAM theory Does not require much regularity
  - The big gaps problem gets completely eliminated. (becomes irrelevant).

For the sake of convenience, I will use maps in the exposition.

It all works for flows.

Going to maps allows to explain results in 4-D maps. This requires less cheating than explaining 6-D flows.

The results are true in all higher dimensions; indeed they become easier the higher the dimension.

Main tool used in this lecture:

Normally hyperbolic manifolds (NHIM) with homoclinic intersections.

NHIM: Invariant manifold so that the normal perturbations grow/decrease at exponential rates. The normal rates are larger than the rates of growth decrease of tangent perturbations.

Using the result described by Prof. Seara, we will see that all the assumptions we need can be formulated as transversality properties of manifolds (and of foliations).

Overview of abundance

The transversality properties can be verified by a finite calculation in a concrete system. In the perturbative case, they can be verified by first order perturbation theory.

This automatically leads to C^1 openess of the assumptions.

With some extra work it can also lead to C^ω density of the assumption.

The diffusion happens in the projection of the intersections of stable/unstable manifolds established above.

In families, this automatically gives increases in action of size $O(1)$ for all $0 < |\varepsilon| \leq \varepsilon_0$ but the size of the increase in actions could depend on the family.

(we need transversality properties. For ONE intersection, they could fail in a codimension one set).

Since there are infinitely many intersections, (generically, they are all transversal) Using SEVERAL generic intersections, the projections overlap.

The formal definition of NHIM Fenichel, Hirsch-Pugh-Shub

$\Lambda \subset M$ is a NHIM if it is invariant and

There exists a splitting of the tangent bundle of TM into Df-invariant sub-bundles

$$TM = E^u \oplus E^s \oplus T\Lambda,$$

and there exist a constant $C > 0$ and rates

$$0 < \lambda_+ < \eta_- \leq 1 \leq \eta_+ \leq \mu_-, \tag{1}$$

such that for all $x \in \Lambda$ we have

$$\begin{aligned} v \in E^s_x &\Leftrightarrow \|Df^k_x(v)\| \leq C\lambda_+^k \|v\| \text{ for a ll } k \geq 0, \\ v \in E^u_x &\Leftrightarrow \|Df^k_x(v)\| \leq C\mu_-^{-k} \|v\| \text{ for al l } k \leq 0, \\ v \in T_x\Lambda &\Leftrightarrow \|Df^k_x(v)\| \leq C\eta_+^k \|v\|, \quad \|Df^{-k}_x(v)\| \leq C\eta_-^{-k} \|v\|, \text{ for all } k \geq 0. \end{aligned} \tag{2}$$

Assume moreover that the angle between the bundles is bounded from below.

Note: There are versions for locally invariant manifolds. For example, center manifolds.

These manifolds were studied extensively in the 70's. (Jarnich-Kurzweil, Fenichel, Hirsch-Pugh-Shub).

Somewhat later, infinite dimensional versions of the results above: (Hale, Chow, Bates, Zeng, Lu, etc.)

A new generation of proofs that that are very constructive and can produce rigorous results very close to the true range of validity. These proofs can cooperate with numerical computations.

See book by Haro, Canadell, Figueras, Luque – functional analysis methods – and several papers by M. Capinski-Zglyczinski (topological methods) M. Capinski, H. Kubica manage to obtain results without rate conditions.

Some classical results on NHIMS (stated informally):

- NIHIM's are *smooth* (the order of smoothness depends on the rates and is finite. One cannot expect any better in general).
- They are persistent.
- When subject to small C^r perturbations, they depend smoothly on the perturbation. (It is possible to get power series perturbative expansions.)
- They posses stable/unstable manifolds.

$$\begin{aligned} W^s_\Lambda &= \{y \,|\, d(f^n(y), \Lambda) \to_{n\to\infty} 0\} \\ &= \{y \,|\, d(f^n(y), \Lambda) \le C_y \lambda_+^n, n > 0\} \\ &= \cup_{x\in\Lambda} W^s_x \\ W^s_x &= \{y \,|\, d(f^n(y), f^n(x)) \le C_y \lambda_+^n, n > 0\} \end{aligned}$$

- W^s_Λ are *smooth*, W^s_x are as smooth as the map. They depend *smoothly* on parameters.
- Same results for unstable manifolds.

The results are sophisticated (somewhat subtle fixed point theorems) and very non-trivial, but they are considered *soft* analysis.

In particular, they are independent of the dimension and they work even in infinite dimensions.

We also note that the decomposition $W^s_\Lambda = \cup_{x\in\Lambda} W^s_x$ is a foliation.

$$W^s_x \cap W^s_y = \emptyset, x \neq y$$

Invariance properties.

$$f(W^s_\Lambda) = W^s_\Lambda$$
$$f(W^s_x) = W^s_{f(x)}$$

If a point converges to Λ in the future, its future orbit is exponentially – with rate λ_+ asymptotic to the orbit of a (unique!!) point in Λ

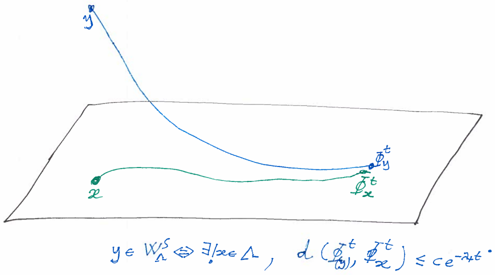

Figure : Trajectories which converge to Λ are asymptotic to the trajectory of a point

Note that the fact that we can describe the asymptotic behavior (which involves infinite time!) by smooth manifolds is amazing.

Even more amazing that it depends smoothly on parameters.

Important fact:

Stable and unstable manifolds of a NHIM can intersect.

This gives origin to very interesting dynamics.

An important result is the Λ-lemma. Many variants (Fenichel, Hirsch-Pugh-Shub some recent ones: Fontich-Martin, Cresson, El Sabagh).

Rougly:

- If Σ close to W^u_x, then, $f^n(\Sigma) \approx W^u_{f^n(x)}$.
- If Ψ transversal to W^s_x, then, $f^n(\Psi) \approx W^{cu}_{f^n(x)}$. (We omit a precise

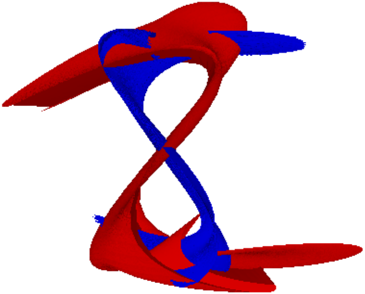

Figure : NHIM's in an oscillating double well: L. Zhang, R. de la Llave Comm. Nonlin. Sci. Num. Anal. (2018)

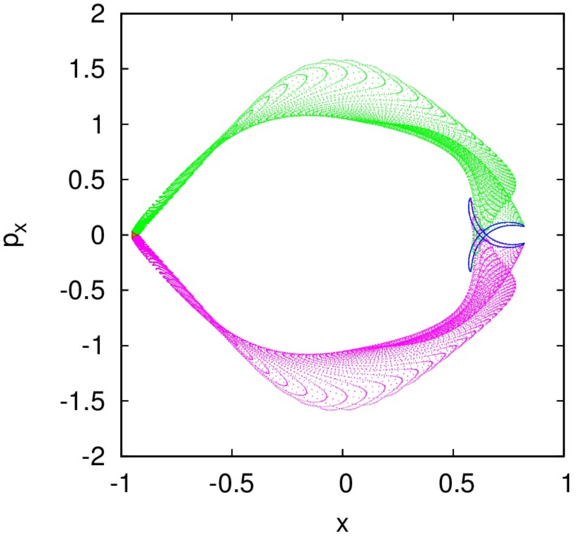

Figure : Stable/unstable manifolds of Lyap orbits near L2 (Sun-Jupiter) M. Capinski, M. Gidea, R. de la Llave Nonlinearity (2017)

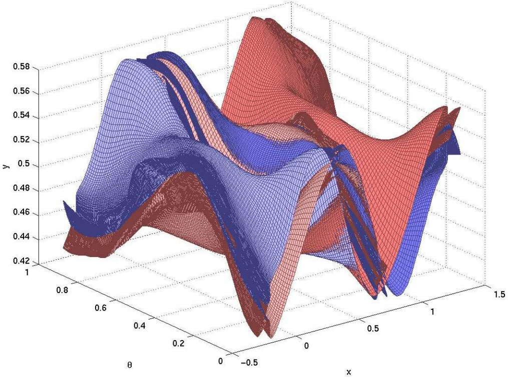

Figure : Stable/unstable manifolds of a quasi-periodic standard map. A. Haro, R. de la Llave SIADS (2006)

Main traditional idea (already used in several contexts)

1. Follow a homoclinic excursion when they are favorable;
2. Stay near the NHIM to recover.

This requires analyzing the dynamics of the jumps and the dynamics in the manifold

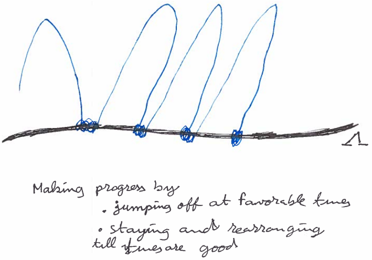

Figure : Heuristic of many instabibliy mechanisms.

Main new development:

We can dispense with analyzing the behavior near the NHIM and use ONLY the dynamics of the excursions

Then, one uses mainly soft methods and dispenses completely with the hard methods (averaging, KAM, Aubry-Mather....). Therefore, many difficult hypothesis (differentiability, convexity, low dimensions, positive definite, etc.).

Even Hamiltonian hypothesis can be weakened.

- Results by Peralta-Salas, Luque on instability in magnetic fields (SIADS 2016)
- Results by A. Granados on coupled piezoelectric oscillators (very dissipative) but applied to energy haversters (Phys. D. 2017)

We need to quantify the jumps very carefully.

Use language similar to the ones used in Mathematical scattering theory.

- Wave maps
- Scattering map

Using the theory of NHIM's we will discover that these objects have very good smoothness/geometric properties.

It was observed long ago (Wheeler, Heisenberg,etc.) that these objects have extremely nice cancellations and give a very concise description.

Alternative

There is an alternative approach (the separatrix map). Introduced by G. Zavslasky, extended substantially by D. Treshev.

- Has the dimension of the full space
  - $+$ Gives global information of all orbits.
  - - Requires more work
- The main terms are singular
- Depends on the dynamics of the map on the manifold (hard to deal with resonances in inner map)

Wave maps

To a point y converging to the manifold Λ, we associate the unique point in Λ whose orbit is exponentially asymptotic to the orbit of y.

$$\Omega_+ y = x \iff y \in W^s_x$$
$$\Omega_- y = x \iff y \in W^u_x$$

The theory of NHIM's guarantees $\Omega_\pm$ are smooth and depend smoothly on parameters.

• We consider transversal intersection Γ of Stable/Unstable manifolds to a manifold Λ.

•We also require that the intersection is transversal to the foliation by unstable manifolds of points.

• Ω_- is a diffeo on its range. Then, we define.

$$S^\Gamma(x) = \Omega_+ \circ (\Omega_-^\Gamma)^{-1}$$

Equivalently

$$\begin{aligned} S^\Gamma(x_-) = x_+ &\iff \exists z \in \Gamma; z \in W^s_{x_+} \cap W^u_{x_-} \\ &\iff \exists z \in \Gamma; d(\phi^t(x_\pm), \phi^t(z) \le C\lambda_\pm^{|t|} t \to \pm\infty \end{aligned}$$

The scattering map gives the asymptotic orbit in the future as a function

Note that it depends on the channel Γ used.

It is defined only on $\Omega_-(\Lambda)$, which in general is a strict subset Λ

In applications, rhere are many such channels. **This is a feature, not a bug.** We can use many ways of moving.

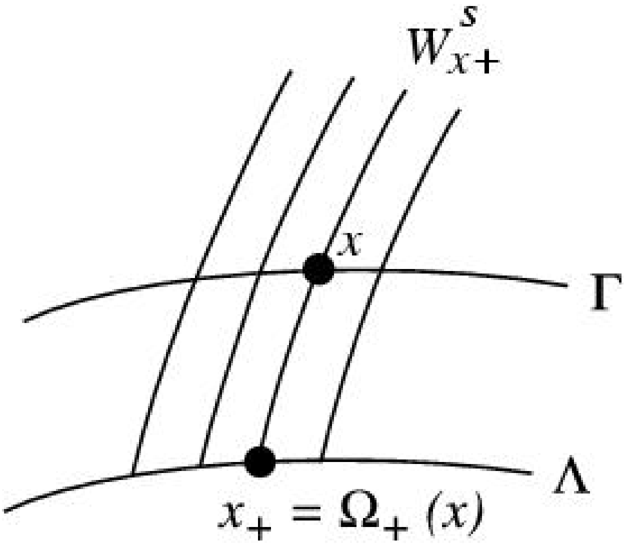

Figure : Illustration of the wave maps and homoclinic channels.

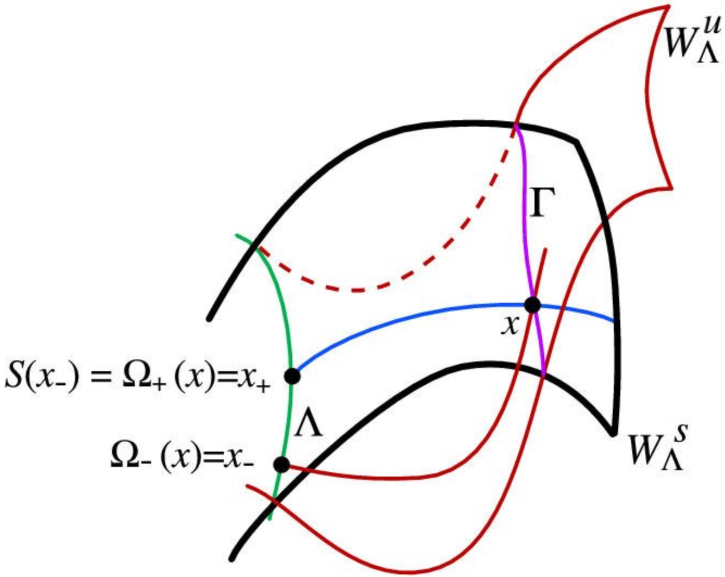

Figure : Illustration of the wave maps and homoclinic channels.

From the general theory of NHIM, we get several remarkable properties of Scattering map.

- Map on Λ (lower dimensional).
- It is smooth.
- It depends smoothly on parameters.
- It can be defined for C^1 open neighborhoods of maps. Even for not symplectic maps.
- If maps are symplectic, Λ is symplectic, then the scattering maps are symplectic
- Smooth dependence on parameters + symplectic properties $\implies$ very effective deformation theory; variational principles, etc.
- It can be numerically computed in examples.

Even when the intersections are globally transversal and the wave maps are locally invertible, there are topological reasons why one may have many scattering maps.

Even in the problem of geodesics in $\mathbb{T}^2$ one gets monodromy.

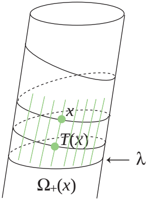

Figure : Illustration of a homoclinic channel with monodromy. Not a bug, a

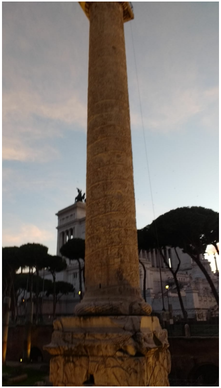

Figure : A very ancient precedent of monodromy in homoclinic intersections

Main result

Theorem

Consider a Λ with several scattering maps $S^{\Gamma^1}, \dots, S^{\Gamma^n}$ (defined in different domains). $\forall \delta > 0$, given any sequence (possibly infinite) of points

$$y_{n+1} = f^{j_n} \circ S^{\Gamma^{a_n}} \circ f^{l_n}\ y_n$$

j_n , l_n sufficiently large depending on the previous ones (and on δ)

Main result

Theorem

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j_n , l_n sufficiently large depending on the previous ones (and on δ)

$\implies \exists$ an orbit of f which passes

at a distance smaller than δ from all the y_n

(i.e. there exists $z_{n+1} = f^{j_n + l_n}(z_n)$, such that $d(z_n, y_n) \leq \delta$)

Note that the assumption that we can construct the pseudo orbit requires that the orbits land in the domain of the subsequent scattering map. This is not a big assumption, specially in perturbative case.

Note that this construction has many choices.

Starting with y_i, we have many choices for n_i, and several choices of the scattering map to use next. Hence many choices for y_{i+1}. This is consistent with the intution of *diffusion* and allows to reach many points.

For a small perturbation, we can make more elaborate choices the smallest the perturbation to accomplish effects independent of the size of perturbation.

Note:

- The only assumption is the existence of transversal intersections of stable and unstable manifolds.
  This can be verified explicitly in concrete models (Using e.g. perturbation theory, variational methods, numerics, etc.)
  The verification of the assumptions is easier in the symplectic volume preserving case (e.g. in Magnetic fields, fluids).
- No assumptions on convexity, symplectic, dynamics in the manifold.
- The hypothesis are true in C^1 open sets of maps.
- In the symplectic (or volume preserving) case, the hypothesis are C^ω dense in the space of mappings.
- It allows to consider infinite orbits.
- Allows to work in infinite dimensional maps (indeed, works better in higher dimensions).

Corollary

Assume that $f|_\Lambda$ has dense non-wandering orbits (in particular, if it satisfies Poincaré recurrence).

Given any finite sequence

$$y_n = S^{\sigma_{\sigma n}} y_{n+1}$$

it can be δ shadowed by a true orbit of the map.

Corollary

Assume that $f|_\Lambda$ preserves volume and that there is a finite sequence

$$y_n = S^{\Gamma_{\sigma n}} y_{n+1}$$

that moves a certain distance.

Then, either there are sets of pints that move away by $f|_\Lambda$ or there are orbits that shadow y_n.

Application to Celestial Mechanics: Capinski, Gidea, R.L.

We consider the *planar, circular restricted thee body problem* PCR3BP for the values of the masses Sun, Jupiter.

nn One studies it in a co-rotating frame (Synodic system).

- There is a conserved quantity (Jacobi constant)
- (Lagrange) There are 5 equilibria.
  For the values of Sun/Jupiter, L_2 has a center directions (hence a center manifold) and hyperbolic directions.
- (Lyapunov) The center manifold is filled with periodic orbits.

If the Sun-Jupiter move not exactly as circles, the system is not autonomous in the rotating frame (energy is not conserved).

Will the effects of the tides accumulate?

Application to Celestial Mechanics: Capinski, Gidea, R.L.

Theorem Nonlinearity (2017)

In the Sun-Jupiter system.

Assume some explicit expression (computed in orbits of the PCR3BP) is not identically zero.

We can find $e_0, E^* > 0$. such that if the primaries (Sun-Jupiter) move in an orbit of eccentricity e, $0 < e \leq e_0$, then there are orbits near L_3 that gain energy E^*.

Numerical Computation

The assumption of the above theorem is true

Application to Celestial Mechanics: Capinski, Gidea

`www.arxiv.org//1812003665`

Theorem: (Capinski, Gidea)

The above computations are true for several values of the masses appearing in celestial mechanics of the solar system.

Moreover, Very explicit ranges of values of the perturbation allowed.

Theorem: Stochastic description (Capinski-Gidea)

For each stochastic process in a class, there is a set of initial conditions of positive Hausdorff dimension and a prob measure on them so that the solutions in this set converges to the paths of the stochastic process.

Work in progress (Anderson, Gidea, Kumar, R.L)

Supplement the above mechanism with small thrusts to make the gain of energy happen in reasonable time.

Idea: the thrusts allow you to clip the time spent in the NHIM. A small thrust allows you to jump from stable to unstable when you are close to NHIIM.

Note that the thrusts are not hamiltonian. Other non-hamiltonian effects are incorporated (albedo, solar radiation).

The optimization in this slightly non-hamiltonian context leads to many interesting mathematical questions such as equations satisfied by the optimal results.

It is interesting to compare it with some methods in classical astrodynamics (patched conics + optimization through massive simulation). For example `Copernicus`.

`https://www.nasa.gov/centers/johnson/copernicus/index.html`

Rather than using conics, we use the invariant objects produced by the new class of algorithms.

Application to the a-priori unstable model

Theorem (Gidea, Seara, R. L)

Consider the models described by the Hamiltonian

$$H(p, q, I, \phi, t) = \sum_i \pm(p_i^2/2 + V(q_i)) + h_0(I) + \varepsilon H_1(p, q, I, \phi, t; \varepsilon)$$

Assume

- $V_i'(0) = 0$, $V_i'' \neq 0$, no other critical point same energy.
- Some mild regularity assumptions.

Application to the a-priori unstable model (theorem continued)

- Define

$$L(\tau, I, \phi, s) = -\int_{-\infty}^{\infty} \Big[H_1(p^0(\tau + t\bar{1}), q^0(\tau + t\bar{1}), I, \phi + \omega(I)t, s + t; 0) - H_1(0, 0, I, \phi + \omega(I)t, s + t; 0)\Big]\, dt. \tag{3}$$

  where $\omega(I) = \partial h_0/\partial I$.
  Assume that, L has non-degenerate critical point as a funtion of τ.
- Hence we can (by implicit function theorem) find non-degenerate critical points os the form $\tau^*(I, \phi, s)$.
  Define

$$\mathcal{L}(I, \phi, s) = L(\tau^*(I, \phi, s), I, \phi, s), \quad \mathcal{L}^*(I, \theta) = \mathcal{L}(I, \theta, 0). \tag{4}$$

  Assume that for some (I, ϕ, s).

$$\partial_\phi \mathcal{L}(I, \phi, s) \neq 0$$

Application to the a-priori unstable model (theorem continued)

Then, you get instability of order 1.

If there are several critical points , and they satisfy some non-degeneracy condition.

Then, either you get diffusion just along the manifold

OR

for every path in action space, there are trajectories that follow the action space up to an error $O(\varepsilon^{1/2})$ (approximate transitivity.)

Note:

- No assumptions on the integrable part h_0.
- Arbitrary dimensions (even ∞, work in progress).
- The assumptions are C^2 open, analytic dense (work in progress)

The assumptions made are transversality conditions (of the jets).

The first one implies that for all $0 < |\varepsilon| \leq \varepsilon_0$, we get transversality of intersections.

The second one implies that we get that the scattering map moves the actions.

All of those are first order perturbation theories. Very similar to Melnikov functions (but the usual treatment of Melnikov function assumes that the orbits have a special motion). The Melnikov function considered here is global, convergent.

You get an alternative either diffusion inside Λ – cannot happen in compact manifolds – or you get approximate transitivity.

The proof follows from the argument in Prof. Seara's talk.

No inner diffusion $\to$ Poincare recurrence $\to$ you can shadow orbits of scattering map.

Similar results were obtained in this model with different assumptions:

Treschev (2012)

- h_0 is non-degenerate
- The variables p, q are 1-D
- To some orders in perturbation, the system remains a product system.
- No resonances in the first order perturbation theory. ‘
- Obtained estimates on time.
    - An example by D. Turaev, shows that in the non-product case, the estimates on time have to be more complicated than in the product case.
    - An interesting question for applications to astrodynamics is how to shorten times of transit by adding small thrusts (with a finite total budget of fuel).

Delshams, Seara R.L. (2016)

- h_0 is non-degenerate
- Polynomial perturbation
- Some non-degeneracy assumptions near resonances

Some work in progress

- (Q. Chen and R.L) The conditions in the theorem for a-priori unstable system are C^ω dense.
  (C-Q Cheng and Q. Chen had shown C^∞ genericity in positive definite systems; Nonlinearity (2018)).
  In many proofs, of the above results one gets that the range of the instability depends on the perturbing family. We believe that one can get a uniform range of instability for an analytic generic set of H^1.

- (M. Gidea, T. Seara, R.L.) The conditions for transitivity can be relaxed to the conditions of Chow's theorem.
  That is: if there exist scattering maps whose generators $\mathcal{S}_i$ satisfy that

  $$\text{Span}(\nabla\mathcal{S}_i, \nabla\{\mathcal{S}_{i_1}, \mathcal{S}_{i_2}\}, \nabla\{\{\mathcal{S}_{i_1}\mathcal{S}_{i_2}\}, \mathcal{S}_{i_3}\}, \ldots \nabla\{\{\mathcal{S}_{i_1}\mathcal{S}_{i_2}\}, \cdots, \mathcal{S}_{i_L}\} = T\Lambda$$

  These conditions are C^ω dense, even for 2 scattering maps.
  The idea is the same as Chow's. We can move along commutators. A new idea is that we can obtain the inverse of scattering map by shadowing (it is automatic if there is reversibility as in the application to PCR3BP done before).

- The first order resonance model

$$H(p,q,I,\phi,t) = \sum_i A_i(I) p_i^2/2 + V(q_i, I) + h_0(I) + \varepsilon H_1(p,q,I,\phi,t;\varepsilon)$$

  (Very similar to the a-priori unstable model, but the properties of the penduli depend on the actions. This is motivated by the normal forms near resonances discussed by Prof. Efthymiopoulos)

- More complicated time depencence in the perturbations (e.g. quasi-periodic, almost periodic, random).

Work in Progress

Estimates on time required and on Hausdorff dimension of diffusing orbits.

The method as explained here produces not so good estimates on time, but it seems possible that some variations will lead to better estimates on time.

Work in Progress

Normally hyperbolic Laminations

If there is a transversal intersection, one gets a normally hyperbolic

lamination. Hence infinitely many scattering maps.

NOTE: Had been done assuming non-resonance in the inner map. We want to do it more generally.

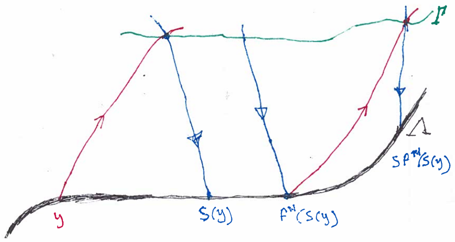
IDEA: Use Horseshoe theorem in spaces of embedding.

Work in Progress

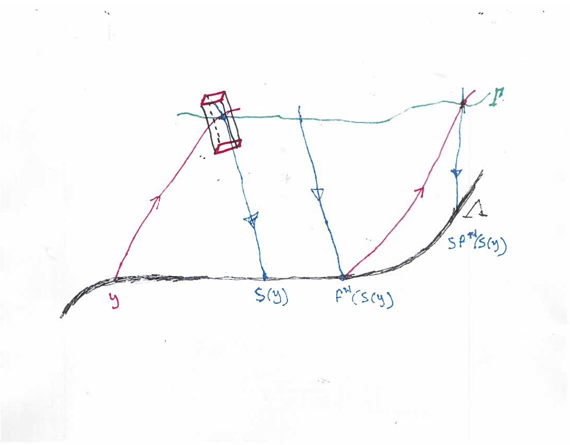
Applications to infinite dimensional systems

- Lattice dynamical systems
- PDE's

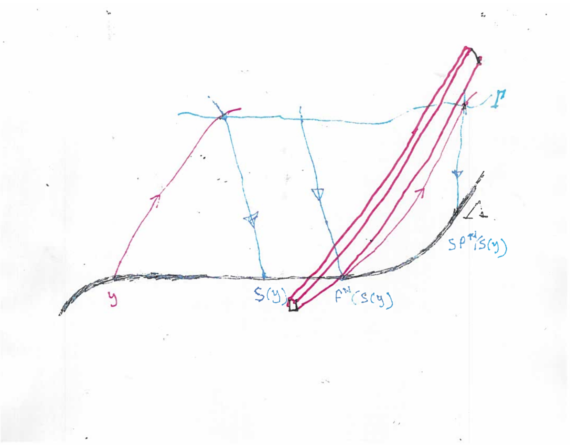
Some idea of a topological proof of the main theorem



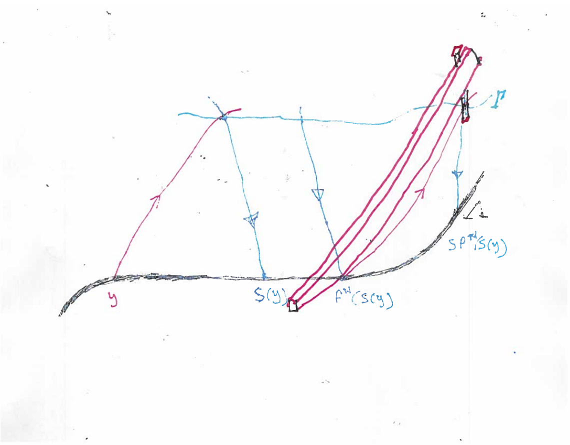
Some idea of a topological proof of the main theorem



Some idea of a topological proof of the main theorem

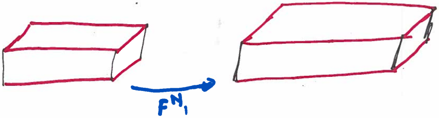


Some idea of a topological proof of the main theorem

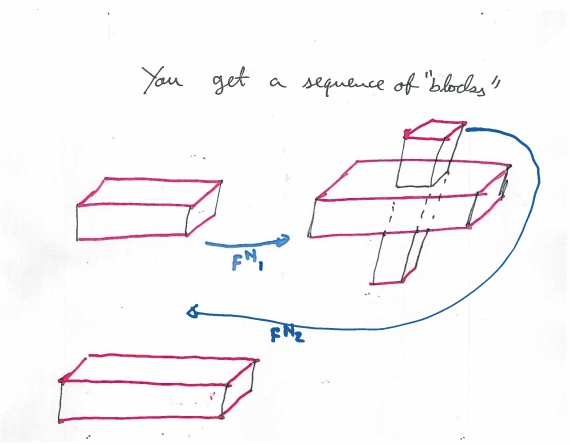


Some idea of a topological proof of the main theorem

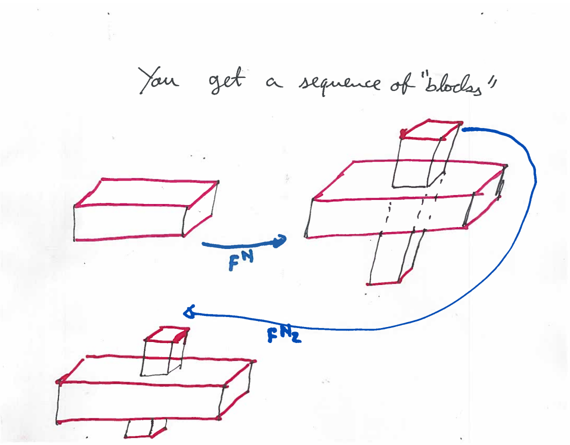
You get a sequence of "blocks"



Some idea of a topological proof of the main theorem



Some idea of a topological proof of the main theorem



Thank you for your attention